



IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

IEEE Ultrasonics, Ferroelectrics and Frequency Control Society

Developed by the
IEEE Ultrasonics, Ferroelectrics and Frequency Control Committee

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(Revision of IEEE Std 1139-2008)



STANDARDS

IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

Developed by the

IEEE Ultrasonics, Ferroelectrics and Frequency Control Committee
of the
IEEE Ultrasonics, Ferroelectrics and Frequency Control Society

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IEEE SA Standards Board

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Abstract: Methods of describing random instabilities of importance to frequency and time metrology are covered in this standard. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain deviations of frequency fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude, and phase instabilities, especially with regard to the recording of experimental parameters, experimental conditions, and calculation techniques.

Keywords: AM noise, amplitude instability, FM noise, frequency domain, frequency instability, frequency metrology, frequency modulation, IEEE 1139™, noise, phase instability, phase modulation, phase noise, PM noise, time domain, time metrology

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³ Available at: <https://ieeexplore.ieee.org/browse/standards/collection/ieee>.

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Participants

At the time this draft standard was completed, the Working Group for Revision to IEEE Std 1139 Working Group had the following membership:

Elizabeth Donley, *Chair*
Patrizia Tavella, *Vice Chair*

Neil Ashby
Claudio Eligio Calosso
Magnus Danielson
Lorenzo Galleani
Dave Howe
Attila Kinali
Paul Koppang

Judah Levine
Calvin Lin
Robert Lutwak
Demetrios Matsakis
Peter Mitchell
Aaron Partridge

Alexander Radnaev
Daniele Rovera
Enrico Rubiola
Jennifer Taylor
François Vernotte
John Vig
Gregory Weaver

The following members of the individual Standards Association balloting group voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

Claudio Eligio Calosso
Lee Cosart
Magnus Danielson
Elizabeth Donley
Daphna Enzer

Christine Hackman
Werner Hoelzl
Piotr Karocki
Sivaraman P

Laura Popa
Stefano Ruffini
Marco Siccardi
Lisa Ward
Zuo-Guang Ye

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Ted Burse, *Vice Chair*
Gary Hoffman, *Past Chair*
Konstantinos Karachalios, *Secretary*

Edward A. Addy
Ramy Ahmed Fathy
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Guido R. Hiertz
Yousef Kimiagar
Joseph L. Koepfinger*
Thomas Koshy
John D. Kulick

Johnny Daozhuang Lin
Kevin Lu
Daleep C. Mohla
Andrew Myles
Damir Novosel
Annette D. Reilly
Robby Robson
Jon Walter Rosdahl

Mark Siira
Dorothy V. Stanley
Lei Wang
F. Keith Waters
Karl Weber
Sha Wei
Philip B. Winston
Daidi Zhong

*Member Emeritus

Introduction

This introduction is not part of IEEE Std 1139-2022, IEEE Standard for Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities.

Techniques to characterize and measure the frequency, phase, and amplitude instabilities in frequency and time devices are of fundamental importance to all manufacturers and users of frequency and timing signals.

In 1964, Standards Coordinating Committee 14 and, in 1966, the Technical Committee on Frequency and Time were formed to prepare an IEEE standard on frequency stability. In 1969, these committees completed a document proposing definitions for measures of frequency and phase stabilities (Barnes [B8]). In 1988, an updated IEEE standard on frequency stability was published as IEEE Std 1139-1988, *IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology*. Later on, revisions of this standard were published as IEEE Std 1139-1999 and IEEE Std 1139-2008, with the 1999 and 2008 revisions named *IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology – Random Instabilities*. The recommended measures of instabilities in frequency generators have gained acceptance among frequency and time users throughout the world.

This standard is a revision of IEEE Std 1139-2008, which was prepared by a previous Standards Coordinating Committee 27, consisting of Eva S. Ferre-Pikal, Chair; John R. Vig, Vice Chair; James C. Camparo, Christopher R. Ekstrom, Charles A. Greenhall, Lute Maleki, Victor Reinhardi, William J. Riley, Fred L. Walls, and Joseph D. White.

Most of the major manufacturers now specify instability characteristics of their standards in terms of the recommended measures. This standard thus defines and formalizes the general practice.

Contents

1. Overview	10
1.1 Scope	10
1.2 Purpose	10
1.3 Word usage	11
2. Normative references	11
3. Definitions, acronyms, and abbreviations	11
3.1 Definitions	11
3.2 Acronyms and abbreviations	12
4. Standards for characterizing or reporting measurements of frequency, amplitude, and phase instabilities	13
4.1 Relevant measurement and analysis parameters shall be given	13
4.2 Any preprocessing shall be indicated	14
4.3 Nonrandom phenomena	14
Annex A (informative) Measures of frequency, amplitude, and phase instabilities	16
A.1 General terms	16
A.2 Frequency domain	17
A.3 Time domain	21
A.4 Systematic instabilities	31
A.5 Time error prediction	31
Annex B (informative) Power laws and conversion between frequency and time domain	34
B.1 Power-law spectral densities	34
B.2 Conversion between frequency and time domain	35
B.3 The effect of frequency spurs	36
Annex C (informative) Examples of computation of deviations	39
C.1 Introduction	39
C.2 Allan deviation $\sigma_y(\tau)$ examples	39
C.3 Modified Allan deviation $Mod\sigma_y(\tau)$	42
C.4 Parabolic deviation example	44
C.5 Total deviation example	45
C.6 Dynamic Allan deviation $\sigma_y(t, \tau)$ example	47
Annex D (informative) Other deviations used to describe frequency instabilities in the time domain	48
Annex E (informative) Confidence limits of measurements	49
E.1 Frequency Domain	49
E.2 Time domain confidence intervals	49
E.3 Bayesian approach in the time domain	52
Annex F (informative) Bibliography	55

IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities

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1. Overview

1.1 Scope

This standard covers the fundamental metrology for describing random instabilities of importance to frequency and time metrology. Quantities covered include frequency, amplitude, and phase instabilities; spectral densities of frequency, amplitude, and phase fluctuations; and time-domain deviations of frequency and phase fluctuations. In addition, recommendations are made for the reporting of measurements of frequency, amplitude, and phase instabilities, especially with ~~in~~ regard to calculation techniques, but also including ~~the recording of~~ experimental parameters, ~~experimental conditions, and calculation techniques~~. The annexes cover basic concepts and definitions, time prediction, and confidence limits when estimating deviations and spectral densities from a finite data set. The annexes also cover translation between the frequency domain and time domain instability measures, examples on how to calculate the time-domain measures of frequency and phase fluctuations, and an extensive bibliography of the relevant literature. Systematic instabilities, such as environmental effects and aging, are discussed in IEEE Std 1193™ [B39]^{1, 2}~~2003 [B54]~~.¹

1.2 Purpose

This standard defines and formalizes the techniques and physical quantities used to measure and characterize frequency, phase, and amplitude instabilities in time and frequency instruments and systems.

¹ Notes in text, tables, and figures are given for information only and do not contain requirements needed to implement the standard.

² The numbers in brackets correspond to those of the bibliography in Annex F.

1.3 Word usage

The word *shall* indicates mandatory requirements strictly to be followed in order to conform to the standard and from which no deviation is permitted (*shall* equals *is required to*).^{3, 4}

The word *should* indicates that among several possibilities one is recommended as particularly suitable, without mentioning or excluding others; or that a certain course of action is preferred but not necessarily required (*should* equals *is recommended that*).

The word *may* is used to indicate a course of action permissible within the limits of the standard (*may* equals *is permitted to*).

The word *can* is used for statements of possibility and capability, whether material, physical, or causal (*can* equals *is able to*).

2. Normative references

The following referenced documents are indispensable for the application of this document (i.e., they must be understood and used, so each referenced document is cited in text and its relationship to this document is explained). For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

There are no normative references in this standard.

³ The use of the word *must* is deprecated and cannot be used when stating mandatory requirements; *must* is used only to describe unavoidable situations.

⁴ The use of *will* is deprecated and cannot be used when stating mandatory requirements; *will* is only used in statements of fact.

3. Definitions, acronyms, and abbreviations

3.1 Definitions

For the purposes of this ~~document~~ ~~standard~~, the following terms and definitions apply. The ~~Authoritative Dictionary of IEEE Standards~~ ~~Dictionary Online Terms [B51]~~ should be ~~consulted~~ ~~referenced~~ for terms not defined in this clause.⁵~~If ambiguities are created between the narrative definition given here and the mathematical equation given in the text, the equation has priority.~~

amplitude spectrum $S_a(f)$: One-sided spectral density of the normalized amplitude fluctuations, as defined in ~~fractional~~ ~~normalized~~ **amplitude fluctuations** $a(t)$.

bias: The difference between the ~~average measured~~ ~~expected~~ value of an estimator of a statistic and its correct value.

confidence interval: An interval of uncertainty associated with an estimate of an instability measure from a finite number of measurements. The endpoints of a confidence interval are called *confidence limits*.

frequency spectrum $S_y(f)$: One-sided spectral density of the normalized frequency fluctuations, ~~as~~ defined in ~~terms of fractional~~ ~~normalized~~ **frequency fluctuations** $y(t)$.

estimated frequency: The estimated value of the nominal signal frequency ν_0 found by weighting phase or frequency measurements over a measurement interval. The measured frequency is in general time dependent and is represented by $\nu(t)$.

~~fractional frequency~~ ~~normalized amplitude~~ **fluctuations** $a(t)$: Instantaneous normalized amplitude departure from a nominal amplitude.

⁵IEEE Standards Dictionary Online is available at: <http://dictionary.ieee.org>. An IEEE Account is required for access to the dictionary, and one can be created at no charge on the dictionary sign-in page.

fractional frequency fluctuations $y(t)$: Normalized difference between the instantaneous frequency $\nu(t)$ and the nominal signal frequency ν_0 , expressed as

$$y(t) = \frac{\nu(t) - \nu_0}{\nu_0}$$

~~2.6 normalized frequency fluctuations $y(t)$: Instantaneous, normalized frequency departure from a nominal frequency.~~

phase fluctuations $\phi(t)$: Instantaneous phase ~~deviation~~ departure from a nominal phase.

phase noise $L(f)$: One-half of the **phase spectrum** $S_\phi(f)$ ~~as defined in phase spectrum $S_\phi(f)$.~~

phase spectrum $S_\phi(f)$: One-sided ~~power~~ spectral density of the phase fluctuations.

time fluctuations $x(t)$: Instantaneous time ~~deviation~~ departure from a nominal time. ~~$x(t)$ is also sometimes referred to as the phase (with units of seconds), or simply as the “time deviation.”~~

time interval error (TIE): The time difference between a real clock and an ideal uniform time scale following a ~~time~~ period of time t after ~~perfect~~ synchronization.

time spectrum $S_x(f)$: ~~A~~ one-sided spectral density of the time fluctuations.

two-sample deviation $\sigma_y(\tau)$: Also called the *Allan deviation* ~~and abbreviated by ADEV~~; the square root of the **two-sample variance** $\sigma_y^2(\tau)$ ~~.ADEV is most commonly used to characterize the instability of frequency measurements through $\sigma_y(\tau)$, but can be used to characterize the instability of other quantities, such as $\sigma_a(\tau)$ to characterize amplitude instability.~~

two-sample variance $\sigma_y^2(\tau)$: Also called the *Allan variance* ~~and abbreviated by AVAR~~; one-half the time average of the square of the difference between the averages of ~~fractional~~ ~~normalized~~ frequency fluctuations over two adjacent time intervals of length τ with no dead time between the two averaging intervals.

3.2 Acronyms and abbreviations

ADEV Allan deviation

AVAR Allan variance

4. Standards for characterizing or reporting measurements of frequency, amplitude, and phase instabilities

The [most common standard](#) measure [used](#) for characterizing frequency and phase instabilities in the frequency domain is $L(f)$ (pronounced “script-ell of f ”), defined as one half of the one-sided spectral density of phase fluctuations:

$$\mathcal{L}(f) = \frac{1}{2} S_{\phi}(f). \quad (1)$$

[The dimensions of \$S_{\phi}\(f\)\$ and of \$L\(f\)\$ are squared angle divided by a frequency, which is \$\text{rad}^2/\text{Hz}\$ in the case of \$S_{\phi}\(f\)\$. However, the unit of \$L\(f\)\$ does not have a name in the International System of Units. It is common practice to express \$L\(f\)\$ in \$\text{dBc}/\text{Hz}\$, which refers to the historical definition of \$L\(f\)\$, where the noise in 1 Hz bandwidth was given in terms of “dB below the carrier.” However, this interpretation does not hold in cases of strong phase modulation. With the above definition of \$L\(f\)\$ in terms of \$S_{\phi}\(f\)\$, the unit “dBc” is still used but does not generally mean “dB below the carrier.” \[A.2\]\(#\) provides further details.](#)

[A frequency standard is most often characterized in the frequency domain by a plot or a table of \$L\(f\)\$ versus Fourier frequency \$f\$.](#)

[The most common](#) ~~When expressed in decibels, the units of $L(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). A device shall be characterized by a plot of $L(f)$ versus Fourier frequency f . In some applications, providing $L(f)$ versus discrete values of Fourier frequency is sufficient. (See Annex A and Annex B for further discussion).~~ The ~~standard~~ measure [used](#) for characterizing amplitude instability in the frequency domain is one half of the one-sided spectral density of the normalized amplitude fluctuations, $\frac{1}{2} S_a(f)$. [The units of \$S_a\(f\)\$ are \$1/\text{Hz}\$.](#) ~~When expressed in decibels the units of $\frac{1}{2} S_a(f)$ are dBc/Hz (dB below the carrier in a 1 Hz bandwidth). See Annex A for a detailed discussion on spectral densities.~~

In the time domain, the standard measure of frequency and phase instabilities is the [two-sample deviation](#) $\sigma_y(\tau)$, also called the Allan deviation (ADEV). ~~$\sigma_y(\tau)$~~ . A [frequency standard device](#) shall be characterized by [the two-sample deviation and presented in](#) a plot [or table](#) of $\sigma_y(\tau)$ versus [the integration period](#), ~~averaging time~~ τ . The cutoff frequency (f_H). ~~In some cases, providing discrete values of $\sigma_y(\tau)$ versus τ is sufficient. (See Annex A and Annex B for further discussion.) The measurement system bandwidth (f_H) and the total measurement period time shall be indicated. For some applications, other instability measures are appropriate (see Annex A for further discussion).~~

~~In addition~~ The provisions in [4.3.1](#), [4.2](#), and [4.3](#) shall be followed ~~3.2 are recommended~~ when reporting measurements on frequency and phase instabilities.

4.1 Relevant measurement and analysis parameters shall be given

[For the measurement of variances and deviations in the time domain, relevant measurement and analysis parameters include the following:](#)

- [a\) The nominal signal frequency \$\nu_0\$.](#)
- [b\) The method of measurements or the instrumentation used.](#)
- [c\) The cutoff frequency \$f_H\$ set by the signal source instrumentation or measurement system.](#)
- [d\) The number of contiguous frequency measurements \(data samples\) \$M\$ of period \$\tau_0\$ or time deviation measurements \$N = M + 1\$, resulting in a total measurement period \$M\tau_0\$ and a sampling rate of \$1/\tau_0\$.](#)
- [e\) The characteristics of the reference signal\(s\). In the case of one reference, equal or lower noise than the signal under test is needed. In the case of two or more references, the rejection of the noise of the references provided by the method or by the instrument shall be reported.](#)
- [f\) The calculation techniques \(see Annex A\).](#)
- [g\) The confidence interval \(or uncertainty\) of the estimate and its statistical probability \(e.g., \$1\sigma\$ for 68%, \$2\sigma\$ for 95%\). See Annex E.](#)

h) The environment during measurement.

For the measurement of power spectral densities in the frequency domain, relevant measurement and analysis parameters include the following:

a) The nominal signal frequency ν_0 .

b) The method of measurements or the instrumentation used.

c) The resolution bandwidth, i.e., the bandwidth associated to each bin of the plot.

d) The number of averaged spectra or cross spectra for each segment of the plot.

e) The characteristics of the reference signal(s). In the case of one reference, equal or lower noise than the signal under test is needed. In the case of two or more references, the rejection of the noise of the references provided by the method or by the instrument shall be reported.

f) The impedance matching condition at the carrier frequency ν_0 and in the analysis bandwidth $\pm B$ centered at ν_0 , where B is the maximum Fourier frequency in the plot of $L(f)$.

g) The minimum detectable noise of the instrument or method in the measurement conditions (power, frequency, etc.).

h) The environment during measurement.

4.2 Any preprocessing shall be indicated

Random instabilities of the frequency standard under test should be characterized without any spurious effects given by artifacts and measurement errors. To this aim, an initial visualization and inspection of the data is always recommended.

In some cases, it is advisable to remove trends from aging or systematic environmental instabilities to focus on pure random noises (A.4).

Any pretreatment on measured data shall be stated together with the results of the analysis.

4.3 Nonrandom phenomena ~~should be recognized~~

The following nonrandom phenomena should be recognized:

~~In particular:~~

- a) Any observed time dependence of the statistical measures should be stated.
- b) Known environmental sensitivities should be stated (for example, the dependence of frequency and/or phase on temperature, magnetic fields, barometric pressure, and vibration). Environmental sensitivities are the topic of IEEE Std 1193™ [B39].
- c) The method of modeling any systematic behavior should be specified (e.g., an estimate of the linear frequency drift was obtained from the coefficients of a linear least-squares regression to M frequency measurements, each with a specified averaging or sample period τ_0 and corresponding cutoff frequency ~~time t and measurement bandwidth f_H~~ .)
- d) Periodic disturbances should be listed with frequency and relative strength.
- e) ~~The environmental sensitivities should be stated (for example, the dependence of frequency and/or phase on temperature, magnetic field, barometric pressure, and vibration).~~

~~3.2 Relevant measurement or specification parameters should be given~~

~~Relevant measurement or specification parameters include the following:~~

- a) ~~The nominal signal frequency ν_0 .~~
- b) ~~The method of measurements.~~
- c) ~~The measurement system bandwidth f_H and the corresponding low-pass filter response.~~
- d) ~~The total measurement (data sample) time and the number of measurements N .~~
- e) ~~The characteristics of the reference signal (equal noise or much lower noise assumed).~~

- ~~f) For averaging times which exceed 10% of the total measurement time using $\hat{\sigma}_{y,TOTAL}(\tau)$ to estimate $\sigma_y(\tau)$ is recommended (see Howe [B41], [B42] and Howe and Greenhall [B44]).~~
- ~~g) The calculation techniques [e.g., details of the window function when estimating power spectral densities from time domain data, or the assumptions about effects of dead time when estimating the two sample deviation $\sigma_y(\tau)$].~~
- ~~h) The confidence interval (or uncertainty) of the estimate and its statistical probability (e.g., 1σ for 68%, 2σ for 95%). See Annex E.~~
- ~~i) The environment during measurement.~~

Annex A

(informative)

Measures of frequency, amplitude, and phase instabilities

A.1 General terms

~~A.1 Measures of frequency, amplitude, and phase instabilities~~

The instantaneous signal output ~~voltage~~ of a precision oscillator ~~can be~~ expressed here in terms of a voltage, can be written as

$$V(t) = (V_0 + \varepsilon(t)) \cos(2\pi v_0 t + \phi(t)), \quad (\text{A.1})$$

(New)

$$v(t) = (V_0 + \varepsilon(t)) \sin(2\pi v_0 t + \phi(t)) \quad (\text{A.1})$$

~~(Deleted)~~

where

V_0 is the nominal peak voltage amplitude

$\varepsilon(t)$ is the deviation from the nominal amplitude

v_0 is the nominal frequency

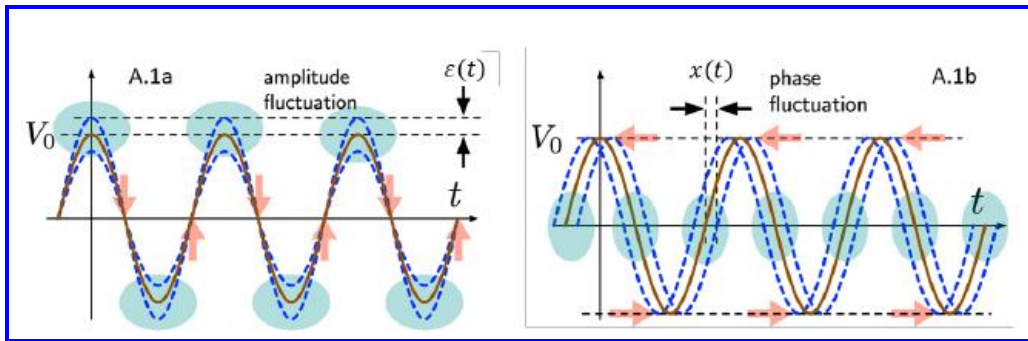
$\phi(t)$ is the phase deviation from the nominal phase $2\pi v_0 t$

From $\phi(t)$, we can also derive the instantaneous time deviation $x(t) = \frac{\phi(t)}{2\pi v_0}$ so that Equation (A.1) can be expressed as

$$V(t) = (V_0 + \varepsilon(t)) \cos(2\pi\nu_0(t + x(t))). \quad (\text{A.2})$$

(New)

Figure A.1a illustrates a signal with amplitude instability resulting in fluctuations of the peak voltage. Figure A.1b illustrates a signal with pure phase instability, with the phase fluctuations translating into to fluctuations in the period of oscillation and the zero crossings (the points where the waveform crosses zero). The pink arrows emphasize the quantities that are not affected by noise, which for AM noise are the zero crossings and for PM noise are the amplitudes. The blue ovals emphasize where the noise appears strongest for the different instability types (See Rubiola, [B59] and Rubiola and Vernotte [B60].)

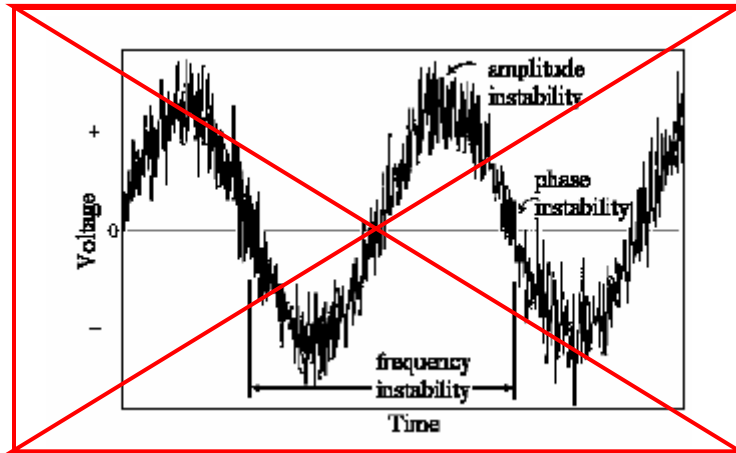


(New)

Figure A.1—Illustration of pure amplitude noise (A.1a) and pure phase (frequency) noise (A.1b) applied to a pure sinusoid (shown in brown)

~~Figure A.1² illustrates a signal with frequency, amplitude, and phase instabilities. As shown, frequency instability is the result of fluctuations in the period of oscillation. Fluctuations in the phase result in instability of the zero crossing. Fluctuations in the peak value of the signal (V_{peak}) result in amplitude instability.~~

²~~In the signal shown in Figure A.1, the frequency components of the noise are higher than the carrier frequency. The higher frequency components are used for illustration purposes only. In general, this standard applies to the frequency components of amplitude, phase, and frequency instabilities that are lower in frequency than the carrier frequency.~~



(Deleted)

Figure A.1—Instantaneous output voltage of an oscillator

Frequency instability of a precision oscillator is defined in terms of the instantaneous, [fractional](#) ~~normalized~~ frequency deviation, $y(t)$, [which can also be expressed in terms of the time derivative of the instantaneous phase deviation](#) as ~~follows~~

$$y(t) \equiv \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0}, \quad (\text{A.3})$$

where $\nu(t)$ is the instantaneous frequency ~~(time derivative of the phase divided by 2π)~~, and

$$\dot{\phi}(t) = \frac{d\phi(t)}{dt}. \quad (\text{A.4})$$

[In terms of the instantaneous time deviation, the instantaneous, fractional frequency deviation is](#)

$$y(t) = \frac{dx(t)}{dt}. \quad (\text{A.5})$$

[\(New\)](#)

Amplitude instability is defined in terms of the instantaneous [fractional](#), ~~normalized~~ amplitude deviation

$$a(t) \equiv \frac{\varepsilon(t)}{V_0}. \quad (\text{A.6})$$

Phase instability, defined in terms of the instantaneous phase deviation $\phi(t)$, can also be expressed in units of time, as

$$x(t) = \frac{\phi(t)}{2\pi\nu_0} \quad (\text{A.5})$$

(Deleted)

With this definition, the instantaneous, normalized frequency deviation is

$$y(t) = \frac{dx(t)}{dt} \quad (\text{A.6})$$

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Other random phenomena observed in certain oscillators include ~~are~~ frequency jumps, that is, discontinuities in the frequency of oscillation. These phenomena are not repetitive or well understood, and cannot be characterized by standard statistical methods.

A.2 Frequency domain

In the frequency domain, frequency, amplitude, and phase instabilities can be defined or measured in terms of ~~by~~ one-sided power spectral densities of the fluctuations of these quantities (see Table A.1). From a purely mathematical point of view, the formal definitions of these instabilities as applied to frequency control are quite complicated. From an engineering point of view, the definitions can be straightforward and related to measurements of the amplitude, frequency, and phase values and the measurement bandwidth. For more background into the expressions in this section, see references Leeson [B44], Howe et al. [B33], Allan and Barnes [B3], and Stein [B64].

The most widely used method for evaluating the one-sided power spectral density is with a Fast Fourier transform (FFT) algorithm (see Welch [B77]). The output of an FFT algorithm, $Q(f)$, is the Fourier transform of the data record $q(t)$ and is a complex vector. $Q(f)$ can be used to calculate the power spectral density of $q(t)$ through

$$S_q(f) = \frac{2}{T} Q(f) Q^*(f), \quad (\text{A.7})$$

(New)

where T is the total acquisition period used to acquire the data record $q(t)$. The factor of two in Equation (A.7) corrects the total power after deleting the negative (redundant) frequencies of $Q(f)$. The frequency resolution of the FFT is equal to $1/T$. Corrections for various window functions for discrete Fourier transforms are accounted for in software [B26]. Formulations of Equation (A.7) for fractional frequency, phase, and time deviation are used to compute the power spectral densities of these quantities (see Stein [B64]).

Expressed in basic mathematical terms, the power ~~The measure of frequency instability is the~~ spectral density of normalized frequency fluctuations, $S_y(f)$ ~~is~~ given by

$$S_y(f) = \frac{1}{\Delta f_k} \langle y^2(f) \rangle, \quad (\text{A.8})$$

(New)

$$S_y(f) = y_{rms}^2(f) \frac{1}{BW} \quad (\text{A.7})$$

(Deleted)

where

~~$y_{rms}(f)$ is the measured root mean squared (rms) value of normalized frequency fluctuations in a band of Fourier frequencies containing frequency f~~

~~BW is the width of the frequency band in Hz~~

where $\langle y^2(f) \rangle$ is the mean-squared fractional frequency fluctuation in a band Δf_k of Fourier frequencies centered on the frequency f . The units of $S_y(f)$ are 1/Hz. The subscript k indicates that the bandwidth can be variable depending on the measurement resolution.

This expression for $S_y(f)$ in Equation (A.8) can be derived from the Fourier transform relation between power spectral density and autocorrelation function:

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T y^2(t) dt = \langle y^2 \rangle = \int_0^\infty S_y(f) df, \quad (\text{A.9})$$

(New)

so that for sufficiently narrow Fourier frequency bands (such that $S_y(f)$ is approximately constant over the bandwidth) we have

$$\langle y^2 \rangle \cong \sum_{k=1}^{\infty} S_y(f_k) \cdot \Delta f_k = \sum_{k=1}^{\infty} \langle y^2(f_k) \rangle. \quad (\text{A.10})$$

(New)

Similarly, the measure of amplitude instability is the power spectral density of fractional amplitude fluctuations $S_a(f)$, given by

$$S_a(f) = \frac{1}{\Delta f_k} \langle a^2(f) \rangle. \quad (\text{A.11})$$

(New)

The units of $S_a(f)$ are 1/Hz. Here, $\langle a^2(f) \rangle$ is the mean-squared fractional amplitude fluctuation at an offset or Fourier frequency f from the carrier and Δf_k is the bandwidth of the measurement system.

~~The units of $S_y(f)$ are 1/Hz. This expression for $S_y(f)$ can be derived from the Fourier transform relation between power spectral density and autocorrelation function:~~

~~$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |y(t)|^2 dt = y_{rms}^2 = \int_0^\infty S_y(f) df$$~~

~~(Deleted)~~

~~so that for sufficiently narrow Fourier frequency bands (such that $S_y(f)$ is approximately constant over the bandwidth) we have~~

~~$$y_{rms}^2 \cong \sum_{k=1}^{\infty} S_y(f_k) BW = \sum_{k=1}^{\infty} y_{rms}^2(f_k)$$~~

~~(Deleted)~~

The measure of amplitude instability is the spectral density of normalized amplitude fluctuations, $S_a(f)$, given by

$$S_a(f) = a_{rms}^2(f) \frac{1}{BW} \quad (A.8)$$

(Deleted)

The units of $S_a(f)$ are 1/Hz. Again, $a_{rms}(f)$ is to be understood as an rms value in a specific Fourier frequency band of width BW .

Phase instability can be characterized by the [power](#) spectral density of phase fluctuations, $S_\phi(f)$, given by

$$S_\phi(f) = \frac{1}{\Delta f_k} \langle \phi^2(f) \rangle. \quad (A.12)$$

[\(New\)](#)

$$S_\phi(f) = \phi_{rms}^2(f) \frac{1}{BW} \quad (A.9)$$

(Deleted)

The units of $S_\phi(f)$ are rad²/Hz. Here $\langle \phi^2(f) \rangle$ ~~too,~~ $\phi_{rms}(f)$ is [the mean-squared phase fluctuation](#) ~~an~~ [rms value](#) in a specific Fourier frequency band.

~~As mentioned previously, $S_y(f)$, $S_a(f)$, and $S_\phi(f)$, are one-sided spectral densities that, and~~ apply over a Fourier frequency f range from 0 to ∞ . They are equivalent to [twice the sums of](#) the two-sided or single sideband spectral densities (see Table A.1), which are the complex Fourier transforms of their respective autocorrelation functions, for both positive and negative frequencies f and $-f$. ~~This equivalence arises because the complex Fourier transform of a real time-domain process is both real and symmetric in f .~~

$S_{\phi}(f)$ ~~is the quantity that~~ has been historically utilized (see Cutler and Searle [B17 B29]) in frequency metrology; however, over the years $L(f)$ has become the most used prevailing measure of phase noise among manufacturers and users of frequency standards for characterizing both frequency and phase instabilities in the frequency domain. ~~According to the old definition (see Kartaschoff [B59]),~~ $L(f)$ ~~is the ratio of the power in one sideband due to phase modulation (PM) by noise (for a 1 Hz bandwidth) to the total signal power (carrier plus sidebands); that is,~~

$$\mathcal{L}(f) = \frac{\text{power density in one phase noise modulation sideband, per Hz}}{\text{total signal power}} \quad (\text{A.10})$$

(Deleted)

The ~~old~~ definition of $L(f)$ is related to $S_{\phi}(f)$ by

$$\mathcal{L}(f) \equiv \frac{S_{\phi}(f)}{2}. \quad (\text{A.13})$$

Usually, $L(f)$ is expressed in decibels (dB) as $10 \log_{10} L(f)$, and its units are given dB below the carrier in a 1 Hz bandwidth, generally abbreviated as dBc/Hz. While “dB” and “Hz” have their usual meaning, “c” should be taken as an arbitrary unit, related to the historical practice of measuring the noise in 1 Hz bandwidth in reference to the carrier power. For small values of the mean-squared phase deviation, has the simple interpretation of being equal to the power density in one phase noise modulation sideband (per Hz) divided by the total signal power. This interpretation holds only in the case of additive noise, which splits equally between amplitude noise and phase noise, and becomes meaningless in the case of strong modulation.

Estimates of the uncertainty of $L(f)$ from measurement parameters are summarized by Hati et al. [B25]. For an oscillator in use in the field, for example on a vehicle, $L(f)$ is often dominated by phase noise from vibrations. Phase noise from vibrations is discussed in detail in IEEE Std 1193™ [B39].

~~This relationship breaks down when the mean-squared phase deviation, $\langle \phi^2(t) \rangle =$ the integral of $S_{\phi}(f)$ from f to ∞ , exceeds about 0.1 rad². To circumvent this difficulty, $L(f)$ is redefined as~~

$$\mathcal{L}(f) \equiv \frac{S_{\phi}(f)}{2} \quad (\text{A.12})$$

(Deleted)

This redefinition is intended to avoid difficulties in the use of $L(f)$ in situations where the small angle approximation is not valid. $L(f)$, as defined by Equation (A.12), should be designated as the standard measure of phase instability in the frequency domain. The reasons are the following:

- It can always be measured unambiguously, and
- It conforms to the prevailing usage.

Definitions of spectral densities and their relations are given in Table A.1.

Table A.1 — Spectral density definitions

Name	Classical variance	Relationships assuming even symmetry with $S(f) = S(-f)$
One-sided spectral density $S_{1-sided}(f)$	$\int_0^{\infty} S_{1-sided}(f) df$	$S_{1-sided}(f) = 2S_{2-sided}(f)$
Two-sided spectral density $S_{2-sided}(f)$	$\int_0^{\infty} S_{2-sided}(f) df$	$S_{2-sided}(f) = \frac{1}{2} S_{1-sided}(f)$
$\mathcal{L}(f)$	$\int_0^{\infty} 2 \mathcal{L}(f) df$	$\mathcal{L}(f) = \frac{1}{2} S_{\phi,1-sided}(f)$ $\mathcal{L}(f) = S_{\phi,2-sided}(f)$ for $f > 0$

(New)

Name	Error variance	Relationships (assumes $\neq$ symmetry)
One-sided spectral density $S_{1-sided}(f)$	$\int_0^{\infty} df S_{1-sided}(f)$	$S_{1-sided}(f) = S_{DSB}(f) = 2S_{2-sided}(f) = 2S_{SSB}(f) = 2\mathcal{L}(f)$
Two-sided spectral density $S_{2-sided}(f)$	$\int_{-\infty}^{\infty} df S_{2-sided}(f)$	$S_{2-sided}(f) = S_{SSB}(f) = \mathcal{L}(f) = \frac{1}{2}S_{1-sided}(f) = \frac{1}{2}S_{DSB}(f)$
Single sideband spectral density $S_{SSB}(f)$	$\int_{-\infty}^{\infty} df S_{SSB}(f)$	$S_{SSB}(f) = S_{2-sided}(f) = \mathcal{L}(f) = \frac{1}{2}S_{1-sided}(f) = \frac{1}{2}S_{DSB}(f)$
Double sideband spectral density $S_{DSB}(f)$	$\int_0^{\infty} df S_{DSB}(f)$	$S_{DSB}(f) = S_{1-sided}(f) = 2S_{2-sided}(f) = 2S_{SSB}(f) = 2\mathcal{L}(f)$
"Script ell of f" $\mathcal{L}(f)$	$2\int_0^{\infty} df \mathcal{L}(f)$	$\mathcal{L}(f) = S_{SSB}(f) = S_{2-sided}(f) = \frac{1}{2}S_{1-sided}(f) = \frac{1}{2}S_{DSB}(f)$

(Deleted)

Phase instability can also be ~~normalized so that it is~~ expressed as time instability by $S_x(f)$, the one-sided power spectral density of the phase fluctuations expressed in units of time ($x(t)$):

$$S_x(f) = \frac{1}{\Delta f_k} \langle x^2(f) \rangle, \quad (\text{A.14})$$

(New)

$$S_x(f) = x_{rms}^2(f) \frac{1}{BW} \quad (\text{A.13})$$

(Deleted)

where $\langle x^2(f) \rangle$ is the mean-squared phase fluctuation in a band of Fourier frequencies centered on the frequency f with the phase expressed in units of time. The units of $S_x(f)$ are s^2 / Hz . $S_\phi(f)$ and $S_x(f)$ are related by

~~where, again, $x_{rms}(f)$ is defined in a fashion similar to that of $y_{rms}(f)$. From Equation (A.5), $S_\phi(f)$ and $S_x(f)$ are related by~~

$$S_x(f) = \frac{1}{(2\pi\nu_0)^2} S_\phi(f). \quad (\text{A.15})$$

Since angular frequency is the time derivative of the phase, the spectral densities of frequency and phase fluctuations are related. By inserting the relationship for $y(t) = \frac{1}{2\pi\nu_0} \frac{d\phi}{dt}$ into Equation (A.8) and noting from the properties of Fourier transforms that $\frac{dq(t)}{dt} = \frac{i\omega}{2\pi} \int_{-\infty}^{\infty} Q(\omega) e^{i\omega t} d\omega$ (with $i^2 = -1$), one can show

$$S_y(f) = (2\pi f)^2 S_x(f) = \frac{f^2}{\nu_0^2} S_\phi(f). \quad (\text{A.16})$$

Other quantities related to phase instability are phase jitter and wander. The International Telecommunication Union (ITU) defines ~~timing~~ jitter and wander as variations ~~(deviations)~~ of the significant instants of a timing signal from their ideal positions in time excluding frequency offsets and drifts. ~~where~~ The jitter consists of variations with Fourier frequency above 10 Hz, ~~and~~ The wander consists of variations with Fourier frequency below 10 Hz (see ITU-T Recommendation G.810 [B41] and G.8260 [B42]). ~~(08/96) [B56]). One can similarly define phase jitter through Equation (A.9).~~ We define the phase jitter deviation by

$$\phi_{jitter} = \left[\int_{f_1}^{f_2} S_\phi(f) df \right]^{1/2}, \quad (\text{A.17})$$

which is the square root of the integral of $S_\phi(f)$ over the Fourier frequencies between a low frequency cutoff f_1 and high-frequency cutoff f_2 determined by the filtering properties of the application being considered. The use of 10 Hz for f_1 is useful for standardizing ~~products of~~ time and frequency equipment, but care should be exercised because this cutoff is not necessarily related to user requirements, and other cutoff frequencies might be relevant in some contexts. ~~It is also noted that it is~~ also noted that it is necessary to specify f_2 ~~in order to~~ properly define phase jitter deviation. Equation (A.17 ~~16~~) assumes infinitely sharp filters. In practice, this is difficult ~~hard~~ to achieve and there can be contributions to the measured jitter deviation from phase noise outside the region of interest if the filter skirts are not steep enough.

Similarly, one can define phase wander deviation as the square root of the integral of $S\phi(f)$ over Fourier frequencies below f_1 , although ~~it is noted that~~ this quantity diverges when negative power law noise (including flicker phase noise, white frequency noise, flicker frequency noise, and random-walk frequency noise) is present. It is also ~~noted that is~~ not possible to obtain $S\phi(f)$ from the phase jitter deviation, unless the shape of $S\phi(f)$ is known.

A.3 Time domain

A.3.1 Two-Sample (Allan) deviation

In the time domain, a standard measure for defining a frequency standard's frequency uncertainty (also called the ~~an oscillator's~~ frequency instability) is ~~the defined by a~~ two-sample deviation $\sigma_y(\tau)$. The two-sample deviation, also known as ~~called~~ the Allan deviation or ADEV, ~~which~~ is the square root of a two-sample variance $\sigma_y^2(\tau)$, also known as ~~called~~ the Allan variance. In metrology, it is common practice to estimate the uncertainty of the average of a series of N measurements by dividing the standard deviation by $\sqrt{N}$. This approach is correct if the frequency measurement is only affected by white, uncorrelated noise. The two-sample deviation was introduced by Allan [B1] to estimate the uncertainty of clocks and oscillators because the more common standard deviation does not converge for several important noise processes, such as flicker noise and random walk. The Allan variance also has the property that it corresponds to the classical variance when the only type of noise present in a data set is white frequency noise. This correspondence is an underlying motivation for the use of the Allan deviation as an instability estimator. The two-sample deviation also has the property that it can distinguish between different types of noise, and thus can be a valuable tool for optimization, estimation, smoothing, and prediction. A thorough yet concise reference for the two-sample variance and related instability measures was published by Riley [B56]. ~~This variance $\sigma_y^2(\tau)$ assumes no dead time between adjacent average frequency samples. (Dead time refers to the time between time-ordered data sets when no measurement of frequency is taken.)~~

For the averaging time τ ,

$$\sigma_y(\tau) = \left[\frac{1}{2} \langle [\bar{y}_{k+1} - \bar{y}_k]^2 \rangle \right]^{1/2}. \quad (\text{A.18})$$