



IEEE Guide for Measurement of Environmental Sensitivities of Standard Frequency Generators

IEEE Ultrasonics, Ferroelectrics, and Frequency Control Committee

Developed by the
Ultrasonics, Ferroelectrics, and Frequency Control Committee

IEEE Std 1193™-2022
(Revision of IEEE Std 1193-2003)

STANDARDS

IEEE Guide for Measurement of Environmental Sensitivities of Standard Frequency Generators

Developed by the

Ultrasonics, Ferroelectrics, and Frequency Control Committee
of the
IEEE Ultrasonics, Ferroelectrics, and Frequency Control Society

Approved 7 December 2022

IEEE SA Standards Board

Abstract: Guidelines for the measurement and specification of environmental sensitivities of frequency standards and references are presented, including considerations specific to a variety of atomic and mechanical frequency sources.

Keywords: atomic clock, atomic frequency standard, environmental sensitivities, frequency standard, IEEE 1193™, oscillator, quartz crystal oscillator, standard frequency generator

The Institute of Electrical and Electronics Engineers, Inc.
3 Park Avenue, New York, NY 10016-5997, USA

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PDF: ISBN 978-1-5044-9574-5 STD26058
Print: ISBN 978-1-5044-9575-2 STDPD26058

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Introduction

This introduction is not part of IEEE Std 1193-2022, IEEE Guide for Measurement of Environmental Sensitivities of Standard Frequency Generators.

Techniques to characterize and measure the frequency and phase instabilities of frequency and time devices are of fundamental importance to all manufacturers and users of frequency and time technology.

In 1988, the IEEE Standards Coordinating Committee 27 (SCC27) Time and Frequency, issued IEEE Std 1139TM-1988,⁶ Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology, which defined and confirmed those measures of instability in frequency generators that had gained general acceptance by researchers, designers, and users throughout the world. In 1999, the SCC27 issued a revision of this standard, IEEE Std 1139TM-1999.

After issuing IEEE Std 1139-1988, SCC27 embarked on a much more ambitious effort aimed not only at codifying proper terminology but also at providing guidelines for the characterizations and use of frequency and time standards in realistic environments. In 1994, the SCC27 issued the result of this work, IEEE Std 1193TM-1994, which covered all important environmental conditions to which time and frequency devices are normally exposed. IEEE Std 1193-1994 aided the designer and manufacturer in characterizing their product and helped the user to properly accept, test, and confirm the specified behavior of devices in a variety of environmental conditions. The standard was revised in 2003 to IEEE Std 1193TM-2003.

This standard is a revision of IEEE Std 1193-2003, which was prepared by a working group consisting of Eva S. Ferre-Pikal, Chair; John R. Vig; James C. Camparo; Leonard S. Cutler; Christopher Ekstrom; Lute Maleki; Victor S. Reinhardt; William J. Riley; Fred L. Walls; and Joseph D. White. Some sections of the 2003 standard remain unchanged.

⁶ Information on references can be found in Clause 2

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IEEE Guide for Measurement of Environmental Sensitivities of Standard Frequency Generators

1. Overview

1.1 Scope

The basic principle governing the physical measurement of time is demarcation of equal time intervals by observation of a repeating process such as oscillations of a quartz crystal or of the electromagnetic field that excites a transition in an atom (e.g., cesium). Such a repeating periodic process can be used as a frequency standard. Frequency standards include atomic frequency standards, laser and cavity resonators, and mechanical oscillators [including, e.g., quartz, dielectric resonator (DROs), cryogenic sapphire, micro-electro-mechanical system (MEMS), and thin-film resonator (TFR) oscillators]. All types of frequency standards can be influenced by environmental sensitivities.

The following three different areas of concern exist for the environmental testing and specifications of frequency standards:

- a) Fitness of a frequency standard for actual user needs in specific environments (tests attempt to mimic the anticipated environments)
- b) Characterization of a frequency standard's environmental sensitivities (tests attempt to provide distinct sensitivity coefficients for variations in independent environmental parameters)
- c) Reliability and survival (tests attempt to stress the unit by either going to extremes of operating ranges or by repeated application of stimuli, e.g., cycling)

This document puts emphasis on item b). It provides guidance and a conceptual framework rather than a prescription of procedures that must be followed. It emphasizes proper methodology and practice and cautions against pitfalls. In summary, this IEEE guide is not a specification document but a resource document for deriving specification statements.

This document often refers to the influence of random frequency fluctuations on the measurement and characterization of environmental sensitivities. Readers unfamiliar with frequency fluctuations, phase noise, and Allan deviations should refer to IEEE Std 1139.

1.2 Purpose

This document describes the nature of the environmental effects, as well as the test methods to evaluate, quantify, and report (i.e., in specifications) the sensitivity of a frequency standard's output frequency to environmental influences such as magnetic fields, atmospheric pressure, humidity, vibration, acceleration, temperature, ionizing radiation, and intermittent operation. Its primary purpose is to aid in defining specifications and to verify specified performance through measurement. In addition, this document will help to assure consistency and repeatability of environmental sensitivity measurements, as well as the use of consistent methods across the various segments of the time and frequency community.

1.3 Word usage

The word *shall* indicates mandatory requirements strictly to be followed in order to conform to the standard and from which no deviation is permitted (*shall* equals *is required to*).^{7, 8}

The word *should* indicates that among several possibilities one is recommended as particularly suitable, without mentioning or excluding others; or that a certain course of action is preferred but not necessarily required (*should* equals *is recommended that*).

The word *may* is used to indicate a course of action permissible within the limits of the standard (*may* equals *is permitted to*).

The word *can* is used for statements of possibility and capability, whether material, physical, or causal (*can* equals *is able to*).

1.4 Summary

The broad scope of this guide makes it desirable to briefly summarize the many individual environmental phenomena that are presented. Following this introduction (Clause 1) and the normative references (Clause 2), the following list summarizes the content of this guide by clause and annex:

- Clause 3: Framework for expressing environmental sensitivities

Foundational definitions of time and frequency metrology are introduced, and the standard formalism used to break down environmental sensitivities and determine sensitivity coefficients with Taylor series expansions is presented. The important concepts of time error prediction and clock holdover are also presented in detail.

- Clause 4: Environmental sensitivities, measurement methods, and error budgets

Environmental effects on precision frequency standards can be evaluated by identification of relevant parameters and transducing factors through correlation and spectral analyses, control or removal of systematic effects (through curve-fitting, differentiation, etc.), and evaluation of residual errors by means of two-sample variances and covariances and an error budget analysis. Measurements may be complicated by nonlinear responses, intercorrelations, differing time constants, transient effects, hysteresis, and aging.

- Clause 5: Acceleration effects

The effects of acceleration on frequency standards and other precision frequency sources are reviewed, and guidelines are provided for the specification and testing of acceleration sensitivities. The discussion includes steady-state acceleration effects, gravitational change effects, vibration effects, and shock. Vibration effects include sinusoidal vibration, random

⁷ The use of the word *must* is deprecated and cannot be used when stating mandatory requirements; *must* is used only to describe unavoidable situations.

⁸ The use of *will* is deprecated and cannot be used when stating mandatory requirements; *will* is only used in statements of fact.

vibration, and acoustic noise effects. Also discussed are the effects of frequency multiplication and modulation index, the effects on short-term instability, and spectral responses at other than the vibration frequency.

— Clause 6: Temperature, humidity, and pressure

Variations in temperature, humidity, and/or pressure (THP) are commonly specified environmental perturbations. In general, the effects of THP on frequency are nonlinear and interdependent. For example, the temperature coefficient of a frequency standard is often dependent on the level of humidity. Hence, providing a complete multidimensional specification for even a single device, let alone a whole model line, would be prohibitively complex. Rather, this guide seeks to provide tractable guidelines and precautions for test methods used in determining the dependence of the output frequency of precision frequency standards on THP. These guidelines and precautions encourage consistency and repeatability for measurement and specification of these environmental sensitivities.

— Clause 7: Electromagnetic and relativistic effects

Guidelines are formulated for test methods leading to practical specifications for systematic effects on frequency standards due to electric and magnetic fields. Relativistic effects from time dilation and the gravitational red shift are also described.

— Clause 8: Ionizing and particle radiation

Characterization of the response of frequency standards to ionizing and particle radiation, such as those encountered aboard a spacecraft, requires a thorough understanding of the radiation environment (proton, electron, neutron, flash X-ray radiation, and single-event upset) and radiation scenarios (dose/anneal cycle and combined environments). Emphasis is placed on flux, fluence, dose rate levels, and interaction mechanisms that pertain to realistic radiation exposure scenarios. In addition, selection criteria for radiation sources are presented, including dosimetry and procedures for the radiation testing of frequency standards.

— Clause 9: Aging, initialization, equilibration, and retrace

While not strictly environmental sensitivities, aging, warm-up time, and frequency retrace are characteristics of frequency standards that must be specified to determine suitability for a particular application. This clause addresses the definition, characterization, and specification of these properties.

— Annex A: Measuring the temperature coefficient of a frequency standard

A detailed example of the measurement and computation of total temperature sensitivity of a frequency standard is presented, along with an error estimate of the measurement. The example shows how to determine the total temperature sensitivity over a range and determine its uncertainty. A temperature sensitivity coefficient is also presented for the special case that assumes a linear response. Temperature is chosen for convenience, but the methods described in this annex would apply equally well to any environmental sensitivity measurement.

— Annex B: Interactions between environmental sensitivities and sources of error

References for various pitfalls that may be encountered during the measurement of environmental sensitivities are provided, including common interactions encountered between environmental sensitivities.

2. Normative references

The following referenced documents are indispensable for the application of this document (i.e., they must be understood and used, so each referenced document is cited in text and its relationship to this document is explained). For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

IEEE Std 1139™, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities.^{9, 10}

3. Framework for expressing environmental sensitivities

3.1 Foundational definitions for time and frequency metrology

Frequency instability of a precision frequency standard is commonly defined in terms of the instantaneous, fractional frequency deviation $y(t)$, which can also be expressed in terms of the time derivative of the instantaneous phase deviation, as shown in Equation (1).

$$y(t) \equiv \frac{\nu(t) - \nu_o}{\nu_o} = \frac{\dot{\phi}(t)}{2\pi\nu_o} \quad (1)$$

Here, ν_o is the carrier frequency, $\nu(t)$ is the instantaneous frequency, and the time derivative of the instantaneous phase deviation is expressed in Equation (2).

$$\dot{\phi}(t) = \frac{d\phi(t)}{dt} \quad (2)$$

Phase instability, defined in terms of the instantaneous phase deviation $\phi(t)$, can also be expressed in units of time as the instantaneous time deviation, as shown in Equation (3).

$$x(t) = \frac{\phi(t)}{2\pi\nu_o} \quad (3)$$

With this definition, the instantaneous, fractional frequency deviation can be expressed as shown in Equation (4).

$$y(t) = \frac{dx(t)}{dt} \quad (4)$$

The two-sample deviation $\sigma_y(\tau)$ (also called the “Allan deviation”) is used to present the instability of the fractional frequency output over averaging interval τ . The two-sample deviation is often referred to by the abbreviation ADEV. The reader shall refer to IEEE Std 1139 for a complete discussion of these quantities, especially regarding the analysis of contributions from random instabilities to $y(t)$.

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¹⁰ IEEE publications are available from The Institute of Electrical and Electronics Engineers (<https://standards.ieee.org/>).

This guide focuses on deterministic environmental sensitivities that cause a predictable change in frequency in response to a change in an environmental parameter, like temperature. Since the best prediction of the stochastic fluctuations of $y(t)$ is zero, stochastic fluctuations do not have a direct effect on environmental sensitivities. But random fluctuations will affect the measurement process, including how long a measurement of an environmental sensitivity takes and how a measurement is performed (see Annex A). Random fluctuations will also affect the confidence of the measurement of environmental sensitivities.

Focusing on deterministic effects and ignoring noise terms, at any time t the fractional frequency deviation of a frequency standard can be written as shown in Equation (5).

$$y(t) = y_0 + A_1 \cdot t + y_{nl}(t) + y_{env}(t) \quad (5)$$

where

- y_0 is the initial fractional frequency offset of the standard
- A_1 is the component of aging or drift that is linear with respect to time (measured under constant environmental conditions)
- $y_{nl}(t)$ includes higher order components of aging

The final term $y_{env}(t)$ includes perturbations to the frequency output due to environmental stimuli, which are the topic of this guide. The concept of aging is discussed in detail in Clause 9; in short, aging describes a change of a frequency standard's output frequency over time under constant environmental conditions.

3.2 Environmental sensitivity coefficients

Frequency standards potentially exhibit sensitivity to any or all external environmental variations, which may adversely impact the stability of $y(t)$ on all timescales and may dominate the error budget for certain applications in challenging environments. Due to the high number of environmental variables, it is impossible to fully characterize a class of devices for all possible application environments. Thus, it is convenient to separately identify and characterize the environmental sensitivities as follows.

The environmental perturbations result from sensitivities on many variables, as shown in Equation (6).

$$y_{env}(t) = y_{env}(V, A, T, H, P, S, B, E, \dots) \quad (6)$$

In Equation (6), the environmental stimuli include, at least, *V*ibration, *A*cceleration, *T*emperature, *H*umidity, *P*ressure, power *S*upply, magnetic fields (*B*), and *E*lectric fields, respectively.

For notational convenience, we define the generic environmental parameter χ^j where the full set of $\{\chi^j\} = \{V, A, T, H, P, S, B, E, \dots\}$ and the environmental conditions at time t are represented by $\{\chi^j(t)\} = \{V(t), A(t), T(t), H(t), P(t), S(t), B(t), E(t), \dots\}$.

Then we can write Equation (7).

$$y_{env}(t) = y_{env}(\{\chi^j(t)\}) \quad (7)$$

Given this complexity and the potentially nonlinear interdependence of the environmental sensitivities, it is generally not possible to obtain a complete model for a given type of device. Our purpose here is to arrive at tractable guidelines and precautions for test methods used in determining the dependence of the output frequency of precision frequency standards on external environmental perturbations.

To calculate an environmental sensitivity coefficient, first define some nominal operating values of the environmental parameters χ^j . Call these χ_0^j . Then, expand the fractional frequency about these nominal values using a Taylor series (see Arfken discussion of multivariate Taylor expansion [B8]),¹¹ as shown in Equation (8).

$$y_{env} = y_{env}(\{\chi_0^j\}) + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\left(\sum_m \delta\chi^m \frac{\partial}{\partial\chi^m} \right)^n y_{env}(\{\chi^j\}) \right]_{\{\chi_0^j\}} \quad (8)$$

In Equation (8), we have implicitly defined a small perturbation in environmental parameter χ^m as $\delta\chi^m$ and the partial derivative with respect to environmental parameter χ^m as $\frac{\partial}{\partial\chi^m}$. Since $y_{env}(\{\chi^j\})$ includes environmental sensitivities for the full set of $\{\chi^j\}$, cross terms are preserved in Equation (8).

We define the small shift in the fractional frequency due to a small perturbation in the environment around the nominal values of $\{\chi_0^j\}$ as the perturbed fractional frequency deviation δy given by Equation (9).

$$\delta y = y_{env} - y_{env}(\{\chi_0^j\}) = \sum_{n=1}^{\infty} \frac{1}{n!} \left[\left(\sum_m \delta\chi^m \frac{\partial}{\partial\chi^m} \right)^n y_{env}(\{\chi^j\}) \right]_{\{\chi_0^j\}} \quad (9)$$

Breaking out the first-order $n = 1$ term, we find Equation (10).

$$\delta y = \sum_m \delta\chi^m \cdot \frac{\partial}{\partial\chi^m} y_{env}(\{\chi^j\}) \Big|_{\{\chi_0^j\}} + \sum_{n=2}^{\infty} \frac{1}{n!} \left[\left(\sum_m \delta\chi^m \frac{\partial}{\partial\chi^m} \right)^n y_{env}(\{\chi^j\}) \right]_{\{\chi_0^j\}} \quad (10)$$

For small values of $\delta\chi^m$, the terms of $n = 2$ and higher can often be ignored, but to account for the interactions among environmental factors, terms of $n = 2$ and higher in the sum may need to be retained. However, for many applications, it is sufficient to include only the leading term in the expansion.

For convenience, we define the sensitivity coefficients according to Equation (11) and Equation (12).

¹¹ The numbers in brackets correspond to those of the bibliography in Annex C.

$$\alpha_j \equiv \frac{\partial y}{\partial \chi^j} \text{ at constant } \chi^{i \neq j} \quad (11)$$

and

$$\beta_{jk} \equiv \frac{\partial^2 y}{\partial \chi^j \partial \chi^k} \text{ at constant } \chi^{i \neq j,k} \quad (12)$$

As an example, consider a frequency standard whose primary environmental sensitivities are to temperature (T) and, to a lesser extent, humidity (H) or pressure (P). Out to second order ($n = 2$), Equation (10) becomes Equation (13).

$$\begin{aligned} \delta y = & \delta T \left. \frac{\partial y(T, H, P)}{\partial T} \right|_{T_o, H_o, P_o} + \delta H \left. \frac{\partial y(T, H, P)}{\partial H} \right|_{T_o, H_o, P_o} + \delta P \left. \frac{\partial y(T, H, P)}{\partial P} \right|_{T_o, H_o, P_o} \\ & + \frac{1}{2} \left(\delta T \frac{\partial}{\partial T} + \delta H \frac{\partial}{\partial H} + \delta P \frac{\partial}{\partial P} \right)^2 y(T, H, P) \Big|_{T_o, H_o, P_o} \end{aligned} \quad (13)$$

In the case where $\partial y / \partial T$ dominates, then in expanding to second order, terms of order δH^2 , $\delta H \delta P$, and δP^2 can be ignored compared with terms of order δT^2 , $\delta T \delta H$, and $\delta T \delta P$. In this case, Equation (13) can be approximated by Equation (14).

$$\delta y \equiv \left(\alpha_T + \frac{1}{2} \beta_T \delta T \right) \delta T + (\alpha_H + \beta_{TH} \delta T) \delta H + (\alpha_P + \beta_{TP} \delta T) \delta P \quad (14)$$

In Equation (14), we have retained terms to second order in δT but first order in δH and δP .

Depending on the type of frequency standard and target application environment, it may be useful to specify either or both of the following sensitivities:

- **Total Sensitivity:** The maximum excursion of the frequency standard output over the specified operating range of the environmental parameter χ^j . Total sensitivity has the units of fractional frequency, with the range of operation explicitly noted.
- **Sensitivity coefficient:** The coefficient α_j for the differential change of the frequency standard output over some interval of the environmental parameter χ^j . The sensitivity coefficient has the units of fractional frequency divided by the units of χ^j , with the range of applicability explicitly noted.

Note that all reported sensitivity coefficients must specify the nominal value of χ_0^j , as well as the interval $\delta \chi^j$, for example, “temperature sensitivity coefficient over a 1 °C interval around 25 °C.”

In addition, for many applications, it is useful to specify:

- **Worst-case sensitivity coefficient:** The worst-case sensitivity coefficient $\Delta y / \Delta \chi_i$ is specified over a particular interval across the specified operating range, for example, “worst-case temperature sensitivity coefficient over any 1 °C interval between -40 °C and +85 °C.” The worst-case sensitivity coefficient has the units of fractional frequency divided by the units of χ^j .

3.3 Time error prediction and clock holdover

Predictions of the time deviation $x(t)$ and its uncertainty for free-running frequency standards are required for determining their holdover specifications, where holdover is defined as the maximum value of time interval error accumulated by a free-running clock over a specified period of time (see Equation (18)).

When observed over a prediction interval of period $\tau_p = t_p - t_0$ starting at time t_0 and ending at t_p , the time deviation may be predicted from the sum of an initial time deviation $x(t_0)$ and an integral over the fractional frequency (Panfilo and Tavella [B124]), as shown in Equation (15).

$$x(t_p) = x(t_0) + \int_{t_0}^{t_p} y(t') dt' \quad (15)$$

Equation (15) can be used to estimate clock time deviations that arise from environmental sensitivities, which are usually expressed in terms of frequency shifts.

Integrating over fractional frequency deviations for the initial frequency offset and accounting only for the linear component of aging [ignoring $y_{nl}(t)$ in Equation (5)] gives Equation (16).

$$\hat{x}(t_p) = x(t_0) + y(t_0)\tau_p + \frac{1}{2}A_1\tau_p^2 + \int_{t_0}^{t_p} y_{env}(t') dt' \quad (16)$$

In Equation (16), $y_{env}(t)$ is the time-dependent fractional frequency deviation arising from environmental sensitivities, which must be integrated over the period from t_0 to t_p to determine the environmental contribution to $\hat{x}(t_p)$. Estimates of $\hat{x}(t_p)$ that include environmental sensitivities involve many assumptions and variables and can become complex, but Equation (16) can be solved analytically in simple cases and can be verified through experimental environmental testing. Accounting for nonlinear aging in Equation (16) can be similarly complex, but it can be included as a term that is the integral over $y_{nl}(t)$ if nonlinear aging is significant. Such a term is not included here for simplicity and to keep the focus on environmental sensitivities.

Because the random, or stochastic, fluctuations are equally likely to be positive or negative, the best prediction of the stochastic fluctuations of $y(t)$ is zero. Thus, random fluctuations have no effect on the best prediction of $x(t_p)$ but will affect the confidence of the prediction. An approximate estimate of the stochastic uncertainty can be found from the value of the Allan deviation at τ_p with the expression $\tau_p\sigma_y(\tau_p)$, where the linear aging A_1 has been removed from the data before calculating $\sigma_y(\tau_p)$. Estimating the stochastic uncertainty as a sum of terms from different noise types can also be done (IEEE Std 1139).

The estimate of the value of the time error, shown in Equation (16), should be accompanied by an estimate of its uncertainty. In addition to the degradation in the optimal prediction from stochastic noise, uncertainties in the deterministic terms also introduce uncertainty in the optimum prediction of $\hat{x}(t_p)$ since the estimates of the deterministic parameters have errors. Including uncertainties from the deterministic and stochastic parts gives the squared uncertainty, as shown in Equation (17).

$$\sigma_{\hat{x}(t_p)}^2 = \sigma_{x_0}^2 + \sigma_{y_0}^2\tau_p^2 + \frac{1}{4}\sigma_{A_1}^2\tau_p^4 + \sigma_{y,env}^2(t_p) + \tau_p^2\sigma_y^2(\tau_p) \quad (17)$$

The time deviation $\hat{x}(t_p)$ is expected to be within the range $\hat{x}(t_p) \pm \sigma_{\hat{x}(t_p)}$ with 67% confidence. The deterministic uncertainty terms depend on the uncertainty in the initial synchronization σ_{x_0} , the uncertainty in the initial frequency value σ_{y_0} , the uncertainty in the linear component of aging σ_{A_1} , and the uncertainty in the environmental contribution to the time deviation $\sigma_{y,env}$ given the known environmental sensitivities and any changes in the environment during the holdover period. Estimates of σ_{x_0} , σ_{y_0} , σ_{A_1} , and $\sigma_y(\tau_p)$ can be determined from a period of “learning” measurements leading up to t_0 and tested for consistency by repeated measurements (Vernotte et al. [B164]). The length of the learning period can be a dominant factor in holdover performance. $\sigma_{y,env}(t)$ can be determined by error propagation and analysis of the integral in Equation (16), as well as through repeated measurements over a range of environmental parameters.

The difference in time intervals measured over a period t between a free-running clock and a reference timescale is known as time interval error $TIE(t)$, which is related to a clock’s holdover specification. When observed over a period starting at time t_0 and ending at $t_0 + t$, the time interval error is defined as (ITU-T G.810 [B81]) shown in Equation (18).

$$TIE(t; \tau) = x(t + \tau) - x(t) \quad (18)$$

$TIE(t)$ can be predicted by finding $x(t + \tau)$ from the deterministic terms as in Equation (16), with $t_p = t + \tau$. The random noise will contribute an uncertainty to the estimate of $TIE(t)$, and can be determined from the Allan deviation as in Equation (17).

Environmental sensitivities dominate $TIE(t)$ for some types of frequency standards, including quartz and MEMS oscillators. For rubidium standards, the stochastic and deterministic terms are typically of similar magnitude, and for cesium standards, $TIE(t)$ is dominated by the stochastic terms.

Holdover is defined as the maximum value of $TIE(t)$ accumulated by a free-running clock over a specified holdover period. Holdover specifications typically depend on the performance and environmental requirements of the application. A complete holdover specification must include both the maximum value of $TIE(t)$ and the environmental conditions under which it was measured. To avoid misleading suppression of error estimates, holdover specifications should avoid averaging of environmental sensitivities over periodic changes in conditions.

Examples of the minimum requirements for clocks used in synchronizing network equipment that follow the Synchronous Digital Hierarchy can be found in ITU-T G.813 [B82]. Specific examples of holdover specifications for enhanced primary reference time clocks can be found in ITU-T G.8272.1/Y.1367.1 [B83].

4. Environmental sensitivities, measurement methods, and error budgets

4.1 General introduction

Resonance frequencies of mechanical resonators such as quartz crystals and MEMS oscillators are determined by the material characteristics and mechanical geometry, excitation and detection methods, and other factors. Environmental perturbations affect the resonator dimensions and thereby introduce frequency shifts. Similarly, although isolated atoms at rest have fixed resonant frequencies, environmental factors acting on them inside an atomic frequency standard can cause a frequency shift of the resonance. In practice, manufacturers of both mechanical and atomic frequency standards strive

to reduce the impact of variations of environmental conditions on the frequency inaccuracy of the standard, although it is inevitable that sensitivity will always appear at some level.

In passive atomic frequency standards such as a cesium beam frequency standard or a rubidium frequency standard, a voltage-controlled quartz crystal oscillator is typically locked to an atomic resonator, which generates a highly stable frequency reference based on an atomic transition. Of the many atomic transitions available, frequency standards operate on those least sensitive to environmental disturbances and to which a crystal oscillator may be conveniently locked (Vanier and Audoin [B160]). The environmental sensitivity of such a frequency standard is theoretically given by the sensitivity of the crystal oscillator, atomic resonator, and dynamics of the crystal's sensitivity modified by the transfer function between them (Box and Jenkins [B23] and Breakiron et al. [B27]). For a well-tuned system, the short-term instability is generally determined by the crystal oscillator and the long-term instability by the atomic resonator, with the crossover point determined by the design and frequency response of the feedback control loop (Vanier and Audoin [B160] and De Marchi et al. [B41]).

The environmental sensitivity of frequency standards may restrict their applicability and performance under some environmental conditions. Predicting a frequency standard's performance under expected operating conditions requires knowledge of the frequency standard's sensitivity to each external influence. Accordingly, manufacturers endeavor to employ standard test methods and evaluation criteria that can be used to predict performance in a specific operating environment. Standardization of environmental test methods enables system designers to predict performance and optimize device selection and system trade-offs to improve frequency standard performance in their specific application. The need for improved definition and standardization of environmental sensitivities continues to increase with increased deployment of timing, communications, and positioning systems in the extreme environments encountered in land- and sea-mobile, airborne, and spaceborne applications.

To measure the environmental sensitivity coefficients of a frequency standard, a reference frequency standard is required (see 4.2.1), which should be maintained in a stable environment while the device under test (DUT) is subjected to controlled environmental perturbations.

Instabilities in frequency standards are analyzed both deterministically and stochastically. In the deterministic part of the analysis, systematic effects arising from environmental sensitivities are modeled by an analytic method such as curve-fitting the time or frequency deviations relative to a reference frequency standard as environmental parameters are varied. Analysis of the frequency deviations yields the average frequency deviation, its derivative, and higher order, or otherwise nonlinear trends. After the systematic variations are documented and removed, random errors can be quantified by measures such as the two-sample deviation in the time domain (Allan [B2], IEEE Std 1139, and Rutman [B137]) or the power spectral density function in the frequency domain (IEEE Std 1139, Percival [B130], and Greenhall [B63]).

4.1.1 Characterizing deterministic environmental sensitivities

Deterministic trends are usually characterized through curve-fitting a set of frequency deviation measurements to a series of polynomials, consistent with the Taylor series approach described in 3.2. Most modern data analysis and visualization software includes routines for performing least-squares fitting, including analysis of the confidence intervals. Often these polynomial fits are based on basic physical models for effects like thermal expansion, Zeeman shifts, light shifts, and pressure shifts. After all relevant environmental influences have been identified, estimates for the sensitivity coefficients can be determined from a least-squares fit analysis of the frequency standard's frequency deviations δy as a function of the magnitude of the relevant environmental parameters χ_j . Including only first-order sensitivities, the change in frequency deviation from a change in environmental parameters may be expressed as Equation (19).

$$\delta y = \sum_j \frac{\partial y}{\partial \chi_j} \delta \chi_j \quad (19)$$

For a frequency standard sensitive to temperature in first order and magnetic field in second order, the model for its environmental sensitivities would be written as shown in Equation (20).

$$\delta y \cong \alpha_T \delta T + \frac{1}{2} \beta_B (\delta B)^2 \quad (20)$$

The first-order temperature sensitivity coefficient α_T can be found by holding all other environmental parameters constant, measuring the frequency deviation as a function of temperature in a temperature-controlled enclosure, and fitting the resulting curve to a straight line. The slope of the linear fit gives α_T . Likewise, the second-order sensitivity coefficient to magnetic field β_B can be found by measuring the frequency deviation as a function of the magnetic field and fitting the curve to a quadratic function.

Some variables act through transducing factors, for example, a hydrogen maser cavity's frequency variation may be related to the cavity's dimension D and temperature T as shown in Equation (21).

$$\delta y = \frac{\partial y}{\partial D} \frac{\partial D}{\partial T} \delta T \quad (21)$$

If these influences can be measured or predicted and their effects modeled, y can be corrected for them. For example, the frequency of a temperature-compensated crystal oscillator (TCXO) is automatically corrected for thermal effects based on a predetermined temperature dependence (Stein and Vig [B144]).

The Taylor series expansion is only valid when the frequency deviation as a function of environmental parameters is smooth, and the system does not pass through any phase transitions around the operating point. As with all Taylor approximations, significant nonlinearity occurs, higher order terms are required for a faithful approximation.

4.1.2 Characterization of random errors

The characterization of random errors is usually performed with two-sample deviations in the time domain and the power spectral density function in the frequency domain. Readers shall refer to IEEE Std 1139 for a thorough review of these techniques.

The power spectral density function, discussed in depth in IEEE Std 1139, is important for characterizing environmental sensitivities when environmental perturbations occur over a spectrum of frequencies, adding phase noise to the frequency standard. The best example is with acceleration sensitivity, where vibrations can induce sidebands on a frequency standard's spectrum, thereby degrading its performance. This issue is discussed in detail in Clause 5. For completeness and as it relates to the discussion of vibration sensitivity, the spectral density function is briefly introduced here.

Phase instability can be characterized by the power spectral density of phase fluctuations $S_\phi(f)$ given by Equation (22).

$$S_\phi(f) = \frac{1}{\Delta f_k} \langle \phi^2(f) \rangle \quad (22)$$

$S_{\phi}(f)$ is a one-sided spectral density and applies over a Fourier frequency range from 0 to ∞ , with units of rad^2/Hz . $\langle \phi^2(f) \rangle$ is the mean-squared phase fluctuation in a specific Fourier frequency band Δf_k of Fourier frequencies centered on the frequency f .

More often, the spectral density is expressed in terms of the quantity $\mathcal{L}(f)$, which is defined to be one half of the spectral density of phase fluctuations $S_{\phi}(f)$. For small modulation indices, $\mathcal{L}(f)$ is equal to the power density in one phase noise modulation sideband per Hz divided by the total power. For a detailed discussion of $\mathcal{L}(f)$ and $S_{\phi}(f)$, the reader is referred to IEEE Std 1139.

For a frequency standard on a moving platform, $\mathcal{L}(f)$ is often dominated by phase noise from vibrations (Clause 5).

4.2 Measurement methods

The environmental sensitivity of a frequency standard cannot be measured more precisely than as limited by its instability over the measurement interval. Since instability generally depends on the sampling period, test procedures should allow sufficient measurement time before and after application of environmental stimuli for the frequency standard to reach a given instability level (IEEE Std 1139 and Howe et al. [B78]). This limitation can be overcome somewhat by repeated measurements and use of correlation techniques.

4.2.1 Reference frequency standard

All frequency measurements depend on a reference frequency, which should be as stable and accurate as required in view of the performance being measured. If possible, the reference frequency standard should have superior noise properties to the DUT on all relevant timescales, as determined by its two-sample deviation. The reader shall refer to IEEE Std 1139 for complete discussion of the instability of frequency standards, especially regarding the analysis of contributions from random instabilities.

For long-term measurements, the two-sample deviation of the reference clock can increase due to environmental influences and aging. Over long sampling periods that require a reference with excellent long-term stability, one may need to refer to a “paper timescale” computed as a filtered average from an ensemble of clocks (Allan et al. [B4], Percival [B131], and Stein and Evans [B143]) or utilize a reference signal such as common-view GPS (Lewandoski and Thomas [B99]) from an official timing center whose reference clock is steered in frequency toward a paper timescale. In the latter case, the effects of steering may have to be removed by comparison to International Atomic Time (TAI) through the Bureau International des Poids et Mesures (BIPM) *Circular T*,¹² which serves as the source of traceability to Coordinated Universal Time (UTC) for local realizations of UTC maintained by national institutes. *Circular T* provides clock difference data with 5 day resolution in monthly reports. Since 2013, the BIPM has also provided a rapid realization of UTC called UTCr,¹³ which provides daily values of the clock differences on a weekly basis.

Neglecting correlations, the combined instability of time or frequency differences between pairs of frequency standards is the square root of the sum of the squares of their individual instabilities. An “ n -cornered-hat analysis” of two-sample variances may be used to separate individual instabilities, assuming measurements are simultaneous and statistically independent and neglecting noise added by the measurement system. Alternatively, one can analyze two sample covariances if correlations are

¹² <https://www.bipm.org/en/time-ftp/circular-t>

¹³ <https://www.bipm.org/en/time-ftp/utcr>

present. For n frequency standards, the variance σ_i^2 of a frequency standard i may be computed from Equation (23).

$$\sigma_i^2 = \frac{1}{n-2} \left(\sum_{j=1}^n \sigma_{ij}^2 - A \right) \quad (23)$$

In Equation (23), $\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2$ is the variance of the differences between frequency standards i and j , $\sigma_{ii}^2 = 0$, and the quantity A is expressed in Equation (24) (Allan [B3], Barnes [B13], and Gray and Allan [B62]).

$$A = \frac{1}{n-1} \sum_{k=1}^n \sum_{j < k} \sigma_{jk}^2 \quad (24)$$

The method works best when the frequency standards are comparable in instability. For $n = 3$, the variance of frequency standard 1 is $\sigma_1^2 = \frac{1}{2} [\sigma_{12}^2 + \sigma_{13}^2 - \sigma_{23}^2]$

4.2.2 Frequency/phase detection

The fractional frequency deviation $y(t)$ of a precision frequency standard with respect to the reference frequency standard is often measured with a system consisting of frequency conditioning through frequency multipliers, dividers, and mixers and a time interval and/or frequency counter or phasemeter. If the environmental sensitivity occurs over a range of frequencies, the influence of the environmental sensitivity can be characterized with a spectrum analyzer. Phase noise may be measured with a double-balanced mixer and a phase-locked reference oscillator or a commercial phase-noise measurement system. In cases with very low phase noise, it may be necessary to use a two-channel cross-correlation technique to measure the phase noise (Howe et al. [B78], Stein [B142], Walls et al. [B172], and Walls [B173]). Control of experimental conditions is critical to obtaining reproducible results.

4.2.3 Correlation techniques

Analyzing a time series of frequency measurements with autocorrelation functions can be a useful tool for identifying and analyzing environmental sensitivities. A sinusoidal appearance to the autocorrelogram indicates the presence of periodic phenomena, either internal or environmental in origin. Internal variations and statistical noise inherent to the system can be characterized and calibrated in steady state and subsequently applied to environmental test data to isolate environmental sensitivity (Box and Jenkins [B23]).

Cross-correlation techniques can also be used to deduce environmental sensitivities (Box and Jenkins [B23] and Breakiron et al. [B27]). The method may be compromised if different (e.g., thermal) time constants are involved (Clause 6). An example of determining a temperature sensitivity coefficient with cross correlations is presented in Annex A.

Any positive correlation between two frequency standards reduces the deviations between their frequencies, resulting in an underestimation in environmental sensitivities. Interactions between adjacent frequency standards (e.g., a test unit and its local reference or a field unit and its backup) may result from common-mode electric field coupling, magnetic field coupling, common ground currents, common vibrations, temperature, and feedback through control loops where circuit isolation is deficient (Jaduszliwer et al. [B84]).

Intercorrelations among the frequency standards and errors associated with the variance determinations may result in the computation of negative variances (Stein [B142] and Yoshimura [B178]), which may be avoided by taking explicit account of the intercorrelations (Tavella and Premoli [B153], Premoli and Tavella [B132], Torcaso et al. [B158], Ekstrom and Koppang [B45]). Measurement system noise may be rejected by an equivalent method that utilizes cross-correlation (Gros Lambert et al. [B67]).

4.2.4 Isolating measurements of environmental sensitivities

When a precision frequency standard is under test, it is powered on and allowed sufficient time to stabilize. An initial baseline of its resonance frequency and phase noise are measured. Then, as ambient conditions are varied, measurements are made of the frequency standard's characteristics (e.g., its frequency vs. temperature curve or its phase noise; Kinsman and Rydback [B93]).

Typically, it is best to carry out environmental sensitivity measurements by varying the environmental parameter being studied on a timescale different from other time-dependent variations in the DUT. For instance, if the frequency standard has an unavoidable diurnal variation due to a systematic effect unrelated to the environmental effect being studied, then the diurnal effect should first be removed, or the environmental study should be carried out over a time duration or with a repetition period very short or very long compared with 1 day.

It is commonly difficult to effectively measure the sensitivity of the DUT rather than a sensitive property of the environmental sensor or transducer (e.g., oven). Consequently, all sensors must be independently calibrated and redundant where possible. In addition, the sensor and transducer should be as closely coupled to the frequency standard as possible or, alternatively, should be located and/or buffered to duplicate as closely as possible the experience of the frequency standard as it will be deployed in actual field use. This process is particularly important whenever there are gradients in the environmental parameter. For instance, the temperature at the base plate supporting the frequency standard or the ambient air surrounding the DUT may be very different from the temperature somewhere inside the device itself and the difference may be time dependent. In this case, the location of the temperature sensor must be clearly stated in the reported temperature coefficient.

If more than one environmental factor proves to be of significance, all other such factors should be held constant while the dependence of each is independently determined. Since the relevant factors are not always obvious, and since unrecognizable influences can obscure the effect of a given parameter, it is best to control as many of the ambient conditions as possible, for example, by isolating the frequency standard and measurement system in a controlled environmental chamber with good thermal isolation, humidity control, air circulation, and magnetic shielding. To separate and determine the dominant effect, the variables (e.g., T , H , and P) are typically allowed to assume high and low values in a defined pattern (Bhote and Keki [B20]). For an example of determination of a temperature sensitivity, see Annex A.

The testing of mathematical models for the environmental dependences should start with the simplest form (in number and order of terms) and proceed with the addition of single terms (in decreasing order of importance) and retaining those parameters whose coefficients exceed their associated standard errors. The most consistently successful models reflect using adjustable parameters sparingly (Box and Jenkins [B23]).

If error residuals are normally distributed, the values of the sensitivity coefficients and their standard errors may be determined by least-squares fit. In the case of errors that have non-Gaussian but well-determined distributions, the coefficients may be estimated by the maximum likelihood method or the method of moments (Hazelton [B72]). If individual measurements differ significantly in precision and different types of measurements are combined in a solution, each type should be weighted by its respective variance.

If any variables are significantly correlated, the covariance matrix should be included in the solution. Although multivariate analytical methods exist for the circumvention of covariance determination, the

correlations are too interesting in themselves to recommend these. If a correlation between two quantities arises because they are functions of some other quantity, it might be preferable (depending on the acceptable number of parameters) to incorporate the latter quantity in the model, thereby eliminating the correlation. Interactions between environmental sensitivities can also complicate the analysis. Some examples of interactions among environmental stimuli are listed in Annex B.

Other factors that can complicate evaluation of sensitivity coefficients are changes due to hysteresis and, in the long term, aging. In the case of nonlinearities, hysteresis, and dynamic effects, plots may be more informative than models.

4.2.5 Transient effects and aging

Specifications of environmental sensitivities assume quasi-static conditions, for which the frequency standard has stabilized and is in equilibrium with its surroundings before an ambient change occurs, and after application of a change in environmental conditions, the frequency standard has reached a new equilibrium. When enough time is not allowed between the application of an environmental change and a measurement of the effect of the change, the measurement results can depend on the temporal profile of the stimulus and the initial condition of the frequency standard (Hellwig [B75]).

Most frequency standards exhibit changes in output frequency over time without any application of environmental stimuli through equilibration after turn-on (Lu and Tsuzuki [B105]) and aging (see Clause 9). Frequency standards also exhibit an equilibration period after the application of a change in environmental conditions where the output frequency changes exponentially to a new value.

To avoid artifacts in the measurement of environmental sensitivities, frequency measurements for environmental tests should be made when the DUT has warmed up beyond any significant equilibration period and been allowed to reach equilibrium after a change in ambient conditions. A period of at least two exponential time constants after turn-on or application of an environmental stimulus should be allowed for before measurement. After equilibration, enough data for a statistically meaningful result must be acquired for each measurement (Hellwig [B75]).

Because most types of frequency standards exhibit aging, modulation of the environmental conditions under study over many cycles can mitigate uncontrolled frequency changes from aging such that it can be measured and corrected (Annex A).

If the user is specifically interested in the effect of a dynamic environment, test methods would ideally simulate the exact profile of the stimulus in question. Analysis via the dynamic Allan deviation (DADEV) may be employed to monitor for transient effects on the measurement precision (Galleani and Tavella [B57], [B58]).

4.3 Error budgets

A published environmental sensitivity measurement should always be accompanied by its uncertainty, and to help ensure that both are meaningful and reproducible, the physical and statistical methods involved should be clearly documented, including all limitations, correction factors, and error sources. The values of all other environmental parameters during the measurement should also be stated.

In principle, variation of all the parameters on which the results of a measurement depend should yield the total variation in the measured value. The total variance would be just the vector sum of the variances and covariances of the parameters and of any other sources of error. However, time and resource constraints usually require that the measurement uncertainty be evaluated using a mathematical model and the law of propagation of errors. Assuming the errors are random, if a variable f is a function of other variables $x_1, x_2, \dots, x_j, \dots, x_p$, then the law of propagation results in a total error squared as shown in Equation (25).

$$\sigma^2(f) \approx \sum_{j=1}^p \left[\frac{\partial f}{\partial \chi_j} \right]^2 \sigma^2(\chi_j) + 2 \sum_{j=1}^p \sum_{k < j} \frac{\partial f}{\partial \chi_j} \frac{\partial f}{\partial \chi_k} \phi(\chi_j, \chi_k) \quad (25)$$

or results in an approximate error as shown in Equation (26).

$$\sigma^2(f) \approx \sum_{j=1}^p \left[\frac{\partial f}{\partial \chi_j} \right]^2 \sigma^2(\chi_j) + 2 \sum_{j=1}^p \sum_{k < j} \frac{\partial f}{\partial \chi_j} \frac{\partial f}{\partial \chi_k} r(\chi_j, \chi_k) \sigma(\chi_j) \sigma(\chi_k) \quad (26)$$

where

$$\begin{aligned} \sigma^2(\chi_j) & \text{ is the variance of } \chi_j \\ \phi(\chi_j, \chi_k) & \text{ is the covariance between } \chi_j \text{ and } \chi_k \\ r(\chi_j, \chi_k) & \text{ is the linear correlation coefficient between } \chi_j \text{ and } \chi_k \end{aligned}$$

In the special case where the derivatives are unity and the intercorrelations are zero, Equation (26) becomes Equation (27).

$$\sigma^2(f) = \sum_{j=1}^p \sigma^2(\chi_j) \quad (27)$$

Furthermore, if f is strictly a product and a quotient of the input quantities, then all the variances may be expressed as relative variances as follows in Equation (28).

$$\frac{\sigma^2(f)}{f^2} = \sum_{j=1}^p \left[\frac{\sigma^2(\chi_j)}{\chi_j} \right]^2 \quad (28)$$

In Equation (25), let f be the frequency, let $\chi_j, \chi_k, \dots$ be the environmental parameters measured at time $m\tau$, where τ is the measurement interval, and let $\sigma^2(\chi_j)$ be the two-sample variance of n observations of χ_j , as shown in Equation (29).

$$\sigma^2(\chi_j) = \frac{1}{2(n-1)} \sum_{m=1}^{n-1} [\chi_j(m+1) - \chi_j(m)]^2 \quad (29)$$

The two-sample covariance is shown in Equation (30).

$$\phi(\chi_j, \chi_k) = \frac{1}{2(n-1)} \sum_{m=1}^{n-1} [\chi_j(m+1) - \chi_j(m)] [\chi_k(m+1) - \chi_k(m)] \quad (30)$$

The two-sample variance and covariance can be plotted as a function of measurement period τ (see IEEE Std 1139). One could then use Equation (25), Equation (29), and Equation (30) to predict how variations in the ambient conditions transduce into variations in the frequency, as well as to predict how intercorrelations between the parameters affect the frequency. Also, one could use this method to measure environmental effects stochastically.

To properly evaluate the overall instability of a system in the presence of all the known sources of error, one should perform an “error (or uncertainty) budget” analysis. After identifying all significant error sources through, say, correlation studies, the individual contributions to the combined uncertainty $\sigma^2(f)$ by each parameter variance $\sigma^2(\chi_j), \sigma^2(\chi_k), \dots$ and its covariances $\phi(\chi_j, \chi_k), \dots$, which may be evaluated through measurement. Some error sources are not available or amenable to repeated measurements and statistical analysis. Still, variances for them should be estimated as well as possible based on published information or experience (as should be stated).

Equation (25) allows the apportioning of an error budget among the relevant contributing sources. Any detailed report about uncertainties should consist of a complete list of the error components, the numerical value of each standard error, and the method used to determine it. If the measured quantities are significantly correlated, the correlation coefficient matrix, the covariance matrix, or preferably both should be given (GUM:1995 [B69]).

The total uncertainty may be expressed either as a tolerance (a specified number of standard errors) or as a confidence limit (a specified probability percentage). For the former to be meaningful and for the latter to be determinate at all, the probability distribution of the total error residuals must be known, at least approximately. Hence, it is preferable to correct the data first for systematic (nonstochastic) effects so the residuals will be normally (Gaussian) distributed, which simplifies further analysis. But for the analysis to be valid, the distribution of the residuals needs to be evaluated and checked carefully. Special care must also be taken for very small data sets (see, for example, the use of Student’s t distribution in Meyer [B112]). The hypothesis that the residuals are randomly distributed may be tested, for example, with a chi-square test or normal probability plot. The presence of uncorrected systematic effects may be recognized in residuals that depart significantly from a normal distribution. The corrections for systematic effects have their own uncertainties that should be included in any error budget (GUM:1995 [B69]).

White noise frequency modulation (FM) tends to be the predominant noise model for integration times of the order of a second out to several thousand seconds in both cesium and rubidium frequency standards. In the case of cesium, white FM noise may dominate for integration times of the order of a day and even longer. Measuring sensitivity coefficients in the presence of this kind of noise presents a practical signal-to-noise problem. As the optimum estimate of the mean of a white process is the simple mean, when measuring the effect of changes in these environmental parameters, it is best to hold them constant and average the frequency for an interval such that the $\sigma_y(\tau)$ curve starts changing from $\tau^{-1/2}$ toward a flattening (flicker floor), where τ is the period over which the frequency is averaged. Then the environmental parameter being evaluated should be changed and the integration time should be repeated to measure the frequency change. The precision with which the change in frequency with a change in an environmental parameter can be determined may be improved by reiterating the above process several times (following the above rule for integration time). In principle, if N is the number of changes back and forth, then the confidence on the frequency change is the value of $\sigma_y(\tau)N^{-1/2}$. One should respect the settling times after changing a parameter’s value, as well as other systematics affecting the measurement.

For assessing and understanding the instability and accuracy of a frequency standard, the concepts of Type A and Type B uncertainties are often used. Type A uncertainties are classified as uncertainties that are evaluated by statistical measurements. Type B uncertainties are evaluated by other means. Their definitions and explanations as they apply to specific types of measurements can be found in GUM:1995 [B69]. A simple correspondence does not always occur between the classification of uncertainty components into categories A and B and the commonly used classification of uncertainty components as “random” and “systematic” (Taylor and Kuyatt [B155]).