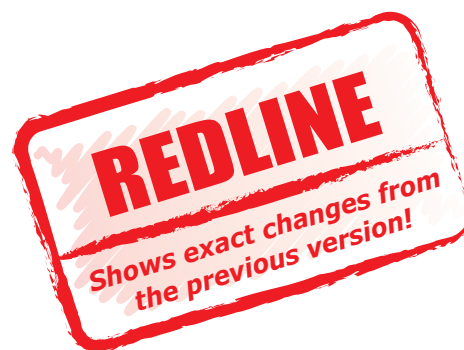




IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

IEEE Power & Energy Society

Sponsored by the
Power System Instrumentation and Measurements Committee



IEEE
3 Park Avenue
New York, NY 10016-5997, USA

19 March 2010

IEEE Std 1459™-2010
(Revision of
IEEE Std 1459-2000)

This is a Redline Document produced by Techstreet, a business of Thomson Reuters.

This document is intended to provide users with an indication of changes from one edition to the next. It includes a full-text version of the new document, plus an indication of changes from the previous version.

Redlines are designed to save time and improve efficiencies by using the latest software technology to find and highlight document changes. More professionals are using valuable new technologies like redlines, to help improve outcomes in a fastpaced global business world.

Because it may not be technically possible to capture all changes accurately, it is recommended that users consult previous editions as appropriate. In all cases, only the current base version of this publication is to be considered the official document.

Redline Processing Notes:

1. ~~Red Text~~ - Red strikethrough text denotes deletions.
2. Blue Text - Blue underlined text denotes modifications and additions.

IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

Sponsor

Power System Instrumentation and Measurements Committee

of the

IEEE Power & Energy Society

Approved 2 February 2010

IEEE-SA Standards Board

Figure 1 © 1983 IEEE. Reprinted, with permission, from the IEEE and R. H. Stevens.

Abstract: Definitions used for measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions are provided in this standard. Mathematical expressions that were used in the past, as well as new expressions, are listed, as well as explanations of the features of the new definitions.

Keywords: active power, apparent power, nonactive power, power factor, reactive power, total harmonic distortion

The Institute of Electrical and Electronics Engineers, Inc.
3 Park Avenue, New York, NY 10016-5997, USA

Copyright © 2010 by the Institute of Electrical and Electronics Engineers, Inc.
All rights reserved. Published 19 March 2010. Printed in the United States of America.

IEEE is a registered trademark in the U.S. Patent & Trademark Office, owned by the Institute of Electrical and Electronics Engineers, Incorporated.

PDF: ISBN 978-0-7381-6058-0 STD95967
Print: ISBN 978-0-7381-6059-7 STDPD95967

IEEE prohibits discrimination, harassment and bullying. For more information, visit <http://www.ieee.org/web/aboutus/whatis/policies/p9-26.html>.
No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

IEEE Standards documents are developed within the IEEE Societies and the Standards Coordinating Committees of the IEEE Standards Association (IEEE-SA) Standards Board. The IEEE develops its standards through a consensus development process, approved by the American National Standards Institute, which brings together volunteers representing varied viewpoints and interests to achieve the final product. Volunteers are not necessarily members of the Institute and serve without compensation. While the IEEE administers the process and establishes rules to promote fairness in the consensus development process, the IEEE does not independently evaluate, test, or verify the accuracy of any of the information or the soundness of any judgments contained in its standards.

Use of an IEEE Standard is wholly voluntary. The IEEE disclaims liability for any personal injury, property or other damage, of any nature whatsoever, whether special, indirect, consequential, or compensatory, directly or indirectly resulting from the publication, use of, or reliance upon this, or any other IEEE Standard document.

The IEEE does not warrant or represent the accuracy or content of the material contained herein, and expressly disclaims any express or implied warranty, including any implied warranty of merchantability or fitness for a specific purpose, or that the use of the material contained herein is free from patent infringement. IEEE Standards documents are supplied “**AS IS.**”

The existence of an IEEE Standard does not imply that there are no other ways to produce, test, measure, purchase, market, or provide other goods and services related to the scope of the IEEE Standard. Furthermore, the viewpoint expressed at the time a standard is approved and issued is subject to change brought about through developments in the state of the art and comments received from users of the standard. Every IEEE Standard is subjected to review at least every five years for revision or reaffirmation, or every ten years for stabilization. When a document is more than five years old and has not been reaffirmed, or more than ten years old and has not been stabilized, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art. Users are cautioned to check to determine that they have the latest edition of any IEEE Standard.

In publishing and making this document available, the IEEE is not suggesting or rendering professional or other services for, or on behalf of, any person or entity. Nor is the IEEE undertaking to perform any duty owed by any other person or entity to another. Any person utilizing this, and any other IEEE Standards document, should rely upon his or her independent judgment in the exercise of reasonable care in any given circumstances or, as appropriate, seek the advice of a competent professional in determining the appropriateness of a given IEEE standard.

Interpretations: Occasionally questions may arise regarding the meaning of portions of standards as they relate to specific applications. When the need for interpretations is brought to the attention of IEEE, the Institute will initiate action to prepare appropriate responses. Since IEEE Standards represent a consensus of concerned interests, it is important to ensure that any interpretation has also received the concurrence of a balance of interests. For this reason, IEEE and the members of its societies and Standards Coordinating Committees are not able to provide an instant response to interpretation requests except in those cases where the matter has previously received formal consideration. A statement, written or oral, that is not processed in accordance with the IEEE-SA Standards Board Operations Manual shall not be considered the official position of IEEE or any of its committees and shall not be considered to be, nor be relied upon as, a formal interpretation of the IEEE. At lectures, symposia, seminars, or educational courses, an individual presenting information on IEEE standards shall make it clear that his or her views should be considered the personal views of that individual rather than the formal position, explanation, or interpretation of the IEEE.

Comments for revision of IEEE Standards are welcome from any interested party, regardless of membership affiliation with IEEE. Suggestions for changes in documents should be in the form of a proposed change of text, together with appropriate supporting comments. Recommendations to change the status of a stabilized standard should include a rationale as to why a revision or withdrawal is required. Comments and recommendations on standards, and requests for interpretations should be addressed to:

Secretary, IEEE-SA Standards Board
445 Hoes Lane
Piscataway, NJ 08854
USA

Authorization to photocopy portions of any individual standard for internal or personal use is granted by The Institute of Electrical and Electronics Engineers, Inc., provided that the appropriate fee is paid to Copyright Clearance Center. To arrange for payment of licensing fee, please contact Copyright Clearance Center, Customer Service, 222 Rosewood Drive, Danvers, MA 01923 USA; +1 978 750 8400. Permission to photocopy portions of any individual standard for educational classroom use can also be obtained through the Copyright Clearance Center.

Introduction

This introduction is not part of IEEE Std 1459-2010, IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions.

The definitions for active, reactive, and apparent powers that are currently used are based on the knowledge developed and agreed on during the 1940s. Such definitions served the industry well, as long as the current and voltage waveforms remained nearly sinusoidal.

Important changes have occurred in the last 50 years. The new environment is conditioned by the following facts:

- a) Power electronics equipment, such as Adjustable Speed Drives, Controlled Rectifiers, Cycloconverters, Electronically Ballasted Lamps, Arc and Induction Furnaces, and clusters of Personal Computers, represent major nonlinear and parametric loads proliferating among industrial and commercial customers. Such loads have the potential to create a host of disturbances for the utility and the end-user's equipment. The main problems stem from the flow of nonactive energy caused by harmonic currents and voltages.
- b) New definitions of powers have been discussed in the last 30 years in the engineering literature (Filipski and Labaj [B9]^a). The mechanism of electric energy flow for nonsinusoidal and/or unbalanced conditions is well understood today.
- c) The traditional instrumentation designed for the sinusoidal 60/50 Hz waveform is prone to significant errors when the current and the voltage waveforms are distorted (Filipski and Labaj [B9]).
- d) Microprocessors and minicomputers enable today's manufacturers of electrical instruments to construct new, accurate, and versatile metering equipment that is capable of measuring electrical quantities defined by means of advanced mathematical models.
- e) There is a need to quantify correctly the distortions caused by the nonlinear and parametric loads, and to apply a fair distribution of the financial burden required to maintain the quality of electric service.

This standard lists new definitions of powers needed for the following particular situations:

- When the voltage and current waveforms are nonsinusoidal
- When the load is unbalanced or the supplying voltages are asymmetrical
- When the energy dissipated in the neutral path due to zero-sequence current components has economical significance

The new definitions were developed to give guidance with respect to the quantities that should be measured or monitored for revenue purposes, engineering economic decisions, and determination of major harmonic polluters. The following important electrical quantities are recognized by this standard:

- The power frequency (60/50 Hz or fundamental) of apparent, active, and reactive powers. These three basic quantities are the quintessence of the power flow in electric networks. They define what is generated, transmitted, distributed, and sold by the electric utilities and bought by the end users. This is the electric energy transmitted by the 60/50 Hz electromagnetic field. In poly-phase systems, the power frequency positive-sequence powers are the important dominant quantities. The power frequency positive-sequence power factor is a key value that helps determine and adjust the flow of power frequency positive-sequence reactive power. The

^a The numbers in brackets correspond to those of the bibliography in Annex C.

fundamental positive-sequence reactive power is of utmost importance in power systems; it governs the fundamental voltage magnitude and its distribution along the feeders and affects electromechanical stability as well as the energy loss.

- The effective apparent power in three-phase systems is $S_e = 3V_e I_e$, where V_e and I_e are the equivalent voltage and current. In sinusoidal and balanced situations, S_e is equal to the conventional apparent power $S = 3V_{\ell n} I = \sqrt{3}V_{\ell \ell} I$, where $V_{\ell n}$ and $V_{\ell \ell}$ are the line-to-neutral and the line-to-line voltage, respectively. For sinusoidal unbalanced or for nonsinusoidal balanced or unbalanced situations, S_e allows rational and correct computation of the power factor. This quantity was proposed in 1922 by the German engineer Buchholz [B1] and in 1933 was explained by the American engineer Goodhue [B11].
- The non-60 Hz or nonfundamental apparent power is S_N (for brevity, 50 Hz power is not always mentioned). This power quantifies the overall amount of harmonic pollution delivered or absorbed by a load. It also quantifies the required capacity of dynamic compensators or active filters when used for nonfundamental compensation alone.
- Current distortion power D_I identifies the segment of nonfundamental nonactive power due to current distortion. This is usually the dominant component of S_N .
- Voltage distortion power D_V separates the nonfundamental nonactive power component due to voltage distortion.
- Apparent harmonic power S_H indicates the level of apparent power due to harmonic voltages and currents alone. This is the smallest component of S_N and includes the harmonic active power P_H .

To avoid confusion, it was decided not to add new units. The use of the watts (W) for instantaneous and active powers, volt-amperes (VA) for apparent powers, and (var) for all the nonactive powers maintains the distinct separation among these three major types of powers.

There is not yet available a generalized power theory that can provide a simultaneous common base for

- Energy billing
- Evaluation of electric energy quality
- Detection of the major sources of waveform distortion
- Theoretical calculations for the design of mitigation equipment such as active filters or dynamic compensators

This standard is meant to provide definitions extended from the well-established concepts. It is meant to serve the user who wants to measure and design instrumentation for energy and power quantification. It is not meant to help in the design of real-time control of dynamic compensators or for diagnosis instrumentation used to pinpoint to a specific type of annoying event or harmonic.

These definitions are meant to serve as a guideline and as a useful benchmark for future developments.

Notice to users

Laws and regulations

Users of these documents should consult all applicable laws and regulations. Compliance with the provisions of this standard does not imply compliance to any applicable regulatory requirements. Implementers of the standard are responsible for observing or referring to the applicable regulatory

requirements. IEEE does not, by the publication of its standards, intend to urge action that is not in compliance with applicable laws, and these documents may not be construed as doing so.

Copyrights

This document is copyrighted by the IEEE. It is made available for a wide variety of both public and private uses. These include both use, by reference, in laws and regulations, and use in private self-regulation, standardization, and the promotion of engineering practices and methods. By making this document available for use and adoption by public authorities and private users, the IEEE does not waive any rights in copyright to this document.

Updating of IEEE documents

Users of IEEE standards should be aware that these documents may be superseded at any time by the issuance of new editions or may be amended from time to time through the issuance of amendments, corrigenda, or errata. An official IEEE document at any point in time consists of the current edition of the document together with any amendments, corrigenda, or errata then in effect. In order to determine whether a given document is the current edition and whether it has been amended through the issuance of amendments, corrigenda, or errata, visit the IEEE Standards Association web site at <http://ieeexplore.ieee.org/xpl/standards.jsp>, or contact the IEEE at the address listed previously.

For more information about the IEEE Standards Association or the IEEE standards development process, visit the IEEE-SA web site at <http://standards.ieee.org>.

Errata

Errata, if any, for this and all other standards can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/updates/errata/index.html>. Users are encouraged to check this URL for errata periodically.

Interpretations

Current interpretations can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/interp/index.html>.

Patents

Attention is called to the possibility that implementation of this standard may require use of subject matter covered by patent rights. By publication of this standard, no position is taken with respect to the existence or validity of any patent rights in connection therewith. The IEEE is not responsible for identifying Essential Patent Claims for which a license may be required, for conducting inquiries into the legal validity or scope of Patent Claims or determining whether any licensing terms or conditions provided in connection with submission of a Letter of Assurance, if any, or in any licensing agreements are reasonable or non-discriminatory. Users of this standard are expressly advised that determination of the validity of any patent rights, and the risk of infringement of such rights, is entirely their own responsibility. Further information may be obtained from the IEEE Standards Association.

Participants

At the time this standard was submitted to the IEEE-SA Standards Board for approval, the Non-Sinusoidal Situations Working Group had the following membership:

Alexander E. Emanuel, *Chair*
Eddy So, *Sponsor*

Jose Policarpo Abreu
Rejean Arseneau
Santiago Barcon
Andrew Berrisford
Yahia Baghzouz
Keneth B. Bowes
James A. Braun
Antonio Cataliotti
David Cooper
Valentina Cosentino
Mikey D. Cox
Roger H. Daugherty
Soni Devendra
Dario Di Cara
William Dickerson
Alexander Domijan

David Elmore
Gaetan Ethier
Erich Gunther
Dennis Hansen
Ernst Hanique
Gilbert C. Hensley
John Houdek
Roberto Langella
Michael Lowenstein
William Moncrief
Alexander McEachern
Dalgerti Milanez
Thomas L. Nelson
Vuong Nguyen
Daniel Nordell
Salvatore Nuccio

Slobodan Pajic
Lorenzo Peretto
Johan H. C. Pretorius
Paulo Ribeiro
Daniel Sabin
Kalyan Sen
Piet H. Swart
Donald Tandon
Alfredo Testa
Grazia Todeschini
Daniel Ward
Scott Weikel
Stephan Weiss
Douglas Williams
Jacques L. Willems
Daan van Wyk

The following members of the individual balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

William J. Ackerman
Ali Al Awazi
David Baron
Steven Brockschink
William Brumsickle
Gustavo Brunello
Yunxiang Chen
John Cooper
Tommy Cooper
John Crouse
Roger H. Daugherty
Gary L. Donner
Neal Dowling
Dana Dufield
Gearold O. H. Eidhin
Alexander E. Emanuel
Gary Engmann
Paul Forquer
Marcel Fortin
Randall Groves
Gary Heuston

Werner Hoelzl
Randy Horton
Innocent Kamwa
Piotr Karocki
Jon Kay
Tanuj Khandelwal
Yuri Khersonsky
Harold Kirkham
Joseph L. Koepfinger
Jim Kulchisky
Federico Lopez
Michael Lowenstein
Keith Malmedal
Jose Marrero
Kenneth Martin
William McBride
Kenneth Mcclenahan
Gary Michel
Charles Morse
Jerry Murphy
Bruce Muschlitz

Michael S. Newman
David Nichols
Ulrich Pohl
Iulian Profir
Michael Roberts
Charles Rogers
Bob Saint
Steven Sano
Bartien Sayogo
Thomas Schossig
Kenneth Sedziol
Ahmed El Serafi
James E. Smith
Aaron Snyder
Eddy So
Michael Swearingen
David Tepen
John Vergis
Scott Weikel
James Wilson
Ahmed Zobaa

When the IEEE-SA Standards Board approved this standard on 2 February 2010, it had the following membership:

Robert M. Grow, *Chair*
Tom A. Prevost, *Vice Chair*
Steve M. Mills, *Past Chair*
Judith Gorman, *Secretary*

John Barr
Karen Bartelson
Victor Berman
Ted Burse
Richard DeBlasio
Andrew Drozd
Mark Epstein

Alexander Gelman
James Hughes
Richard H. Hulett
Young Kyun Kim
Joseph L. Koepfinger*
John Kulick

David J. Law
Ted Olsen
Glenn Parsons
Ronald C. Petersen
Narayanan Ramachandran
Jon Walter Rosdahl
Sam Sciacca

*Member Emeritus

Also included are the following nonvoting IEEE-SA Standards Board liaisons:

Howard L. Wolfman, *TAB Representative*
Michael Janezic, *NIST Representative*
Satish K. Aggarwal, *NRC Representative*

Lorraine Patsco
IEEE Standards Program Manager, Document Development

Matthew J. Ceglia
IEEE Standards Program Manager, Technical Program Development

Contents

1. Overview	1
1.1 Scope	1
1.2 Purpose	2
2. Normative references.....	2
3. Definitions	2
3.1 Single phase.....	2
3.2 Three-phase systems.....	13
Annex A (informative) Theoretical examples	30
Annex B (informative) Practical studies and measurements: A detailed explanation of apparent power components.....	34
Annex C (informative) Bibliography.....	39

IEEE ~~Trial-Use~~ Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

IMPORTANT NOTICE: This standard is not intended to ensure safety, security, health, or environmental protection in all circumstances. Implementers of the standard are responsible for determining appropriate safety, security, environmental, and health practices or regulatory requirements.

This IEEE document is made available for use subject to important notices and legal disclaimers. These notices and disclaimers appear in all publications containing this document and may be found under the heading “Important Notice” or “Important Notices and Disclaimers Concerning IEEE Documents.” They can also be obtained on request from IEEE or viewed at <http://standards.ieee.org/IPR/disclaimers.html>.

1. Overview

This ~~trial-use~~ standard is divided into three clauses. Clause 1 lists the scope of this document. Clause 2 lists references to other standards that are useful in applying this standard. Clause 3 provides the definitions, among which there are several new expressions.

The preferred mathematical expressions recommended for the instrumentation design are marked with a || sign. The additional expressions are meant to reinforce the theoretical approach and to facilitate a better understanding of the explained concepts.

1.1 Scope

This document provides definitions ~~used for measurement~~ of electric power to quantify the flow of electrical energy in single-phase and three-phase circuits under sinusoidal, nonsinusoidal, balanced, ~~or~~ and unbalanced conditions. ~~It lists the mathematical expressions that were used in the past, as well as new expressions, and explains the features of the new definitions.~~

1.2 Purpose

This document provides organizations with criteria for designing and using metering instrumentation

~~This trial-use standard is meant to provide organizations with criteria for designing and using metering instrumentation.~~

2. Normative references

~~This trial use standard shall be used in conjunction with the following publications. If the following publications are superseded by an approved revision, the revision shall apply.~~

The following referenced documents are indispensable for the application of this document (i.e., they must be understood and used, so each referenced document is cited in text and its relationship to this document is explained). For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

DIN 40110-1997, Quantities Used in Alternating Current Theory.¹

~~IEEE Std 280-1985 (Reaff 1997), IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering.²~~

IEC 80000-6:2008, ISO 31-5:1992 Quantities and Units—Part 6:5: Electricity and Magnetism.³
Electromagnetism.²

3. Definitions

For the purposes of this document, the following terms and definitions apply. The IEEE Standards Dictionary: Glossary of Terms & Definitions should be referenced for terms not defined in this clause.³

NOTE—Mathematical expressions that are considered appropriate for instrumentation design are marked with the sign ||. When the sign || appears on the right side, it means that the last expression that is listed is favored. Each descriptor of a power type is followed by its measurement unit in parentheses.⁴

3.1 Single phase

3.1.1 Single-phase sinusoidal

A sinusoidal voltage source

$$v = \sqrt{2}V \sin(\omega t)$$

supplying a linear load will produce a sinusoidal current (assumed lagging the voltage) of

$$i = \sqrt{2}I \sin(\omega t - \theta)$$

where

V is the rms value of the voltage (V)

I is the rms value of the current (A)

¹DIN publications are available from ~~the~~ [DIN](http://www.din.de) Deutsches Institut für Normung, ~~Burggrafenstrasse 6, Postfach 1107, 12623 10787~~ Berlin 30, Germany (011 49 30 260 1362), Germany

²~~IEEE~~ (<http://www.din.de>). ³IEC publications are available from the [Institute Sales Department](http://standards.ieee.org/) of ~~Electrical and Electronics Engineers~~, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA (<http://standards.ieee.org/>).

³~~ISO publications are available from the ISO Central Secretariat~~ [the International Electrotechnical Commission](http://www.iso-iec.ch/), Case Postale 56, 131, 3 rue de Varembe, CH-1211, Genève 20,

Switzerland/Suisse (<http://www.iso-iec.ch/>). ~~ISO IEC~~ publications are also available in the United States from the Sales Department, American National Standards Institute, 11 West 42nd Street, 13th Floor, New York, NY 10036, USA.

⁴The *IEEE Standards Dictionary: Glossary of Terms & Definitions* is available at <http://shop.ieee.org/>.

⁴ Notes in text, tables, and figures of a standard are given for information only and do not contain requirements needed to implement this standard.

- ω is the angular frequency $2\pi f$ (rad/s)
- f is the power system frequency (Hz)
- θ is the phase angle between the current and the voltage (rad)
- t is the time (s)

For more information on symbols and units, see IEEE Std 280™-1985 [B13] and IEC 80000-6:2008.

3.1.1.1 Instantaneous power (W)

The instantaneous power p is given by

$$p = vi$$

$$p = p_a + p_q$$

where

$$p_a = VI \cos \theta [1 - \cos(2\omega t)] = P[1 - \cos(2\omega t)]; \quad P = VI \cos \theta$$

$$p_q = -VI \sin \theta \sin(2\omega t) = -Q \sin(2\omega t); \quad Q = VI \sin \theta$$

NOTE 1—The component p_a is the instantaneous active power. It is produced by the active component of the current (i.e., by the component that is in phase with the voltage). The instantaneous active power p_a is the rate of flow of the energy

$$w_a = \int_{t_0}^t p_a dt = P(t - t_0) - \frac{P}{2\omega} [\sin(2\omega t) - \sin(2\omega t_0)]$$

This energy flows unidirectional from the source to the load. Its steady-state rate of flow is not negative, $p_a \geq 0$.

~~2—The instantaneous power p_q is produced by the reactive component of the current, i.e., the component that is in quadrature with the voltage. It is the rate of flow of the energy~~

NOTE 2—The instantaneous active power has two terms: the active or real power P and ~~This type of energy oscillates between the source inductances, capacitances; the intrinsic power $-P \cos(2\omega t)$.~~ The intrinsic power is always present when net energy is transferred to the load; however, this oscillating component does not cause power loss in the supplying lines.

NOTE 3—The component p_q is the instantaneous reactive power. It is produced by the reactive component of the current (i.e., the component that is in quadrature with the voltage). The instantaneous reactive power p_q is the rate of flow of the energy

Authorized licensed use limited to: Tsinghua University. Downloaded on February 24, 2026 at 16:25:01 UTC from IEEE Xplore. Restrictions apply.

$$w_q = \int_{t_0}^t p_q dt = \frac{Q}{2\omega} [\cos(2\omega t) - \cos(2\omega t_0)]$$

[This energy component oscillates between the sources and the electromagnetic energy stored within the magnetic field of the inductors and electric field of the capacitors of electrical equipment, as well as the mechanical energy stored in moving masses pertaining to electromechanical systems \(motor and generator rotors, plungers, and armatures\). The average value of this rate of flow is zero, and the net transfer of energy to the load is nil; nevertheless, these power oscillations do cause power loss \(Joule and eddy-current\) in the conductors.](#)

3.1.1.2 Active power (W)

The active power P , [which is also called real power](#), is the ~~mean~~ [average](#) value of the instantaneous power during the ~~observation~~ [measurement](#) time interval τ to $\tau + kT$

$$\begin{aligned} \| P &= \frac{1}{kT} \int_{\tau}^{\tau+kT} p dt \\ \| P &= \frac{1}{kT} \int_{\tau}^{\tau+kT} p dt = \frac{1}{kT} \int_{\tau}^{\tau+kT} p_a dt \end{aligned}$$

where

$T = 1/f$ is the cycle [time](#) (s)
 k is [a positive](#) integer number
 τ is the moment when the measurement starts
 $P = VI \cos \theta$

[NOTE— \$P\$ is also equal to the average of \$p_q\$ over a period, or an integer number of periods, because the average of \$p_q\$ is zero.](#)

3.1.1.3 Reactive power (var)

The [magnitude of the](#) reactive power Q ~~is equals~~ the amplitude of the oscillating instantaneous [reactive](#) power p_q .

$$Q = VI \sin \theta$$

$$\begin{aligned} Q &= \frac{1}{2\pi} \oint v di = \frac{-1}{2\pi} \oint i dv = \frac{1}{kT\omega} \int_{\tau}^{\tau+kT} v \frac{di}{dt} dt = \frac{-1}{kT\omega} \int_{\tau}^{\tau+kT} i \frac{dv}{dt} dt = \frac{-\omega}{kT} \int_{\tau}^{\tau+kT} v \left[\int i dt \right] dt \\ \| Q &= \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i \left[\int v dt \right] dt \end{aligned}$$

NOTE 1— If the load is inductive, then $Q > 0$. If the load is capacitive, then $Q < 0$. [This means that when the current lags the voltage \$\theta > 0\$ and vice versa.](#)

[NOTE 2—The application of the previous definitions to nonsinusoidal conditions is presented in A.2.](#)

3.1.1.4 Apparent power (VA)

The apparent power S is the product of the root-mean-square (rms) voltage and the rms current [\(see The IEEE Standards Dictionary: Glossary of Terms & Definitions⁵\)](#).

$$\|S\| = VI$$

$$S = \sqrt{P^2 + Q^2}$$

NOTE—Instantaneous power p follows a sinusoidal oscillation with a frequency $2f = 2\omega/2\pi$ biased by the active power P . The amplitude of the sinusoidal oscillation is the apparent power S .

3.1.1.5 Power factor

3.1.1.6 Complex power (VA)

$$S = VI^* = P + jQ$$

where

$V = V\angle 0^\circ$ is the voltage phasor,

$$I^* = I\angle -\theta$$

is the conjugated current phasor.

This expression stems from the power triangle, S , P , Q , and is useful in power flow studies. Figure 1 summarizes the conventional power flow directions as interpreted in literature (Stevens [B12]⁴).

3.1.2 Single-Phase nonsinusoidal

For steady-state conditions a nonsinusoidal instantaneous voltage or current has two distinctive components: the power system frequency components v_1 and i_1 , and the remaining terms v_H and i_H that contains all integer and noninteger number harmonics.

$$v_1 = 2V_1 \sin(\omega t - \alpha_1)$$

$$i_1 = 2I_1 \sin(\omega t - \beta_1)$$

$$v_H = 2 \sum_{h \neq 1} V_h \sin(h\omega t - \alpha_h)$$

$$i_H = 2 \sum_{h \neq 1} I_h \sin(h\omega t - \beta_h)$$

⁴The numbers in brackets correspond to those of the bibliography in Annex C.

NOTE 1—The apparent power of a single-phase load can be interpreted as the maximum active power that can be transmitted through the same line while keeping the load rms voltage V constant and the supplying line power loss constant (i.e., the rms current I constant). This is an ideal condition, for which the process of energy conversion at the load remains unchanged, but the utilization of the supplying line is improved (i.e., the thermal stress of the line or cable remains the same while the amount of energy transmitted through the supplying line is increased). This concept implies that an additional load with unity power factor can be connected in parallel with the original load compensated by means of a shunt capacitance or an active compensator.

NOTE 2—The instantaneous power p follows a sinusoidal oscillation with a frequency $2f = 2\omega / 2\pi$ biased by the active power P . The amplitude of the sinusoidal oscillation is the apparent power S .

⁵The IEEE Standards Dictionary: Glossary of Terms & Definitions is available at <http://shop.ieee.org/>.

3.1.1.5 Power factor

$$\text{PF} = \frac{P}{S}$$

NOTE 1—The power factor can be interpreted as the ratio between the energy transmitted to the load over the maximum energy that could be transmitted provided the line losses are kept the same.

NOTE 2—For a given S and V , maximum utilization of the line is obtained when $P = S$; hence, the ratio P/S is a utilization factor indicator.

NOTE 3—When a load, or a cluster of loads, is to be compensated to a higher power factor, the load voltage will increase by a certain increment. If the new voltage is larger than the recommended value, the load voltage can be reduced and brought within recommended range by means of voltage regulators, transformer tap-changers, or other methods of voltage control.

3.1.1.6 Complex power (VA)

The complex power is a complex quantity in which the active power is the real part and the reactive power is the imaginary part

$$S = P + jQ = \mathbf{V}\mathbf{I}^*$$

where according to 3.1.1

$\mathbf{V} = V / \theta^\circ$ is the voltage phasor

$\mathbf{I} = I / -\theta$ is the current phasor

$\mathbf{I}^* = I / \theta$ is the complex conjugate of the current phasor

This expression stems from the power triangle, S , P , and Q , and is useful in power flow studies. Figure 1 summarizes the conventional power flow directions as interpreted in literature (see Stevens [B19]). The angle θ is the phase angle of the equivalent complex impedance $\mathbf{Z} / \theta = \mathbf{V} / \mathbf{I}$.

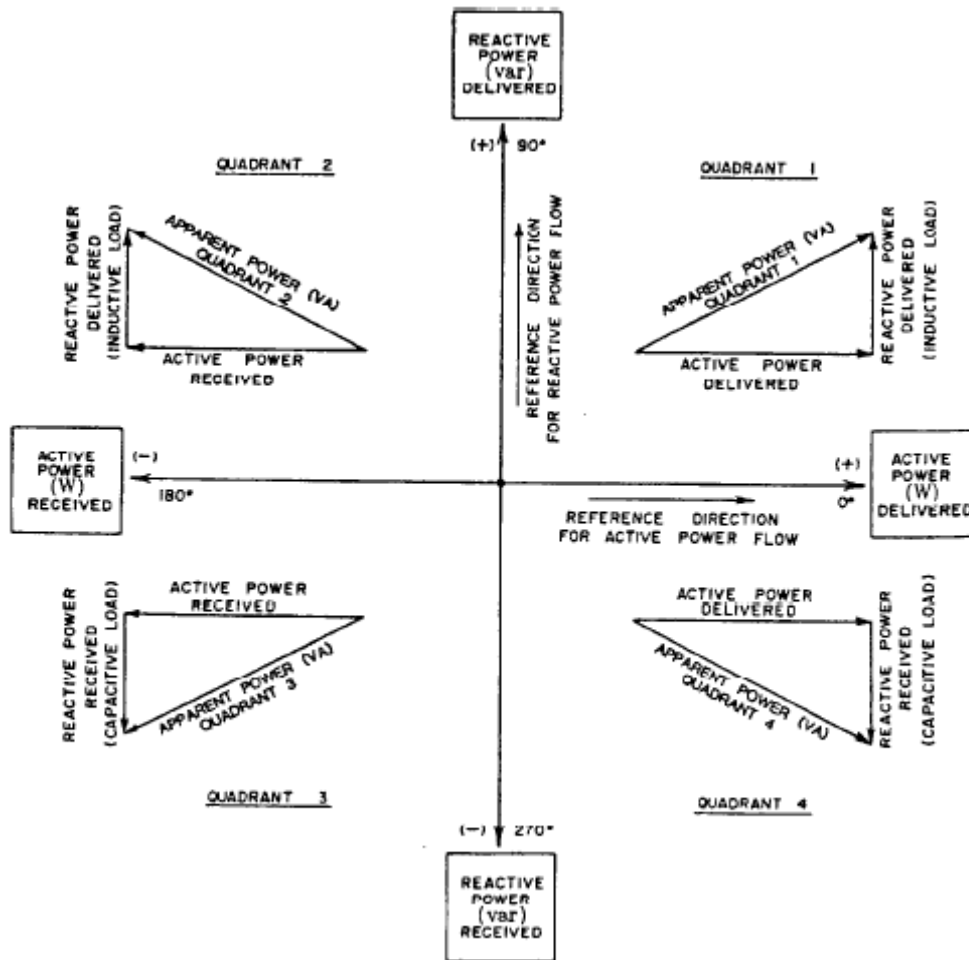


Figure 1—Four-quadrant power flow directions
 (© 1983 IEEE. Reprinted, with permission, from the IEEE and R. H. Stevens [B19])

3.1.2 Single-phase nonsinusoidal

For steady-state conditions, a nonsinusoidal periodical instantaneous voltage or current has two distinct components: the power system frequency components v_1 and i_1 and the remaining term v_H and i_H , respectively.

$$v = v_1 + v_H \text{ and } i = i_1 + i_H$$

where

$$v_1 = \sqrt{2V_1} \sin(\omega t - \alpha_1)$$

$$i_1 = \sqrt{2I_1} \sin(\omega t - \beta_1)$$

$$v_H = V_0 + \sqrt{2} \sum_{h \neq 1} V_h \sin(h\omega t - \alpha_h)$$

$$i_H = I_0 + \sqrt{2} \sum_{h \neq 1} I_h \sin(h\omega t - \beta_h)$$

The corresponding rms values squared are as follows:

$$V^2 = \frac{1}{kT} \int_{\tau}^{\tau+kT} v^2 dt = V_1^2 + V_H^2$$

$$I^2 = \frac{1}{kT} \int_{\tau}^{\tau+kT} i^2 dt = I_1^2 + I_H^2$$

where

$$V_H^2 = V_0^2 + \sum_{h \neq 1} V_h^2 = V^2 - V_1^2 \quad ||$$

and

$$I_H^2 = I_0^2 + \sum_{h \neq 1} I_h^2 = I^2 - I_1^2 \quad ||$$

are the squares of the rms values of v_H and i_H , respectively.

NOTE 1—The direct voltage and the direct current terms V_0 and I_0 , must be included in V_H and I_H .

~~They correspond to a hypothetical $\alpha_0 = \beta_0 = -45^\circ$; $(\sin(-\alpha_0) = \sin(-\beta_0) = \sin 45^\circ = 1/\sqrt{2})$.~~ Significant direct current (dc) components are rarely present in alternating current (ac) power systems; however, traces of dc are common.

NOTE 2—Distorted waveforms often contain frequency components called interharmonics (see IEC 61000-4-7:2002 [B12]). A special group of interharmonics is characterized by $h < 1$. The components belonging to this group have periods larger than the period T of the fundamental frequency. They are called subsynchronous frequency components or subsynchronous interharmonics (in the earlier documents, they are called subharmonics).

NOTE 3—If the distorted voltage and current waveforms consist of harmonics only, then a measurement time interval kT (see 3.1.1.2) enables the correct measurement of rms and power values. If the monitored waveform contains an interharmonic, the measurement time interval kT , which is needed to correctly measure rms values and powers, is the least common multiple of the periods of the fundamental component and the interharmonic component (i.e., $kT = mT_i$; $T_i = 1/f_i$, where f_i is the interharmonic frequency and $k, m = \text{integer numbers}$). When the measurement time interval kT does not include an integer number of periods T_i (i.e., $kT \neq mT_i$), the rms value of the interharmonic as well as the powers associated with it are incorrectly measured (see Peretto et al. [B17]). This error is also reflected in the measurement accuracy of the total rms and powers values. The error is also compounded by the fact that cross-products between the interharmonic current and harmonic voltages (and vice versa) do not yield instantaneous powers with zero mean value.

If at least one of the interharmonics of order h is an irrational number, then the observed waveform is not periodic (it is called nearly periodic.) In such a case, the measurement time interval kT should be infinitely large to have a correct measurement of the rms value or power. For practical situations when the bulk power is carried by the fundamental components, such errors are small (see A.1). The larger the measurement time kT , the less significant become the errors caused by interharmonics (see Peretto et al. [B16]). The theoretical measurement error created when the active power of an interharmonic is measured is strongly affected by the phase angle between the voltage and the current. The closer the phase angle is to $\pm 90^\circ$, the larger becomes the error (see Peretto et al. [B16]).

Figure 2 presents the envelopes of the maximum errors made when the rms value of an interharmonic is measured in function of the number of cycles m .

For example, if $m = 20$, the rms value of the interharmonic will be measured with a maximum error of $\pm 0.2\%$.