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*Standard*  
*Definitions and Methods*  
*of Measurement*  
*for*  
**PIEZOELECTRIC VIBRATORS**

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# DEFINITIONS AND METHODS OF MEASUREMENT FOR PIEZOELECTRIC VIBRATORS

## INTRODUCTION:

This Standard is a revision of the IRE Standard on Piezoelectric Crystals—The Piezoelectric Vibrator: Definitions and Methods of Measurement, 1957 (57 IRE 14. S1)<sup>1</sup> and a continuation of Standards in this field.<sup>2, 3, 4</sup>

An introductory review of the equivalent electric circuit of a piezoelectric vibrator and its parameters is followed by a discussion of the determination of these parameters by the transmission method. This method was published in 1951<sup>5</sup> and became the basis for the 1957 IRE Standard.<sup>1</sup> Since that time, a thorough investigation of the transmission method has resulted in more precise expressions which permit a more accurate evaluation of the parameters.<sup>6</sup> This method is suitable for frequencies up to about 30 MHz for the commonly encountered ranges of the capacitance ratio  $r$  and the figure of merit  $M_r$ , provided that errors due to instrumentation are taken into account. The equations presented in this Standard have been formulated to correct these errors.

## 1. CLASSIFICATION OF PHENOMENA

### 1.1 The Piezoelectric Vibrator and its Equivalent Electric Circuit

A piezoelectric vibrator consists of an element usually in the form of a plate, bar or ring cut from a piezoelectric material and has electrodes attached to or supported near the element to excite one of its resonance frequencies.

The electrical behavior of a lightly damped mechanical vibrating system which is excited piezoelectrically through electrodes forming a two-terminal network, can be represented in the vicinity of any mechanical resonance by an equivalent electric circuit (Figure 1) which consists of a capacitance  $C_1$ , inductance  $L_1$ , and resistance  $R_1$  in series, shunted by the parallel capacitance  $C_0$ . The parameters are independent of frequency for isolated modes of motion. Generally, the mode in question is sufficiently isolated from other modes to permit this assumption. When this is not true, the equations and measuring methods outlined herein do not apply. For identification of symbols used in this Standard, see Table 1A.

### 1.2 Parameters of Piezoelectric Vibrators

The fundamental parameters  $C_1$ ,  $L_1$ ,  $R_1$ , and  $C_0$ , define the equivalent electric circuit shown in Figure 1 and all other parameters may be de-

rived from them. At a given frequency the parameters of the equivalent electric circuit generally approach constant values as the amplitude of vibration approaches zero. The amplitude which can be tolerated before the parameters are appreciably affected varies widely between vibrators of various types and can only be determined by experiment.

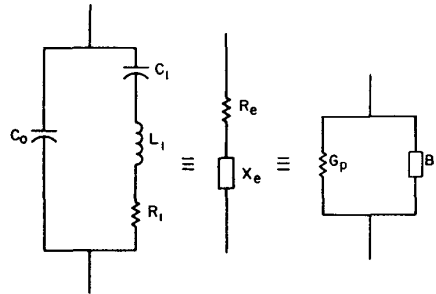


FIGURE 1

Equivalent Electric Circuit of a Piezoelectric Vibrator Near a Resonance.

The equation for the impedance  $Z$  or admittance  $Y$

$$Z = \frac{1}{Y} = \frac{j}{\omega C_0} \frac{\Omega - j\delta}{1 - \Omega + j\delta} \quad (1)$$

of the equivalent electric circuit of the piezoelectric vibrator is the basic equation describing the relationships between the various parameters. In equation (1)

$$\Omega = \frac{f^2 - f_s^2}{f_p^2 - f_s^2} \quad \text{and} \quad \delta = 2\pi f C_0 R_1$$

are the normalized frequency factor and the normalized damping factor, respectively. See Table 1A for definitions of  $f_p$ ,  $f_s$ , and the other symbols used in equation (1) and for other essential parameters. The characteristic frequencies of equation (1) are defined in Table 2.

The magnitude of the impedance of the equivalent electric network ( $|Z|$ ), its resistive component ( $R_e$ ), its reactive component ( $X_e$ ), and the reactance  $X_1$  of the  $L_1$ ,  $C_1$ ,  $R_1$  branch are plotted as functions of frequency in Figure 2, for the purpose of defining the different characteristic frequencies.  $Z_m$  and  $Z_n$  denote minimum and maximum impedance respectively, and  $R_n$ ,  $R_a$  the impedances at zero phase angle. These curves, however, have only qualitative character

and do not represent a particular piezoelectric vibrator.<sup>7</sup>

For further clarification, the impedance and admittance circles of a piezoelectric vibrator are reproduced in Figure 3. However, the circle representation of the impedance or admittance of a piezoelectric vibrator is valid only if the circle diameter of the admittance diagram is large compared with the change of  $2\pi fC_0$  in the resonance range or if  $r \ll Q^2$ , which is fulfilled in most vibrators. If the latter conditions are not fulfilled, the admittance curve shows a cisoidal character. Throughout the remainder of this Standard, it is assumed that the impedance (or admittance) of the vibrator can be represented by a circle diagram. Table 3 gives data for  $Q$ ,  $r$ , and  $Q^2/r$  for various types of vibrators, indicating that this assumption is valid for all practical cases.

It is necessary to make approximations in deriving practical equations for general use. It is the error of these approximations, in addition to the errors of instrumentation, that govern the overall accuracy of the experimentally derived parameters.

As a first approximation sufficient for many

practical purposes, the following assumptions can be made:  $f_m = f_r = f_s$  and  $f_a = f_p = f_n$ .

More exact relations between the characteristic frequencies  $f_m$ ,  $f_r$ ,  $f_a$ ,  $f_p$ ,  $f_n$  and the series resonance frequency  $f_s$  of a vibrator, valid for the figure of merit  $M > 10$  and the capacitance ratio  $r > 10$ , are shown in Table 4. These relationships have been derived by various authors<sup>8,9</sup> under the assumption that  $M \gg 1$ .

The separation between parallel and series resonance frequencies is given by:

$$\frac{f_p^2 - f_s^2}{f_s^2} = \frac{C_1}{C_0} = \frac{1}{r} \quad (2)$$

The approximation

$$\begin{aligned} \frac{f_p - f_s}{f_s} &= \sqrt{1 + r^{-1}} - 1 \\ &= \frac{1}{2r} \left(1 - \frac{1}{4r} + \dots\right) \approx \frac{1}{2r} \\ &= \frac{1}{2} \frac{C_1}{C_0} \end{aligned} \quad (3)$$

can be used for larger values of  $r$  (for example, when  $r$  is greater than 25 the error is less than 1 percent.)

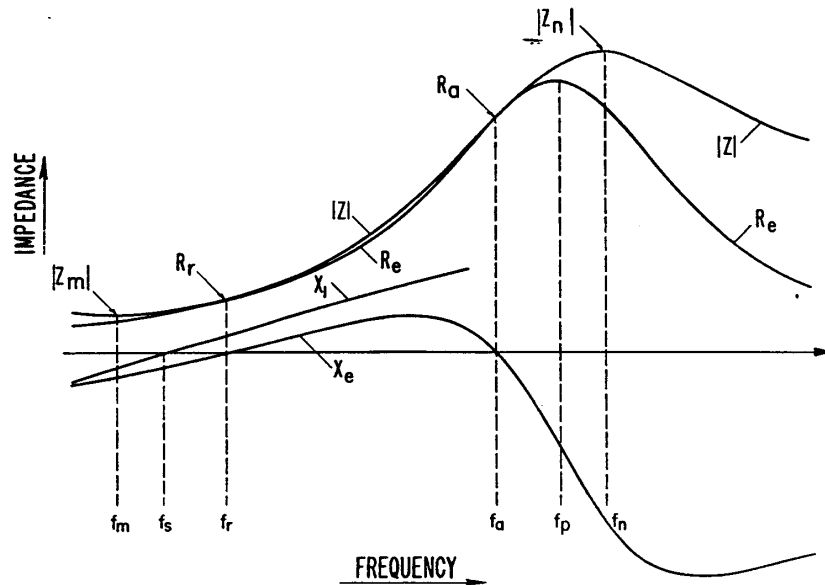


FIGURE 2

Impedance  $|Z|$ , Resistance  $R_e$ , Reactance  $X_e$ , and Series Arm Reactance  $X_1$  of a Piezoelectric Vibrator as a Function of Frequency.  $Z_m$  and  $Z_n$  denote minimum and maximum impedance,  $R_r$  and  $R_a$  the impedances at zero phase angle. For the meaning of the different frequencies, see Table 1A and Figure 2.