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IRE STANDARDS ON PIEZOELECTRIC CRYSTALS: DETERMINA-
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CONSTANTS—THE ELECTROMECHANICAL COUPLING
FACTOR, 1958

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IRE Standards on Piezoelectric Crystals: Determination of the Elastic, Piezoelectric, and Dielectric Constants—The Electromechanical Coupling Factor, 1958*

(58 IRE 14. S1)

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INTRODUCTION

IN 1949, Standards on Piezoelectric Crystals [1] were issued covering the definition of axes for piezoelectric crystals and their relation to rectangular axes, standards for specifying crystal plate orientation and nomenclature referring to the piezoelectric relations, symbols, and units. In 1957, the IRE Standards on Piezoelectric Crystals [2] was issued covering the piezoelectric vibrator: definition and measurements. It is the object of these present Standards to specify a method for determining the basic elastic, piezoelectric, and dielectric constants of crystals. In addition, the electromechanical coupling factor and the relations to these constants are considered. Among the several known methods for the determination of the elastic and piezoelectric constants, the resonance method has been proven simple and reliable. These constants are derived from the motional parameters of the equivalent electric circuit of piezoelectric vibrators made from the material considered. The motional parameters and their measurements are specified in the 1957 IRE Standards [2].

The expressions relating measured quantities to basic coefficients are different for crystals of various symmetry classes. A complete presentation covering all symmetry classes is rather complex. Fortunately, most piezoelectric materials of practical interest belong to crystal classes having several symmetry elements, so that the problem of determining the elastic and piezoelectric constants is considerably simplified when compared to the most general case of the asymmetric class. Determination of the constants may be further simplified by choosing cuts specially oriented in relation to the symmetry planes or axes of the material considered. Therefore it is not necessary to use the theoretical relationships in their complete tensor form. Reference is made to the general elastic and piezoelectric equations in the 1949 IRE Standards and several textbooks (see [3]–[5]).

1. THE PIEZOELECTRIC EQUATIONS

The piezoelectric equations usually are expressed as relations between the six tensor components of the stress T_p and the strain S_q as elastic variables, the three vector components of the electric field E_i and the electric displacement D_j as electric variables, and the scalars temperature θ and entropy σ as thermal variables. In the notation used in these Standards, the symbols for subscripts numbered from 1 to 6 will be represented by p, q indicating second-order tensors, and those for subscripts numbered from 1 to 3 by i, j indicating vectors.

For the present purpose, one of the thermal scalars, the entropy, is taken as constant, and only the elastic and electric properties under adiabatic conditions are considered.

Four sets of equations for any piezoelectric crystal can be given depending upon which of the elastic quantities (stress, strain) or electric quantities (electric field, electric displacement) are taken as independent vari-

ables. Each form consists of a set of nine equations comprising a group of six elastic equations (corresponding to $p=1$ to 6), together with three electric equations (corresponding to $i=1$ to 3).

Two sets of equations refer to controlled field conditions, the other two to open circuit conditions. For the present purpose of determination of the elastic and piezoelectric constants, only the equations referring to controlled field conditions are of interest.

The piezoelectric equations of state for the most general cast (triclinic crystal, hemihedral class C_1), where the components of stress and electric field are taken as independent variables (controlled field conditions), while the components of strain and electric displacement are taken as dependent variables, are

$$\begin{aligned}
 S_1 &= s_{11}^E T_1 + s_{12}^E T_2 + s_{13}^E T_3 + s_{14}^E T_4 + s_{15}^E T_5 \\
 &\quad + s_{16}^E T_6 + d_{11} E_1 + d_{21} E_2 + d_{31} E_3 \\
 S_2 &= s_{12}^E T_1 + s_{22}^E T_2 + s_{23}^E T_3 + s_{24}^E T_4 + s_{25}^E T_5 \\
 &\quad + s_{26}^E T_6 + d_{12} E_1 + d_{22} E_2 + d_{32} E_3 \\
 S_3 &= s_{13}^E T_1 + s_{23}^E T_2 + s_{33}^E T_3 + s_{34}^E T_4 + s_{35}^E T_5 \\
 &\quad + s_{36}^E T_6 + d_{13} E_1 + d_{23} E_2 + d_{33} E_3 \\
 S_4 &= s_{14}^E T_1 + s_{24}^E T_2 + s_{34}^E T_3 + s_{44}^E T_4 + s_{45}^E T_5 \\
 &\quad + s_{46}^E T_6 + d_{14} E_1 + d_{24} E_2 + d_{34} E_3 \\
 S_5 &= s_{15}^E T_1 + s_{25}^E T_2 + s_{35}^E T_3 + s_{45}^E T_4 + s_{55}^E T_5 \\
 &\quad + s_{56}^E T_6 + d_{15} E_1 + d_{25} E_2 + d_{35} E_3 \\
 S_6 &= s_{16}^E T_1 + s_{26}^E T_2 + s_{36}^E T_3 + s_{46}^E T_4 + s_{56}^E T_5 \\
 &\quad + s_{66}^E T_6 + d_{16} E_1 + d_{26} E_2 + d_{36} E_3 \\
 D_1 &= d_{11} T_1 + d_{12} T_2 + d_{13} T_3 + d_{14} T_4 + d_{15} T_5 \\
 &\quad + d_{16} T_6 + \epsilon_{11}^T E_1 + \epsilon_{12}^T E_2 + \epsilon_{13}^T E_3 \\
 D_2 &= d_{21} T_1 + d_{22} T_2 + d_{23} T_3 + d_{24} T_4 + d_{25} T_5 \\
 &\quad + d_{26} T_6 + \epsilon_{12}^T E_1 + \epsilon_{22}^T E_2 + \epsilon_{23}^T E_3 \\
 D_3 &= d_{31} T_1 + d_{32} T_2 + d_{33} T_3 + d_{34} T_4 + d_{35} T_5 \\
 &\quad + d_{36} T_6 + \epsilon_{13}^T E_1 + \epsilon_{23}^T E_2 + \epsilon_{33}^T E_3
 \end{aligned} \tag{1}$$

where s_{pq}^E are the elastic compliances at constant field strength, d_{jp} the piezoelectric strain constants, and ϵ_{ij}^T the dielectric permittivities at constant stress.

In matrix notation the equations of state (1) can be more simply written [1]

$$\begin{aligned}
 S &= s^E T + d_t E \\
 D &= d T + \epsilon^T E.
 \end{aligned} \tag{2}$$

Alternatively, when the strain S and the electric field E are taken as independent variables,

$$\begin{aligned}
 T &= c^E S - e_t E \\
 D &= e S + \epsilon^S E.
 \end{aligned} \tag{3}$$

where the subscript t signifies a transposed matrix. In (2), s^E are the elastic compliances at constant field strength, d the piezoelectric strain constants, and ϵ^T the dielectric permittivities at constant stress. In (3), c^E are the elastic stiffnesses at constant field strength, e the piezoelectric stress constants, and ϵ^S the dielectric permittivities at constant strain. The matrices s , c , and ϵ are symmetric; e.g., $s_{pq} = s_{qp}$; $c_{pq} = c_{qp}$; and $\epsilon_{ij} = \epsilon_{ji}$. Eq. (1) through (3) are written in rationalized 4π -free units. The relationship between the elastic stiffnesses and compliances, and between the piezoelectric stress constants and strain constants, can be found in the 1949 IRE Standards [1].

1.1 The Elasto-Piezo-Dielectric Matrix

A simplified presentation of the elasto-piezo-dielectric matrix for all seven crystal systems and thirty-two classes, using both Hermann-Mauguin's and Schoenflies' notation (the former in parentheses), is shown in Chart I (pp. 767-768) [6]. The tetragonal system and the trigonal system are divided into two groups designated as (a) and (b). For the tetragonal system IV(b), s_{16} equals zero; for the trigonal system V(b), s_{26} equals zero. Of all the thirty-two crystal classes, twenty are piezoelectric and are denoted by the following Schoenflies' symbols

$$\begin{aligned} C_1, C_{1v} (=C_s) \\ C_n, C_{nv}, D_n \quad n = 2, 3, 4, 6 \\ S_4, D_{2d}, C_{3h}, D_{3h}, T, T_d. \end{aligned}$$

The twenty schemes of piezoelectric constants reduce to sixteen independent schemes, since the symmetry operations for $n=4$ or $n=6$ have the same effect on the piezoelectric scheme and the schemes for classes T and T_d are identical. The arrangement of the classes in Chart I is in accordance with generally accepted practice. The numbers on the right side of each scheme indicate, from top to bottom, the number of the independent elastic, piezoelectric, and dielectric constants.

Table I gives some known examples of crystals belonging to various crystal systems for which the elastic and piezoelectric coefficients have been determined. Elastic and piezoelectric constants of crystals belonging to the classes of the tetragonal system IV(a) and the trigonal system V(a) have not been determined.

The symmetry type of polarized polycrystalline ceramic material; e.g., barium titanate ceramics, is associated with the crystallographic class C_{6v} of the hexagonal crystal system in regard to all those physical properties that are described by tensors of ranks up to four and which include dielectric, piezoelectric, and elastic phenomena.

2. THE RESONANCE METHOD FOR DETERMINATION OF THE ELASTIC AND PIEZOELECTRIC CONSTANTS

2.1 Resonance Method—General

The method for determining the values for the elastic and piezoelectric constants of piezoelectric crystals

TABLE I
SOME KNOWN PIEZOELECTRIC CRYSTALS

Crystal System and Class			Material
II	Monoclinic	C_2	Ethylene diamine tartrate (EDT). Potassium tartrate (DKT). Lithium sulphate monohydrate.
III	Orthorhombic	D_2	Rochelle salt.
IV (b)	Tetragonal	C_{4v} D_{2d}	Barium titanate single crystal. Ammonium dihydrogen phosphate (ADP).
V (b)	Trigonal	D_3	Alpha-quartz.
VI	Hexagonal	D_6	Beta-quartz.
VII	Isometric	T	Sodium bromate. Sodium chlorate.

should be simple and reliable. To determine the elastic and piezoelectric constants, the simplest method is the resonance method (see also [5], [7], [15]).

The main properties of a piezoelectric vibrator; e.g., the frequency and the parameters of the equivalent electric circuit, are expressed in terms of elastic, piezoelectric, and dielectric constants. Therefore, the values for these constants can be derived from measurements of resonance frequency, parameters of the equivalent electric circuit, and the dimensions and density of a suitably oriented specimen, when the theory for the mode of motion for the specimen is known. The relation between the elastic and piezoelectric constants and the measurable quantities of the piezoelectric vibrator are best expressed in terms of the frequency constant N , the motional capacitance constant Γ , and the capacitance ratio r . Further, each of these constants in turn can be described by a general expression including a factor adaptable to any mode, even though the modes have different strain-stress systems. These factors are derived from the differential equations of vibration under the appropriate boundary conditions.

2.2 Frequency Constant (N)

The frequency constant N of a vibrator is defined as the product of the series resonance frequency f_s and the linear dimension h which determines the frequency of the specimen; e.g., length l for the length-extensional mode of bars and contour modes of square plates, diameter ϕ for contour modes of disks, thickness t for thickness modes (see Section 2.5).

$$N = f_s h = \frac{F}{2} \sqrt{\frac{q}{\rho}} \quad (4)$$

where q is the effective elastic modulus, ρ the density of the material, and

$$F = \frac{2\kappa}{\pi}, \quad (5)$$

F being a factor which is related to the eigen value κ of the mode.

CHART I*
ELASTO-PIEZO-DIELECTRIC MATRIX OF VARIOUS CRYSTAL CLASSES

I TRICLINIC SYSTEM	C_1 (1) C_i ($\bar{1}$) 21 18 6 45 0 6 27
II MONOCLINIC SYSTEM	C_2 (2) 13 8 4 25 C_s (m) 13 10 4 27 C_{2h} ($\frac{2}{m}$) 13 0 4 17
III ORTHORHOMBIC SYSTEM	D_2 (2 2 2) 9 3 3 15 C_{2v} (2 m m) 9 5 3 17 D_{2h} ($\frac{2}{m} \frac{2}{m} \frac{2}{m}$) 9 0 3 12
IV TETRAGONAL SYSTEM	C_4 (4) 7 4 2 13 S_4 ($\bar{4}$) 7 4 2 13 C_{4h} ($\frac{4}{m}$) 7 0 2 9 D_4 (4 2 2) 6 1 2 9 C_{4v} (4 m m) 6 3 2 11 D_{2d} ($\bar{4} 2 m$) 6 2 2 10 D_{4h} ($\frac{4}{m} \frac{2}{m} \frac{2}{m}$) 6 0 2 8

SEE CHART 1 (CONT.) FOR KEY

* The numbers on the right side of each scheme indicate, from top to bottom, the number of the independent elastic, piezoelectric, and dielectric constants.