

American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology

Sponsor

**IEEE Standards Coordinating Committee 14 on
Quantities, Units, and Letter Symbols**

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Abstract: Signs and symbols used in writing mathematical text are defined. Special symbols peculiar to certain branches of mathematics, such as non-Euclidean Geometries, Abstract Algebras, Topology, and Mathematics of Finance, which are not ordinarily applied to the physical sciences and engineering, are omitted.

Keywords: letter symbol, mathematical notation, mathematical sign, mathematical symbol, mathematics, operation symbol, quantity symbol, unit symbol

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Introduction

(This Introduction is not part of ANSI/IEEE Std 260.3-1993, American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology.)

This Standard is a revision of ANSI Y10.20-1975, the original edition of *Mathematical Signs and Symbols for Use in Physical Sciences and Technology*. The purpose of this second edition is twofold: to serve as an authoritative national standard for mathematical notation and, as promised in the first edition, to include symbols of those lesser-known branches of mathematics that are increasingly being applied to the physical sciences. Added to this revision are signs and symbols used in Symbolic Logic, Set Theory, Arithmetic, Differential Geometry, Matrices, Probability and Statistics.

The Table of Signs and Symbols has been reorganized and the format modified by insertion of an additional column to exhibit, if appropriate, the application of each sign or symbol, with its meaning in that context, under the Description heading. The former subclause on Trigonometry has been renamed Circular Functions and now includes principal values of the inverse functions. To encourage the use of roman type, rather than italic type, to symbolize specific mathematical functions, the subclause on Special Functions in the original edition now occupies five subclauses; the total number of items being increased from 8 to 54.

Incorporated in this Standard is a revision of ANSI Y10.17-1961 (R 1988), *Guide for Selection of Greek Letters Used as Letter Symbols for Engineering Mathematics*. Clause 10 of this Standard supersedes ANSI Y10.17-1961 entirely.

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American National Standard Mathematical Signs and Symbols for Use in Physical Sciences and Technology

1. Scope

Only signs and symbols used in writing mathematical text are contained in this Standard. Special symbols peculiar to certain branches of mathematics, such as non-Euclidean Geometries, Abstract Algebras, Topology, and Mathematics of Finance, which are not ordinarily applied to the physical sciences and engineering, have been omitted. Because there is no consensus in the literature for signs and symbols used in tensor analysis, the subject of tensors is relegated to future editions when there is general agreement among authorities in the field.

2. Letter Symbols

Letter symbols¹ include symbols for physical quantities (*quantity symbols*), symbols for units in which these quantities are measured (*unit symbols*), and symbols for operators on, and functions of, these quantities, as well as special symbols for frequently used words and phrases (*operation symbols*).

A quantity symbol is, in general, a single letter,² e.g., I , to represent an electric current, modified, when appropriate, by one or more subscripts or superscripts, e.g., I_i , to represent input current. A symbol assigned to denote a quantity in a treatise should be used consistently for that quantity throughout the work.

¹“Letter symbol” as a technical term does not have the same meaning as either “name” or “abbreviation”. An abbreviation is a letter or combination of letters (sometimes with apostrophe(s) or period) that, by convention, represents a *word* or a *name* in a particular language; hence, an abbreviation may be different in another language. A symbol represents a *quantity*, a *unit*, or an *operation*, and should be independent of language (except, by tradition, some unit symbols and their prefixes have Latin or Greek origins, and many operation symbols are Latin derivatives), e.g., the symbol for the quantity: electromotive force is “ E ”, whereas the abbreviation is “emf” in English, “fem” in French, and “EMK” in German. The word for the unit of electric current “ampere” is often abbreviated “amp”, but the symbol for this unit is “A”. The international standard symbol for the circular function, “sine” is “sin”, although, for example, the word for “sine” in Spanish is “seno”.

²Quantity symbols comprising two letters are sometimes used for dimensionless transport parameters, e.g., Prandtl Number, “Pr”, and Reynolds Number, “Re”, not to be confused with the complex variables function, “Re”, meaning “real part of” (qv 9.10.2, p 26).

A unit symbol³ is a letter or group of letters, e.g., “m” for meter(s) and “Hz” for hertz, or a special sign, such as ° for degree(s), that may be used in place of the name of the unit.

An operation symbol is a letter, a group of letters, or special sign(s) that represents a mathematical operator, a specific mathematical function or relationship, a word, or a phrase.

3. Alphabets and Typography

Letter symbols are restricted primarily to the English and Greek alphabets.⁴ Script, black letter, or other special fonts to distinguish between possible conflicting uses of the same letter for different quantities should not be used.

Symbols for physical quantities, mathematical variables, indices, and general functions⁵ are printed in *italic* type, e.g.,

A	area
e	eccentricity of a conic section
x, y, z	Cartesian coordinates
i, j, k	indices
$f(x)$	f function of x

Symbols used for physical units, as well as mathematical constants, specific mathematical functions, operators, and all numerals are printed in roman (upright) type, e.g.,

cm	centimeter(s)
i	imaginary unit: $\sqrt{-1}$
$\sin 2\theta$	sine of the angle: $2 \times \theta$
$a \times b + c$	a times b plus c
dx	differential of x

All punctuation,⁶ including grouping symbols, such as parentheses, brackets, and braces, are also printed in roman type, e.g.,

$F(a, b; c; z)$	hypergeometric function
$[x]$	integer function of x
$[abc]$	triple scalar product of vectors
$\{a, b\}$	LCM of a and b
$n!$	factorial n

Subscripts and superscripts are governed by the above principles. Those that are letter symbols for physical quantities, mathematical variables, or for indices are printed in italic type, whereas others are printed in roman type, e.g.,

$\sin^p x$	p^{th} power of $\sin x$
a_{ij}, a_{45}	matrix elements
I_i, I_o	input, output currents
$B_x(\alpha, \beta)$	incomplete beta function

³It was once common to treat unit symbols in the same manner as general abbreviations, but the recommendations of the International Organization for Standardization (ISO) and many other international and national bodies concerned with standardization, emphasize the symbolic character of these designations and rigidly prescribe the manner in which they shall be treated. The concept of the unit symbol is therefore adopted in this Standard.

⁴Greek letters that are easily confused with English letters should be avoided. Clauses 8 and 10 provide a guide for selecting Greek letters to be used as symbols.

⁵The term “general functions” is used here to contrast with “specific mathematical functions”, discussed below.

⁶It should be noted that the commas, semicolons, brackets, braces, and exclamation point in these examples are mathematical operators and, consistent with the previous paragraph, should be displayed in roman type.

πr^2 area enclosed by a circle

To indicate the vector character of a quantity, boldface type is used, italic for general vectors, roman for unit vectors and symbols for special vector functions, e.g.,

F_i force on the i^{th} element
grad f gradient of the scalar, f
 div F divergence of the vector, F
 $\mathbf{i}, \mathbf{j}, \mathbf{k}$ orthogonal unit vectors
 k_n normal curvature vector

The gradient symbol is boldface because its operation results in a vector, but “div” is not, as its operation results in a scalar. Ordinary italic type may be used to represent the magnitude of a general vector quantity.

4. Quantity Symbols

Quantity symbols may be used in mathematical expressions in any way consistent with good mathematical usage. The product of two scalar quantities a and b is indicated by writing ab . The quotient may be indicated by writing any of the following:

$\frac{a}{b}$, a/b , $a \div b$, or ab^{-1}

When more than one solidus (/) is used in an algebraic expression, grouping symbols shall be inserted to remove any ambiguity. Thus, one may write $(a/b)/c$, or $a/[b/c]$, but *not* $a/b/c$.

Subscripts and superscripts are commonly used with quantity symbols. Several subscripts or superscripts, sometimes separated by commas, may be attached to a single letter; but, unless logical clarity dictates it, subscripts and superscripts should not be attached to other subscripts or superscripts.⁷ A symbol that has been modified by a superscript shall be enclosed in grouping symbols before an exponent is appended.

Care should be taken not to assign the same symbol for different quantities in the same work. Use of different symbols or appending subscripts to distinguish the symbols, is recommended.

5. Unit Symbols

Unit symbols are written in lowercase letters, except the initial letter is capitalized whenever the unit is derived from a proper name.⁸ The distinction between uppercase and lowercase letters shall be followed even when the symbols appear in applications where the other lettering is in uppercase style. A unit symbol is printed in roman type, the form being the same for both singular and plural. A final period (.) shall *not* be part of a unit symbol.

⁷There are acceptable exceptions exhibited in Clause 9: Item Numbers 9.4.18, 9.4.19, 9.4.23, 9.4.24, 9.5.2.1.2, 9.8.10, 9.11.4.4.9, 9.11.4.4.10, 9.11.5.4, 9.12.10, 9.12.11.

⁸To prevent confusion with the numeral, 1, the uppercase letter, L, rather than the lowercase letter, l, is the symbol for liter. Prefixes are considered separately. Some examples of frequently used unit symbols are listed below.

picofarad	pF	decibel	dB
nanosecond	ns	volt	V
microampere	μA	kilopascal	kPa
milliliter	mL	megawatt	MW
centimeter	cm	gigahertz	GHz

When a compound unit is formed by multiplication of two or more other units., the symbol consists of the symbols for the separate units joined by a raised dot (preferred) or by a space, e.g., N·m (or N m) for newton meter. The dot or space may be omitted in the case of familiar compounds such as watt-hour (symbol: Wh) if no confusion would result.⁹ Hyphens shall not be used in symbols for compound units; however, exponents may be applied to unit symbols.

Should a compound unit be formed by division of one unit by another, its symbol consists of the symbols for the separate units, either separated by a solidus or conjoined using negative powers, e.g., m/s or m·s⁻¹ for meter(s) per second. In simple cases, use of the solidus is preferred, but in no case shall more than one solidus on the same line, or a solidus followed by a product, be included in such a combination unless grouping symbols, such as parentheses, are inserted to avoid ambiguity. In complicated cases, use of negative powers is recommended.¹⁰

The following prefixes from the International System of Units (SI) are used to indicate decimal multiples or submultiples of units:

FACTOR	SI PREFIX	SYMBOL
10 ²⁴	yotta	Y
10 ²¹	zetta	Z
10 ¹⁸	exa	E
10 ¹⁵	peta	P
10 ¹²	tera	T
10 ⁹	giga	G
10 ⁶	mega	M
10 ³	kilo	k
10 ²	hecto	h
10	deka	da
10 ⁻¹	deci	d
10 ⁻²	centi	c
10 ⁻³	milli	m
10 ⁻⁶	micro	μ
10 ⁻⁹	nano	n
10 ⁻¹²	pico	P
10 ⁻¹⁵	femto	f
10 ⁻¹⁸	atto	a
10 ⁻²¹	zepto	z
10 ⁻²⁴	yocto	y

⁹It may also be omitted where adjacent symbols are separated by an exponent, as in $V \equiv \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \cdot \text{A}^{-1}$.

¹⁰The notation for products and quotients of unit symbols is intentionally made more explicit than that given in Clause 4 for quantity symbols because many unit symbols consist of more than one letter, cf Reference 3 in Subclause 7.1.

Values that are outside the range of the above prefixes should be expressed with powers of ten applied to the base unit(s). In any case, compound prefixes shall not be used, e.g.,

tera	T	<i>not</i>	megamega	MM,
giga	G	<i>not</i>	kilomega	kM,
nano	n	<i>not</i>	millimicro	mμ,
pico	p	<i>not</i>	micromicro	μμ.

6. Operation Symbols

In this Standard “operation symbols” include signs and symbols for mathematical operators, specific mathematical functions, mathematical relations between quantities, and precisely defined symbols for words and phrases commonly used in mathematical works; e.g., the operation symbol, ∇ , represents

$$i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

where the signs, ∂ , $+$, and $—$ (division sign) are also operation symbols, and **i, j, k**, as well as x, y, z , are quantity symbols. Other examples of operation symbols are “exp” for the exponential function and the symbol “ $\ni$ ”, which is commonly used in place of the phrase “such that”.

A compilation of operation symbols is listed in Clause 9, with their definitions and, where appropriate, typical use of the symbol and a description of its meaning in that application.

7. Reference Documents

The signs and symbols tabulated in Clause 9 are, for the most part, in accord with general usage. The original edition of this Standard and Reference 1¹¹ generally follow the rules stated in Clause 3 regarding the use of roman type for symbols to designate mathematical constants, operators, and specific mathematical functions. However, the other references do *not* generally conform to those precepts, often using e for the base of natural logarithms, i for the imaginary unit, $\sqrt{-1}$, d for the differential operator, and, among other symbols for functions, $J_n(z)$ for the Bessel function of the 1st kind of order n .

In addition to the references below and the handbooks listed in the bibliography, many modern textbooks were consulted to obtain a consensus for each sign and symbol; the font character being changed whenever necessary, to comply with the aforementioned rules. In some cases subjective judgment was used, but in all cases the main concern was to avoid ambiguity.

Although most of the signs and symbols depicted in Reference 1 agree in meaning with those of this Standard, there are some differences dictated by tradition in the United States. For example, the International Organization for Standardization (ISO) prefers a comma (,) to denote the decimal sign; however, in the United States and some other countries, a dot on the baseline (.) is used.

¹¹Information on references can be found in Subclause 7.1.

In general, this Standard favors the notation, definition, and meaning given in Reference 2 wherever disagreement exists with Reference 1. Since Reference 2 was published by the National Bureau of Standards¹² and is a standard reference, even for other references, it is appropriate that this Standard be consistent with Reference 2 except where type styles need to be modified to conform to the rules in Clause 3 of this Standard. Where an alternative sign or symbol displayed in Reference 1 agrees with this Standard, the two Standards are assumed to be in accord.¹³ Those symbols in Reference 1 that belong to special fonts, violating the principles of Clause 3, are not included in this Standard.

Some minor discrepancies between ISO and symbols of this Standard (ANS) exist for circular and hyperbolic cotangents as well as their inverses (ISO 11–9; ANS 9.5.2.2, 9.5.2.3). In this Standard the cotangent functions are symbolized by “ctn” to make them more distinguishable from “cos” than “cot”. Then each of the six basic three-letter symbols has no more than one letter positionally in common with any of the other five. Also, the valid ranges in the principal values of the inverse secants and cosecants for negative arguments differ in the two Standards.

Two items in Reference 1, viz, ISO 11–11.3 and 11–13.4, use italic type, where, by the rules of Clause 3, roman type is mandatory; qv ANS 9.8.4 and 9.7.1, respectively. The combinatorial symbol, ISO item 11–6.16, uses subscripts and superscripts for arguments, but to accommodate complicated literal values for them, an ordered pair is used in this Standard, qv ANS 9.5.1.18.

The symbol, $\ni$, in Reference 1 (11–4.3) has the meaning “contains the element”, but means “such that” in ANS (9.3.7). Although zero belongs to the set, $\mathbb{N}$, in Reference 1 (11–4.9), in ANS (9.3.15) the set, $\mathbb{N}$, consists only of the natural numbers, i.e., the positive integers, which does not include zero. The ISO symbol for equality by definition (11–5.3) is language dependent (cf ANS 9.2.16). The order of the arguments in the symbols for elliptic integrals, as well as the sign of the characteristic, n , are different for the two Standards. Although the symbols used for polar, cylindrical, and spherical coordinates in Reference 1 are the same as those used in this Standard, the definitions of all of these symbols, except for z , disagree in the two documents (cf ISO 11–12, ANS 9.6.2).

The most serious variances are in the meanings attached to the associated Laguerre polynomial (ISO 11–14.13; ANS 9.11.1.5) and the exponential integral (ISO 11–14.21; ANS 9.11.4.4.3). The coefficient of y in the differential equation satisfied by the Laguerre polynomial in ISO is $n - m$, where in ANS, the (equivalent) coefficient of $f(x)$ is n . The difference is reflected in the relationship between the associated Laguerre polynomial and the derivatives of the basic Laguerre polynomial, as shown by the formula in Item Number 11–14.13 of ISO 31-11:1992:

$$L_n^m(x) = \frac{d^m}{dx^m} L_n(x) \quad (m, n \in \mathbb{N}; m \leq n)$$

This equation differs from its counterpart in Item Number 9.11.1.5 of this Standard, which agrees with Reference 2, [B4],¹⁴ and [B9]. The exponential integral given in Reference 1, according to Reference 2, [B4], and [B5], is equivalent to $-\text{Ei}(-x)$ in the notation of those references and is denoted by $E_1(x)$ in Reference 2 and [B9]. Again, this Standard subscribes to the meaning given in Reference 2.

7.1 References

- [1] International Standard ISO 31-11:1992. *Mathematical signs and symbols for use in the physical sciences and technology*. 2nd ed. International Organization for Standardization.
- [2] Abramowitz, Milton, and Stegun, Irene A. *Handbook of Mathematical Functions*, N. B. S. Applied Mathematics Series • 55. Washington: National Bureau of Standards, 1972.
- [3] ANSI / IEEE Std 260-1978 (Reaffirmed 1991). IEEE Standard Letter Symbols for Units of Measurement.

¹²The National Bureau of Standards is now known as the National Institute of Standards and Technology.

¹³Reference 1, Introduction

¹⁴Brackets identify treatises listed in bibliography, Clause 11.

8. Conventions

Many conventions for quantity symbols have been adopted by authorities in various fields of the physical sciences and technology. Most authors adhere to these conventions even though they are not mandatory. Because they facilitate comprehension by the reader who is familiar with the field, conventional symbols should be used whenever feasible. Some of the more common conventions are described here for reference.

Constants are generally denoted by the first few lowercase letters of the English alphabet: $a, b, c, \dots$, while variables are designated by the last few letters: $\dots, x, y, z$. Integers and indices are usually indicated by the lowercase letters: i, j, k, l, m , and n . Lowercase Greek letters often symbolize dimensionless quantities, such as ratios or angles.¹⁵ Some physical quantities are also represented by Greek letters, e.g., λ to denote wave length, ρ to indicate the density of a substance, and ω to signify the magnitude of angular velocity.¹⁶

When describing the equations of motion of a physical body, such as a ship, an aircraft, a missile, etc, it is customary to use reference triads, i.e., three mutually orthogonal axes oriented in inertial space, atmosphere, earth, body, etc. A generally accepted convention for orienting the body triad assigns the $+x$ -axis to the forward direction in the plane of symmetry (if practicable) parallel to the waterline (or some arbitrary plane), the $+y$ -axis toward starboard, normal to the plane of symmetry, and the $+z$ -axis pointed in a downward direction, perpendicular to the x - y plane. General translational motion can be described along these x, y, z coordinate axes.¹⁷ Speeds are usually designated by u, v , and w along x, y , and z respectively. Full six-degree-of-freedom equations require rotations about each of the three primary body axes. The rotations about the axes x, y , and z are commonly denoted by ϕ (roll), θ (pitch), and ψ (yaw), respectively, where positive rotations are in the right-handed sense, and time rates of change of ϕ, θ , and ψ are specified by p, q , and r , respectively.

9. Signs and Symbols

The following table has been categorized into the various branches of mathematics¹⁸ to facilitate its use, primarily, but also because some signs and symbols are common to two or more branches; e.g., $\equiv$ generally means “is equivalent to”, as described in Item Number 9.2.13, but means “is congruent to” in Arithmetic (Item Number 9.4.3). Also, Item Numbers 9.1.4, 9.4.24, and 9.11.2.2, each describe the letter symbol, “M”, but all have different meanings. However, confusion is not very likely to result from these ambiguous definitions; the meaning should be clear by the context in which it is used, if not by the subscript, superscript, or argument(s) attached to it. Some specific quantity symbols that are associated with the various branches of mathematics are also included.¹⁹

¹⁵Angles are ratios when measured in radians or steradians.

¹⁶When necessary to distinguish the parameters, subscripts are used, e.g., ρ_0 to designate air density at sea level, and ω_e for the earth's rotational rate with respect to inertial space.

¹⁷Correlatives of x, y , and z are sometimes specified by the lowercase Greek letters ξ, η , and ζ , respectively.

¹⁸Not all branches are represented; qv Clause 1.

¹⁹There are certain mathematical quantities that are represented by generally accepted symbols. One purpose of this Standard is to document such symbols. Some of the items in Subclauses 9.1, 9.3, 9.4, 9.5.1, 9.6.1, 9.7, 9.8, 9.10, 9.12, as well as all of Subclauses 9.6.2, 9.6.3, consist of quantity symbols.