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IN THE FREQUENCY DOMAIN

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CONTENTS

	PAGE
1. INTRODUCTION	5
1.1 Preamble	5
1.2 Definitions and Symbols	5
1.3 Spectral Analysis of a Burst	6
1.4 Application of Spectral Analysis	6
2. MEASUREMENT OF THE ENERGY DENSITY SPECTRUM	6
2.1 The Spectral Sampling Theorems	6
2.2 Outline of Measurement Methods	7
2.3 Discussion of Measurement Methods	8
3. REPRESENTATION OF THE ENERGY DENSITY SPECTRUM	12

APPENDICES

	PAGE
APPENDIX A—THE SPECTRUM OF A SIGNAL	13
APPENDIX B—MAXIMUM OF THE FRACTION OF BURST ENERGY CONTAINED IN A LIMITED FREQUENCY	14
APPENDIX C—THE SAMPLING THEOREM IN THE FREQUENCY DOMAIN	15
APPENDIX D—MATHEMATICAL FOUNDATION OF SPECTRAL SAMPLING METHODS	18
APPENDIX E—RECONSTITUTION OF THE CONTINUOUS SPECTRUM	20

Technical Report on Recommended Practices for BURST MEASUREMENTS IN THE FREQUENCY DOMAIN

1. INTRODUCTION

1.1 Preamble

This document is a companion to *Technical Committee Report on Recommended Practices for Burst Measurements in the Time Domain*, IEEE No. 257, May 1964. In the time domain document, bursts are defined and particular attention is placed on their duration and magnitude.

The use of additional characteristics may prove desirable when investigating both cause and effect of a burst. Mathematical transformations have been widely used to bring out particular characteristics of signals. Perhaps the one most commonly used is the Fourier Transform, which defines the spectrum of signals. The energy density spectrum of a burst, a quantity derived from the Fourier Transform, is the subject of this report. Other transformations such as those of Hilbert or Hankel, may be used to display different characteristics of a burst but they will not be considered here.

Sampling the energy density spectrum is the key concept of this document. It constitutes the basis by which this spectrum can be characterized comprehensively by a practical number of measurements. The sampling theorems in the frequency domain are, therefore, given detailed consideration in Appendices to the extent necessary for understanding the measurement methods to be discussed.

1.2 Definitions and Symbols

1.2.1 Definitions

The term "burst", as well as some important characteristics of a burst, are defined in the companion document. Other terms which are used when considering measurement methods in the frequency domain include the following:

Bandwidth—The smallest frequency interval outside of which the integral of the energy spectrum is less than some designated fraction of the total energy of the burst

Center Frequency—The arithmetic mean of the two frequencies that define the bandwidth of a filter

Convolution Function—The integral of the function $x(\tau)$ multiplied by another function $y(-\tau)$ shifted in time by t :

$$\int_{-\infty}^{\infty} x(\tau) \cdot y(t - \tau) d\tau$$

Energy Density Spectrum—The square of the magnitude of the Fourier Transform of a burst

Interpolation Function—A function which may be used to obtain additional values between sampled values

Modified Autocorrelation Function—The time integral of a function $x(t)$ multiplied by itself shifted in time by τ :

$$\Gamma_x(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t + \tau) dt$$

1.2.2 Symbols

The following symbols will be used:

Symbol	Definition
b_n, c_n	Complex Fourier coefficients
B	Equivalent positive frequency bandwidth
E_{x_0}	Energy of $x_0(t)$
f, f_n	Frequency, in cycles per second
$h(t)$	Impulse response
$H(f)$	Fourier transform of $h(t)$
$ H(f) ^2$	Energy density spectrum of $h(t)$
$\text{Im} [\]$	Imaginary part of expression in brackets
$\text{Re} [\]$	Real part of expression in brackets
t	Time
T	Burst duration or a time interval
$x(t)$	The burst, or a specified time function
$x_0(t)$	Filter output as a result of an input $x(t)$
$x_p(t)$	Periodic continuation of $x(t)$
$X(f)$	Fourier transform of $x(t)$
$X_p(f)$	Fourier transform of $x_p(t)$
$X^*(f)$	The complex conjugate of $X(f)$
$ X(f) ^2$	Burst energy density spectrum
y	Arbitrary variable
$\Gamma_x(t), \Gamma_h(t)$	The modified autocorrelation function of $x(t)$ and $h(t)$ respectively

Symbol	Definition
δ	Dirac function:
	$X(t_0) = \int_{-\infty}^{\infty} X(t) \cdot \delta(t-t_0) dt$
ϕ	The upper limit of the fraction of the energy confined in a bandwidth B of a signal of duration T
τ, s	Dummy time variables

1.3 Spectral Analysis of a Burst

Certain conditions must be satisfied for the energy density spectrum of a burst to exist. As discussed in Appendix A, these conditions are met in practice. It should be observed that the power density spectrum of a burst has little value since it vanishes as the finite burst energy is averaged over all times.

A burst is by definition time limited and equal to zero outside of the burst duration. Thus the integration interval for computing the Fourier Transform does not have to extend to infinity.

Because of the time-limited nature of a burst, its energy density spectrum is continuous and extends over all frequencies. The shorter the time duration of a burst, the wider must be the bandwidth to accommodate the major portion of its energy density spectrum. This relationship can be expressed quantitatively by examining the upper limit of the fraction of the energy contained within a frequency interval extending from $-B$ to $+B$. This upper limit as a function of the product of B and the burst duration T is given in Figure 1. For a more extensive discussion of this subject consult Appendix B.

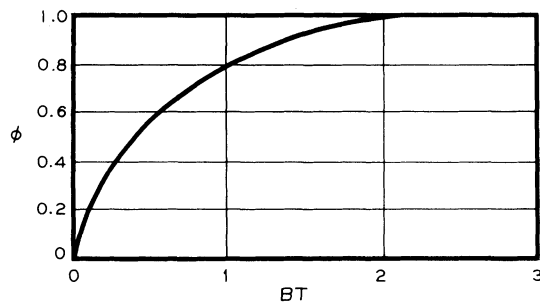


FIGURE 1

The upper limit (ϕ) of the fraction of the energy contained in a bandwidth B , due to a burst of duration T .

In an attempt to make the analysis more comprehensive, the concept of an "instantaneous spectrum", as sometimes used in speech analysis, was considered. Unfortunately, there is neither a unique nor a widely accepted definition of "in-

stantaneous spectrum". The concept of a time-varying spectrum is therefore not pursued in this document.

1.4 Applications of Spectral Analysis

Spectral analyses have long been recognized as a convenient tool for studying stochastic processes, noise or signal, that are passed through a time-invariant linear filter. In many situations it has been found that the energy density spectrum contains all the information needed to derive meaningful answers to specific questions. For example, the amount of burst energy passed through a system can often be computed by integrating the product of the energy density spectrum of the burst and the system frequency response, if both are known. If multiple bursts occur, it is necessary that the system respond rapidly enough to allow the system to return to rest between bursts. Otherwise, the spectrum of the series of bursts must be considered in the computation.

Analysis of the energy density spectrum of bursts is performed in a number of applications. These include the determination of the amplitude response of transducers, filters, and transmission media which are excited by impulse type signals or trains of such signals. It has been recognized that in these instances the energy density spectrum is a highly powerful means for interpreting the burst waveform.

The measurement methods discussed here are specifically tailored to examining the energy density spectra of time limited bursts. Nevertheless, these methods are also useful for measuring energy density spectra in general, such as time varying spectra and spectra of stationary as well as non-stationary stochastic processes. Examples include the measurement of spectra in the analysis of speech, using a sound spectrograph, and the measurement of noise spectra in sonar systems.

2. MEASUREMENT OF THE ENERGY DENSITY SPECTRUM

2.1 The Spectral Sampling Theorems

The problem in performing a spectral analysis is to relate the energy spectral density of a burst to a finite set of practical physical measurements. As derived in Appendix C1, the Fourier Transform can be resolved into distinct independent components which are measurable and specify the spectral function completely. To be specific, complex samples spaced $1/T$ cycles per second taken from the Fourier Transform of a time function of duration T are necessary and sufficient to determine this continuous transform uniquely. Thus, the energy