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*IEEE Recommended Practice For
Power System Analysis*



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An American National Standard

**IEEE Recommended Practice for
Industrial and Commercial
Power Systems Analysis**



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Std 399-1980

An American National Standard

**IEEE Recommended Practice for
Industrial and Commercial
Power Systems Analysis**

Sponsor

**Power System Technologies Committee
of the
IEEE Industry Applications Society**

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IEEE Standards Board

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American National Standards Institute

Approved December 20, 1979
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Foreword

(This Foreword is not a part of IEEE Std 399-1980, IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis.)

This Recommended Practice is the product of about ten years of effort by a working group of the Power System Technologies Committee of IEEE Industry Applications Society. It is intended as a practical, general treatise on the theoretical basis of power system analysis, and as a reference work on the analytical techniques most commonly applied to electric power systems in industrial plants and commercial buildings.

IEEE Recommended Practice for Industrial And Commercial Power System Analysis, the IEEE Brown Book, joins the series of *Color Books* sponsored by the Power System Technologies Committee of IEEE Industry Applications Society. It is both complementary and supplementary to the other color books, extending the coverage of some topics which they introduce as well as discussing some entirely new material.

Comments, corrections, and suggestions for the next revision of the Brown Book are welcome and should be submitted to the

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Industrial and Commercial Power Systems Analysis

1st Edition

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1. Introduction

1.1 General Discussion. IEEE Std 399-1980, IEEE Recommended Practice for Industrial and Commercial Power System Analysis, commonly known as the IEEE Brown Book, is published by the Institute of Electrical and Electronics Engineers (IEEE) as a reference source for plant electrical engineers in making power system studies. The IEEE Brown Book can also be helpful in preparing system modeling and data acquisition for an outside engineering consultant to perform necessary engineering studies prior to designing a new system or expanding an existing power system. Such information will help ensure high standards of power system reliability and maximize the utilization of capital investment.

The IEEE Brown Book has been prepared on a voluntary basis by engineers and designers functioning as the Power System Analysis Working Group within the IEEE, under the Industrial Power Systems Department of The Industry Applications Society. This recommended practice is not intended as a replacement for the many excellent texts available in this field. The IEEE Brown Book comple-

ments the other IEEE color book standards, and emphasizes up-to-date techniques in system studies which are most applicable to industrial and commercial power systems. Today, such techniques are mostly computer oriented.

1.2 History of Power System Studies. The planning, design, and operation of a power system require continual and comprehensive analyses to evaluate current system performance and to ascertain the effectiveness of alternative plans for system expansion.

The computational work to determine power flows and voltage levels resulting from a single operating condition for even a small network is all but insurmountable if performed by manual methods. The need for computational aids led to the design of a special purpose analog computer (AC Network Analyzer) as early as 1929. It provided the ability to determine load flows and system voltage during normal and emergency conditions and to study the transient behavior of the system resulting from fault conditions and switching operations.

The earliest application of digital computers to power system problems dates back to the late 1940s. Most of the early applications were limited in scope, however, because of the small capacity of the punched card calculators in use during that period. The large scale digital computers became available in the middle 1950s. The initial success of the load flow program led to the development of programs for short circuit and transient stability calculations.

Today, the digital computer is an indispensable tool in power system planning where it is necessary to predict future growth and simulate day-to-day operations over periods of 20 years or more.

1.3 Applying Power System Analysis Techniques to Industrial and Commercial Power Systems. As computer technology has advanced, so has the complexity of industrial and commercial power systems. These power systems have grown in recent decades with capacities far exceeding that of a small electric utility system.

Today, plant or building management personnel are highly concerned with system interruptions and their effect on overall operation. Therefore, they demand assurances of maximum return on capital investment for any sizable expansion. The complexity of modern industrial and commercial power systems has made manual performance of power system studies difficult and time consuming, if not impossible. However, through the use of digital computers these studies can be made with relative ease. Answers to many perplexing questions regarding impact of expansion on the system, short circuit capacity, stability, load distribution, etc, can be intelligently obtained.

The recent advances in computer

technology now enable engineers to communicate with the computer and change or modify the system to meet whatever design criteria may arise.

1.4 Purposes of this Recommended Practice

1.4.1 Why a Study. As is stated in Section 2, the planning, design, and operation of industrial or commercial power systems require several studies to assist in the evaluation of the initial and future system performance, reliability, safety, and ability to grow with production or operating requirements. The studies most likely needed are load-flow studies, short-circuit studies, stability studies, and motor-starting studies. A brief summary of such studies and what each can accomplish are given in Section 2 to help a plant engineer determine overall system needs before he proceeds with specific planning and engineering design.

Additional studies of transients, reliability, grounding, and harmonics may also be required. The plant engineer in charge of system design must decide which studies are needed to ensure that the system will operate safely, economically, and efficiently over the expected life of the system.

1.4.2 How to Prepare for a Power System Study. For a plant engineer to solve a power system analysis problem, he must be thoroughly familiar with valid electrical engineering fundamentals, such as the Thevenin equivalent circuit, the phasor representation, the Fourier series, symmetrical components, etc. He can then analyze the problem and prepare necessary equivalent circuits and basic system data before proceeding with a computer program. Failure to use a valid analytical procedure to establish a sound basic approach to the problem could

lead to disastrous consequences in both the design and operation of a system.

Section 3 offers an excellent review of the most essential fundamentals in a system study.

To set up a computer program for system analysis, certain basic data must be gathered with accuracy and proper presentation. System modeling is a *must* technique. The extent of system representation, choosing the swing and infinite bus, restrictions in terms of nodes (buses) and branches (lines and transformers), balanced three-phase network and a single-phase equivalent network, single line diagram, impedance diagram, etc, are all important inputs to a meaningful system study.

Section 4 deals with system modeling and data requirements to illustrate how these basic inputs for a study can be properly prepared or organized.

Once the basic preparations are completed, the next step is to look for an actual computer program, whether it be in-house or a commercial computing service. Today, many types of computing services are available, such as batch, time-sharing, and consulting services. A plant engineer must select the most suitable configuration for his needs.

Section 13 discusses the basic computation methods, various types of computer systems and their requirements, and the availability of commercial computing services and their capabilities. Section 13 gives a plant engineer the basic knowledge and direction of approach whenever he is called upon to perform a power system study with the aid of a computer program.

1.4.3 The Most Important System Studies. For the plant engineer, the fol-

lowing sections represent the most needed studies for the design or operation of an industrial or commercial power system: Section 5, Load Flow; Section 6, Short Circuit; Section 7, Stability; Section 8, Motor Starting; Section 9, Harmonics; Section 10, Switching Transients; Section 11, Reliability; and Section 12, Ground Mat. Each of the above sections contains a sample study which includes a computer input data file and a computer print-out for that specific study. The purpose of each study and what can be achieved by it are briefly explained.

After studying these sections, a plant engineer should be better equipped in preparing necessary data and criteria for a specific computer study if the necessity arises. The study can be performed in-house or by an outside consultant. There is a growing number of consulting firms that specialize in system studies with reasonable costs if the plant engineer can supply the necessary system data with a fair degree of correctness and accuracy.

1.5 Standard References. The following standards were used as references in the preparation of this standard:

ANSI/IEEE Std 142-1972, IEEE Recommended Practice for Grounding of Industrial and Commercial Power Systems

IEEE Std 141-1976, IEEE Recommended Practice for Electric Power Distribution for Industrial Plants

IEEE Std 241-1974, IEEE Recommended Practice for Electric Power Systems in Commercial Buildings

IEEE Std 242-1975, IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems

2. Applications of Power System Analysis

2.1 Introduction. The planning, design and operation of industrial power systems require several studies to evaluate the current system performance, reliability, safety, and ability to grow with production requirements. Plant management is concerned with system outages, their effect on production, and in maximizing utilization of capital investment. Management requests justifications before decisions are made regarding the addition of equipment in the system and its impact on future system operation. The complexity of modern industrial power systems makes such studies difficult, tedious and time consuming to perform manually. By using digital computers, however, these computational tasks have been extremely simplified.

Modern Digital Computers. The modern digital computer offers power system engineers a powerful tool to perform more effective studies of any industrial power system. Computers help engineers design optimum power systems at minimum cost, regardless of system complexity. Recent advances in computer technology, and in particular, the intro-

duction of time-sharing techniques, have helped to reduce computing costs. Time-sharing terminals can be installed in any office. Some are portable and can be carried to the industrial site. By means of a telephone line, the terminal can be connected to a large computer system located many miles away. By way of the terminal, the engineer has access to the computer and the power system analysis programs stored in its memory. The user does not have to be familiar with computer programming or languages. Program manuals describe to the engineer, in a simple, logical way, how to feed information into the computer, what kind of input formats to use, the output options and how to store his master file for further system studies, changes and modifications. The cost of running these programs and the speedy way in which the results are obtained is very surprising. In fact, the engineer can converse with the computer to change or modify the system to meet whatever design criteria are imposed. Power system studies given below can be performed digitally using the time-sharing computer access method.

2.2 Load Flow Studies. Load flow studies determine the voltage, current, power, and power factor or reactive power at various points in a power system under existing or contemplated conditions of normal and emergency operation. Load flow studies are essential in planning the system. They are used to determine the best operating procedure for the system, especially in the event of a loss of one or more generating units or transmission lines. They offer valuable information regarding system losses, data for equipment specifications, overall system capability and limitations, proper settings of transformers in the system, and optimization of circuit usage in the system. Load flow studies can be used to study an industrial power system under suddenly applied loads (impact loads). Impact load studies are essential in determining whether this type of load can be imposed on an existing system design. Any changes needed in system design to ensure the successful operation of the overall system are also determined. Load flow studies also determine the most suitable location of capacitors in the system for power factor improvement. Because of the complexity of load flow calculations, digital computers are used extensively in such studies. Digital computer models have been developed to study practically any industrial power system under any loading condition and to provide the initial system data (initial system power flows, voltages at various system nodes, and initial machines' electrical angles) for transient stability studies.

2.3 Fault and Short-Circuit Studies. The current which flows in different parts of an industrial power system immediately after a fault differs from that flowing a few cycles later, just before circuit

breakers are called upon to open the line on both sides of the fault. Both these currents differ widely from the current that would flow under steady-state conditions if the fault were not isolated from the rest of the system by the operation of circuit breakers. The proper selection of circuit breakers and switchgear depends on two factors, the current flowing immediately after the fault occurs and the current that the breaker must interrupt. Short-circuit calculations consist of determining these currents for various types of faults at various locations in the system. The data obtained from fault calculations also determine the settings of relays which control the circuit breakers.

Digital computers are used effectively in the calculation of the three-phase short-circuit levels at various points in the system, and in determining the momentary and interrupting ratings of circuit breakers. Line-to-ground faults and double-line short circuits can also be digitally analyzed. In this aspect, the method of symmetrical components used in the computer model represents the system sequence networks and enables the calculation of unsymmetrical faults in the system. Without digital computers, such calculations are tedious and time consuming.

2.4 Stability Studies. In industrial power systems, the current which flows in an ac generator or synchronous motor depends on:

- (1) The magnitude of its generated, or internal voltage
- (2) The phase angle of its internal voltage with respect to the phase angle of the internal voltage of every other machine in the system
- (3) The characteristics of the network and loads

The problem of stability is maintaining the synchronous operation of generators and motors in the system.

Stability studies are usually complex, but by using digital computers, such studies can be performed effectively at a relatively reasonable cost. Depending on the program used, the synchronous generators are simulated to include voltage regulators, excitation systems and governor response. Motors are represented by their dynamic characteristics and system disturbances are simulated digitally. Through analysis, a thorough understanding of the power system performance under various system disturbances can be determined.

2.5 Motor Starting Studies. Starting large synchronous or induction motors in an industrial power system can be a problem. It is preferable to start these motors across the line, if possible. This can cause severe voltage dips in the system however, and, in certain circumstances, the motor may not be able to *break away* from standstill or might stall during acceleration.

By using digital computer techniques, all such problems can be predicted in advance before the installation of the motor in the system. If a starting device is needed, its characteristics and rating are determined by this computer motor start-up simulation technique.

Available computer programs simulate the dynamic characteristics of the motor digitally. From these programs, the motor's voltage, current, speed, torque, and acceleration time can be computed as well as system voltage dips and currents at various time intervals from motor starting to full load speed. If a reactor or other starting device is required for motor starting, the speed at which switching should occur can be determined.

2.6 System Transients Studies. Industrial power systems that include arc furnaces and capacitor banks, resonance and switching transients can impose serious problems. By using modern digital analysis techniques, all such problems can be detected early in the design stage. These computer programs are also used in investigating the effect of adding capacitor banks to an existing system, the switching surges in a system, system response to lightning surges, lightning arrestor applications, ferro-resonance problems and system transient recovery voltage.

2.7 Reliability Analysis. When comparing various industrial power system design alternatives, system reliability and cost are essential factors in selecting optimum design. Using probability and statistical analysis techniques, the system reliability can be studied in depth with digital simulation techniques.

Interruptions. Reliability is expressed as the frequency of interruptions and expected number of hours of interruption during one year of system operation. Momentary and sustained system interruptions, component failures and outage rates are used in available digital computer reliability programs to compute overall system reliability indexes at any node in the system, and to investigate their sensitivity to parameter changes. With these results, management can select the optimum power system design and justify the decision when requested.

2.8 Power Generation Planning. In planning industrial power systems, management is faced with certain difficult questions. One problem is the decision of whether to build a local power plant or to purchase the required power from a neighboring utility system. In case of local generation expansion, the unit type,

size, and time of availability in the system is another problem. By using modern digital computer analysis techniques, a thorough investigation of all such problems can be achieved.

Digitally Simulated Loads. The industrial load is simulated digitally. For specific unit outage rates, the loss of load probability associated with having insufficient generation to meet the load is calculated. Whether power is purchased or generated locally, load carrying capability of the system for various specified and tolerable loss of load probabilities in the system is then computed. The economics of both generation systems are then investigated. Depending on the type of units used in local generation (for example, diesel driven, gas turbine or

fossil unit), the fuel costs, operation and maintenance costs and capacity factors are computed. Demand charges on purchased power plus other operating parameters are calculated to arrive at a comparative cost for the cases under investigation. The digital analysis model proceeds to use present worth mathematics to plant investment costs to determine the present worth costs. Yearly and total plant life fixed charges on investment, plus other fixed costs associated with the type of generation, are computed to find a final plant cost for comparison purposes. Management can now justify not only local generation or purchased power schemes, but also the type of units to be used in system expansion.

3. Analytical Procedures

3.1 Introduction. With the development of the digital computer and advanced computer programming techniques, power system problems of the most complex types can be rigorously analyzed. Previously solutions were usually only approximate and errors were introduced by many simplifying assumptions necessary to permit classical longhand calculating procedures. For progress to be realized in using the computer for power system analysis work, it has been necessary for the specialist involved in the creation of power system analysis computer programs to understand thoroughly the application of basic analytical solution methods that apply. It is also important for those concerned with assembling and preparing data for input to a power system analysis computer program and those interpreting and applying results generated by such a program to understand the application of analytical solution methods.

This section attempts, first, to identify and document the basic analytical solution methods that are valid for determining the voltage and current relationships

which exist during various power system network events and operating conditions. Secondly, these basic analytical solution methods will be demonstrated where not otherwise self-evident. Finally, critical restraints that must be respected to avoid serious error in applying analytical solution methods will be discussed.

Whether a power system analysis problem is to be solved directly or by a computer program, proper application of sound analytical solution methods is essential for three reasons. First, accuracy of the solution to each individual problem being considered will be directly affected. Second, and perhaps the most important because of the significant expense involved, accuracy of the solution determines the validity and effectiveness of any remedial measures suggested. Finally, extension of erroneous results to related problems or to what appears to be a trivial modification of the original problem, possibly in combination with other misapplied or misunderstood techniques, can lead to a compounding of initial error and a progression of incorrect conclusions.

The most common causes of errors in circuit analysis work are:

(1) Failure to use a valid analytical procedure because the analyst is unaware of its existence or applicability, or both

(2) Careless or improper use of *cook-book* methods that have neither a factual basis, nor support in the technical literature, nor a valid place in electrical engineering discipline

(3) Improper use of a valid solution method due to application beyond limiting boundary restraints or in combination with an inaccurate simplifying assumption

Many situations occur in industrial and commercial power systems that illustrate some or all of these common causes of error, as well as the resulting evils. Any problem investigated as a part of the general types of power-system analysis studies covered in other sections of this recommended practice and described as follows would qualify.

- (1) Short circuit studies
- (2) Load analysis studies
- (3) Load flow studies
- (4) Stability studies
- (5) Motor starting studies
- (6) Harmonic studies
- (7) Reliability studies
- (8) Grounding mat studies
- (9) Switching transient studies

3.2 The Fundamentals. The following identify the more important analytical solution methods that are either available as or are the basis for valid techniques in solving power system network circuit problems.

- (1) Linearity
- (2) Superposition
- (3) The Thevenin Equivalent Circuit
- (4) The Sinusoidal Forcing Function
- (5) The Phasor Representation
- (6) The Fourier Representation
- (7) The Single-Phase Equivalent Circuit

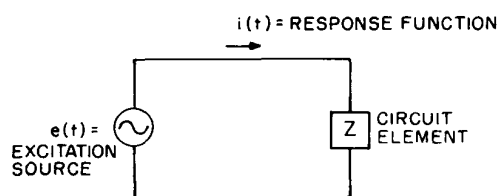
(8) The Symmetrical Component Analysis

(9) The Per Unit Method

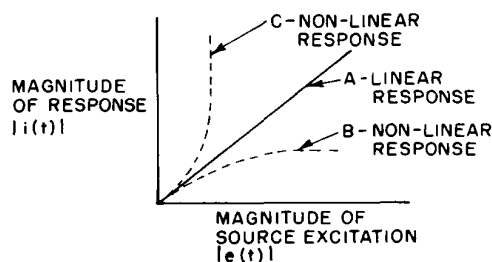
Rigorous treatment of these analytical techniques is available in several circuit analysis texts [1]¹, [2], [3], [4], [5], [6], and is beyond the scope of this discussion. In the following sections, however, a brief qualitative explanation of each principle is presented, along with a review of major benefits and restraints associated with the use of each principle.

3.2.1 Linearity. Probably the simplest concept of all, linearity is also one of the most important because of its influence on the other principles. Linearity is best understood by examination of Fig 1.

Fig 1
Linearity



(a)



(b)

¹Numbers in brackets correspond to those in the References at the end of each section.

The simplified network represented by the single-impedance element Z in Fig 1(a) is linear for the chosen excitation and response functions, if a plot of response magnitude (current) versus source excitation magnitude (voltage) is a straight line.

This is the situation shown for case A (solid line) in Fig 1(b). When linearity exists, the plot applies either to steady state value of the excitation and response functions or to the instantaneous value of the functions at a specific time.

When linear dc circuits are involved, the current doubles if the voltage is doubled. The same holds for linear ac circuits if the frequency of the driving voltage is held constant. In a similar manner, it is possible to predict the response of a *constant* impedance circuit (that is, constant R , L , and C elements) to any magnitude of dc source excitation or *fixed frequency* sinusoidal excitation based on the known response at any other level of excitation. For the chosen excitation function of voltage and the chosen response function of current, both dotted curves B and C are examples of the response characteristic of a nonlinear element.

With the circuit element represented by any of the response curves shown in Fig 1 (including the linear element depicted by curve A) the circuit will, in general, become nonlinear for a different response function, for example, power. If, for example, the element was a constant resistance (which would have a linear voltage-current relationship), the power dissipated would increase by a factor of 4 if voltage were doubled ($P = I^2 R$).

An important limitation of linearity, therefore, is that it applies only to responses that are linear for the circuit conditions described (that is, a constant

impedance circuit will yield a current that is linear with voltage). This restraint must be recognized in addition to the previously mentioned limitations of constant source excitation frequency for ac circuits and constant circuit element impedances for ac or dc circuits. Excitation sources, if not independent, must be linearly dependent. This restraint forces a source to behave just as would a linear response (which, by definition, is also linearly dependent).

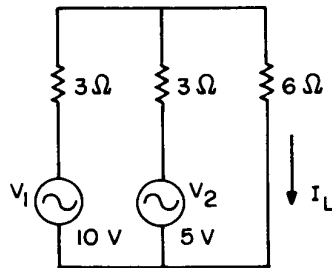
3.2.2 Superposition. This very powerful principle is a direct consequence of linearity and can be stated as follows:

In any linear network containing several dc or fixed frequency ac excitation sources (that is, voltages), the total response (that is, current) can be calculated by algebraically adding all the individual responses caused by each independent source acting alone (that is, voltage sources shorted; current sources opened).

An example which illustrates this principle is shown in Fig 2. The equation written is for the sum of the currents from each individual source V_1 and V_2 . Although Fig 2 also illustrates a way this principle might actually be used, more often its main application is in support of other calculating methods. The only restraint associated with superposition is that the network should be linear. All limitations associated with linearity apply.

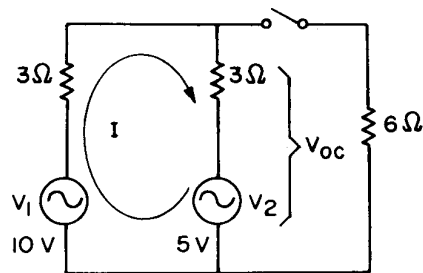
The nonapplicability of superposition is why all but the very simplest nonlinear circuits are almost impossible to analyze using hand calculations. Although most real circuit elements are nonlinear to some extent, they can often be accurately represented by a linear approximation. Solutions to network problems involving such elements can be readily obtained.

Problems involving complex networks having substantially nonlinear elements

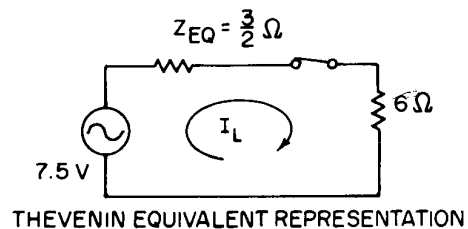


$$\begin{aligned}
 I_L &= i_{v_1} + i_{v_2} \\
 &= \frac{10}{\left(3 + \frac{6 \cdot 3}{6+3}\right)} \left(\frac{6 \cdot 3}{6+3}\right) \cdot \frac{1}{6} + \frac{5}{\left(3 + \frac{6 \cdot 3}{6+3}\right)} \left(\frac{6 \cdot 3}{6+3}\right) \cdot \frac{1}{6} \\
 &= \frac{10}{5} \cdot 2 \cdot \frac{1}{6} + \frac{5}{5} \cdot 2 \cdot \frac{1}{6} \\
 &= \frac{2 \cdot 2}{6} + \frac{2}{6} = \frac{2}{3} + \frac{1}{3} = 1 \text{ A}
 \end{aligned}$$

Fig 2
Superposition



$$\begin{aligned}
 V_{OC} &= I \cdot 3 + 5 \\
 &= \frac{10-5}{6} \cdot 3 + 5 \\
 &= \frac{5}{2} + 5 = \frac{15}{2} \\
 &= 7.5 \text{ V} \\
 Z_{EQ} &= \frac{3 \cdot 3}{3+3} = \frac{9}{6} = \frac{3}{2} \Omega
 \end{aligned}$$



$$\begin{aligned}
 I_L &= 7.5 \cdot \frac{1}{6 + \frac{3}{2}} \\
 &= \frac{15}{2} \cdot \frac{2}{12+3} \\
 &= 15 \cdot \frac{1}{15} = 1 \text{ A}
 \end{aligned}$$

Fig 3
The Thevenin Equivalent

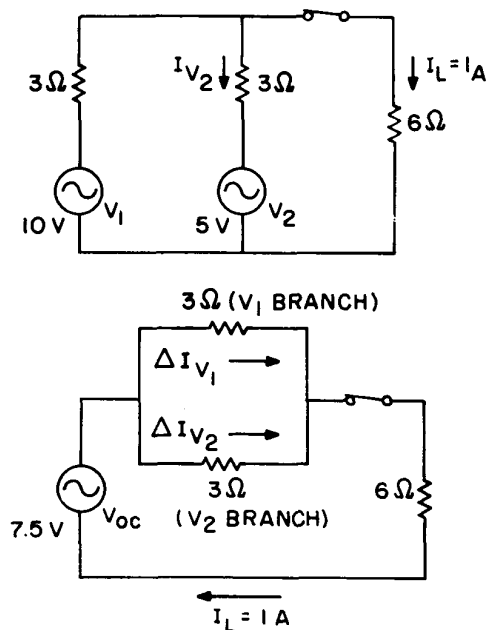
can practically be solved only through the use of certain simplification procedures, or through adjustment of calculated results to correct for nonlinearity. Both of these approaches can potentially lead to significant inaccuracy. Tiresome iterative calculations performed in an instant by the digital computer make accurate solutions possible when an equation can be written mathematically to describe the nonlinear circuit elements.

3.2.3 The Thevenin Equivalent Circuit. This powerful circuit analysis tool is based on the fact that any active linear network, however complex, can be represented by a single voltage source equal to the open-circuit voltage across any two terminals of interest, in series with the equivalent impedance of the network viewed from the same two terminals with all sources in the network inactivated (that is, voltage sources shorted and current sources opened). Validity of this representation requires only that the network be linear. Existence of linearity is a necessary restraint. Application of the Thevenin equivalent circuit can be appreciated by referring to the simple circuit of Fig 2 and developing the Thevenin equivalent for the network with the switch in the open position as illustrated in Fig 3. After connecting the $6\ \Omega$ load to the Thevenin equivalent network by closing the switch, the solution for I_L is the same as before, 1 A. Use of the simple Thevenin equivalent shown for the entire left side of the network makes it easy to examine circuit response as the load impedance value is varied.

The Thevenin equivalent circuit solution method is equally valid for complex impedance circuits. It is the type of representation shown in Fig 3 that is the basis for making per unit short-circuit calculations, although the actual values

for the source voltage and branch impedances would be substantially different from those used in this case. (The circuit property of linearity would, incidentally, allow them to be scaled up or down.) The network shown in Fig 3(a), with the $6\ \Omega$ resistance shorted and the other resistances visualized as reactances, might well serve as an oversimplified representation of a power system about to experience a bolted fault with the closing of the switch.

The V_1 branch of the circuit would correspond to the utility supply while the V_2 branch might represent a large motor running unloaded, immediately adjacent to the fault bus, and highly idealized so as to have no rotor flux leakage. For such a model, the 5 V source corresponds to the pre-fault, air-gap voltage behind a stator leakage (subtransient) reactance of $3\ \Omega$ [7]. In a more realistic situation where rotor leakage is evident, a model that accurately describes the V_2 branch in detail before and after switch closing is much more difficult to develop, because the air-gap voltage decreases (exponentially) with time and varies (linearly) with the steady-state rms magnitude of the motor stator current following application of the fault. The problem of accounting for motor internal behavior is avoided altogether by use of a Thevenin equivalent. This permits the V_2 branch to be represented by the apparent motor reactance (or, more generally, impedance) effective at the time following switch closure. In shunt with the equivalent impedance for the remainder of the network, the Thevenin equivalent impedance for the motor (at any point in time of interest) is simply connected in series with the pre-fault open-circuit voltage to obtain the corresponding current response to switch closing.



$$I_{V_2} = (1 \cdot 6 - 5) \cdot \frac{1}{3}$$

$$= \frac{1}{3} \text{ A}$$

$$\Delta I_{V_2} = \Delta I_{V_2} = \frac{1}{2} \text{ A}$$

$$I_{V_2} = \Delta I_{V_2} - I_{V_2} \text{ (SWITCH OPEN)}$$

$$= \frac{1}{2} - \frac{5}{6}$$

$$= -\frac{2}{6} = -\frac{1}{3} \text{ A (INTO BRANCH)}$$

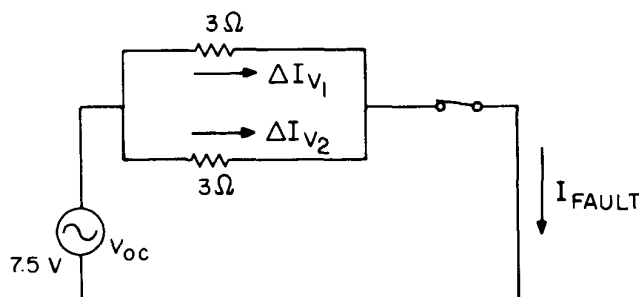
Fig 4
Current Flow of a Thevenin
Equivalent Representation

The current response obtained in each branch of a network using a Thevenin equivalent circuit solution represents the change of current in that branch. The actual current that flows is the vector sum of currents before and after the particular switching event being considered. See Fig 4.

In Fig 4A the current flowing in the V_2 branch circuit is shown to be $\frac{1}{3}$ A. A more detailed representation of the Thevenin equivalent circuit previously examined in Fig 3 is shown in Fig 4B. Here, the solution for the same current I_{V_2} is determined by subtracting the cur-

rent flowing in the V_2 branch prior to closing the switch ($\frac{5}{6}$ A from inspection of the circuit in Fig 3A) from the current $I_{V_2} = \frac{1}{2}$ A, calculated to be flowing in the Thevenin equivalent for this V_2 branch.

In the branch of the circuit defined by the switch itself, the *change of current* due to closing is normally the response of interest. This means the solution to the Thevenin equivalent is sufficient. The resultant current in the other branches, however, cannot be determined by the solution to the Thevenin equivalent network alone.



$$I_{\text{FAULT}} = \frac{15}{2} \cdot \frac{2}{3} = 5 \text{ A}$$

$$\Delta I_{V_1} = \Delta I_{V_2} = 2.5 \text{ A} = \frac{5}{2} = \frac{15}{6}$$

$$I_{V_2} = \Delta I_{V_2} - I_{V_2} \text{ (SWITCH OPEN)}$$

$$I_{V_2} = \frac{15}{6} - \frac{5}{6} = \frac{10}{6} \text{ A}$$

Fig 5
Fault Flow

In the case where the V_2 branch represents a motor switched onto a bolted fault, the motor contribution is the locked-rotor current minus the pre-fault current as illustrated in Fig 5 and not just the locked-rotor current as it is so often carelessly described. As a rule, this effect is never as significant as the example suggests, even when the motor is loaded prior to the fault; the load current is much smaller than the locked-rotor current and almost 90° out of phase with it.

A Norton equivalent which consists of a current source in parallel with a (different) equivalent impedance can alternatively be developed for the Thevenin equivalent circuit. This representation is not generally as useful in power system analysis work.

3.2.4 The Sinusoidal Forcing Function.

It is a most fortunate truth in nature that the excitation sources (that is, driving voltage) for electrical networks, in general, have a sinusoidal character and can be represented by a sine wave plot of the type illustrated in Fig 6. There are two important consequences of this circumstance. First, although the response, that is, current, for a complex R, L, C network represents the solution to at least one second-order differential equation, the result will also be a sinusoid of the same frequency as the excitation and different only in magnitude and phase angle. The relative character of the current with respect to the voltage for simple R, L , and C circuits is also shown in Fig 6.

The second important concept is that

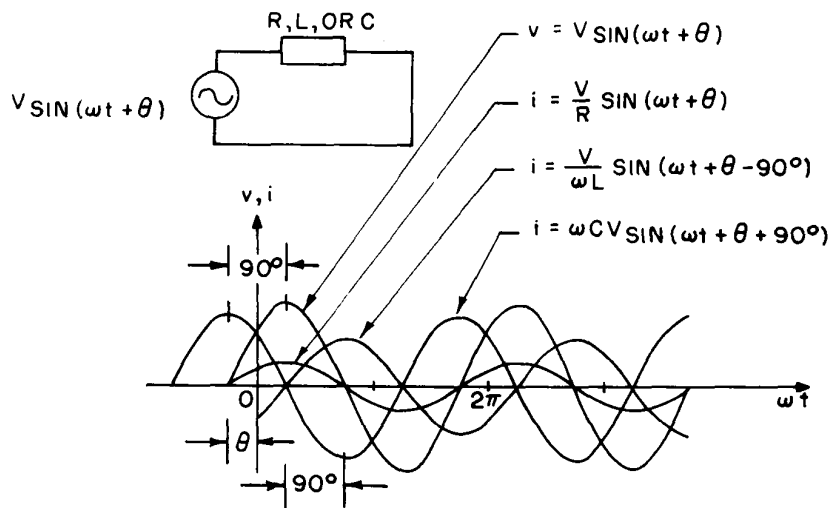


Fig 6
The Sinusoidal Forcing Function

when the sine wave shape of current is forced to flow in a general impedance network of R , L , and C elements, the voltage drop across each element will always exhibit a sinusoidal shape of the same frequency as the source. The sinusoidal character of all the circuit responses makes the application of the superposition technique to a network with multiple sources surprisingly manageable. The necessary manipulation of the sinusoidal terms is easily accomplished using the laws of vector algebra, which evolve from the next technique to be reviewed.

The only restraint associated with the use of the sinusoidal forcing function concept is that the circuit must be comprised of linear elements, that is, R , L , and C are constant as current or voltage varies.

3.2.5 Phasor Representation. Phasor representation allows any sinusoidal forcing function to be represented as a phasor in a complex coordinate system as shown in Fig 7. As indicated, the expression for the phasor representation of a sinusoid can assume any of the following shorthand forms:

Exponential: $E e^{j\theta}$

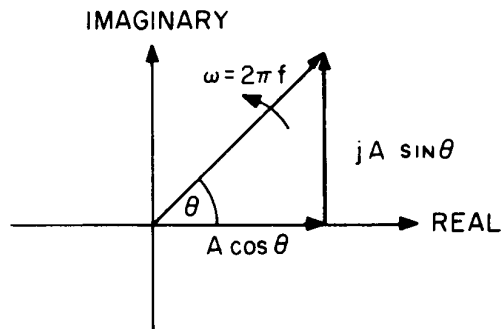
Rectangular: $E \cos \theta + jE \sin \theta$

Polar: $E \angle \theta$

For most calculations, it is more convenient to work in the *frequency domain* where any angular velocity associated with the phasor is ignored, which is equivalent to assuming the coordinate system rotates at a constant angular velocity of ω .

The impedances of the network can likewise be represented as phasors using the vectorial relationships shown. As illustrated, the circuit responses, that is,

$$E e^{j\theta} = E \cos \theta + jE \sin \theta = E \angle \theta$$



$$I = \frac{E}{Z} = \frac{|E| e^{j\theta}}{|Z| e^{j\phi}} = \frac{|E|}{|Z|} e^{j(\theta - \phi)}$$

$$Z = R + j \omega L - j \frac{1}{\omega C} = R + j X$$

$$X_L = \omega L$$

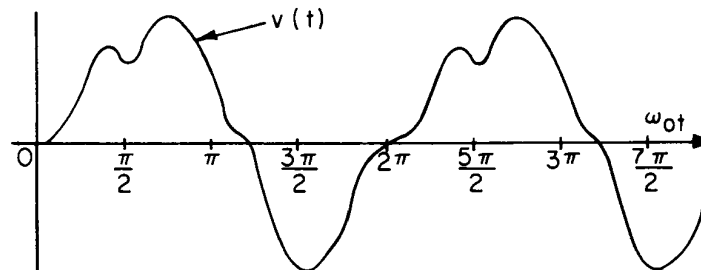
$$X_C = \frac{1}{\omega C}$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\phi = \tan^{-1} \frac{X}{R}$$

R = resistance
 X_L = inductive reactance
 X_C = capacitive reactance
 Z = impedance

Fig 7
The Phasor Representation



$$V(t) = V_0 + V_1 \cos \omega_0 t + V_2 \cos 2\omega_0 t + \dots$$

$$+ V_1' \sin \omega_0 t + V_2' \sin 2\omega_0 t + \dots$$

Fig 8
The Fourier Representation

current, can be obtained through the simple vector algebraic manipulation of the quantities involved. The need for solving complex differential equations to determine the circuit responses is completely eliminated.

The restraints that apply are:

- (1) The sources must all be sinusoidal
- (2) The frequency must remain constant
- (3) The circuit R , L and C elements must remain constant (that is, linearity must exist)

3.2.6 The Fourier Representation. This powerful tool allows any nonsinusoidal periodic forcing function, of the type plotted in Fig 8, to be represented as the sum of a dc component and a series (infinitely long, if necessary) of ac sinusoidal forcing functions. The ac components have frequencies that are an integral *harmonic* of the periodic function (fundamental) frequency. The general mathematical form of the so-called *Fourier series* is also shown in Fig 8.

The importance of the Fourier representation is immediately apparent. The response to the original driving function can be determined by first solving for the response to each Fourier series component forcing function and summing all the individual solutions to find the total superposition. Since each component response solution is readily obtained, the most difficult part of the problem becomes the determination of the component forcing function. The individual harmonic voltages can be obtained, occasionally in combination with numerical integration approximating techniques through several well-established mathematical procedures. Detailed discussion of their use is better reserved for the many excellent texts [2], [3] that treat the subject.

There are several abstract mathematical

conditions that must be satisfied to use a Fourier representation. The only restraints of practical interest to the power systems analyst are that the original driving function must be periodic (repeating) and the network must remain linear.

3.2.7 The Single-Phase Equivalent Circuit. The single-phase equivalent circuit is a powerful tool for simplifying the analysis of balanced three-phase circuits, yet its restraints are probably most often disregarded. Its application is best understood by examining a three-phase diagram of a simple system and its single-phase equivalent, as shown in Fig 9. Also illustrated is the popular one-line diagram representation commonly used to describe the same three-phase system on engineering drawings.

If a three-phase system has a perfectly balanced symmetrical source excitation (voltage) and load, as well as equal series and shunt system and line impedances connected to all three phases (see Fig 9(a)), imagine a conductor (shown dotted) carrying no current connected between the effective neutrals of the load and the source. Under these conditions, the system can be accurately described by either Fig 9(b) or Fig 9(c).

The single-phase equivalent circuit is particularly useful since the solution to the classical loop equations is much easier to obtain than for the more complicated three-phase network. To determine the complete solution, it is only necessary to realize that the other two phases will have responses that are shifted by 120° and 240° but are otherwise identical to the reference phase.

Anything that upsets the balance of the network renders the model invalid. A subtle way this might occur is illustrated in Fig 10. If the switching devices operate independently in each of the three

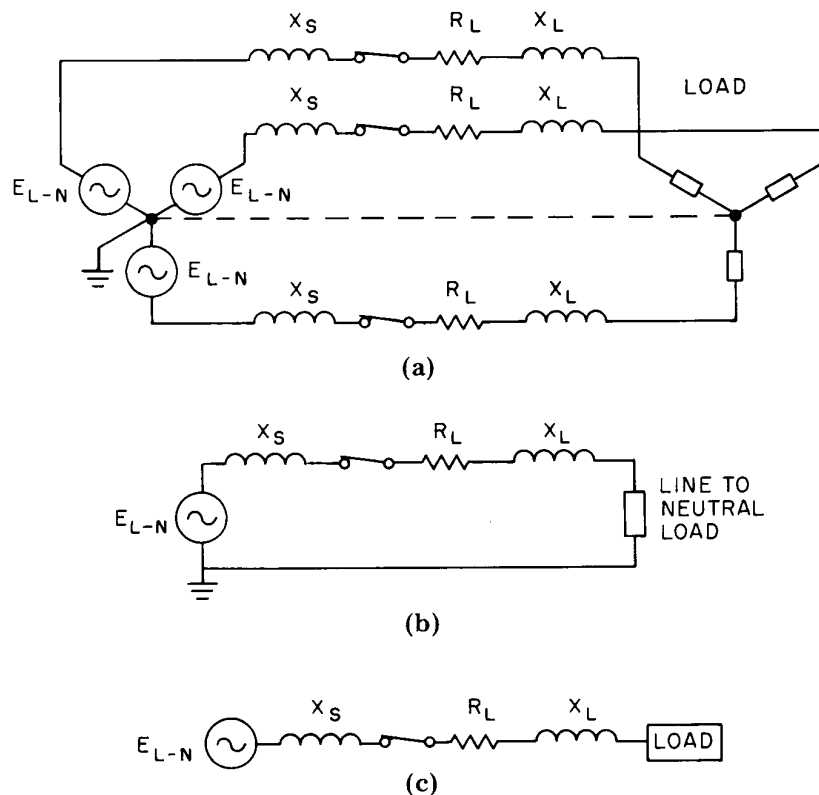


Fig 9
 (a) Three-Phase Diagram, (b) Single-Phase Equivalent, and (c) One-Line Diagram

poles, and for some reason the device in phase A becomes opened, the balance or symmetry of the circuit is destroyed. Neither the single-phase equivalent nor the one-line diagram representation is valid. Even though the single-phase and the one-line diagram representations would imply that the load has been disconnected, it continues to be energized by single-phase power. This can cause serious damage to motors and result in unacceptable operation of certain load apparatus.

More importantly, if only one switch-

ing device operates in response to a fault condition in the same phase, as depicted at location X, the system sources would continue to supply fault current from the other unopened phases through the impedance of the load. The throttling effect of the normally substantial load impedance, possibly in combination with additional arc impedance, can reduce the level of the current to a point where detection may not occur in phases (b) and (c). Needless to say, substantial damage can result before the fault finally burns enough to involve the other phases

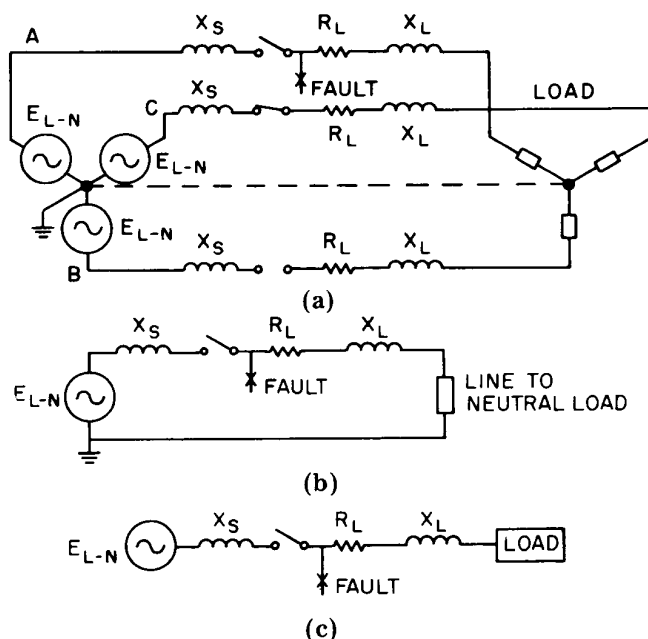


Fig 10
(a) Three-Phase Diagram, (b) Single-Phase
Equivalent, and (c) One-Line Diagram

directly and accomplish complete interruption. Meanwhile, both of the single-line representations fail to recognize the problem, and in fact, suggest that the condition has been safely disconnected. Therefore, the restraints of this calculating aid are:

(1) Symmetry of the electrical system, including all switching devices, and balance of the applied load

(2) Any of the other previously described restraints which apply to the analytical technique being used in combination with the single-phase equivalent

3.2.8 The Symmetrical Component Analysis. This approach comes to the analyst's rescue when he is confronted with an unbalance, the most common circuit condition which invalidates the single-phase equivalent circuit solution

method. The symmetrical component analysis allows the response to any unbalanced condition in a three-phase power system to be investigated and correctly synthesized by the sum of the responses to as many as three separate balanced system conditions. The application of an unbalanced set of voltage phasors, such as displayed in Fig 11, to a balanced downstream system and load is the sum of the responses of the balanced components which vectorially add to form the original unbalanced set.

Similar conclusions apply when the voltages are balanced, but the connected phase impedances or loads, or both, and therefore, the line currents are unbalanced. Here, the unbalanced current phasors are the sum of up to three balanced sets that flow through the balanced

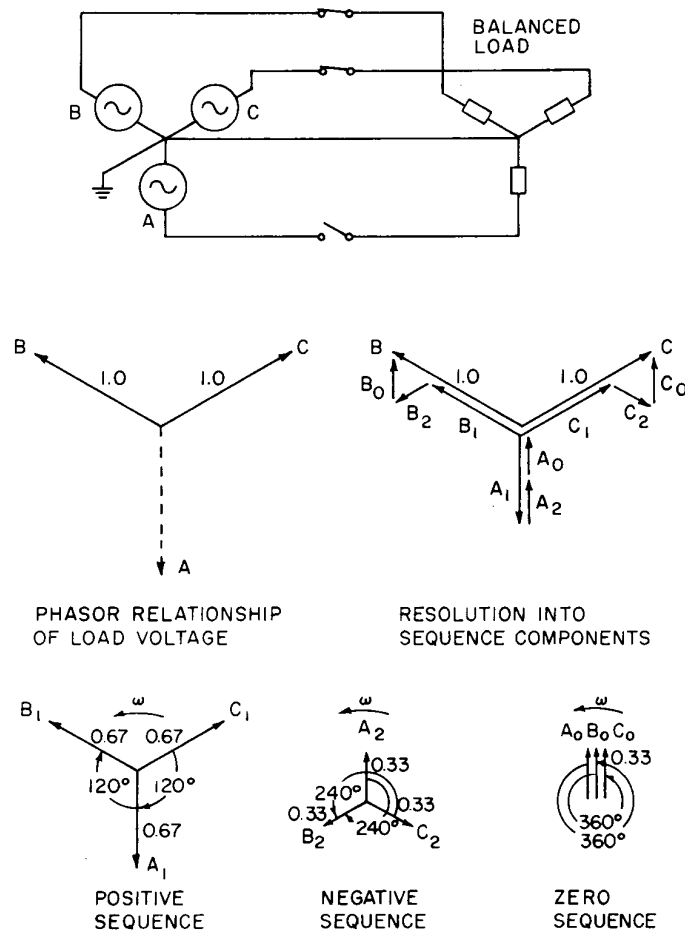


Fig 11
The Symmetrical Component Analysis

system impedances on either or both sides of the unbalance producing voltage drops that satisfy the needs of the applied voltages and the boundary conditions at the point of unbalance.

The mathematical expression for three unbalanced phasors (see Fig 11) as a function of the balanced phasor components is as follows:

$$\bar{A} = \bar{A}_0 + \bar{A}_1 + \bar{A}_2$$

$$\bar{B} = \bar{A}_0 + a^2 \bar{A}_1 + a \bar{A}_2$$

$$\bar{C} = \bar{A}_0 + a \bar{A}_1 + a^2 \bar{A}_2$$

The positive (1), negative (2), and zero (0) sequence vector components of any phase always have the angular relationship with respect to one another as described in the vector diagram of Fig 11.

These are assigned a counter-clockwise direction of rotation in the *time domain* as illustrated. In the *space domain*, the negative sequence phasors will produce exactly the same results as a set of equal magnitude phasors that are displaced from one another by 120° and that rotate clockwise with time.

Although the proof that a set of N unbalanced vectors can be completely represented by N sets of balanced vectors is seldom presented in the texts dealing with the subject of symmetrical components, it is true. First, it is postulated that it might be possible to describe three arbitrary but defined vectors, $\bar{A}$, $\bar{B}$, and $\bar{C}$ by the expressions (previously provided) involving only the three unknowns $\bar{A}_0$, $\bar{A}_1$, and $\bar{A}_2$. Then the terms are rearranged to solve for these unknowns as follows:

$$\bar{A}_0 = 1/3 (\bar{A} + \bar{B} + \bar{C})$$

$$\bar{A}_1 = 1/3 (\bar{A} + a\bar{B} + a^2\bar{C})$$

$$\bar{A}_2 = 1/3 (\bar{A} + a^2\bar{B} + a\bar{C})$$

Here are three independent equations for three unknowns ($\bar{A}_0$, $\bar{A}_1$ and $\bar{A}_2$), which are all that are required to uniquely and completely describe $\bar{A}_0$, $\bar{A}_1$, and $\bar{A}_2$ and, therefore, substantiate their existence. The corresponding $\bar{B}$ and $\bar{C}$ phase components are, then, defined by the vector relationship shown in Fig 11.

The merit of the symmetrical component analysis is that a relatively complicated, and often unwieldy problem, can be solved by simply vectorially summing the solution to no more than three balanced network problems. Several reference texts [7], [1], [5] provide convenient tables showing the network interconnections that must be used to solve for the responses to many commonly encountered system unbalances, as well as certain balanced conditions.

The three impedance networks (positive, negative, and zero sequence) are symbolically represented in short-hand fashion by an empty *block diagram* for the phase most definitive of the condition being studied up to the unbalance or other point of interest. Here, the final interconnections of the networks are shown which satisfy the necessary boundary conditions describing the system at the point of concern. The analyst can fill the *block diagrams* with the proper sources and impedances, including loads, in each sequence network and solve the single-phase loop equations. This, then, produces the three sequence responses, that is, current or voltage, which add vectorially to produce the resultant phase responses. Similarly, the other phase responses can be obtained by adding vectorially the individual sequence solutions shifted by the appropriate multiple of 120° .

One curious and often confounding feature of this solution procedure is that the phase in the system which usually provides the best, and sometimes the only, approach to the solution for an unbalance is the one least actively involved in the event. The unbalance illustrated in Fig 11 is one such example where the solution is obtained through an analysis of the non conducting phase A. A double line-to-ground fault is another, where examination of the open phase gives the most direct access to the network solution.

The symmetrical component analysis always involves the use of superposition as well as most of the other procedures previously discussed. The restraints which apply to these other procedures, therefore, must also govern the use of the symmetrical component analysis. In addition, due to mutual phase winding coupling and other effects, the impedance dis-

played by electrical machines will be different when excited by the different sequence sources. Hence, the *per phase* impedance of the negative and zero sequence networks will, in general, be different from the positive. Currents flowing in the zero sequence network, being in phase, do not add to zero as do both the positive and negative sequence currents and are, therefore, influenced by impedance in this additional circuit path. When harmonic excitation sources are present (requiring the use of the Fourier representation) special care must be exercised in treating the sequence networks. Starting with the fundamental, the harmonic term progressively shifts from the positive to the negative to the zero sequence networks, and then the process repeats.

3.2.9 The Per Unit Method. This method of calculation and its close companion, the percentage method, are well documented in [4], [5] and [6] and are generally well known. As a result, they will only be mentioned in passing here.

Fundamentally, the per unit method and the percentage method amount to a short-hand calculating procedure where all equivalent system and circuit impedances are converted to a common kVA *base*. This permits the ready combination of circuit elements in a network where different system voltages are present without the need to convert impedances each time responses are to be determined at a different voltage level.

Associated with each impedance element and its kVA base is a line-to-line kV base (usually the *nominal* line voltage at which the element is connected to the system), along with the resulting *base impedance*, and *base current* related by the following expressions:

(1) *Three-Phase Network*

$$I_{\text{base}} = \frac{\text{kVA}_{\text{base}}}{\sqrt{3} \text{ kV}_{\text{base}}}$$

$$Z_{\text{base}} = \frac{\text{kV}_{\text{base}}^2}{\text{kVA}_{\text{base}}} \cdot 1000$$

$$\text{kVA}_{\text{base}} = \sqrt{3} \text{ kV}_{\text{base}} I_{\text{base}}$$

(2) *Single-Phase Network*

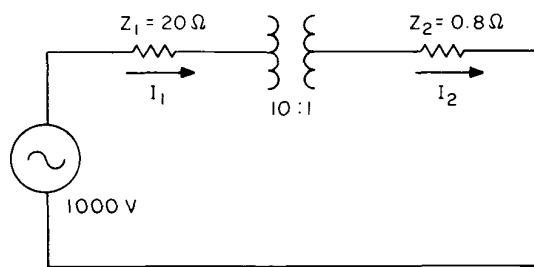
$$I_{\text{base}} = \frac{\text{kVA}_{\text{base}}}{\text{kV}_{\text{base}}}$$

$$Z_{\text{base}} = \frac{\text{kV}_{\text{base}}^2}{\text{kVA}_{\text{base}}} \cdot 1000$$

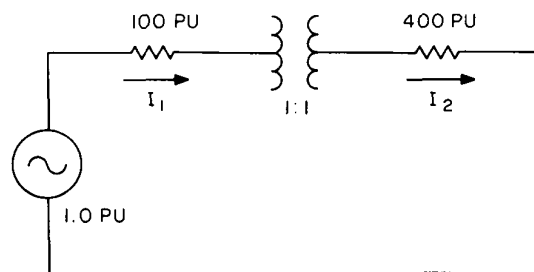
$$\text{kVA}_{\text{base}} = \text{kV}_{\text{base}} I_{\text{base}}$$

The equivalent open-circuit driving voltage ahead of the entire network can then conveniently and properly be expressed as 1.0 per unit (or 100%) if the base kV is selected to match the operating voltage. The method is simply illustrated and compared to the conventional ohmic calculation method for a single phase network in Fig 12 where the equivalent circuit impedances as viewed from the primary and secondary sides at the transformer are identified as Z_p and Z_s , respectively.

Before combining impedances it is essential that the corresponding per unit, or percentage, ratings have been expressed on a common kVA base and that they are connected to a system having a line-to-line voltage equal to the kV base to which the per unit, or percentage, rating is referenced. Since the per unit method of calculation is based on the existence of linearity and is always used in combination with one or more of the other principles, it is necessary to observe all of the associated restraints discussed earlier as they apply.



(a)



(b)

$$Z_P = 20 + \left(\frac{10}{1}\right)^2 \cdot 0.8 = 100 \Omega$$

$$Z_S = 20 \cdot \left(\frac{1}{10}\right)^2 + 0.8 = 1.0 \Omega$$

$$I_b = \frac{1000}{100} = 10 \text{ A}$$

$$I_2 = 1000 \cdot \left(\frac{1}{10}\right) \div 10 = 100 \text{ A}$$

$$\text{base kV}_P = 1$$

$$\text{base kV}_S = \frac{1}{10}$$

$$\text{base MVA} = 5$$

$$Z_1 = 20 \div \frac{1.0^2 \cdot 1000}{5000} = 100 \text{ per unit}$$

$$Z_2 = 0.8 \div \frac{(1/10)^2 \cdot 1000}{5000} = 400 \text{ per unit}$$

$$Z_{EQ} = 100 + 400 = 500 \text{ per unit}$$

$$I_1 = I_2 = \frac{1.0 \text{ per unit volts}}{500 \text{ per unit}} = 0.002 \text{ per unit}$$

$$I_{1\text{base}} = \frac{5\text{MVA}}{1\text{kV}} = 5000 \text{ A}$$

$$I_1 = (5000) \cdot (0.002) = 10 \text{ A}$$

$$I_{2\text{base}} = \frac{5\text{MVA}}{0.1\text{kV}} = 50\,000 \text{ A}$$

$$I_2 = (50\,000) \cdot (0.002) = 100 \text{ A}$$

Fig 12

(a) Classical Ohmic Representation,
(b) Per Unit Representation

3.3 References and Bibliography

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4. System Modeling

4.1 Introduction. This section addresses the following questions:

(1) How can each component or each group of components of an industrial or commercial power system be represented so that an analysis of the system performance can be made

(2) Which of the several possible representations, or *models*, of the given components will best describe the system to meet the objectives of a given study

(3) What mathematical expressions will describe the characteristics of each system element so that it can be quantified and programmed for computer input

There are an infinite number of possible power system configurations and a large variety of study types. Consequently standards cannot be established to dictate specific models for all specific circumstances.² This text will therefore serve as a guide to help the reader make judicious trade-offs in selecting models for his study.

²An exception to this statement are the application standards for circuit breakers where models for sources of short circuit current are specified. See 4.9.2.2 and 4.10.3.

Derivations and proofs of mathematical expressions will not be given. References should be used for such purpose. However, several fundamental relationships of electrical and mechanical quantities will be mentioned in the text. This should save the reader the time needed to locate them in text books or handbooks. It should also help refresh the memories of those who have not been exposed to power studies or academic activities for a long time.

The material is intended to be as basic and simple as permitted by the subject. The emphasis is to correlate real life systems with the abstraction of mathematics in order to communicate with the computer properly.

4.2 Modeling. Scale modeling of power systems as a means of analyzing their performance is impractical. However, scale models of certain mechanical components of power systems are used to evaluate their characteristics. This is often the case with hydraulic sections of hydroelectric plants such as turbine runners, spiral cases, gates, draft tubes,

etc. Nonetheless, much expertise is required to establish scaling and normalization factors, to construct the model, to gather meaningful data by measurement and to interpret and extrapolate the results.

A step in the direction of abstraction is the approach used in ac and dc analyzer boards for the study of power systems. Here the power system elements are modeled by modules of equivalent characteristics but physically much smaller. Most modules do not bear visual resemblance to the components they represent. A 200 mi transmission line module may look like a few potentiometers, inductances and capacitors. The modules are adjustable so the system variables can be modeled easily. These modules can be interconnected, each adjusted to satisfy the conditions of the power system being studied, and the required measurements made and interpreted. Once the board is configured for a base case, the effect of one variable on the overall system can be readily analyzed simply by adjusting the affected module. Most analyzing boards have been abandoned today in favor of the more economical digital method using computers.

Digital computers can be programmed to solve quickly and at relatively low cost a large number of simultaneous equations and can handle the algebra of large matrices. This makes them particularly well suited for applications in power system analysis. An immense variety of programs have been written to study an ever increasing number of problems in the electrical field. These programs are usually set up to receive the problem information in the form of numbers rather than analog settings. This then forces the power system analyst to model the system quantitatively. The programs, designed to maximize their

usefulness are written in general non-specific terms. This exposes the analyst to a choice of program features and alternatives that require decisions to be made every step of the way. Finally the programs are often structured to handle extensive power systems (3000 bus programs are not uncommon). This aspect suggests that the analyst should consider to what extent his system will be modeled to avoid, on the one hand, an expensive overkill and on the other an incomplete problem statement that ensures questionable answers.

4.3 Review of Basics. Power network elements may be classified in two categories, passive elements and active elements.

4.3.1 Passive Elements. The passive elements comprise such components as transmission lines, transformers, reactors, and capacitors. They will, in general, be regarded as linear. They will be modeled by one or more of the following electrical quantities:

Name	Symbol	Unit
resistance	R	ohm
inductance	L	henry
capacitance	C	farad

The voltage across and the current through the element will be governed by these relationships:

$$v = Ri \quad i = \frac{v}{R} \quad (\text{Eq 1})$$

$$v = L \frac{di}{dt} \quad i = \frac{1}{L} \int v dt \quad (\text{Eq 2})$$

$$v = \frac{1}{C} \int i dt \quad i = C \frac{dv}{dt} \quad (\text{Eq 3})$$

where the lowercase letters represent the instantaneous values of voltage and current.

In dc circuits under steady state conditions these equations will reduce to:

$$V = RI \quad I = \frac{V}{R}$$

$$V = 0 \quad (\text{since } \frac{di}{dt} = 0)$$

$$I = 0 \quad (\text{since } \frac{dv}{dt} = 0) \quad (\text{Eq 4})$$

In ac circuits with sinusoidal wave shapes, the equations become:

$$V = RI \quad I = \frac{V}{R}$$

$$V = jX_L I$$

where

$$X_L = 2\pi fL$$

= inductive reactance (Eq 5)

$$V = -jX_C I$$

where

$$X_C = \frac{1}{2\pi fC}$$

= capacitive reactance (Eq 6)

The capital letters for voltages and currents represent their rms values, f is the frequency in hertz, and j the 90° operator ($= \sqrt{-1}$). Inverting and combining these elements in series or parallel will define the set of quantities of Table 1.

It should be noted here that it is customary in ac power circuits to use the

R , X and Z quantities for the series (line) elements and the G , B and Y quantities for the shunt (line to neutral) elements.

Note also that Z and Y are complex quantities that can be expressed in the rectangular form above or the polar form $Z = |Z| \angle \theta$ or $Y = |Y| \angle \theta$. Most computer programs accept the Z and Y values in the rectangular form.

A final remark concerns the sign ahead of the reactances and susceptances. The four diagrams of Fig 13 are self explanatory. The wise analyst will verify the program instructions to make sure that the computer will interpret the input data properly.

4.3.2 Active Elements. The active elements of a power system comprise such components as motors, generators, synchronous condensers, other loads like furnaces, adjustable speed drives, etc. The active elements will be regarded as non-linear, although some of the components may behave linearly under certain circumstances.

One or more of the parameters of a model of an active element will vary as a function of time, phase angle, frequency, speed, etc.

The four expressions for power quantities given in Table 2 can be used to model non linear elements. Given any two of the four values, the remaining two can be defined. Power can also be expressed in polar form: $S = |S| \angle \theta$ which yields these relationships: $\text{PF} = \cos \theta$, $P = S \cos \theta$ and $Q = S \sin \theta$.

Table 1
Equation References for Conductance, Susceptance, Impedance and Admittance

Name	Symbol	Unit	Defining Equation
conductance	G	S (mho)	$1/R$
inductive susceptance	B	S (mho)	$1/X$
capacitive susceptance	B	S (mho)	$1/X$
impedance	Z	S (ohm)	$(R + jX)$
admittance	Y	S (mho)	$(G + jB)$

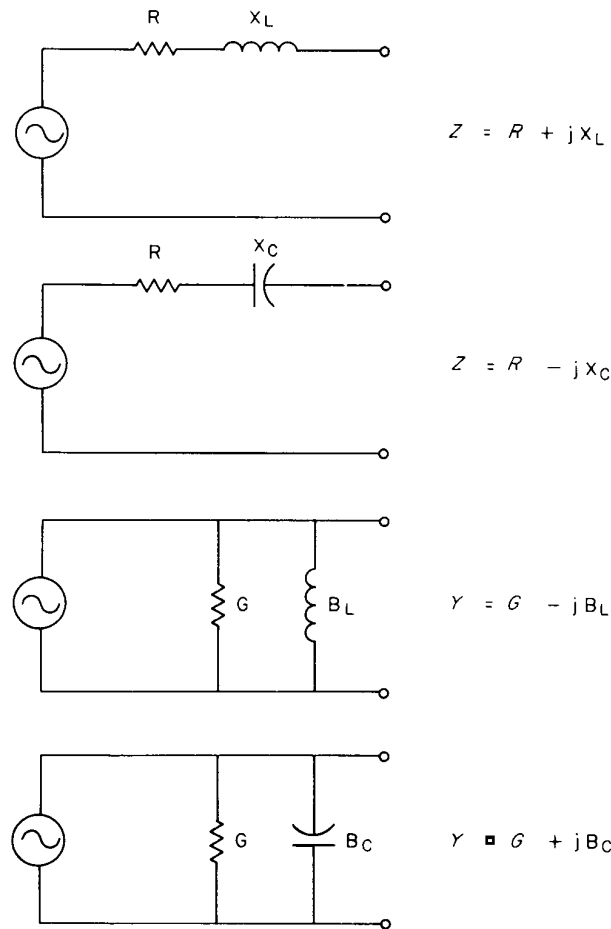


Fig 13
Squirrel Cage Induction Motor Model

Table 2
Four Expressions for Power Quantities

Name	Symbol	Unit	Expression
complex power	S	VA	$S = P + jQ$
active power	P	W	$P = \sqrt{S^2 - Q^2}$
reactive power	Q	var	$Q = \sqrt{S^2 - P^2}$
power factor	PF	per unit	$PF = \frac{P}{S}$

Note that the signs of P or Q may be positive or negative. By convention, the positive sign of Q is used for inductive loads, that is, the current will lag the voltage applied to a load that *consumes* vars. In this sense it is said that capacitors *generate* vars (current leads the voltage) and that squirrel cage induction motors *absorb* vars.

By convention also, the sign of P is positive for a load that consumes energy or a source that generates energy. Thus a load with a negative sign for P could be used to represent a generator, and vice versa.

Some of the power system components can best be expressed in terms of current or voltage. For instance, an infinite bus may be specified by a voltage source of constant magnitude and angle, and a particular load may be described as a constant current element. The current and voltage quantities may be complex numbers, in which case they have to be described in terms of a reference vector that may be a voltage or current quantity. This introduces the phase angle concept:

$$I = |I| \angle \theta = I_x + jI_y = I(\cos \theta + j \sin \theta)$$

$$V = |V| \angle \theta = V_x + jV_y = V(\cos \theta + j \sin \theta)$$

where the x axis of the coordinate system is taken as the reference shown in Fig 14.

So far we have reviewed most of the quantities used in the type of studies that require network solutions at a single instant: a *snapshot* of the network in equilibrium. This instant shows the system in a steady state or during a transient, at an instant for which all network parameters are known, say $\frac{1}{2}$ cycle after a short circuit has occurred.

Some studies, of which motor starting and transient stability are examples, require that a complete period of time be covered to assess the effects of a disturbance on system performance. This requirement for a certain period of time to be considered introduces the need for mechanical quantities.

The fundamental quantities of mechanics are space, matter and time. Length measures space and mass measures matter. Other mechanical quantities are derived from these three. Some of the derived quantities officially recognized

Fig 14
Section of a Typical
Industrial Plant Impedance Diagram

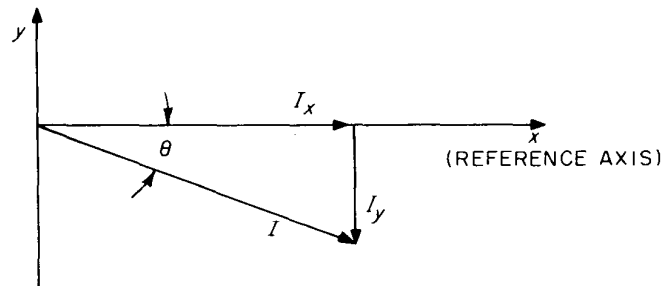


Table 3
Fundamental Equations for
Translation and Rotation

Name	Symbol	Unit	Defining Equation
Fundamental			
length	l	meter (m)	
mass	m	kilogram (kg)	
time	t	second (s)	
Translation			
velocity	v	m/s	$v = \frac{dl}{dt}$
acceleration	a	m/s ²	$a = \frac{d^2l}{dt^2}$
force	F	newton (N)	$F = ma$
work	W	joule (J)	$W = \int F dl$
power	P	watt (W)	$P = \frac{dw}{dt}$
momentum	M'	N/s	$M' = mv$
Rotation			
radius	r	m	
circular arc	s	m	
moment of inertia	I	kg-m ²	$I = \int r^2 dm$
angle	θ	radian (rad)	$\theta = \frac{s}{r}$
angular velocity	ω	rad/s	$\omega = \frac{d\theta}{dt}$
angular acceleration	α	rad/s ²	$\alpha = \frac{d^2\theta}{dt^2}$
torque	T	N·m	$T = rF$
work	W	J	$W = \int T d\theta$
power	P	W	$P = T\omega$
angular momentum	M	J s/rad	$M = I\omega$

for electrical engineering work have been tabulated in Table 3, with the MKS system of units and the defining equations.

4.4 Power Network Solution. Before dealing with the detailed models of power system components, it is important to review what constitutes the solution of a network.

It can be said that a network is resolved if all the bus voltages and the relative phase angles between these voltages are known. This of course requires that the impedance between the buses be known.

It also requires that a reference voltage and angle be specified for one bus.

Consider for instance Fig 15 which shows a small section of the typical plant electrical system of Section 5, Load Flow. Assume that the voltages at buses 2, 4 and 24 (also called A, B and C, to simplify notations) and the impedances of T2 and line 8 are known. They are summarized on Fig 16 and listed as follows:

$$\begin{aligned} V_2 &= V_A = 69.00 \text{ kV } \angle 0^\circ \\ V_4 &= V_B = 13.60 \text{ kV } \angle -1.6^\circ \\ V_{24} &= V_C = 13.51 \text{ kV } \angle -1.82^\circ \end{aligned}$$

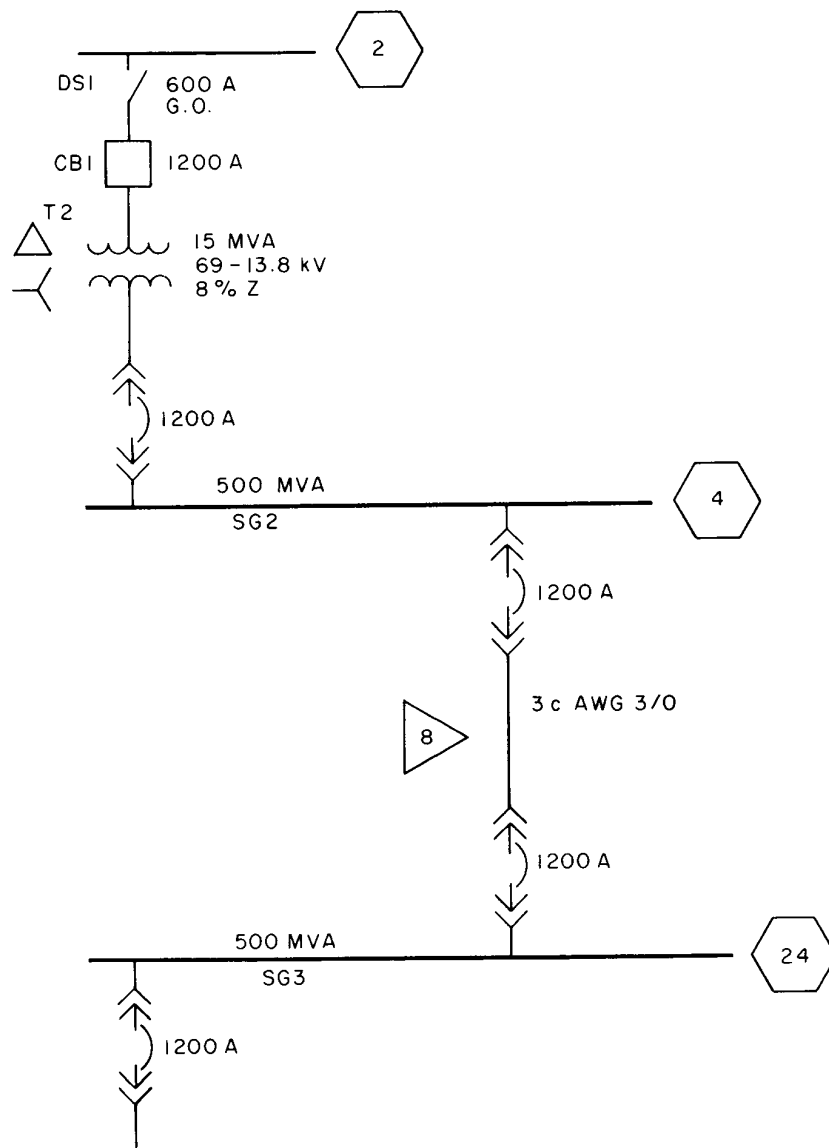


Fig 15
Single Line Diagram

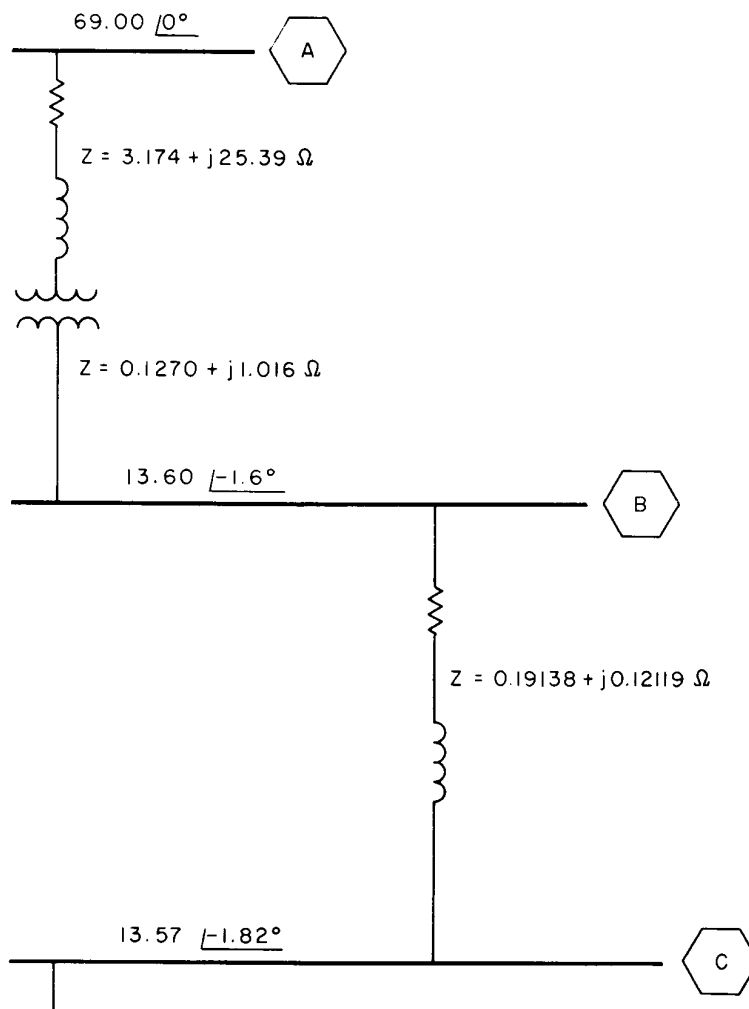


Fig 16
Impedance Diagram