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IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis

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**Power Systems Engineering Committee
of the
Industrial and Commercial Power Systems Department
of the
IEEE Industry Applications Society**

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Abstract: This Recommended Practice is a reference source for engineers involved in industrial and commercial power systems analysis. It contains a thorough analysis of the power system data required, and the techniques most commonly used in computer-aided analysis, in order to perform specific power system studies of the following: short-circuit, load flow, motor-starting, cable ampacity, stability, harmonic analysis, switching transient, reliability, ground mat, protective coordination, dc auxiliary power system, and power system modeling.

Keywords: cable ampacity, dc power system studies, ground mat studies, harmonic analysis, load flow studies, motor-starting studies, power system analysis, power system modeling, power system studies, protective coordination studies, reliability studies, short-circuit studies, stability studies, switching transient studies.

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Introduction

(This introduction is not a part of IEEE Std 399-1997, IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis.)

This Recommended Practice, commonly known as the “Brown Book,” is intended as a practical, general treatise on power system analysis theory and as an engineer’s reference source on the techniques that are most commonly applied to the computer-aided analysis of electric power systems in industrial plants and commercial buildings. The Brown Book is a useful supplement to several other power system analysis texts that appear in the references and bibliography subclauses of the various chapters of this book. The Brown Book is both complementary and supplementary to the rest of the Color Book series.

One new and important chapter has been added: Chapter 16, entitled “DC auxiliary power system analysis.” All the other chapters in this new edition have been revised and updated—in some cases quite substantially—to reflect current technology.

To many members of the working group who wrote and developed this Recommended Practice, the Brown Book has become a true labor of love. The dedication and support of each individual member is clearly evident in every chapter of the Brown Book. These individuals deserve our many thanks for their excellent contributions.

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IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis

Chapter 1 Overview

1.1 Scope and general information

This Recommended Practice, commonly known as the IEEE Brown Book, is published by the Institute of Electrical and Electronics Engineers, Inc. (IEEE) as a reference source to give plant engineers a better understanding of the purpose for and techniques involved in power system studies. The IEEE Brown Book can also be a helpful reference source for system and data acquisition for engineering consultants performing necessary studies prior to designing a new system or expanding an existing power system. This Recommended Practice will help ensure high standards of power system reliability and maximize the utilization of capital investment.

The IEEE Brown Book emphasizes up-to-date techniques in system studies that are most applicable to industrial and commercial power systems. It complements the other IEEE Color Books, and is intended to be used in conjunction with, not as a replacement for, the many excellent texts available in this field.

The IEEE Brown Book was prepared on a voluntary basis by engineers and designers functioning as a Working Group within the IEEE, under the Industrial and Commercial Power Systems Department of the Industry Applications Society.

1.2 History of power system studies

The planning, design, and operation of a power system requires continual and comprehensive analyses to evaluate current system performance and to establish the effectiveness of alternative plans for system expansion.

The computational work to determine power flows and voltage levels resulting from a single operating condition for even a small network is all but insurmountable if performed by manual methods. The need for computational aids led to the design of a special purpose analog computer (ac network analyzer) as early as 1929. It provided the ability to determine flows and voltages during normal and emergency conditions and to study the transient behavior of the system resulting from fault conditions and switching operations.

The earliest application of digital computers to power system problems dates back to the late 1940s. Most of the early applications were limited in scope because of the small capacity of the punched card calculators in use during that period. Large-scale digital computers became

available in the mid-1950s, and the initial success of load flow programs led to the development of programs for short-circuit and stability calculations.

Today, the digital computer is an indispensable tool in power system planning, in which it is necessary to predict future growth and simulate day-to-day operations over periods of twenty years or more.

1.3 Applying power system analysis techniques to industrial and commercial power systems

As computer technology has advanced, so has the complexity of industrial and commercial power systems. These power systems have grown in recent decades with capacities far exceeding that of a small electric utility system.

Today's intensely competitive business environment forces plant or building management personnel to be very aware of the total owning cost of the power distribution system. Therefore, they demand assurances of maximum return on all capital investments in the power system. The use of digital computers makes it possible to study the performance of proposed and actual systems under many operating conditions. Answers to many questions regarding impact of expansion on the system, short-circuit capacity, stability, load distribution, etc., can be intelligently and economically obtained.

1.4 Purposes of this Recommended Practice

1.4.1 Why a study?

As is stated in Chapter 2, the planning, design, and operation of industrial and commercial power systems require several studies to assist in the evaluation of the initial and future system performance, system reliability, safety, and the ability to grow with production and/or operating requirements. The studies most likely to be needed are load flow studies, cable ampacity studies, short-circuit studies, coordination studies, stability studies, and routine motor-starting studies. Additional studies relating to switching transients, reliability, grounding, harmonics, and special motor-starting considerations may also be required. The engineer in charge of system design must decide which studies are needed to ensure that the system will operate safely, economically, and efficiently over the expected life of the system.

A brief summary of these studies is presented in Chapter 2, along with a discussion pertaining to data collection and preparation of the data files required to perform the desired study on a digital computer.

1.4.2 How to prepare for a power system study

For a plant engineer to solve a power system analysis problem, he or she must be thoroughly familiar with the fundamentals of power electrical engineering. He or she can then analyze

the problem, prepare the necessary equivalent circuits, and obtain appropriate system data before using a computer program to perform repetitive calculations. Failure to use a valid analytical procedure to establish a sound basic approach to the problem could lead to disastrous consequences in both the design and operation of a system. Furthermore, a basic understanding of power engineering is essential to correctly interpret the results of computer calculations. It is important to emphasize the need for a thorough foundation and base of experience in power system engineering in addition to modern, effective computer software. Power system analysis engineering software is an excellent tool for studying power systems, but it cannot be used as a substitute for knowledge and experience. Chapter 3 offers an excellent review of the most essential fundamentals in a system study.

To set up a computer program for system analysis, certain basic data must be gathered with accuracy and proper presentation. The extent of system representation, restrictions in terms of nodes (buses) and branches (lines and transformers), balanced three-phase network and single-line diagram, impedance diagram, etc., are all important inputs to a meaningful system study. Chapter 4 deals with system modeling and data requirements to illustrate how these basic data for a study can be properly prepared or organized.

Once the basic preparations are completed, the next step is to look for an actual computer program. Programs are available—written for personal computers (PCs)—that allow in-house calculation for the studies outlined in this standard. Chapter 5 discusses basic computation methods, various types of computer systems and their requirements, and availability of commercial computing services and their capabilities.

1.4.3 The most common system studies

The following chapters address the most common studies for the design or operation of an industrial or commercial power system:

- Chapter 6, Load flow studies
- Chapter 7, Short-circuit studies
- Chapter 8, Stability studies
- Chapter 9, Motor-starting studies
- Chapter 10, Harmonic analysis studies
- Chapter 11, Switching transient studies
- Chapter 12, Reliability studies
- Chapter 13, Cable ampacity studies
- Chapter 14, Ground mat studies
- Chapter 15, Coordination studies
- Chapter 16, DC auxiliary power system analysis

The purpose of each study and what can be achieved by it are briefly explained in each chapter.

Figure 1-1 is a typical composite single-line diagram for a large industrial power system that is used in Chapters 6 through 15.

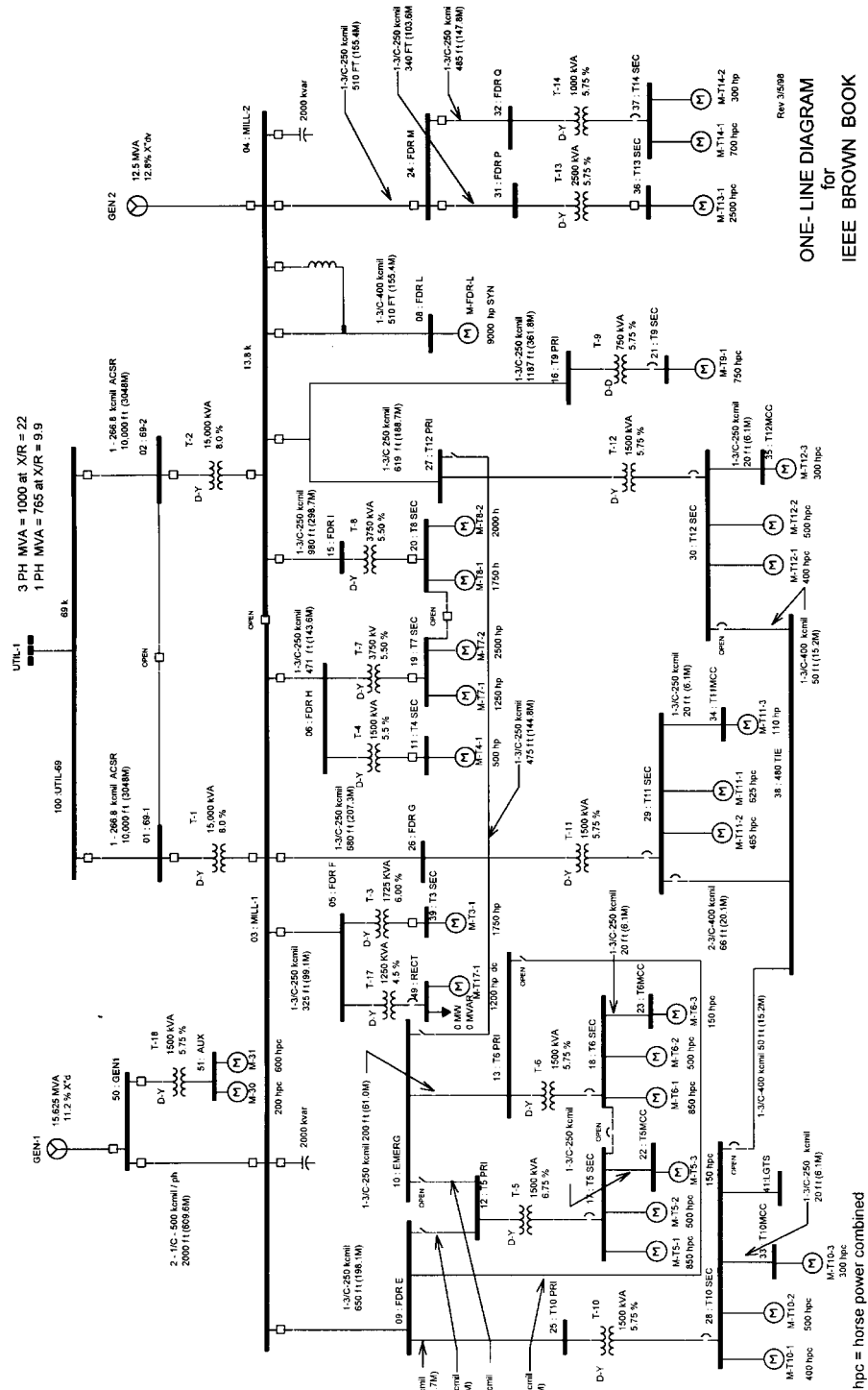


Figure 1-1—Single-line diagram of typical industrial power system for load flow study example

After studying these chapters, an engineer should be better equipped to prepare necessary data and criteria for a specific computer study. The study can be performed in-house or by an outside consultant. There is a growing number of consulting firms that specialize in performing system studies at a reasonable cost.

Studying these chapters will provide the basic understanding of the studies needed to coordinate the data and criteria for specific studies and will also serve as a reference to those analysts for whom studies are a principal activity.

1.5 References

This chapter shall be used in conjunction with the following publications:

IEEE Std 141-1993, IEEE Recommended Practice for Electric Power Distribution for Industrial Plants (IEEE Red Book).¹

IEEE Std 142-1991, IEEE Recommended Practice for Grounding of Industrial and Commercial Power Systems (IEEE Green Book).

IEEE Std 241-1990, IEEE Recommended Practice for Electric Power Systems in Commercial Buildings (IEEE Gray Book).

IEEE Std 242-1986 (Reaff 1991), IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems (IEEE Buff Book).

¹IEEE publications are available from the Institute of Electrical and Electronics Engineers, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA.

Chapter 2

Applications of power system analysis

2.1 Introduction

The planning, design, and operation of industrial and commercial power systems require engineering studies to evaluate existing and proposed system performance, reliability, safety, and economics. Studies, properly conceived and conducted, are a cost-effective way to prevent surprises and to optimize equipment selection. In the design stage, the studies identify and avoid potential deficiencies in the system before it goes into operation. In existing systems, the studies help locate the cause of equipment failure and misoperation, and determine corrective measures for improving system performance.

The complexity of modern industrial power systems makes studies difficult, tedious, and time-consuming to perform manually. The computational tasks associated with power systems studies have been greatly simplified by the use of digital computer programs. Sometimes, economics and study requirements dictate the use of an analog computer—a transient network analyzer (TNA)—which provides a scale model of the power system.

2.1.1 Digital computer

The digital computer offers engineers a powerful tool to perform efficient system studies. Computers permit optimal designs at minimum costs, regardless of system complexity. Advances in computer technology, like the introduction of the personal computer with its excellent graphics capabilities, have not only reduced the computing costs but also the engineering time needed to use the programs. Study work formerly done by outside consultants can now be performed in-house. User-friendly programs utilizing interactive menus, online help facilities, and a graphical user interface (GUI) guide the engineer through the task of using a digital computer program.

2.1.2 Transient network analyzer (TNA)

The TNA is a useful tool for transient overvoltage studies. The use of microcomputers to control and acquire the data from the TNA allows the incorporation of probability and statistics in switching surge analysis. One of the major advantages of the TNA is that it allows for quick reconfiguration of complex systems with immediate results, avoiding the relatively longer time associated with running digital computer programs for these systems.

2.2 Load flow analysis

Load flow studies determine the voltage, current, active, and reactive power and power factor in a power system. Load flow studies are an excellent tool for system planning. A number of operating procedures can be analyzed, including contingency conditions, such as the loss of a generator, a transmission line, a transformer, or a load. These studies will alert the user to

conditions that may cause equipment overloads or poor voltage levels. Load flow studies can be used to determine the optimum size and location of capacitors for power factor improvement. Also, they are very useful in determining system voltages under conditions of suddenly applied or disconnected loads. The results of a load flow study are also starting points for stability studies. Digital computers are used extensively in load flow studies due to the complexity of the calculations involved.

2.3 Short-circuit analysis

Short-circuit studies are done to determine the magnitude of the prospective currents flowing throughout the power system at various time intervals after a fault occurs. The magnitude of the currents flowing through the power system after a fault vary with time until they reach a steady-state condition. This behavior is due to system characteristics and dynamics. During this time, the protective system is called on to detect, interrupt, and isolate these faults. The duty imposed on this equipment is dependent upon the magnitude of the current, which is dependent on the time from fault inception. This is done for various types of faults (three-phase, phase-to-phase, double-phase-to-ground, and phase-to-ground) at different locations throughout the system. The information is used to select fuses, breakers, and switchgear ratings in addition to setting protective relays.

2.4 Stability analysis

The ability of a power system, containing two or more synchronous machines, to continue to operate after a change occurs on the system is a measure of its stability. The stability problem takes two forms: steady-state and transient. Steady-state stability may be defined as the ability of a power system to maintain synchronism between machines within the system following relatively slow load changes. Transient stability is the ability of the system to remain in synchronism under transient conditions, i.e., faults, switching operations, etc.

In an industrial power system, stability may involve the power company system and one or more in-plant generators or synchronous motors. Contingencies, such as load rejection, sudden loss of a generator or utility tie, starting of large motors or faults (and their duration), have a direct impact on system stability. Load-shedding schemes and critical fault-clearing times can be determined in order to select the proper settings for protective relays.

These types of studies are probably the single most complex ones done on a power system. A simulation will include synchronous generator models with their controls, i.e., voltage regulators, excitation systems, and governors. Motors are sometimes represented by their dynamic characteristics as are static var compensators and protective relays.

2.5 Motor-starting analysis

The starting current of most ac motors is several times normal full load current. Both synchronous and induction motors can draw five to ten times full load current when starting

them across the line. Motor-starting torque varies directly as the square of the applied voltage. If the terminal voltage drop is excessive, the motor may not have enough starting torque to accelerate up to running speed. Running motors may stall from excessive voltage drops, or undervoltage relays may operate. In addition, if the motors are started frequently, the voltage dip at the source may cause objectionable flicker in the lighting system.

By using motor-starting study techniques, these problems can be predicted before the installation of the motor. If a starting device is needed, its characteristics and ratings can be easily determined. A typical digital computer program will calculate speed, slip, electrical output torque, load current, and terminal voltage data at discrete time intervals from locked rotor to full load speed. Also, voltage at important locations throughout the system during start-up can be monitored. The study can help select the best method of starting, the proper motor design, or the required system design for minimizing the impact of motor starting on the entire system.

2.6 Harmonic analysis

A harmonic-producing load can affect other loads if significant voltage distortion is caused. The voltage distortion caused by the harmonic-producing load is a function of both the system impedance and the amount of harmonic current injected. The mere fact that a given load current is distorted does not always mean there will be undue adverse effects on other power consumers. If the system impedance is low, the voltage distortion is usually negligible in the absence of harmonic resonance. However, if harmonic resonance prevails, intolerable harmonic voltage and currents are likely to result.

Some of the primary effects of voltage distortion are the following:

- a) Control/computer system interference
- b) Heating of rotating machinery
- c) Overheating/failure of capacitors

When the harmonic currents are high and travel in a path with significant exposure to parallel communication circuits, the principal effect is telephone interference. This problem depends on the physical path of the circuit as well as the frequency and magnitude of the harmonic currents. Harmonic currents also cause additional line losses and additional stray losses in transformers.

Watt-hour meter error is often a concern. At harmonic frequencies, the meter may register high or low depending on the harmonics present and the response of the meter to these harmonics. Fortunately, the error is usually small.

Analysis is commonly done to predict distortion levels for addition of a new harmonic-producing load or capacitor bank. The general procedure is to first develop a model that can accurately simulate the harmonic response of the present system and then to add a model of the new addition. Analysis is also commonly done to evaluate alternatives for correcting problems found by measurements.

Only very small circuits can be effectively analyzed without a computer program. Typically, a computer program for harmonic analysis will provide the engineer with the capability to compute the frequency response of the power system and to display it in a number of useful graphical forms. The programs provide the capability to predict the actual distortion based on models of converters, arc furnaces, and other nonlinear loads.

2.7 Switching transients analysis

Switching transients severe enough to cause problems in industrial power systems are most often associated with inadequate or malfunctioning breakers or switches and the switching of capacitor banks and other frequently switched loads. The arc furnace system is most frequently studied because of its high frequency of switching and the related use of capacitor banks.

By properly using digital computer programs or a TNA, these problems can be detected early in the design stage. In addition to these types of switching transient problems, digital computer programs and the TNA can be used to analyze other system anomalies, such as lightning arrester operation, ferroresonance, virtual current chopping, and breaker transient recovery voltage.

2.8 Reliability analysis

When comparing various industrial power system design alternatives, acceptable system performance quality factors (including reliability) and cost are essential in selecting an optimum design. A reliability index is the probability that a device will function without failure over a specified time period. This probability is determined by equipment maintenance requirements and failure rates. Using probability and statistical analyses, the reliability of a power system can be studied in depth with digital computer programs.

Reliability is most often expressed as the frequency of interruptions and expected number of hours of interruptions during one year of system operation. Momentary and sustained system interruptions, component failures, and outage rates are used in some reliability programs to compute overall system reliability indexes at any node in the system, and to investigate sensitivity of these indexes to parameter changes. With these results, economics and reliability can be considered to select the optimum power system design.

2.9 Cable ampacity analysis

Cable ampacity studies calculate the current-carrying capacity (ampacity) of power cables in underground or above ground installations. This ampacity is determined by the maximum allowable conductor temperature. In turn, this temperature is dependent on the losses in the cable, both I^2R and dielectric, and thermal coupling between heat-producing components and ambient temperature.

The ampacity calculations are extremely complex. This is due to many considerations, some examples of which are heat transfer through the cable insulation and sheath, and, in the case of underground installations, heat transfer to duct or soil as well as from duct bank to soil. Other considerations include the effects of losses caused by proximity and skin effects. In addition, depending on the installation, the cable-shielding system may introduce additional losses. The analysis involves the application of thermal equivalents of Ohm's and Kirchhoff's laws to a thermal circuit.

2.10 Ground mat analysis

Under ground-fault conditions, the flow of current will result in voltage gradients within and around the substation, not only between structures and nearby earth, but also along the ground surface. In a properly designed system, this gradient should not exceed the limits that can be tolerated by the human body.

The purpose of a ground mat study is to provide for the safety and well-being of anyone that can be exposed to the potential differences that can exist in a station during a severe fault. The general requirements for industrial power system grounding are similar to those of utility systems under similar service conditions. The differences arise from the specific requirements of the manufacturing or process operations.

Some of the factors that are considered in a ground-mat study are the following:

- a) Fault-current magnitude and duration
- b) Geometry of the grounding system
- c) Soil resistivity
- d) Probability of contact
- e) Human factors such as
 - 1) Body resistance
 - 2) Standard assumptions on physical conditions of the individual

2.11 Protective device coordination analysis

The objective of a protection scheme in a power system is to minimize hazards to personnel and equipment while allowing the least disruption of power service. Coordination studies are required to select or verify the clearing characteristics of devices such as fuses, circuit breakers, and relays used in the protection scheme. These studies are also needed to determine the protective device settings that will provide selective fault isolation. In a properly coordinated system, a fault results in interruption of only the minimum amount of equipment necessary to isolate the faulted portion of the system. The power supply to loads in the remainder of the system is maintained. The goal is to achieve an optimum balance between equipment protection and selective fault isolation that is consistent with the operating requirements of the overall power system.

Short-circuit calculations are a prerequisite for a coordination study. Short-circuit results establish minimum and maximum current levels at which coordination must be achieved and which aid in setting or selecting the devices for adequate protection. Traditionally, the coordination study has been performed graphically by manually plotting time-current operating characteristics of fuses, circuit breaker trip devices, and relays, along with conductor and transformer damage curves—all in series from the fault location to the source. Log-log scales are used to plot time versus current magnitudes. These “coordination curves” show graphically the quality of protection and coordination possible with the equipment available. They also permit the verification/confirmation of protective device characteristics, settings, and ratings to provide a properly coordinated and protected system.

With the advent of the personal computer, the light-table approach to protective device coordination is being replaced by computer programs. The programs provide a graphical representation of the device coordination as it is developed. In the future, computer programs are expected to use expert systems based on practical coordination algorithms to further assist the protection engineer.

2.12 DC auxiliary power system analysis

The need for direct current (dc) power system analysis of emergency standby power supplies has steadily increased during the past several years in data processing facilities, long distance telephone companies, and generating stations.

DC emergency power is used for circuit breaker control, protective relaying, inverters, instrumentation, emergency lighting, communications, annunciators, fault recorders, and auxiliary motors. The introduction of computer techniques to dc power systems analysis has allowed a more rapid and rigorous analysis of these systems compared to earlier manual techniques.

Chapter 3

Analytical procedures

3.1 Introduction

With the development of the digital computer and advanced computer programming techniques, power system problems of the most complex types can be rigorously analyzed. Previously, solutions were usually only approximate. Limitations and errors were introduced by the many simplifying assumptions necessary to permit classical longhand calculating procedures. For progress to be realized in using the computer for power system analysis, it has been necessary for the creators of power system analysis computer programs to thoroughly understand the application of basic analytical solution methods that apply. It is also important for those concerned with assembling and preparing data for input to a power system analysis computer program and those interpreting and applying results generated by such a program to understand the application of analytical solution methods.

This chapter identifies and documents the basic analytical solution methods that are valid for determining the voltage and current relationships that exist during various power system network events and operating conditions. These basic analytical solution methods are demonstrated in cases where they are not self-evident. Finally, critical restraints that must be respected to avoid serious error in applying analytical solution methods will be discussed.

Whether a power system analysis problem is to be solved directly or by a computer program, proper application of sound analytical solution methods is essential for three reasons. First, accuracy of the solution to each individual problem being considered will be directly affected. Second, and perhaps the most important because of the significant expense involved, accuracy of the solution determines the validity and effectiveness of any remedial measures suggested. Finally, extension of erroneous results to related problems or to what appears to be a trivial modification of the original problem, possibly in combination with other misapplied or misunderstood techniques, can lead to a compounding of initial error and a progression of incorrect conclusions.

The most common causes of errors in circuit analysis work are the following:

- a) Failure to use a valid analytical procedure because the analyst is unaware of its existence or applicability
- b) Careless or improper use of “cookbook” methods that have neither a factual basis, nor support in the technical literature, nor a valid place in the electrical engineering discipline
- c) Improper use of a valid solution method due to application beyond limiting boundary restraints or in combination with an inaccurate simplifying assumption

Many situations occur in industrial and commercial power systems that illustrate some or all of these common causes of error, as well as the resulting evils. Any problem investigated as a

part of the general types of power system analysis studies covered in other sections of this recommended practice and described as follows would qualify.

- Short-circuit studies
- Load analysis studies
- Load flow studies
- Stability studies
- Motor-starting studies
- Harmonic studies
- Reliability studies
- Ground mat studies
- Switching transient studies

3.2 Fundamentals

The following list identifies the more important analytical solution methods that are either available as, or are the basis for, valid techniques in solving power system network circuit problems:

- a) Linearity
- b) Superposition
- c) Thevenin and Norton equivalent circuits
- d) Sinusoidal forcing function
- e) Phasor representation
- f) Fourier representation
- g) Laplace transform
- h) Single-phase equivalent circuit
- i) Symmetrical component analysis
- j) Per unit method

Rigorous treatment of these analytical techniques is available in several circuit analysis texts (Beeman [B1]¹, Close [B3], Hayt and Kemmerly [B7], Stevenson, Jr. [B14], Wagner and Evans [B15], Weedy [B16]) and is beyond the scope of this discussion. In the following sub-clauses of this chapter, however, a brief qualitative explanation of each principle is presented, along with a review of major benefits and restraints associated with the use of each principle.

3.2.1 Linearity

Probably the simplest concept of all, linearity is also one of the most important because of its influence on the other principles. Linearity is best understood by examination of Figure 3-1.

The simplified network represented by the single-impedance element Z in Figure 3-1(a) is linear for the chosen excitation and response functions, if a plot of response magnitude

¹The numbers in brackets correspond to those of the bibliography in 3.3.

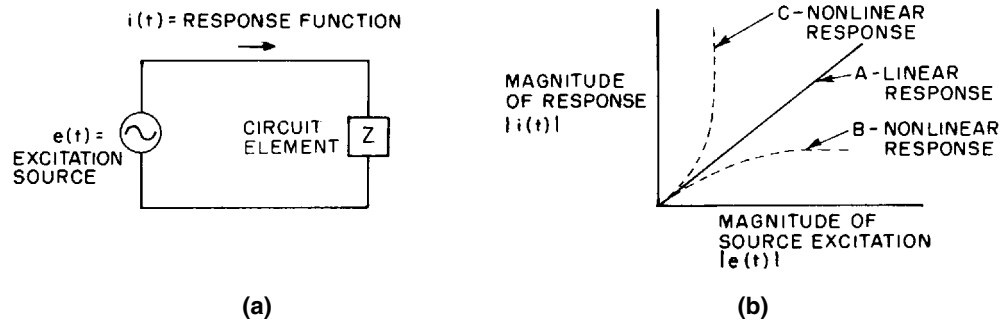


Figure 3-1—Linearity

(current) versus source excitation magnitude (voltage) is a straight line. This is the situation shown for case A in Figure 3-1(b). When linearity exists, the plot applies either to the steady-state value of the excitation and response functions or to the instantaneous value of the functions at a specific time.

When linear dc circuits are involved, the current doubles if the voltage is doubled. The same holds for linear ac circuits if the frequency of the driving voltage is held constant. In a similar manner, it is possible to predict the response of a *constant* impedance circuit (that is, constant R , L , and C elements) to any magnitude of dc source excitation or *fixed frequency* sinusoidal excitation based on the known response at any other level of excitation. For the chosen excitation function of voltage and the chosen response function of current, both dotted curves B and C are examples of the response characteristic of a nonlinear element.

With the circuit element represented by any of the response curves shown in Figure 3-1 (including the linear element depicted by curve A), the circuit will, in general, become nonlinear for a different response function—for example, power. If, for example, the element was a constant resistance (which would have a linear voltage-current relationship), the power dissipated would increase by a factor of 4 if voltage were doubled ($P = I^2 R$).

An important limitation of linearity, therefore, is that it applies only to responses that are linear for the circuit conditions described (that is, a constant impedance circuit will yield a current that is linear with voltage). This restraint must be recognized in addition to the previously mentioned limitations of constant source excitation frequency for ac circuits and constant circuit element impedances for ac or dc circuits. Excitation sources, if not independent, must be linearly dependent. This restraint forces a source to behave just as would a linear response (which, by definition, is also linearly dependent).

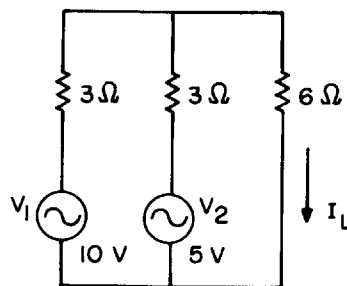
3.2.2 Superposition

This very powerful principle is a direct consequence of linearity and can be stated as follows:

In any linear network containing several dc or fixed frequency ac excitation sources (voltages), the total response (current) can be calculated by algebraically adding all the

individual responses caused by each independent source acting alone, i.e., all other sources inactivated (voltage sources shorted by their internal impedances, current sources opened).

An example that illustrates this principle is shown in Figure 3-2. The equation written is for the sum of the currents from each individual source V_1 and V_2 . Although Figure 3-2 also illustrates a way this principle might actually be used, more often its main application is in support of other calculating methods. The only restraint associated with superposition is that the network should be linear. All limitations associated with linearity apply.



$$\begin{aligned}
 I_L &= I_{V_1} + I_{V_2} \\
 &= \frac{10}{\left(3 + \frac{6 \cdot 3}{6+3}\right)} \left(\frac{6 \cdot 3}{6+3}\right) \cdot \frac{1}{6} + \frac{5}{\left(3 + \frac{6 \cdot 3}{6+3}\right)} \left(\frac{6 \cdot 3}{6+3}\right) \cdot \frac{1}{6} \\
 &= \frac{10}{5} \cdot 2 \cdot \frac{1}{6} + \frac{5}{5} \cdot 2 \cdot \frac{1}{6} \\
 &= \frac{2 \cdot 2}{6} + \frac{2}{6} = \frac{2}{3} + \frac{1}{3} = 1 \text{ A}
 \end{aligned}$$

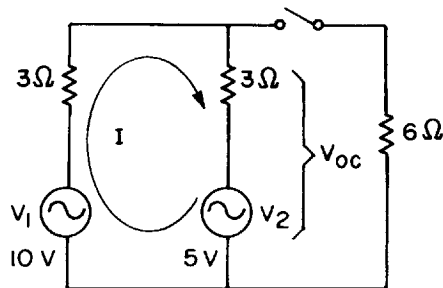
Figure 3-2—Superposition

The nonapplicability of superposition is why all but the very simplest nonlinear circuits are almost impossible to analyze using hand calculations. Although most real circuit elements are nonlinear to some extent, they can often be accurately represented by a linear approximation. Solutions to network problems involving such elements can be readily obtained.

Problems involving complex networks having substantially nonlinear elements can practically be solved only through the use of certain simplification procedures, or through the adjustment of calculated results to correct for nonlinearity. But both of these approaches can potentially lead to significant inaccuracy. Tiresome iterative calculations performed in an instant by the digital computer make more accurate solutions possible when the nonlinear circuit elements can be described mathematically.

3.2.3 Thevenin and Norton equivalent circuits

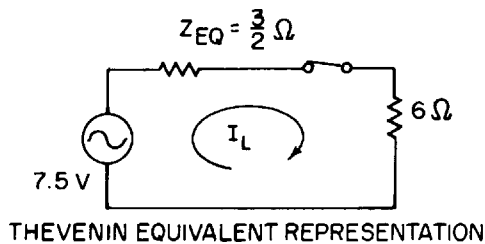
The Thevenin equivalent circuit is a powerful analysis tool based on the fact that any active linear network, however complex, can be represented by a single voltage source, V_{OC} , equal to the open-circuit voltage across any two terminals of interest, in series with the equivalent impedance, Z_{EQ} , of the network viewed from the same two terminals with all sources in the network inactivated (voltage sources shorted by their internal impedances, current sources opened). Validity of this representation requires only that the network be linear. Existence of linearity is a necessary restraint. Application of the Thevenin equivalent circuit can be appreciated by referring to the simple circuit of Figure 3-2 and developing the Thevenin equivalent for the network with the switch in the open position as illustrated in Figure 3-3.



(a)

$$\begin{aligned} V_{OC} &= I \cdot 3 + 5 \\ &= \frac{10-5}{6} \cdot 3 + 5 \\ &= \frac{5}{2} + 5 = \frac{15}{2} \\ &= 7.5 \text{ V} \end{aligned}$$

$$Z_{EQ} = \frac{3 \cdot 3}{3+3} = \frac{9}{6} = \frac{3}{2} \Omega$$



(b)

$$\begin{aligned} I_L &= 7.5 \cdot \frac{1}{6 + \frac{3}{2}} \\ &= \frac{15}{2} \cdot \frac{2}{12+3} \\ &= 15 \cdot \frac{1}{15} = 1 \text{ A} \end{aligned}$$

Figure 3-3—Thevenin equivalent

After connecting the 6Ω load to the Thevenin equivalent network by closing the switch, the solution for I_L is the same as before, 1 A. Use of the simple Thevenin equivalent shown for the entire left side of the network makes it easy to examine circuit response as the load impedance value is varied.

The Thevenin equivalent circuit solution method is equally valid for complex impedance circuits. It is the type of representation shown in Figure 3-3 that is the basis for making per unit short-circuit calculations, although the actual values for the source voltage and branch impedances would be substantially different from those used in this case. (The circuit property of linearity would, incidentally, allow them to be scaled up or down.) The network shown in Figure 3-3(a), with the 6Ω resistance shorted and the other resistances visualized as reactances, might well serve as an oversimplified representation of a power system about to experience a bolted fault with the closing of the switch.

The V_1 branch of the circuit would correspond to the utility supply while the V_2 branch might represent a large motor running unloaded, immediately adjacent to the fault bus, and highly idealized so as to have no rotor flux leakage. For such a model, the 5 V source corresponds to the pre-fault, air-gap voltage behind a stator leakage (subtransient) reactance of 3Ω [B4]. In a more realistic situation where rotor leakage is evident, a model that accurately describes the V_2 branch in detail before and after switch closing is much more difficult to develop, because the air-gap voltage decreases (exponentially) with time and varies (linearly) with the steady-

state rms magnitude of the motor stator current following application of the fault. The problem of accounting for motor internal behavior is avoided altogether by use of a Thevenin equivalent. This permits the V_2 branch to be represented by the apparent motor impedance effective at the time following switch closure. In shunt with the equivalent impedance for the remainder of the network, the Thevenin equivalent impedance, Z_{EQ} , for the motor (at any point in time of interest) is simply connected in series with the pre-fault open-circuit voltage, V_{OC} , to obtain the corresponding current response to switch closing.

The current response obtained in each branch of a network using a Thevenin equivalent circuit solution represents the *change* of current in that branch. The actual current that flows is the vector sum of currents before and after the particular switching event being considered. See Figure 3-4.

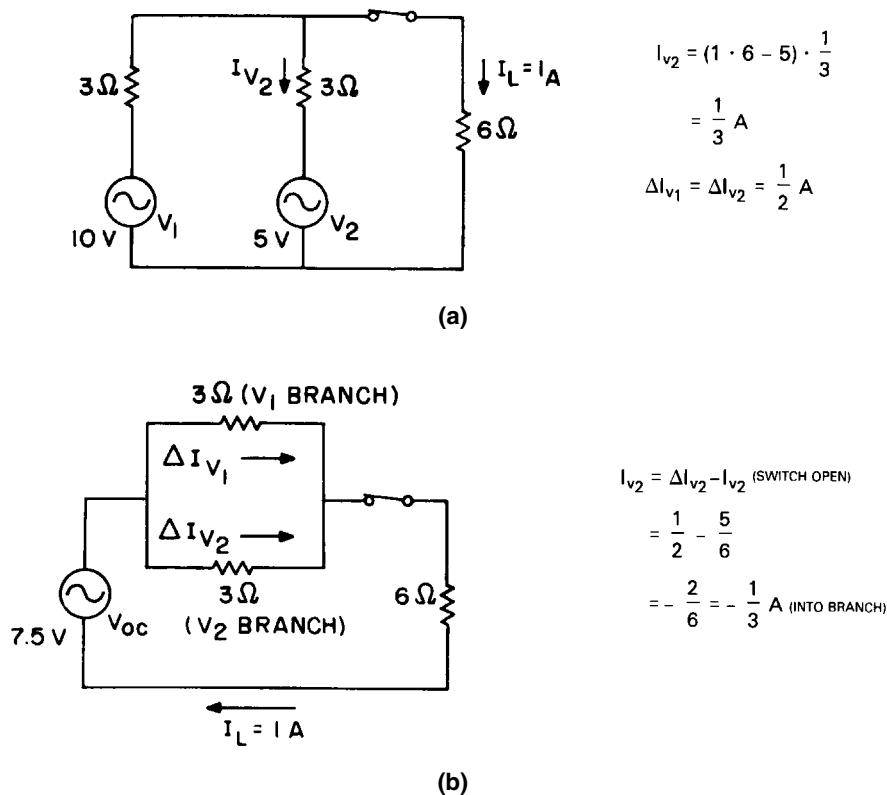


Figure 3-4—Current flow of a Thevenin equivalent representation

In Figure 3-4(a), the current flowing in the V_2 branch circuit is shown to be $1/3$ A. A more detailed representation of the Thevenin equivalent circuit previously examined in Figure 3-3 is shown in Figure 3-4(b). Here, the solution for the same current I_{V2} is determined by subtracting the current flowing in the V_2 branch prior to closing the switch [$5/6$ A from inspection of the circuit in Figure 3-3(a)] from the current $I_{V2} = 1/2$ A, calculated to be flowing in the Thevenin equivalent for this V_2 branch.

In the branch of the circuit defined by the switch itself, the *change of current* due to closing is normally the response of interest. This means the solution to the Thevenin equivalent is sufficient. The resultant current in the other branches, however, cannot be determined by the solution to the Thevenin equivalent network alone.

In the case where the V_2 branch represents a motor switched onto a bolted fault, the motor contribution is the locked-rotor current minus the pre-fault current as illustrated in Figure 3-5 and not just the locked-rotor current as it is so often carelessly described.

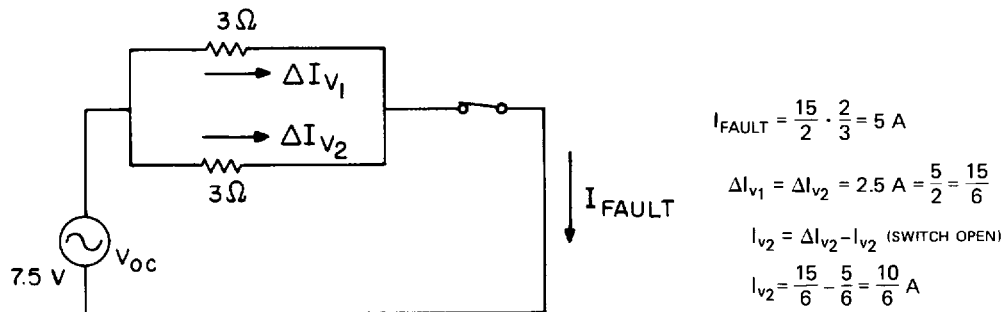


Figure 3-5—Fault flow

As a rule, this effect is never as significant as the example suggests, even when the motor is loaded prior to the fault; the load current is much smaller than the locked-rotor current and almost 90° out of phase with it.

A Norton equivalent circuit, which can be developed directly from the Thevenin equivalent circuit, consists of a current source of magnitude, V_{OC}/Z_{EQ} , to account for the voltage sources in *each* separate branch of the network in *parallel* with the same equivalent impedance for that branch, Z_{EQ} . This representation of circuits can be useful in power system analysis work if some of the sources are true current sources, as is often the case when performing harmonic studies.

3.2.4 Sinusoidal forcing function

It is a most fortunate truth in nature that the excitation sources (driving voltage) for electrical networks, in general, have a sinusoidal character and can be represented by a sine wave plot of the type illustrated in Figure 3-6.

There are two important consequences of this circumstance. First, although the response (current) for a complex R, L, C network represents the solution to at least one second-order differential equation, the result will also be a sinusoid of the same frequency as the excitation and different only in magnitude and phase angle. The relative character of the current with respect to the voltage for simple R, L , and C circuits is also shown in Figure 3-6.

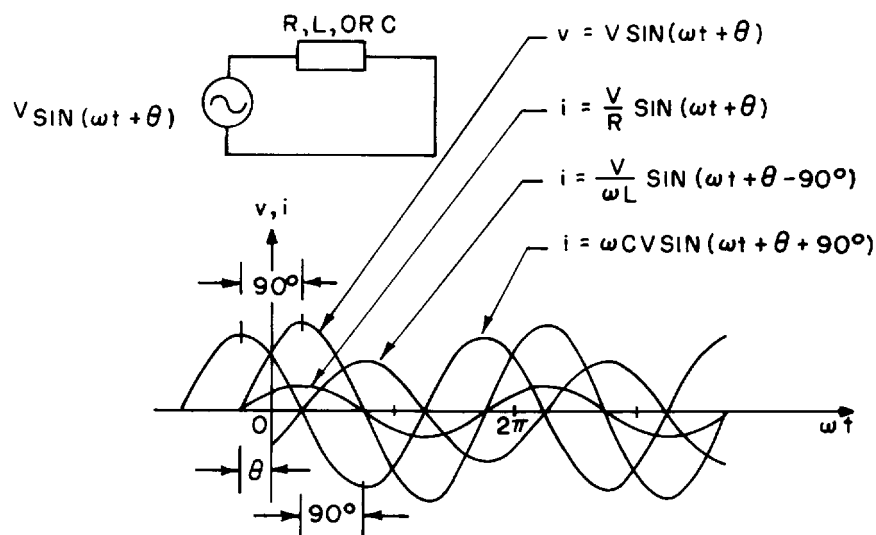


Figure 3-6—Sinusoidal forcing function

The second important concept is that when the sine wavelike shape of current is forced to flow in a general impedance network of R , L , and C elements, the voltage drop across each element will always exhibit a sinusoidal shape of the same frequency as the source. The sinusoidal character of all the circuit responses makes the application of the superposition technique to a network with multiple sources surprisingly manageable. The necessary manipulation of the sinusoidal terms is easily accomplished using the laws of vector algebra, which evolve from the next technique to be reviewed.

The only restraint associated with the use of the sinusoidal forcing function concept is that the circuit must be comprised of linear elements, that is, R , L , and C are constant as current or voltage varies.

3.2.5 Phasor representation

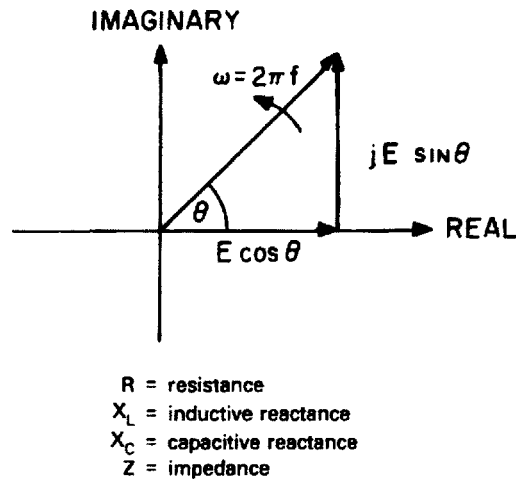
Phasor representation allows any sinusoidal forcing function to be represented as a phasor in a complex coordinate system as shown in Figure 3-7.

As indicated, the expression for the phasor representation of a sinusoid can assume any of the following shorthand forms:

Exponential: $E e^{j\theta}$

Rectangular: $E \cos \theta + jE \sin \theta$

Polar: $E \angle \theta$



$$E e^{j\theta} = E \cos \theta + j E \sin \theta = |E| \angle \theta$$

$$I = \frac{E}{Z} = \frac{|E|}{|Z|} \frac{e^{j\theta}}{e^{j\Phi}} = \frac{|E|}{|Z|} e^{j(\theta - \Phi)}$$

$$Z = R + j \omega L + -j \frac{1}{\omega C} = R + jX$$

$$X = X_L - X_C$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\Phi = \tan^{-1} \frac{X}{R}$$

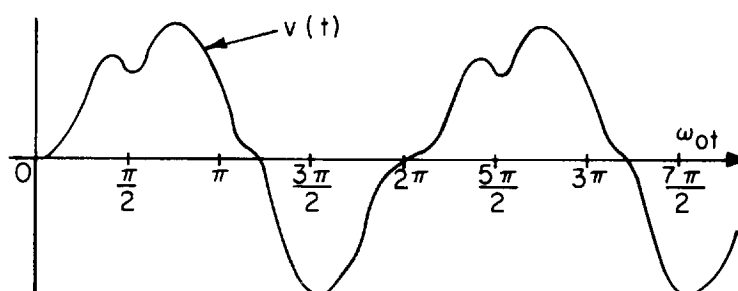
Figure 3-7—Phasor representation

For most calculations, it is more convenient to work in the frequency domain where any angular velocity associated with the phasor is ignored, which is equivalent to assuming the coordinate system rotates at a constant angular velocity of ω . The impedances of the network can likewise be represented as phasors using the vectorial relationships shown. As illustrated, the circuit responses (current) can be obtained through the simple vector algebraic manipulation of the quantities involved. The need for solving complex differential equations to determine the circuit responses is completely eliminated. The restraints that apply are as follows:

- Sources must all be sinusoidal
- Frequency must remain constant
- Circuit R , L , and C elements must remain constant (that is, linearity must exist)

3.2.6 Fourier representation

This powerful tool allows any nonsinusoidal periodic forcing function, of the type plotted in Figure 3-8, to be represented as the sum of a dc component and a series (infinitely long, if necessary) of ac sinusoidal forcing functions. The ac components have frequencies that are integral multiples of the periodic function fundamental frequency. The general mathematical form of the *Fourier series* is also shown in Figure 3-8.



$$V(t) = V_0 + V_1 \cos \omega_0 t + V_2 \cos 2 \omega_0 t + \dots \\ + V_1' \sin \omega_0 t + V_2' \sin 2 \omega_0 t + \dots$$

Figure 3-8—Fourier representation

The importance of the Fourier representation is immediately apparent. The response to the original driving function can be determined by first solving for the response to each Fourier series component forcing function and summing all the individual solutions to find the total superposition. Since each component response solution is readily obtained, the most difficult part of the problem becomes the determination of the component forcing function. The individual harmonic voltages can be obtained, occasionally in combination with numerical integration approximating techniques through several well-established mathematical procedures. Detailed discussion of their use is better reserved for the many excellent texts (Close [B3], Hayt and Kemmerly [B7]) that treat the subject.

There are several abstract mathematical conditions that must be satisfied to use a Fourier representation. The only restraints of practical interest to the power systems analyst are that the original driving function must be periodic (repeating) and the network must remain linear.

3.2.7 Laplace transform

In the solution of circuit transients by classical methods, the models of circuit elements are represented with sets of differential equations. In addition, for a specific problem, a set of initial conditions must be known in order to solve the differential equations for the unknown quantity. An alternative technique for solving a transient problem is by the use of the Laplace transform. The proper use of this technique eliminates the need for the solution of the differential equations and simplifies all mathematical manipulations to elementary algebra.

It is helpful to keep in mind that the concept of mathematical transformations to simplify the solution to a problem is not new. For example, the mathematical operations of multiplication and division are transformed into the simpler operations of addition and subtraction by means of the logarithm transform. Once the addition/subtraction is performed, the solution to the problem is obtained by using the inverse transform, or antilog operation. The transformation is designed to create a new “domain” where the mathematical manipulations are easier to carry out. Once the unknown is found in the new domain, it can be inverse-transformed back to the original domain.

In circuit analysis, the Laplace transform is used to transform the set of differential equations from the time domain (t) to a set of algebraic equations in the new domain called the complex frequency domain or, alternatively, the s -domain. The Laplace transform of a function is given by the expression

$$L\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt \quad (3-1)$$

where the symbol $L\{f(t)\}$ is read as “the Laplace transform of $f(t)$.”

The Laplace transform is also denoted by the notation $F(s)$, that is,

$$F(s) = L\{f(t)\} \quad (3-2)$$

This notation emphasizes that once the above integral has been evaluated, the resulting expression is a function of s . Since the exponent of the e in Equation (3-1) must be dimensionless, s must have the units of reciprocal time, hence the use of the alternate terms “frequency domain” and “ s -domain” to describe the realm of the transformed function.

It can be shown that the “transformation” (or, more briefly, “transform”) described by Equation (3-1) has special mathematical properties. Given an original expression involving both an unknown function (i.e., current, voltage, etc.), and operations on that function (i.e., derivatives, integrals, etc.), the s -domain expression that results when each term is transformed according to Equation (3-1) can be manipulated by ordinary algebraic procedures to yield a solution for the unknown function. The solution for the unknown function in the s -domain can then be transformed back to the time domain to produce the desired result. These mathematical methods can be used to greatly simplify the solution of complex differential equations.

The solution of a system problem involving a linear expression can then be determined in four simple steps, as follows:

- a) Formulate the differential time domain equations for the particular expression, which may contain terms like

$$Ri(t), C \frac{dv}{dt}, \int_0^t i(t) dt, \text{ etc.} \quad (3-3)$$

- b) Find the Laplace transform of the terms in the differential equation, including any initial conditions, according to the definition of the Laplace transform or using Laplace transform tables and equivalent circuit tables such as that shown in Table 3-1 and Table 3-2, respectively.
- c) Solve the transform for the unknown variable. The form of the s function should be manipulated into a form similar to those available in tables of Laplace transform pairs.
- d) From a table, find the inverse Laplace transform of the unknown.

Table 3-1—Laplace transform pairs

$f(t)$ ($t > 0^-$)	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

where

$\delta(t)$ is called the unit impulse function defined as

$$\delta(t) = 0 \text{ for } t \neq 0 \text{ and } \int_{-\infty}^{\infty} \delta(t) dt = 1$$

$u(t)$ is called the unit step function defined as

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

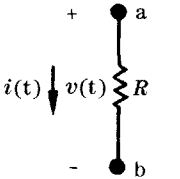
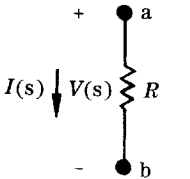
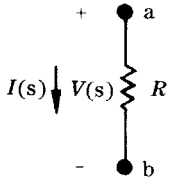
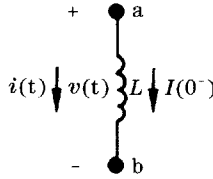
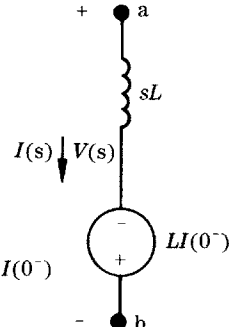
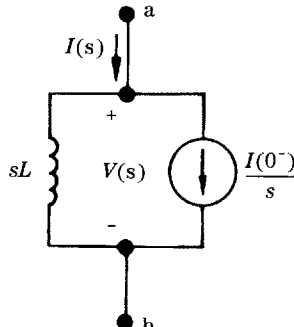
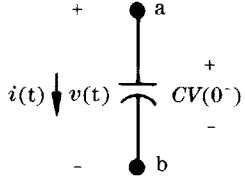
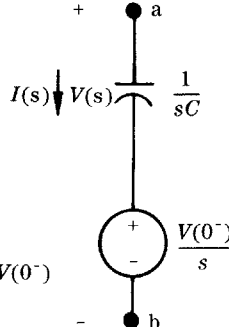
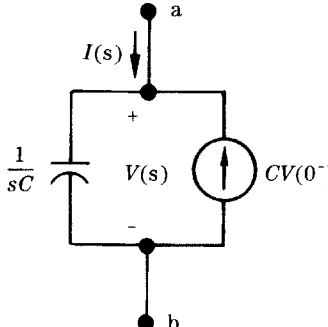
An alternate approach is to interchange steps a) and b) above. In this case, the network is first transformed into the s -domain so that all the advantages of impedance and admittance operations and the use of network solution techniques such as mesh/nodal analysis and Thevenin/Norton equivalent circuits become available.

Applying this transformation to all elements in the system under study, the result will be a network consisting of s -domain equivalent elements. Then, usual circuit analysis will provide the quantities of interest in the s -domain. Finally, from a table of inverse Laplace transforms, the quantity of interest in the time domain can be obtained. In subsequent paragraphs, we will discuss the Laplace transform and its application to the solution of transient problems.

3.2.7.1 Transient analysis by Laplace transforms

The transient analysis of a circuit using Laplace transforms is a straightforward application of elementary algebra. Both the excitation sources and the impedance elements of the system

Table 3-2—*s*-domain equivalent circuits

Time Domain	Frequency Domain	
 $v(t) = Ri(t)$	 $V(s) = RI(s)$	 $I(s) = \frac{V(s)}{R}$
 $v(t) = L \frac{di(t)}{dt}$ $i(t) = \frac{1}{L} \int_{0^-}^t v(x) dx + I(0^-)$	 $V(s) = sLI(s) - LI(0^-)$	 $I(s) = \frac{V(s)}{sL} + \frac{I(0^-)}{s}$
 $i(t) = C \frac{dv(t)}{dt}$ $v(t) = \frac{1}{C} \int_{0^-}^t i(x) dx + V(0^-)$	 $V(s) = \frac{I(s)}{sC} + \frac{V(0^-)}{s}$	 $I(s) = sCV(s) - CV(0^-)$

under study must be replaced with their s -domain equivalent circuits. Table 3-1 contains Laplace transform pairs for commonly used excitation sources. The Laplace transform of the driving voltages and/or currents in the studied circuit must be determined for use in the s -domain equivalent circuit. Using Table 3-2, one of two possible s -domain equivalent circuits must be chosen for each impedance element in the circuit. Once the s -domain network has been developed, the circuit equations can be solved algebraically.

The following examples will illustrate the procedure. They are simple and could very well be solved with less sophistication.

3.2.7.1.1 RL and RC transients

The study of RL transients will begin with a series RL network with a constant (step function) voltage source $V(t)$ applied to the circuit through the closing of a switch. The network is illustrated in Figure 3-9.

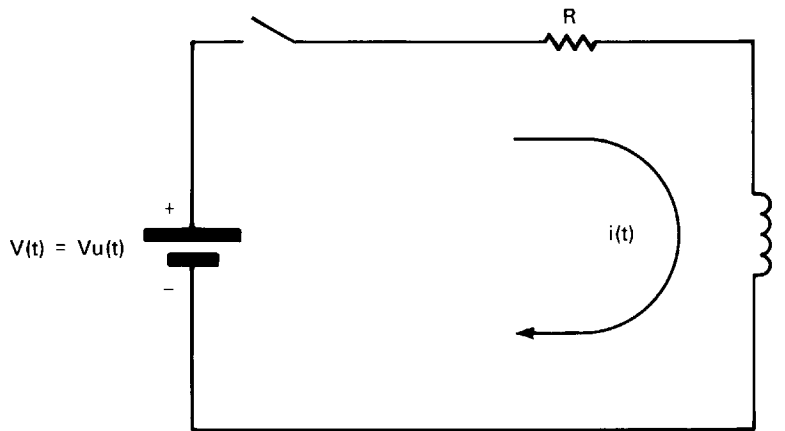


Figure 3-9— RL network

According to Kirchoff's voltage law, at $t = 0^+$ the equation describing the circuit is

$$Vu(t) = Ri(t) + L \frac{di}{dt} \quad (3-4)$$

Converting the terms of the above equation into the s -domain, according to either Table 3-1 or Table 3-2, yields

$$\frac{V}{s} = RI(s) + sLI(s) - LI(0^-) \quad (3-5)$$

Since there could be no current flowing prior to the closing of the switch, the last term on the right can be ignored (initial conditions equal to zero). Solving for the current according to 3.2.7 step c), the result is

$$I(s) = \frac{V}{L} \left[\frac{1}{s \left(s + \frac{R}{L} \right)} \right] = \frac{V}{R} \left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right] \quad (3-6)$$

The transient response is readily obtained from Table 3-1 as

$$i(t) = \frac{V}{R} [1 - e^{-tR/L}] \quad (3-7)$$

The current $i(t)$ is shown graphically in Figure 3-10.

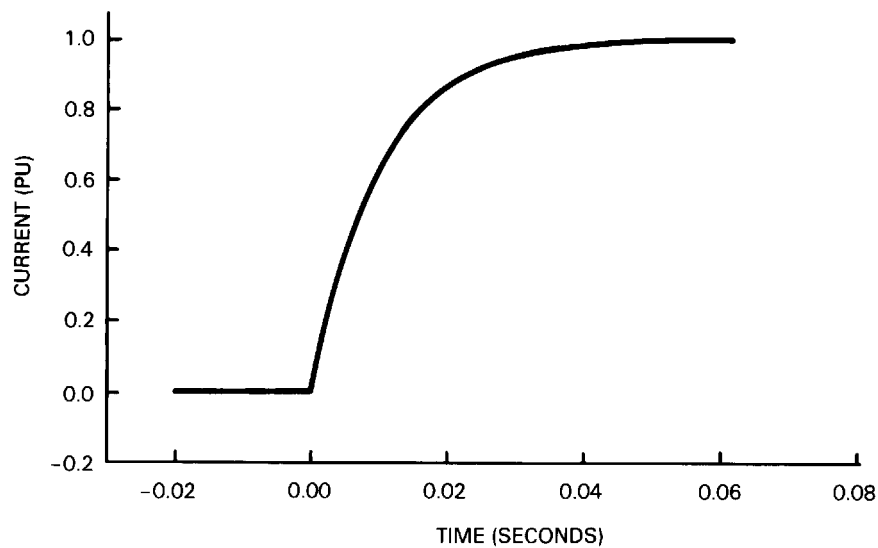


Figure 3-10—Current response (Equation [3-7])

The next circuit to be examined is the RC network depicted in Figure 3-11, already drawn in the symbols of the Laplace transform. The equation describing the circuit is

$$\frac{V}{s} = RI(s) \times \frac{I(s)}{sC} - \frac{V_c(0^-)}{s} \quad (3-8)$$

Assuming there is no initial charge in the capacitor [$V_c(0^-) = 0$] and solving for the current, yields

$$I(s) = \frac{V}{R} \left[\frac{1}{s + 1/RC} \right] \quad (3-9)$$

Again, using the appropriate inverse Laplace transform from Table 3-1, the transient response in the time domain is

$$i(t) = \frac{V}{R} e^{-tR/C} \quad (3-10)$$

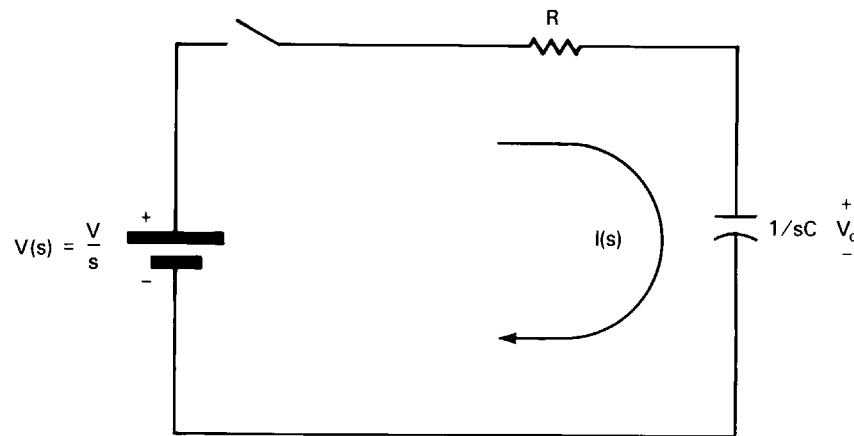


Figure 3-11—RC network

The current response of the RC circuit is shown graphically in Figure 3-12.

If the quantity of interest was the voltage across the capacitor, then from Ohm's law we would have

$$V_{c(s)} = I(s)Z(s) \quad (3-11)$$

Where $I(s)$ is defined by Equation (3-9) and $Z(s)$ is the capacitor impedance, namely

$$Z(s) = \frac{-1}{sC} \quad (3-12)$$

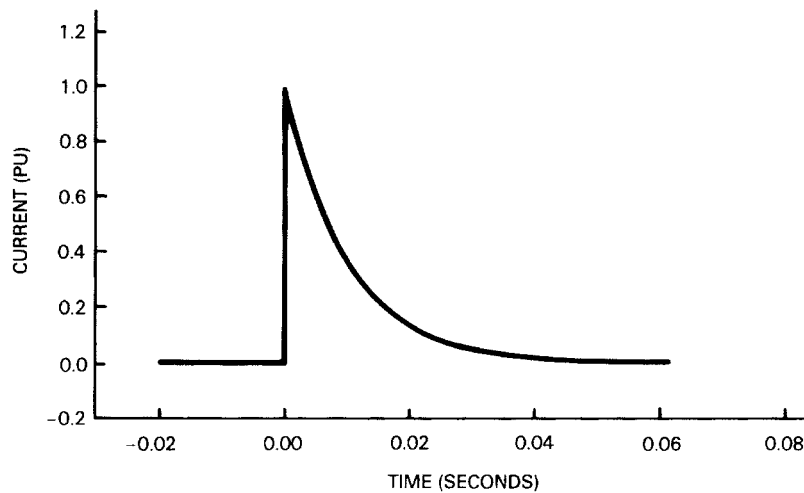


Figure 3-12—Current response (Equation [3-10])

Since, in a series circuit, the current is common to all elements, we can substitute Equation (3-12) into Equation (3-11). After solving for $I(s)$, we substitute the result into Equation (3-9). Rearranging the terms, we obtain

$$V_c(s) = \frac{V}{RC} \left[\frac{1}{s(s + 1/RC)} \right] = V \left[\frac{1}{s} - \frac{1}{s + 1/RC} \right] \quad (3-13)$$

From Table 3-1, the voltage across the capacitor in the time domain is

$$V_c(t) = V(1 - e^{-t/RC}) \quad (3-14)$$

The capacitor voltage described by Equation (3-14) is shown graphically in Figure 3-13.

3.2.7.1.2 Circuits involving all three circuit elements

In cases involving more advanced circuits, the next step is the combination of all three components, R , L , and C , in a common network. This approach is considered in Chapter 11 of this book, where additional circuits with higher degrees of difficulty are investigated using the same basics presented here.

3.2.8 Single-phase equivalent circuit

The single-phase equivalent circuit is a powerful tool for simplifying the analysis of balanced three-phase circuits, yet its restraints are probably most often disregarded. Its application is best understood by examining a three-phase diagram of a simple system and its single-phase equivalent, as shown in Figure 3-14. Also illustrated is the popular single-line diagram representation commonly used to describe the same three-phase system on engineering drawings.

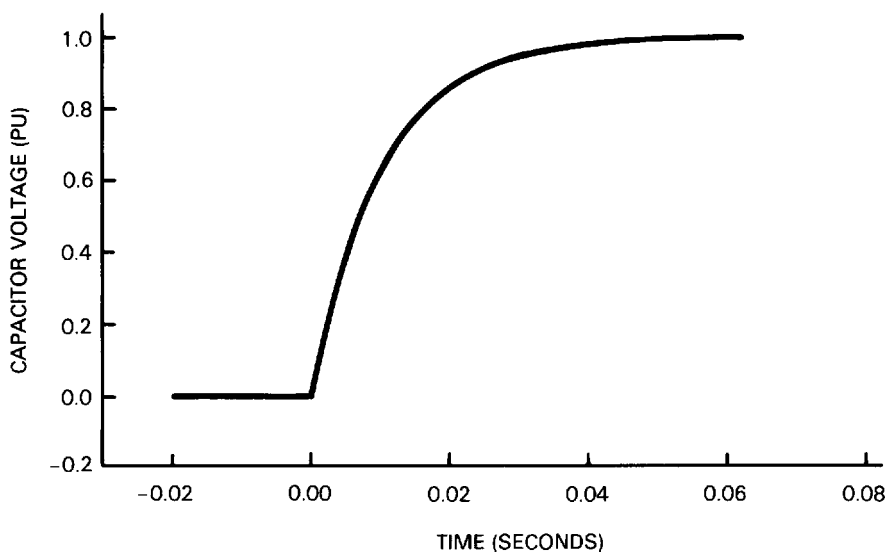


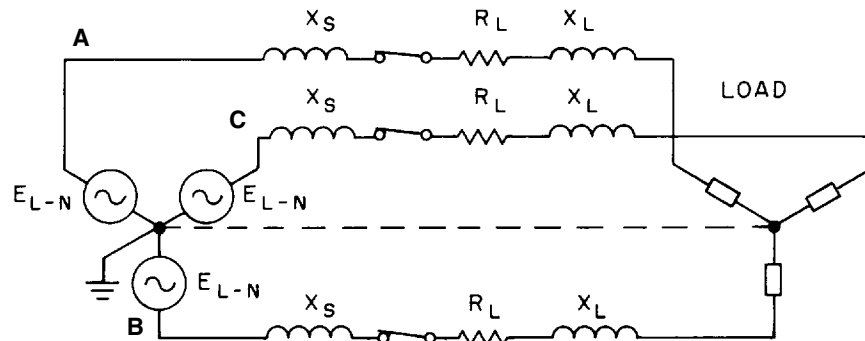
Figure 3-13—Voltage response

If a three-phase system has a perfectly balanced symmetrical source excitation (voltage) and load, as well as equal series and shunt system and line impedances connected to all three phases [see Figure 3-14(a)], imagine a conductor (shown as a dotted line) carrying no current connected between the effective neutrals of the load and the source. Under these conditions, the system can be accurately described by either Figure 3-14(b) or Figure 3-14(c).

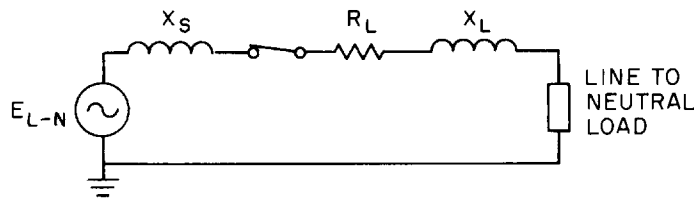
The single-phase equivalent circuit is particularly useful since the solution to the classical loop equations is much easier to obtain than for the more complicated three-phase network. To determine the complete solution, it is only necessary to realize that the other two phases will have responses that are shifted by 120° and 240° but are otherwise identical to the reference phase.

Anything that upsets the balance of the network renders the model invalid. A subtle way this might occur is illustrated in Figure 3-15.

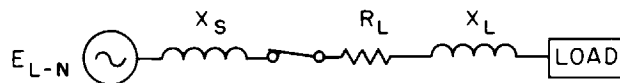
If the switching devices operate independently in each of the three poles, and for some reason the device in phase A becomes opened, the balance or symmetry of the circuit is destroyed. Neither the single-phase equivalent nor the single-line diagram representation is valid. Even though the single-phase and the single-line diagram representations would imply that the load has been disconnected, it continues to be energized by single-phase power. This can cause serious damage to motors and result in unacceptable operation of certain load apparatus.



(a) Three-phase diagram



(b) Single-phase equivalent



(c) Single-line impedance diagram

Figure 3-14—A balanced three-phase system with load connected prior to fault

More importantly, if only one switching device operates in response to a fault condition in the same phase, as depicted at location X, the system sources would continue to supply fault current from the other unopened phases through the impedance of the load. The throttling effect of the normally substantial load impedance, possibly in combination with additional arc impedance, can reduce the level of the current to a point where detection may not occur in phases B and C. Needless to say, substantial damage can result before the fault finally burns enough to involve the other phases directly and accomplish complete interruption. Meanwhile, both of the single-line representations fail to recognize the problem, and in fact, suggest that the condition has been safely disconnected. Therefore, the restraints of this calculating aid are as follows:

- Symmetry of the electrical system, including all switching devices and applied load.
- Any of the other previously described restraints that apply to the analytical technique being used in combination with the single-phase equivalent.

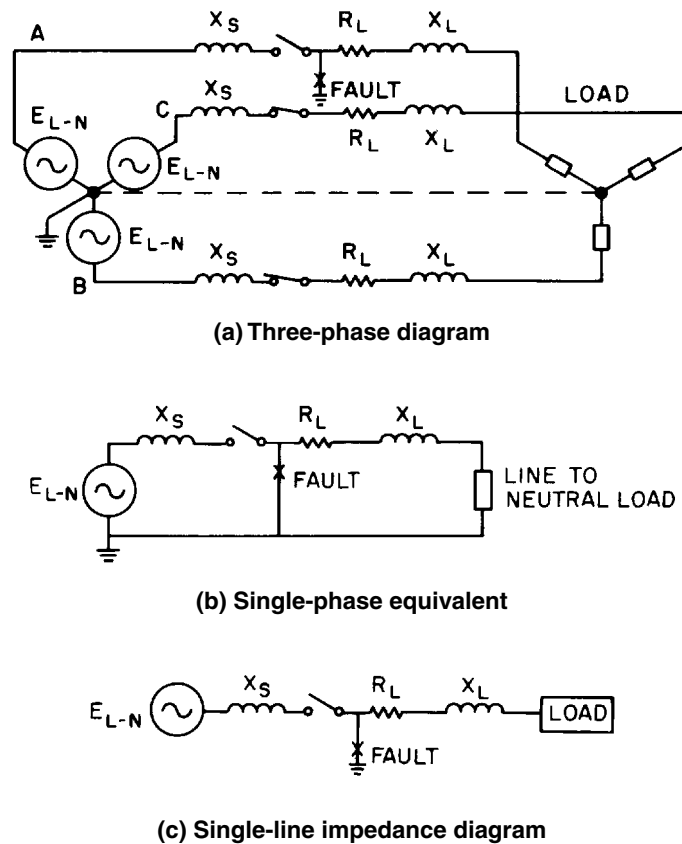


Figure 3-15—A balanced three-phase system with load connected after application of phase A line-ground fault

3.2.9 Symmetrical component analysis

This approach comes to the analyst's rescue when he or she is confronted with an unbalance, the most common circuit condition that invalidates the single-phase equivalent circuit solution method. The symmetrical component analysis allows the response to any unbalanced condition in a three-phase power system to be investigated and correctly synthesized by the sum of the responses to as many as three separate balanced system conditions.

The application of an unbalanced set of voltage phasors, such as those displayed in Figure 3-16, to a balanced downstream load is the sum of the responses of the balanced components that vectorially add to form the original unbalanced set.

Similar conclusions apply when the voltages are balanced, but the connected phase impedances and/or loads and the line currents are unbalanced. Here, the unbalanced current

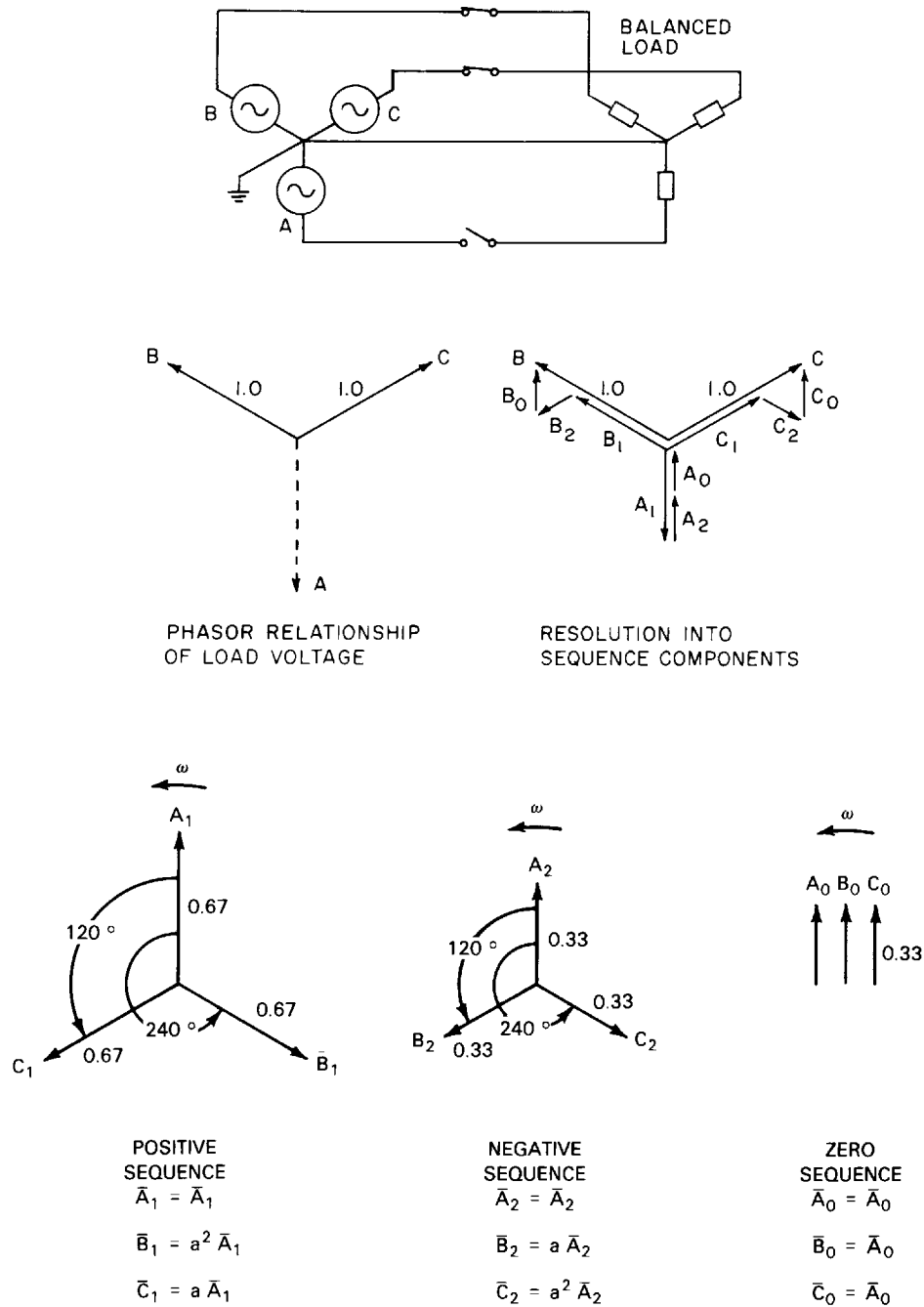


Figure 3-16—Symmetrical component analysis

phasors are the sum of up to three balanced sets that flow through the balanced system impedances on either or both sides of the unbalance, producing voltage drops that satisfy the needs of the applied voltages and the boundary conditions at the point of unbalance.

The mathematical expression for three unbalanced phasors as a function of the balanced phasor components is as follows:

$$\bar{A} = \bar{A}_0 + \bar{A}_1 + \bar{A}_2$$

$$\bar{B} = \bar{B}_0 + \bar{B}_1 + \bar{B}_2$$

$$\bar{C} = \bar{C}_0 + \bar{C}_1 + \bar{C}_2$$

The positive (1), negative (2), and zero (0) sequence vector components of any phase always have the angular relationship with respect to one another as described by the vector diagram and defined by the identities of Figure 3-16.

The operator a causes a counterclockwise rotation through an angle of 120° and is defined as

$$a = 1 \angle 120^\circ = -0.5 + j0.866$$

These are assigned a counterclockwise direction of rotation in the *time domain* as illustrated. In the *space domain*, the negative sequence phasors will produce exactly the same results as a set of equal magnitude phasors that are displaced from one another by 120° and that rotate clockwise with time.

The proof that a set of N unbalanced vectors can be completely represented by N sets of balanced vectors is seldom presented in texts dealing with the subject of symmetrical components. The derivation of symmetrical component analysis is most meaningful when applied to three-phase systems, that is, systems where $N = 3$. First, it is postulated that it might be possible to describe three arbitrary but defined vectors, A , B , and C , by summing their respective symmetrical components (as previously shown). Using the vector identities of Figure 3-16, the set of equations for the unbalanced vectors, A , B , and C , can be simplified into the linear set of three equations involving three unknowns as follows:

$$\bar{A} = \bar{A}_0 + \bar{A}_1 + \bar{A}_2$$

$$\bar{B} = \bar{A}_0 + a^2 \bar{A}_1 + a \bar{A}_2$$

$$\bar{C} = \bar{A}_0 + a \bar{A}_1 + a^2 \bar{A}_2$$

Rearranging, as shown in the following three equations, produces three independent equations for three unknowns (A_0 , A_1 , and A_2), which is everything required to uniquely and com-

pletely describe A_0 , A_1 , and A_2 , and therefore substantiate their existence. The corresponding B and C phase components are defined by the relationships shown in Figure 3-16.

$$\bar{A}_0 = \frac{1}{3}(\bar{A} + \bar{B} + \bar{C})$$

$$\bar{A}_1 = \frac{1}{3}(\bar{A} + a\bar{B} + a^2\bar{C})$$

$$\bar{A}_2 = \frac{1}{3}(\bar{A} + a^2\bar{B} + a\bar{C})$$

The merit of the symmetrical component analysis is that a relatively complicated and often unwieldy problem can be solved by simply vectorially summing the solution to no more than three balanced, independent networks. The three independent networks are referred to as the positive, negative, and zero sequence networks and are illustrated in Figure 3-17 for the system shown in Figure 3-15 prior to application of the line to ground fault. It should be noted in these diagrams that the system voltage source (E_{L-N}) and the load pre-fault open-circuit voltage (V_D) are both balanced, positive sequence voltages, and they appear only in the positive sequence network. Also, the impedance elements the load (R_D , X_D) and the remainder of the system in each network are, as discussed in 3.2.3, intended to represent the Thevenin equivalent impedances at the point in time of interest after the (unbalanced) event.

Typically, the three impedance networks are symbolically represented in shorthand fashion by an empty *block diagram* for the phase most definitive of the condition being studied up to the unbalance or other point of interest, as shown in Figure 3-18. Here, the final interconnections of the networks are shown that satisfy the necessary boundary conditions describing the system at the point of concern in this case, the line-to-ground fault on phase A. In all cases, the analyst can fill the *block diagrams* with the proper sources and impedances, including loads, in each sequence network, properly connect any fault or other impedance involved with the imbalance, and solve the single-phase loop equations. The three current or voltage sequence responses are produced in each of the respective networks, which then add vectorially to produce the resultant responses in the phase represented. The other phase responses then can be obtained by adding vectorially the individual sequence solutions shifted by the appropriate multiple of 120° .

One curious and often confounding feature of this solution procedure is that the phase in the system that usually provides the best, and sometimes the only, approach to the solution for an unbalance is the one least actively involved in the event. The unbalance illustrated in Figure 3-16 is one such example where the solution is obtained through an analysis of the nonconducting phase A. A double line-to-ground fault is another example where examination of the unfaulted phase gives the most direct access to the network solution. With practice evaluating the more commonly encountered unbalances, the analyst can quickly discern the appropriate network interconnection by inspection of the boundary conditions for the event. Several reference texts (Beeman [B1], *Electrical Transmission and Distribution Reference Book* [B4], and Wagner and Evans [B15]) provide convenient diagrams, tables, and other information showing the network interconnections that must be used to solve for the responses to many system unbalances, as well as certain balanced conditions.

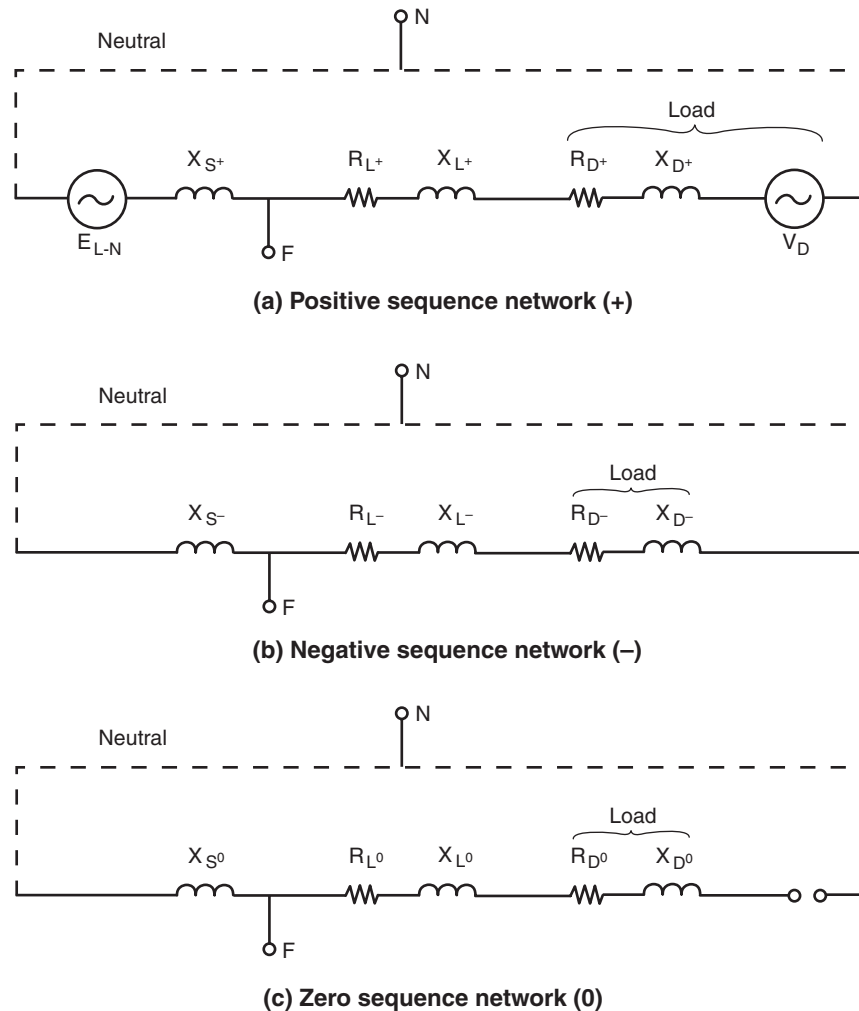


Figure 3-17—Symmetrical component sequence networks for system in Figure 3-15(a) prior to line-ground fault (F) connection

The symmetrical component analysis always involves the use of superposition as well as most of the other procedures previously discussed. The restraints that apply to these other procedures, therefore, must also govern the use of the symmetrical component analysis. In addition, due to mutual phase winding coupling and other effects, the impedance displayed by electrical machines will be different when excited by the different sequence sources. Hence, the *per phase* impedance of circuit elements within the positive, negative, and zero sequence networks will, in general, be different from each other. Currents flowing in the zero sequence network are in phase and do not sum to zero as do the positive and negative sequence currents. Zero sequence currents must therefore flow through the ground circuit and are influenced by any impedance in this circuit path. When harmonic excitation sources are present (requiring the use of the Fourier representation), special care must be exercised in