



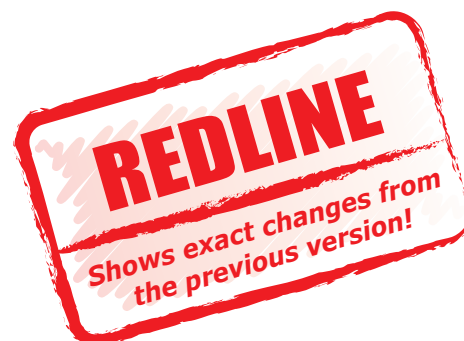
IEEE Recommended Practice for Precision Centrifuge Testing of Linear Accelerometers

IEEE Aerospace and Electronic Systems Society

Sponsored by the
Gyro and Accelerometer Panel

836TM

IEEE
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17 September 2009



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IEEE Recommended Practice for Precision Centrifuge Testing of Linear Accelerometers

Sponsor

Gyro and Accelerometer Panel
of the
IEEE Aerospace and Electronic Systems Society

Approved 17 June 2009

IEEE-SA Standards Board

Abstract: This recommended practice provides a guide to the conduct and analysis of precision centrifuge tests of linear accelerometers and covers each phase of the tests, beginning with the planning. Possible error sources and typical methods of data analysis are addressed. The intent is to provide users involved in centrifuge testing with a detailed understanding of the various factors affecting accuracy of measurement, both factors associated with the centrifuge and factors in the data collection process. Model equations are discussed, both for the centrifuge and for a typical linear accelerometer, each with the complexity needed to accommodate the various identified characteristics and error sources in each. An iterative matrix equation solution is presented for deriving the various model equation coefficients for the accelerometer under test from the centrifuge test data.

Keywords: accelerometer, accelerometer test, centrifuge, linear accelerometer

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Introduction

This introduction is not part of IEEE Std 836-2009, IEEE Recommended Practice for Precision Centrifuge Testing of Linear Accelerometers.

This recommended practice was prepared by the Gyro and Accelerometer Panel of the Aerospace and Electronic Systems Society of the Institute of Electrical and Electronics Engineers (IEEE). It is a guide to the conduct and analysis of precision centrifuge tests of linear accelerometers and includes discussions of possible error sources in those tests. It provides guidance for each phase of the tests, beginning with planning and ending with a discussion of typical methods of data analysis.

A discussion of model equations is provided, both for the centrifuge and for a typical linear accelerometer, each with the complexity needed to accommodate the various identified characteristics and error sources in each. This recommended practice also presents an iterative matrix equation solution for derivation of the various model equation coefficients for the accelerometer under test from the centrifuge test data. The matrix solution provides a two-quadrant best fit of the centrifuge data to the chosen model equation.

This recommended practice is intended to provide users involved in centrifuge testing with a detailed understanding of the various factors affecting accuracy of measurement, both factors associated with the centrifuge and factors in the data collection process. Numerical examples are provided so that each factor can be quantitatively evaluated and its contribution to system error compared with a user's required accuracy. This information allows the user to identify and evaluate the factors that are critical to a specific planned test. The matrix equation method of data reduction also provides a means of determining data validity as well as model equation adequacy and yields the confidence intervals for each of the model equation coefficients.

This recommended practice consists of a main text and ten annexes. The main text provides the recommended practice for precision centrifuge testing following a tutorial clause that is intended to provide users with the understanding of critical factors in centrifuge testing as well as the best methods for taking and reducing data.

Annex A is a compilation of the characteristics of known precision centrifuges from data provided from both users and manufacturers. The listing is intended as a guide to available facilities at the time of publication of this recommended practice. Data provided should be considered as merely informative and not as specifications for the listed machines.

Annex B provides a discussion of evaluation of centrifuges with references provided to clauses of this recommended practice that pertain to the attribute discussed.

Annex C provides a more general discussion of fitting theory with particular regard to the often unique character of centrifuge data and corresponding accelerometer performance requirements. It discusses the application of the method of least squares by which matrix solutions are generated. It also describes the ease of understanding and use. These approximate methods generally address some measure of composite error, rather than directly evaluating model equation coefficients. For example, a common specification limits allowable deviations from a straight line determined by two specified points, such as zero-acceleration and some specific value of input acceleration.

Annex D discusses the limitations of and alternatives to centrifuge testing. Problems produced by the acceleration-gradient, the spin rates required, and the difficulty of producing short high-acceleration input pulses are among the problems discussed. Alternatives such as use of dividing heads and the earth's gravity are discussed, along with sled testing and vibration testing.

Annex E discusses the use of the double centrifuge and rate table with tilted axis of rotation for low-frequency transfer function determination. Annex F presents a refined model of the double centrifuge and rate table with tilted axis of rotation.

Annex G describes the processing of dual-proof-mass vibrating beam accelerometer (VBA) centrifuge data. Annex H discusses scale factor calibration for high-acceleration applications on an ultra-centrifuge. Annex I discusses thermal control and data acquisition on the centrifuge arm. Annex J is a bibliography.

The terminology used in this recommended practice conforms to *The Authoritative Dictionary of IEEE Standards Terms* [B6]^a and IEEE Std 528TM-2001.^b The units used in this recommended practice conform to IEEE Std 268TM-1992 [B9]. In this recommended practice, the symbol *g* is used to denote a unit of acceleration equal in magnitude to the local value of gravity at the test site. This symbol is distinguished from *g*, which is the standard symbol for gram.

The following standards were used in the development of this recommended practice:

- IEEE/ASTM SI 10-1997 [B7]
- IEEE Std 260.1TM-1993 [B8]
- IEEE Std 315TM-1975 [B10]

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^a The numbers in brackets correspond to the numbers of the bibliography in Annex J.

^b Information on references can be found in Clause 2.

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1. Overview

1.1 Scope

This recommended practice describes the conduct and analysis of precision tests that are to be performed on linear accelerometers using centrifuge techniques. The term "precision," in this context, refers to tests that are conducted to evaluate accelerometer parameters, as opposed to tests conducted to establish environmental survivability only. Evaluation may take the form of determining the coefficients of the accelerometer's model equation, except for bias and scale factor, which are most accurately determined by static multiposition tests. Alternatively, evaluation may establish only that the accelerometer output complies with specific error limit criteria.

1.2 Purpose

The principal error sources encountered during precision centrifuge testing of linear accelerometers are described, along with the test practices and data reduction techniques that have been found most efficient in minimizing or compensating for them.

2. Normative references

This recommended practice shall be used in conjunction with the following ~~publications~~ references. In the event of any conflict, the recommended practices of this document shall have precedence over the practices in the listed documents.

The following referenced documents are indispensable for application of this recommended practice (i.e., they must be understood and used; therefore, each referenced document is cited in text and its relationship to this document is explained). For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

IEEE Std 528TM-2001, IEEE Standard for Inertial Sensor Terminology.^{3,4}

~~When the standards superseded by an approved revision, the revision shall apply. IEEE/ASTM SI 10-1997, Standard Use of the International System of Units (SI): The Modern Metric System.¹ IEEE~~

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IEEE Std 260.1-1993, American National Standard Letter Symbols for Units of Measurement (SI Units, Customary Inch-Pound Units, Certain Other Units)

IEEE Std 315-1975, IEEE Graphic Symbols for Electrical and Electronics Diagrams.

IEEE Std 528-2001, IEEE Standard for Inertial Sensor Terminology.

IEEE Std 1293TM-1998, IEEE Standard Specification Format Guide and Test Procedure for Linear, Single-Axis, Non gyroscopic Accelerometers.

3. Definitions, symbols, and abbreviations

For the purposes of this recommended practice, the following terms and definitions apply. IEEE [Std 528-2001](#)⁵ 100, ~~The Authoritative Dictionary of IEEE Standards Terms, Seventh Edition [B6]~~² should be referenced for terms not defined in this clause.

3.1 Definitions relevant to centrifuge testing

arm stretch (R_{AS}): Elongation, due to centrifugal loading, of the mechanical structure connecting an accelerometer to the centrifuge spin axis beyond the static value (R_0).

bucking: An unusually strong interaction between certain terms in a nonorthogonal series due to a similarity in the functions that they represent.

Central Limit Theorem: A theorem stating that under general conditions, the probability distribution of the sum of n statistically independent random variables (or their average) approaches the Gaussian (or normal) distribution as n approaches infinity no matter what the individual distribution functions might be. In so far as the total error in a measurement or in the average of a number of measurements is the sum of a number of small independent errors, total measurement error tends to be Gaussian unless dominated by a single source of error such as quantization noise. *Syn:* **Normal Convergence Theorem.**

centrifuge: A device consisting of a motor or engine-driven arm structure of sufficient dimensions and strength to allow the rigid mounting and subjecting of one or more inertial grade accelerometers to centripetal acceleration levels ranging from ~~fractions of 1 g~~ [a few m/s²](#) to ~~over 200 g~~ [10 000 m/s²](#).

centrifuge arm axis: A line perpendicular to the axis of rotation of a centrifuge through the effective center of mass ([ECM](#)) of the accelerometer under test.

coning: Precession of a centrifuge due to imbalance.

dynamic error: An error that is present when a centrifuge is rotating.

dynamic leveling error: Any angular deflection of the accelerometer mounting fixture from a horizontal line perpendicular to the centrifuge centripetal acceleration vector.

3.1.9 gravity (g_L): See: plumb-bob gravity.

3.1.10 Normal Convergence Theorem: See: Central Limit Theorem.

3.1.11 plumb-bob gravity: The force per unit mass acting on a mass at rest at a point on the earth, not including any reaction force from the suspension. The plumb-bob gravity includes the gravitational attraction of the Earth, the effect of the centripetal acceleration due to the earth rotation, and tidal effects. The direction of the plumb-bob gravity acceleration defines the local vertical down direction, and its magnitude defines the unit of acceleration g used in accelerometer scale factor calibration at a test site (with compensation for tidal effects if accuracy warrants).

²The numbers in brackets correspond to those of the bibliography Annex H

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Normal Convergence Theorem: See: **Central Limit Theorem.**

radial stiffness: A direct measure of the resistance of the centrifuge axis to deflection due to a horizontal force at the center of rotation; it must take into account the effects of all compliances of the structure, including the bearings, connection to the floor, and the foundation upon which the machine is mounted.

static error: An error that is present when a centrifuge is not rotating; it may change with arm position.

static leveling error: An error in the acceleration seen by a test item due to misalignments of the accelerometer input axis ([IA](#)) to the centrifuge and gravity vector.

3.2 Symbols and abbreviations

<i>Symbol</i>	<i>Name</i>	<i>Reference clause</i>
[A]	acceleration matrix	9.3.3, 9.3.4
AFC	amplitude versus frequency characteristic	F.2.2, F.5.3.1, F.5.3.2
$\vec{a}$	general acceleration vector	6.3.3
a_b	centripetal acceleration due to the earth's rotation	5.3
a_c	true average centripetal acceleration of the centrifuge	5.2, 6.1.4, 6.2.2 6.2.3, 6.3.1, 6.3.2 6.3.3, 7.2.6, 9.3.3
$\vec{a}_c$	vector quantity of a_c	5.2
$\hat{a}_c$	measured average acceleration	6.3.1, 6.3.2
a_{c+}	centripetal acceleration of the centrifuge along the +IA	6.1.7
a_{c-}	centripetal acceleration of the centrifuge along the -IA	6.1.7
a_g	component of gravitational acceleration along centrifuge a_c	7.2.4
a_h	horizontal component of Coriolis acceleration	6.3.3, 7.2.2, 9.3.2, 9.3.3
a_i	acceleration along sensor IA	5.1, 6.2.2, 6.2.3, 6.4.1, 7.1
a_{i+}	positive input acceleration	9.3.3
a_{i-}	negative input acceleration	9.3.3
a_{i+}^*	iterated estimate of a_{i+}	9.3.3
a_{i-}^*	iterated estimate of a_{i-}	9.3.3
a_{ik}	kth input acceleration	9.3.3
a_o	acceleration along sensor OA	6.1.2, 6.1.5, 7.1, 7.2.1 7.2.2, 7.2.6
$\vec{a}_T$	vector quantity of total acceleration at ECM	5.2
a_p	acceleration along sensor PA	6.2.3, 7.1
a_r	average total radial acceleration	7.2.6
a_{ra}	radial acceleration in the sensor frame	7.2.6
a_s	indicated sensor output (m/s ²)	5.1, 6.2.3, 6.4.1, 7.1, 9.3.3
a_{Ta}	tangential acceleration in the sensor frame	7.2.6
a_v	average total vertical acceleration	7.2.6
a_{va}	vertical acceleration in the sensor frame	7.2.6
a_0	centripetal acceleration due to the static value of centrifuge radius (m/s ²)	6.1.2, 6.1.5 7.2.1, 7.2.5
[B]	input matrix	9.3.3
BFSL	best-fit straight line	C.5
C	rate of drift in angular velocity	6.3.2, 7.2.2
c_+	positive scale factor error	9.3.3
c_-	negative scale factor error	9.3.3
C_2	fitted quadratic coefficient	9.3.5

C_3	fitted cubic coefficient	9.3.5
C_{AS}	coefficient of arm stretch ($\Delta R/R/(m/s^2)$)	6.1.2, 6.6.3
C_d	deflection coefficient (angle/ a_i)	6.2.2, 6.2.3
C_{dcr}	coefficient of dynamic center of rotation	6.1.5
[D]	data matrix	9.3.3
d_{ii}	diagonal element of [A] ⁻¹	9.3.4
d_k	kth data point	9.3.3
DVM	digital voltmeter	6.6.1, 8.3.2
E	sensor output (output units)	5.1, 7.1
ECM	effective center of mass	6.1.1, 6.1.7, 6.1.8 7.2.1, 8.2.2, 9.3.3
EMF	electromotive force	8.2.2
E_r	average output when centrifuge is rotating	9.1.1
E_s	average output when centrifuge is static	9.1.1
[F]	function matrix	9.3.3
f	number of degrees of freedom	9.3.4
f_w	frequency of periodic variation (wow or flutter)	6.3.1, 7.2.2, 7.2.6
G	gravitational constant ($6.67259 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$)	5.3
g	local value of gravity acceleration	5.3, 6.2.2, 7.2.4, 7.2.6
$\vec{g}$	vector quantity of g	5.2, 5.3
g_N	Newtonian gravitational force per unit mass	5.3
[H]	coefficient matrix	9.3.3
h_j	constant coefficient	9.3.3
IA	input axis	5.1, 6.2.1, 6.2.2, 8.2.2, 8.2.3, 9.1.1, 9.1.4, 9.2.2
k	stiffness	6.1.5
[K]	coefficient matrix	9.3.3
K_i	i th polynomial coefficient	9.3.4
K_{io}	IA–OA cross-coupling coefficient	6.2.3, 7.1, 8.3.1, 9.3.3, 9.3.5
K_{ip}	IA–PA cross-coupling coefficient	6.2.3, 7.1, 8.3.1 9.3.3, 9.3.5
K_{oo}	second-order OA coefficient	7.1, 8.3.1
K_{op}	OA–PA cross-coupling coefficient	7.1, 8.3.1
K_{oq}	odd-quadratic coefficient	6.1.5, 7.1, 9.3.3, 9.3.4
K_{pp}	second-order PA coefficient	6.2.3, 7.1, 8.3.1, 9.3.5
K_{ooo}	third-order OA coefficient	7.1, 8.3.1, 9.3.5
K_{ppp}	third-order PA coefficient	7.1, 8.3.1, 9.3.5
K_s	specific stiffness of centrifuge arm	6.1.2
K_0	bias (m/s^2)	5.1, 6.1.4, 9.5.3
K_{0+}	apparent bias for $a_i > 0$	7.1, 9.3.3
K_{0-}	apparent bias for $a_i < 0$	7.1, 9.3.3
K_1	sensor scale factor (output units/(m/s^2))	5.1, 6.4.1, 7.1, 9.1.1, 9.3.3
K_{1+}	apparent scale factor for $a_i > 0$	7.1
K_{1-}	apparent scale factor for $a_i < 0$	7.1
K_{10}	scale factor at $t = 0$	6.4.1
K_2	second-order coefficient	6.1.5, 6.1.7, 6.2.1 6.2.3, 6.4.3, 6.5.2 7.1, 8.2.2, 8.3.1, 9.3.3 9.3.4, 9.3.5
K_3	third-order coefficient	6.2.1, 6.4.3, 7.1, 8.3.1, 9.3.3 9.3.5
k	stiffness	6.1.5
k_T	scale factor temperature coefficient	6.4.1
m_a	arm mass	6.1.5

M	test object mass	6.1.2
$ M_i $	i^{th} minor of $ A $	9.3.4
N	number of test inputs	9.3.4
$[N]$	noise matrix	9.3.3
N_+	number of positive inputs	9.3.3, 9.3.4
N_-	number of negative inputs	9.3.3, 9.3.4
n_k	noise component	9.3.3
OA	output axis	7.1, 8.2.3, 8.3.1
PA	pendulous axis	7.1, 8.2.3, 8.3.1
PFC	phase versus frequency characteristic	F.2.2, F.5.3.1, F.5.3.2
r	residual	9.3.4
R	effective radius, distance from centrifuge rotation axis to sensor ECM	5.0, 5.2, 6.1.2, 6.1.6 6.3.1, 6.3.2, 6.3.3, 6.5.2, 7.2.1, 9.3.3
$\vec{R}$	vector quantity of R	5.0, 5.2
R_e	equatorial radius of the earth	5.3
R_{cm}	initial center of mass offset due to imbalance	6.1.5
R_0	static value of arm radius	6.1, 6.1.5, 6.1.6, 7.2.1 7.2.2, 7.2.6, 9.1.1, 9.3.3
R_+	radius for acceleration along +IA	6.1.7
R_-	radius for acceleration along -IA	6.1.7
ΔR	total deviation from nominal radius	7.2.1, 7.2.6, 9.3.3
ΔR_a	dynamic radius error proportional to a_c	7.2.1, 7.2.6
ΔR_{AS}	change in arm radius due to centrifugal loading beyond the static value	6.1.2
ΔR_d	nonlinear dynamic error in radius	7.2.1, 7.2.6
ΔR_{der}	shift in dynamic center of rotation	6.1.5
ΔR_+	radius deviation for +IA acceleration	7.2.1
ΔR_-	radius deviation for -IA acceleration	7.2.1
t	Student's t	9.3.4
t	time	6.3.1, 6.3.2, 6.4.1, 7.2.2, 7.2.3
t_1	period over which average centrifuge rate is measured	6.3.1, 6.3.2
$\vec{V}$	tangential velocity of the ECM with respect to the surface of the earth	6.3.3
VBA	vibrating beam accelerometer	5.3, 6.6.2, 8.1.4, Annex G
α	coefficient of thermal expansion of the arm	6.1.6
δ	misalignment of rotation axis from vertical	7.2.4
δ_c	coning angle	6.2.2, 7.2.6
δ_o	misalignment angle between IA and case reference IA about OA (rad)	7.1
δ_p	misalignment angle between IA and case reference IA about PA (rad)	6.2.3, 7.1, 9.3.3
δ_0	average arm misalignment from horizontal	7.2.4, 9.3.3
λ	latitude	5.3, 6.3.3
ϕ	phase of periodic variation at the start of the data sample	6.3.1, 7.2.6
θ_g	projection of θ_n on vertical plane containing IA	9.3.3
θ_{ri}	total misalignment angle between radius and IA	9.3.3
μ	normalized magnitude of largest residual	9.3.4
μ_e	confidence limit check value	9.3.4
ξ_+	random noise component	9.3.3
ξ_-	random noise component	9.3.3

ρ	density of centrifuge arm(mass per unit volume)	6.1.2
$\sigma(r)$	standard deviation of residuals	9.3.4
$\sigma(K_i)$	standard deviation of coefficient K_i	9.3.4
ϕ_T	total sensor misalignment about a tangential axis including sensor and mount	6.2.2, 7.2.5 7.2.6, 9.3.3
ϕ_{To}	initial mount displacement	6.2.2
ϕ_{T+}	static tangential misalignment of +IA	7.2.5
ϕ_{T-}	static tangential misalignment of -IA	7.2.5
$\Delta\phi_{Ta}$	a_c proportional dynamic misalignment	7.2.5, 7.2.6
$\Delta\phi_{Td}$	nonlinear dynamic tangential misalignment	7.2.5, 7.2.6
ϕ_V	sensor misalignment about vertical axis	7.2.5, 6.2.3, 7.2.6, 9.3.3
ϕ_{Vo}	initial alignment angle about vertical axis	6.2.3
ϕ_{V+}	static vertical misalignment for +IA	7.2.5
ϕ_{V-}	static vertical misalignment for -IA	7.2.5
$\Delta\phi_{Va}$	a_c proportional dynamic misalignment	7.2.5
$\Delta\phi_{Vd}$	nonlinear dynamic vertical misalignment	7.2.5
Ω	earth rotation rate, also known as earth rate	5.3, 7.2.6
$\vec{\Omega}$	vector quantity of earth rotation rate	6.3.3
Ω_v	local vertical component of earth rotation rate	6.3.3, 7.2.3, 7.3.6
ω	centrifuge rotational angular velocity	5.2, 6.1.5, 6.3.3, 6.5.2, 7.2.2
$\vec{\omega}$	vector quantity of ω	5.2
$\Delta\omega_d$	deviation from average centrifuge angular velocity	7.2.2, 7.2.6
ω_p	amplitude of centrifuge periodic angular velocity variation	6.3.1, 7.2.2, 7.2.6
ω_s	centrifuge angular velocity at start of data sample	6.3.2, 7.2.2, 7.2.6
$\omega(t)$	instantaneous centrifuge angular velocity	6.3.1, 6.3.2
ω_0	average centrifuge angular velocity	6.3.1, 6.3.2, 7.2.2 7.2.3, 7.2.6, 9.1.1
ω_n	natural resonant frequency of centrifuge structure	6.1.5

4. Summary

The term *centrifuge* is used in this recommended practice to mean a device consisting of a motor- or engine-driven arm structure with sufficient dimensions and strength to allow the rigid mounting and subjecting of one or more inertial-grade accelerometers to centripetal acceleration levels ranging from ~~fractions of 1 g to over 200 g~~ 10 000 m/s². The term *precision* suggests an exactness or advancement in the state- of-the-art over and above environmental survivability centrifuge testing. Precision centrifuge testing is used to calibrate accelerometer model parameters. The accuracy required for this calibration depends on the application of the accelerometer. Typical applications are shown in Table 1, which may be used as a general guideline.

It is standard practice of both manufacturers and users to first characterize inertial accelerometers in the local gravity field through the use of an earth-bound dividing head or precision rotational device of some kind (see IEEE Std 1293-1998). This practice allows “precise” graduations of acceleration to be applied to an accelerometer for the purpose of measuring its scale factor and error coefficients. Unfortunately, the maximum limit (9.8 m/s²)⁶ that can be reached using this method is usually far less than the acceleration level experienced in the application. A common test for force rebalance accelerometers is to insert a current into the accelerometer’s torquer in an attempt to simulate the acceleration levels required in the application. This test, however, demonstrates only that the accelerometer electronics will handle currents associated with the higher acceleration levels expected.

Table 1—Accelerometer performance

Maximum range		Super-high performance (10 ppm CE or better) (See NOTE)	High performance (100 ppm CE)	Medium performance (1000 ppm CE)	Low performance (1% CE)
m/s ²	(g) (informative)				
10–200	1–20	Inertial navigation	Guidance (ship/aircraft/ cruise missile)	Attitude control	Attitude control Switch-stage separation Instrumentation
200–2000	20–200	Ballistic missile	Tactical missile	Ordinance	Instrumentation
1000–3000	100–300	Earth reentry Planetary entry	Earth reentry Planetary entry	Switch-fusing Missile reentry	Car crash Instrumentation
3000–10 ⁶	300–10 ⁵		Jupiter entry	Artillery Rail gun	Car crash Instrumentation Fusing
NOTE—The abbreviation <i>ppm CE</i> means “parts per million composite error” (see IEEE Std 528-2001).					

Linear vibrators and rocket sleds may also, in certain cases, be used to measure accelerometer performance characteristics. The fact remains, however, that centrifuge testing must generally be used to perform precision measurements.

This recommended practice is intended to be useful to a broad spectrum of centrifuge users. Since the needs of a user interested in routine testing of a high-volume 1% accelerometer differ substantially from the needs of a designer doing developmental testing on a highly specialized 10 ppm accelerometer, each clause contains information that pertains to the different levels of precision. Therefore, users may extract the information pertinent to any desired level without becoming deeply involved in considerations that are inappropriate.

In addition to the procedures and supporting theory, which are the heart of this recommended practice, there are ten annexes. Annex A describes the features of some typical centrifuge installations, including a look at hardware designed for different production rates and different levels of precision.

³~~Information on references can be found in Clause 2.~~

Annex B continues this analysis with a description of methods ~~g levels~~ for evaluating a given centrifuge installation. Annex C provides further background for the data reduction clause by presenting a limited treatment of the theory of fitting model equations to noisy test data, including such variations as percentage weighted fits and “min-max” fitting.

Annex D describes the theoretical and practical limitations of centrifuge testing and outlines some alternative test methods that might be used to evaluate certain aspects of accelerometer behavior at high [acceleration levels](#).

[Annex E discusses the use of the double centrifuge and rate table with tilted axis of rotation for low-frequency transfer function determination.](#) [Annex F presents a refined model of the double centrifuge and rate table with tilted axis of rotation.](#)

Annex G describes the processing of dual-proof-mass vibrating beam accelerometer (VBA) centrifuge data. Annex H discusses scale factor calibration for high-[acceleration](#) applications on an ultra-centrifuge. Annex I discusses thermal control and data acquisition on the centrifuge arm. Annex J is a bibliography.

Distributed through this recommended practice are treatments of special subjects (such as the use of supplemental tests and supplemental analyses to support centrifuge testing, dual accelerometer centrifuge

testing, and arm stretch and droop measurements to remove the effects of arm stretch and droop) and recommendations about the use of combined environments.

5. Model equations

Centrifuge testing, while capable of great precision, must be a carefully designed experiment to minimize the many possible error sources peculiar to this type of testing.

In order to develop centrifuge test techniques and understand error mechanisms, mathematical models showing input/output relationships for the centrifuge and accelerometer are developed in this and subsequent clauses. The accelerometer to be modeled is based on the pendulous torque- rebalance type of instrument, but the principles are applicable to other types.

In centrifuge testing, the accelerometer is mounted on the centrifuge arm in a fixture that holds it in fixed alignment with respect to the acceleration vector. Assume a coordinate system X , Y , and Z with the origin at the intersection of the arm and the centrifuge spin axis with X along the arm and Z along the spin axis. The spin axis is aligned with the local vertical, and the directed distance from the spin axis to the effective center of mass (ECM) is defined as R , as shown in Figure 1.

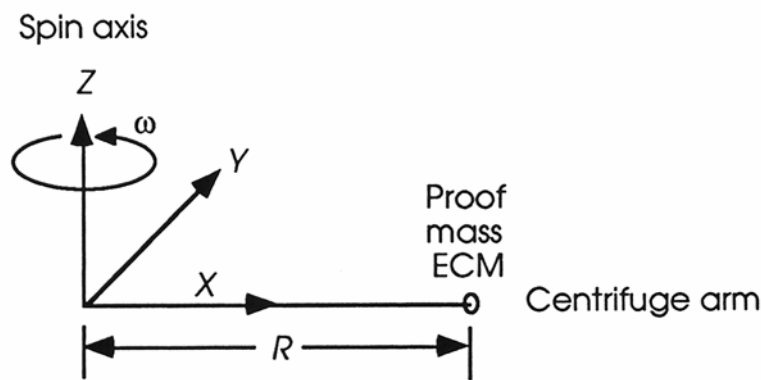


Figure 1—Accelerometer mounted on centrifuge

5.1 Simplified accelerometer model

Ideally, the acceleration indicated by the accelerometer is expressed as

$$a_s = \frac{E}{K_1} = K_0 + a_i \quad (1)$$

where

a_s is the indicated sensor output (m/s^2)

E is the sensor output (output units)

K_1 is the sensor scale factor (output units per m/s^2)

K_0 is the bias (m/s^2)

a_i is the acceleration along sensor input axis (IA) (m/s^2)

~~Although the SI unit of acceleration is the meter per second squared, accelerometer output is often expressed in terms of g , the magnitude of the local plumb-bob gravity (see 3.1.13).~~

This simplified model may be satisfactory for some applications; but for more precise applications, a model equation including nonlinear and cross-axis terms may be required (see Clause 7).

5.2 Simplified centrifuge model

Ideally, the centripetal acceleration of the centrifuge (a_c) imposed on the instrument under test is expressed As

$$\vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{R}) \quad (2)$$

Where

$\vec{\omega}$ is the centrifuge rotational angular velocity (rad/s)

$\vec{R}$ is the directed distance from rotation axis to the ECM

and since $\vec{R}$ is perpendicular to $\vec{\omega}$:

$$\vec{a}_c = -\omega^2 \vec{R} \quad (3)$$

with magnitude

$$a_c = \omega^2 R \quad (4)$$

The total test input (a_T) applied at the ECM is the centripetal acceleration plus the upward reaction to local gravity acceleration ($-g$):

$$\vec{a}_T = \vec{a}_c - \vec{g} \quad (5)$$

The component of a_T along the IA is a_i .

This simplified model may be satisfactory for some applications; but for more precise applications, a detailed model equation of the centrifuge with higher order terms resulting from temperature, mechanical interactions, and instrumentation errors may be required.

These errors are different for every centrifuge and measurement system and must be individually evaluated for significance. An individual model equation containing only the significant error terms must be developed for each setup. In order to develop a custom model equation, typical error equations for each error source identified are listed in Clause 6 so that they can be evaluated and significant terms combined in cookbook fashion to provide a correct equation for each test.

5.3 Simplified gravity model

The term *local gravity* (g) ~~(g_L)~~ as used in this recommended practice refers to “plumb-bob gravity,” which is the force per unit mass exerted by the earth on a mass that is at rest relative to the earth. Its direction defines the local vertical, and its magnitude is that of the vector sum of Newtonian gravitational force per unit mass and the centripetal force arising from the earth’s rotation, divided by the mass.

According to Newton’s Law of Gravity, there exists a mutual attractive force F between any two masses m_1 and m_2 at a distance d between their effective centers so that

where G is the gravitational constant

$$F = \frac{G m_1 m_2}{d^2} \quad (6)$$

$$G = 6.67259 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

Where one of the masses is the earth ($5.975 \times 10^{24} \text{ kg}$), a gravitational field may be defined at the surface so that the force on a test mass is

$$F = m g_N \quad (8)$$

where

g_N is Newtonian gravitation

Since this result has the same form as the inertial law

$$F = ma \quad (9)$$

and the Theory of Relativity states that, in principle, the effects of gravitation and acceleration are indistinguishable, g_N is defined to be gravitational acceleration and has the units of length per unit time squared.

The centripetal acceleration due to the earth’s rotation in inertial space is perpendicular to the earth’s rotation axis and is nominally

$$a_b = R_e \Omega^2 \cos \lambda \quad \text{in magnitude} \quad (10)$$

where

Ω is the earth rotation rate

R_e is the earth’s equatorial radius

λ is the latitude of the mass

At the equator, this amounts to 0.03392 m/s^2 and decreases to zero at the poles. The local acceleration due to gravity g is the vector difference of the gravitational attraction acceleration and the earth’s rotation centripetal acceleration.

The acceleration of gravity also varies because the earth is not spherical and because of density variations in the earth's crust. The acceleration of gravity is further affected by elevation. A typical equation for the variation of g with latitude λ and altitude h (in meters) is approximated by

$$g = 9.780356 \left(1 + 0.0052885 \sin^2 \lambda - 0.0000059 \sin^2 2\lambda \right) - 3.086 \times 10^{-6} h \quad \text{m/s}^2 \quad (11)$$

See *International Critical Tables*, Vol. 4, [B7] [B12]⁷ and *CRC Handbook of Chemistry and Physics* 1997–1998 [B2]. By international agreement, the standard value of gravity is defined as 9.80665 m/s². This value, which corresponds closely to the formula value at 45° latitude and sea level elevation.

The downward force of gravity acting on the proof mass and case of an accelerometer at rest on the earth is balanced by the upward restoring force of the support acting on the case of the accelerometer. For torque or force rebalanced accelerometers, this upward force requires a restoring force of the same polarity on the proof mass to keep the proof mass at its null position. Since the measure of accelerometer output is the restoring force or torque on the proof mass, the output of an accelerometer at rest on the earth therefore indicates an acceleration of g upward. A similar argument for an open loop accelerometer against an elastic restraint or a VBA yields the same result.

In the calibration of an accelerometer scale factor, the local plumb bob gravity acceleration g is frequently used as a reference.

For accelerometer testing, the local value of gravity g should be determined by an organization that specializes in performing such surveys. In the United States, gravity and site location surveys are done by the U.S. Coast and Geodetic Survey and the National Imagery and Mapping Geospatial-Intelligence Agency (formerly Defense Mapping Agency). More information can be obtained from Internet Web sites.

6. Test error sources

This clause endeavors to identify many frequently encountered error sources and to provide a means to evaluate their significance to the test planned. This clause is organized in the fashion of an expansion in partial derivatives or sensitivity coefficients. Each subclause examines one error source and evaluates its contribution to total error as though it were the only error source acting. Subclause 6.6.2 considers the few cases where error sources interact sufficiently that their combined effects may need to be examined as a separate issue. To the extent possible, the forms of the errors are described as an aid to their identification in test data. Error effects are accordingly described in terms of their ability to masquerade as test unit errors when not properly identified and corrected. In particular, they are described in terms of errors in evaluation of the coefficients of a model equation, such as that described in Clause 7.

Measures must be taken to minimize or compensate for significant errors that would be based on the level of precision required in the particular application. These measures can take the form of instrumentation added to a basic centrifuge, or special test procedures (such as multiple-sensor testing to reject common mode errors), or both.

6.1 Radius uncertainty

A major problem of centrifuge testing is radius uncertainty. Factors contributing to this uncertainty include the following:

- a) Uncertainty in the static radius
- b) Dynamic changes in radius

c) Changes in radius due to the environment. ~~Each~~ The contributors to radius uncertainty are discussed in 6.1.1 through 6.1.7 and examples given to show the magnitude of the errors they may cause.

6.1.1 Uncertainty in static radius

To determine the static radius to the ECM of the test accelerometer, it is necessary to know the location of the center of the arm, the location of the ECM relative to the reference surfaces of the accelerometer, and the dimensions and interfaces of all the mechanical components between these points, including test fixtures. For high-precision applications, it may be necessary to know this stack-up of dimensions to an overall accuracy on the order of 1 μm . On a 1m nominal radius machine, for example, 1 μm introduces an error in applied acceleration of 1 ppm.

Few, if any, centrifuges provide a direct means to measure static radius to useful accuracies, even when the assumption is made that the position of the ECM inside the accelerometer is accurately known. Therefore, the static radius must be estimated from the output of the instrument under test.

Many estimation techniques have been used to determine static radius, and the most popular of these are described in Clause 9 and Annex C. In 9.3.3, this recommended practice introduces an algorithm that puts this estimation on a statistically rigorous footing.

A necessary condition for these estimates to be accurate is that the radius must be fixed during the test or that changes in radius during the test are compensated for in the input data.

6.1.2 Arm stretch

Arm stretch R_{AS} is the elongation of the mechanical structure connecting the accelerometer to the centrifuge spin axis beyond the static value (R_0) and is due to centrifugal loading. If the radius change is due only to arm stretch, the equation for radius is

$$R = R_0 + \Delta R_{AS} \quad (12)$$

or

$$R = R_0 \left(1 + \frac{\Delta R_{AS}}{R_0} \right) \quad (13)$$

Because the arm stretch is proportional to load (hence acceleration a_0), one can write

$$R = R_0 (1 + C_{AS} a_0) \quad (14)$$

where C_{AS} is a constant whose dimension is the inverse of acceleration or alternatively, acceleration divided by acceleration squared. Then

$$a_c = R\omega^2 = R_0 (1 + C_{AS} a_0) \omega^2 \quad (15)$$

This error increases the magnitude of the applied acceleration for either polarity and is, therefore, an odd-quadratic term (K_{oq}) in the accelerometer output data. In testing precision accelerometers, every effort should be made to estimate the effect of arm stretch and, if significant, correct the input acceleration data.

The magnitude of this error can be estimated by calculating its value for the simple case of an arm with constant cross-section by integrating the elastic strain over its length. For the component due to loading of the arm by its own mass

$$S(x) = \frac{1}{A} \int_0^x \omega^2 (R - x') \rho A dx' = \rho \omega^2 \left(Rx - \frac{x^2}{2} \right) \quad (16)$$

where

- $S(x)$ is the stress at x
- R is the test radius (arm length)
- x, x' are the distances inward from stress-free end
- A is the cross-sectional area
- ρ is the density (mass per unit volume)

Then, strain = S/Y (dimensionless) and

$$\frac{\Delta R}{R} = \frac{1}{R} \int_0^R \frac{S}{Y} dx = \frac{\rho \omega^2 R^2}{3Y} \quad (17)$$

where Y is Young's Modulus.

It should be noted that the strain is independent of the constant cross-sectional area A . Furthermore, it is also nearly independent of the arm material for the materials commonly used in precision centrifuges, such as steel or aluminum for which

$$\frac{\rho}{Y} = 3.8 \times 10^{-8} \text{ s}^2 \text{m}^{-2} \quad (18)$$

Replacing $\omega^2 R^2$ with $R_0 a_0$ gives

$$\frac{\Delta R}{R} = \frac{R_0 a_0}{3} \frac{\rho}{Y} = C_{AS} a_0 \Rightarrow C_{AS} = \frac{R_0}{3} \frac{\rho}{Y} \quad (19)$$

where C_{AS} is the nonlinearity coefficient caused by stretch of the bare arm. For a 1 m radius, its value is

$$1.27 \times 10^{-8} \left(\text{m/s}^2 \right) / \left(\text{m/s}^2 \right)^2 \cdot \text{s} \quad \text{0.12 mg/gz}$$

The arm stretch due to the test object mass will add to that of the bare arm. The tensile load is the test object mass M multiplied by the acceleration. The corresponding tensile stress is

$$S = \frac{Ma_0}{A} \quad (20)$$

giving

$$\frac{\Delta R}{R} = \frac{1}{R} \int_0^R \frac{S}{Y} dx = \frac{Ma_0}{AY} \quad (21)$$

In this case, both area and arm material are clearly significant. As an example, if the 1m arm is of aluminum with a cross-sectional area of 0.01 m^2 , a 5 kg test object will produce an additional stretch equivalent to $7.04 \times 10^{-9} (\text{m/s}^2) + \frac{(\text{m/s}^2)^2}{g_2} = 0.07 \text{ u/g}_2$.

6.1.3 Accelerometer proof mass deflection

The effects of proof mass deflection under applied acceleration are indistinguishable from the effects of arm stretch since both result in a change of radius to the ECM proportional to applied centripetal acceleration and both will produce an apparent odd quadratic term (K_{oq}) in the accelerometer output data. It differs only in that it is produced by an interaction between test unit and test bed characteristics and cannot be characterized in the same way that a fixed test bed error might be. The magnitude of the deflection can vary from near zero to as much as $0.25 \text{ } \mu\text{m}/(\text{m/s}^2)$, depending on sensor design. Accelerometer packages that contain internal isolators may experience large deflections. Since the deflection is internal to the accelerometer or sensor package, physical measurements of the effect cannot be made on the centrifuge. However, the deflection can be estimated and used to correct the input data if the static compliance of the constraint is known.

For example, if the accelerometer being tested has a proof mass compliance so that the natural frequency is 1000 Hz (equivalent to a translational mass-spring system with a deflection of $0.025 \text{ } \mu\text{m}/(\text{m/s}^2)$), then with a 1 m distance to the center of rotation, the increase in distance per (m/s^2) is 0.025 ppm. This results in an apparent $0.025 \times 10^{-6} (\text{m/s}^2)/(\text{m/s}^2)^2$ odd-quadratic error due to gradient in acceleration applied by the centrifuge. Note that this error is comparable in magnitude to [the error](#) in the arm stretch example of 6.1.2.

6.1.4 Arm bending

Since the arm of a centrifuge has a finite stiffness, it will tend to sag due to the effects of gravity.

Centrifugal forces acting on imbalances of the arm may cause bending in the vertical and/or horizontal Plane

The output of an ideal accelerometer (see 5.1) consists of a bias term (K_0) and a linear term proportional to acceleration along the IA ($K_1 a_i$). When testing an accelerometer on a centrifuge, the bias term will include a component of [the reaction to](#) local gravity if the IA is not perfectly horizontal. Arm bending may also cause this misalignment to change as a function of a_c .

A secondary effect of arm bending is the resulting radius change due to the change of end slope during the test. This [change](#) has the same form of error as arm stretch and will appear as an acceleration-squared term. If a high degree of precision is required, it is important to locate the ECM of the accelerometer as close as possible to the neutral axis of the arm and thereby minimize the effects of arm bending. For typical centrifuges, deflections are on the order of a fraction of a mrad, and the resulting radius change due only to beam curvature is less than 1 ppm. However, for a 1 m radius centrifuge where the ECM of the test accelerometer is mounted 15 cm above the arm neutral axis, a beam deflection of 1 mrad will produce a radius

error of 150 ppm.

6.1.5 Dynamic center of rotation

The centrifuge axis of rotation is defined by a set of bearings that may be ball, roller, hydrodynamic, or other. Good quality bearings typically control the axis of rotation within 0.1 μm at slow speeds. If the arm is balanced along its axis and in the vertical plane, that accuracy will be maintained at the higher speeds required for accelerometer tests. If the centrifuge arm with the accelerometer and its mounting fixture is unbalanced in a vertical plane, the resulting torque will cause the axis of rotation to tilt. If the arm is balanced in the vertical plane but unbalanced radially, the axis of rotation will shift radially and may also tilt depending on the location of the radial bearings and on their individual stiffnesses. Thus, in general, the axis of rotation will move in some orbital fashion that differs from [the axis of rotation](#) defined by the bearings at slow speed.

The simplest case to consider is a planar approximation in which the arm imbalance is static only that is, the arm is loaded symmetrically above and below its neutral axis, but one end is slightly heavier than the other so that the center of mass is shifted slightly along the arm axis by a distance R_{cm} . Then as the arm rotates, it will shift in the direction of the initial offset due to the centrifugal force on the arm. The offset is given approximately by

$$\Delta R_{\text{dcr}} = \frac{(R_{\text{cm}} + \Delta R_{\text{dcr}}) m_a \omega^2}{k} \quad (22)$$

where

ΔR_{dcr} is the shift in dynamic center of rotation

R_{cm} is the initial center of mass offset due to imbalance

m_a is the arm mass

ω is the centrifuge rotational angular velocity

k is the radial stiffness

The term *radial stiffness* is a direct measure of the resistance of the centrifuge axis to deflection due to a horizontal force at the center of rotation and must include the effects of all compliances of the structure, including the bearings, connection to the floor, and the foundation on which the machine is mounted. Then

$$\Delta R_{\text{dcr}} = \frac{R_{\text{cm}} (m_a \omega^2 / k)}{1 - (m_a \omega^2 / k)} = R_{\text{cm}} \frac{m_a \omega^2}{k} \quad (23)$$

This approximation corresponds to the assumption that $m_a \omega^2 / k \ll 1$, or $(\omega / \omega_n)^2 \ll 1$, where $\omega_n^2 = k / m_a$ that is, the rotational frequency is much smaller than the resonant frequency of the arm on its bearings and support structure. Since ω , the centrifuge rotational angular velocity, rarely exceeds 5 r/s, the assumption is usually true.

In this case, ΔR_{dcr} is proportional to ω^2 and, therefore, to the nominal centripetal acceleration a_0 . Substituting the proportionality constant C_{dcr} ,

$$R = R_0 + \Delta R_{\text{dcr}} = R_0 (1 + C_{\text{dcr}} |a_0|)$$

The centripetal acceleration is then

$$a_c = \omega^2 R_0 (1 + C_{\text{dcr}} |a_0|) = a_0 (1 + C_{\text{dcr}} |a_0|) \quad (25)$$

The effect of an imbalance in the centrifuge arm is to introduce a false second-order term. If C_{dcr} remains unchanged when IA is reversed, then only the odd-quadratic nonlinearity coefficient K_{oq} will be affected. If C_{dcr} changes sign but not magnitude when IA is reversed, then only the quadratic coefficient K_{A} will be affected. In either of these cases, the magnitude of the coefficient error will be C_{dcr} .

In general, C_{dcr} will be different for the two polarities so both K_2 and K_{oq} will be affected. The significance of this term is most readily appreciated by an example. Assume

$$\begin{aligned} \Delta R_{\text{dcr}} &= 100 \text{ } \mu\text{m} \\ R_0 &= 1 \text{ m} \\ a_0 &= 300 \text{ m/s}^2 \end{aligned}$$

Then

$$C_{\text{dcr}} = 0.33 \times 10^{-6} \left(\text{m/s}^2 \right) / \left(\text{m/s}^2 \right)^2 \quad (26)$$

The 100 μm motion is unlikely to be noticed without sensitive instrumentation, whereas [the value in Equation \(26\)](#) can be quite significant.

The assumptions used in this simple analysis are valid in a great many cases. Sometimes the signature of imbalance may be more complicated. For example, it may not be true that $\omega \ll \omega_n$, in which case a significant fourth-order term appears and the structure may even pass through resonance where phase effects become significant. In addition, the bearing structure may not be isoelastic, and its compliance will then vary cyclically with arm rotation.

6.1.6 Thermal expansion

Since the acceleration produced by the centrifuge is directly proportional to the arm length, any change of length caused by thermal expansion represents an error source. In its simplest form, where the entire centrifuge arm is operating at a temperature different from the temperature where the arm length was measured, the arm length can be computed as

$$R = R_0 (1 + \alpha \Delta T) \quad (27)$$

where

$$\begin{aligned} \Delta T &\text{ is the temperature rise above calibration temperature} \\ \alpha &\text{ is the coefficient of thermal expansion of the arm} \end{aligned}$$

It should be noted that a number of difficult complications can arise in situations where this error source is

significant, where the temperature of the arm is a function of time as a result of heating or cooling during the centrifuge run itself, or where there is a significant heat flow along the arm causing a temperature gradient to exist. In these cases, the arm temperature must be monitored and the input acceleration data corrected for the changes in R_0 .

As a quantitative example, when the arm is made of cast iron with $\alpha = 13 \text{ ppm/}^\circ\text{C}$ and the arm temperature changes 5°C from the calibration temperature, a change in arm length of 65 ppm results.

6.1.7 Radial asymmetry with acceleration polarity

The radius errors described in 6.1.1 acquire a special importance when the test unit IA is reversed to test both the positive and negative parts of the input range. Since the location of the ECM is typically not known within 0.8 mm, there is almost always a significant change in radius when the IA is reversed. If this change is not compensated for, the resulting output data will appear to have scale factor asymmetry, which can easily corrupt estimates of linearity coefficients.

Even-order coefficients are strongly affected by such asymmetry, and radius estimates based on approximations are frequently not accurate enough to avoid gross errors in these coefficients. The best-known way to minimize this effect is to use the algorithm of 9.3.3, with separate measurements of positive and negative scale factor where needed.

For example, if the ECM location is in error by 1 mm on a 1 m radius centrifuge, the error will be 1000 ppm. At the 500 m/s^2 level, this amounts to $500 \times 1000 \text{ ppm} = 0.5 \text{ m/s}^2$, the same error produced by a K_2 of $2 \times 10^{-6} (\text{m/s}^2)/(\text{m/s}^2)^2$ at 500 m/s^2 .¹² ~~50 g X 1000 ppm = 50,000 μg , the same error produced by a K_2 of 20 $\mu\text{g/g}^2$ at 50 g.~~

6.1.8 Instrumentation methods and errors

The determination of centrifuge radius can be divided into two areas: the determination of the nominal or static radius and the determination of dynamic changes in radius. The static radius can be determined several ways, depending on the accuracy of the test to be performed. Measurements of the radius may be possible with a mechanical arrangement such as an Invar rod and a micrometer. However, the accuracy of measurements by this method is typically limited by endpoint determination to about 1000 ppm. For greater accuracy, it is usually preferred to let the test item measure the static radius. This [method](#) is covered in depth in Clause 9, ~~Data reduction~~.

Several devices and methods are available for measuring dynamic radius changes. Each device must not only measure the stretch of the centrifuge arm, but ~~must~~ also measure any changes in the dynamic center of rotation. If the device itself cannot do this [measurement](#), it must be supplemented with other devices.

¹² [20 \$\mu\text{g/g}^2\$ at 50 g.](#)

A laser interferometer is an optical device that may be mounted either on- or off-board of the centrifuge arm. A major advantage of optical systems is that, ~~assuming when~~ provisions for optical access have been made, the radius growth can be measured right at the test item. Another advantage is that the output of the device, analog or digital, is continuously available for monitoring. The major disadvantages of this system are its sensitivity to temperature and air density, ~~and such sensitivity~~ typically produces errors of several parts per million under normal laboratory conditions. If more precise measurements are required, the optical path should be evacuated and temperature controlled.

Proximity detectors (capacitive, inductive, or laser) are devices that are usually mounted off-board and measure the distance and droop between the wall and the centrifuge arm as it goes by. They do not use any centrifuge slip rings and they are not exposed to forced convection heat transfer currents. They are reliable, easy to use, and inexpensive. However, they do have two major disadvantages. First, their output is valid only as the arm or the test item swings by therefore, the data must be sampled by some kind of fast- sampling

analog-to-digital converter since it is valid for only a few milliseconds. Second, the proximity detectors cannot measure the stretch of the arm at the test package, but must instead look at the surrounding structure, which may not be indicative of [the stretch of the arm](#) at the test package.

Radius measurement devices cannot be used to compensate for the uncertainty in the location of the ECM or for asymmetry with acceleration polarity. These problems are most readily handled during the data reduction and are discussed in Clause 9. Another technique for compensating for arm stretch effects is to use dual accelerometer testing (see 6.6.3).

6.2 Alignment uncertainty

In order to properly define the accelerations along the principal accelerometer axes, it is necessary to know the misalignments between those axes and the centrifuge arm, the axis of rotation, and the gravity vector. In some cases, dynamic terms that cause deflections or changes in the alignment due to centripetal loading or temperature changes must be considered. These error sources must be considered whenever appropriate to achieve the level of precision required.

6.2.1 Static leveling error

Static leveling error refers to errors in the acceleration seen by a test item due to misalignments of the accelerometer IA to the centrifuge and gravity vector. A static error is one that is present when the centrifuge is not rotating and may change with arm position. Typical misalignments are between an accelerometer IA and its mounting surface, the mounting surface and the centrifuge rotation axis, and the rotation axis relative to gravity. As long as the misalignments are truly static (i.e., do not change as a function of the centrifuge [acceleration](#) level), they affect the apparent bias (zero-order term in the data) but have little or no effect on the higher order coefficients (K_2 and K_3). For this reason, a centrifuge test cannot be used to accurately determine the accelerometer bias, but as long as the misalignments are less than about 1° , their effects on the K_2 and K_3 coefficients are negligible.

Tilting of the main centrifuge rotation axis is an error source that requires special consideration in that the acceleration input has a component that varies sinusoidally with each revolution but is independent of centrifuge angular velocity. This error can be eliminated by averaging the accelerometer output over complete revolutions of the arm. At zero rate, a number of readings must be taken and averaged, with the arm stopped at equally spaced intervals through 360° .

6.2.2 Dynamic leveling error

For an accelerometer mounted with its IA nominally parallel to the centrifuge centripetal acceleration vector, any angular deflections of the mounting fixture about a horizontal line perpendicular to the acceleration vector results in a change in the component of gravitational acceleration along the accelerometer IA. A change in the component of centripetal acceleration along the IA also results. Considering this misalignment, the acceleration along the IA can be written as

$$a_i = a_c \cos \phi_T + g \sin \phi_T \quad (28)$$

where

- a_i is the acceleration along sensor IA
- a_c is the centripetal acceleration of the centrifuge
- ϕ_T is the mount deflection angle
- g is the local value of gravity acceleration

For most applications, small angle approximations can be used for $\sin \phi_T$ and $\cos \phi_T$, giving

$$a_i = a_c + g \phi_T \quad (29)$$

If ϕ_T varies linearly with centripetal acceleration, it can be written as

$$\phi_T = \phi_{T_0} + C_d a_c$$

where

ϕ_{T_0} is the initial mount misalignment
 C_d is the deflection coefficient (angle/ a_c)

The acceleration along the sensor IA becomes

$$a_i = a_c (1 + g C_d) + g \phi_{T_0} = \omega^2 [R (1 + g C_d)] + g \phi_{T_0}$$

The proportional term, $a_c g C_d$, produces additional acceleration in the test unit IA, which is indistinguishable from [the acceleration](#) that would be caused by a small change in the test radius, as shown in the second part of the equation. Since the true radius is never known exactly in a centrifuge test and must be estimated by means of the separately measured scale factor, this change produces no error in the indicated nonlinearity. A coefficient of [0.1 arc-second per g is equivalent in/\(m/s²\)](#) ¹³ corresponds to [an apparent](#) radius change of only 5 ppm.

The constant term, $g \phi_{T_0}$, is similar to a change in accelerometer bias in that it produces a small change in indicated acceleration, which is added to all data points. For example, [1" is approximately equal](#) to [5 × 10⁻⁵ m/s²](#).¹⁴ Angles in excess of 0.5 ° are required to produce significant errors in indicated nonlinearity even in precision testing.

However, when the sensor IA is not approximately parallel to the arm, small deflections can strongly affect recovery of the nonlinearity coefficients. For example, when the IA is vertical, the acceleration along its IA is

$$a_i = a_c \sin \phi_T + g \cos \phi_T$$

or, using small angle approximations,

$$a_i = a_c \phi_T + g$$

¹³ [1 □/g.](#)

¹⁴ [5 μg.](#)

If ϕ_T varies linearly with centripetal acceleration, [Equation \(33\)](#) becomes

$$a_i = C_d a_c^2 + \phi_{T_0} a_c + g$$

If not compensated for, [deflections result in](#) errors in determining accelerometer cross-axis sensitivities to the square of acceleration. For example, a deflection of [1" results in approximately](#) a 5×10^{-5} m/s²/(m/s²)² ¹⁵ error in the determination of the coefficients K_{oo} or K_{pp} .

A common source of dynamic leveling error is the mode of motion called *coning*. When a centrifuge turns

very slowly, its axis of rotation defines a line that is fixed in the rotating structure. The line perpendicular to this rotating structure axis through the ECM of the accelerometer under test defines the centrifuge arm axis. As the centrifuge angular velocity is increased, forces due to imbalance can drive the elastic compliance of the bearings and their support structure to cause the structure axis to trace out a cone about the rotation axis as shown in Figure 2. This motion is synchronous with the basic rotation rate, ω .

The radius of the cone as measured at the level of the centrifuge arm is the radius change due to a shift in the dynamic center of rotation, R_{dcr} , which was introduced in 6.1.5. The cone angle δ_c is a contributor to dynamic leveling error. Coning is a prominent feature of many centrifuge analyses for two reasons. First, the displaced structure axis/arm coordinate system is the one to which test fixtures are typically related. Second, the geometry of the coning motion is one of the features commonly measured by auxiliary instrumentation, such as bearing support motion detectors.

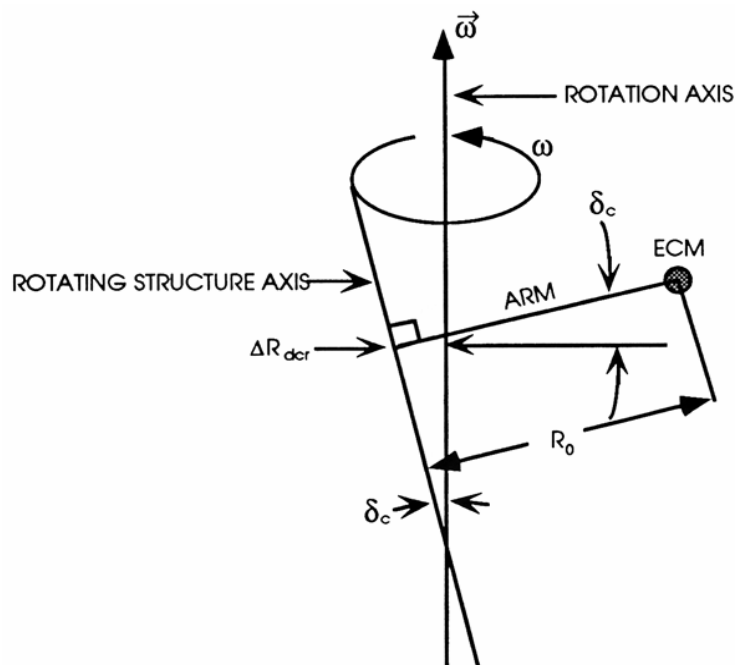


Figure 2—Centrifuge coning

6.2.3 Sensor misalignment about a vertical axis

Misalignments and changes of the IA about a vertical axis have no effect on the coupling of gravity, but they do affect the component of a_c that lies along the sensor IA. Static misalignment creates only a scaling error, which is indistinguishable from radius uncertainty. Changes in misalignment that are a function of applied acceleration due, for example, to arm and fixture compliances, may produce linearity errors in the input acceleration that can masquerade as linearity errors in the sensor output. These linearity effects generally can be identified and compensated for by means of multiple-orientation testing.

[15.5 \$\mu\$ g/g₂](#)

Multiple-orientation testing may require specification of nominal angles with respect to the centripetal acceleration vector that are too large to be treated by small angle approximations. The format of the rotation angle ϕ_v , accordingly, is somewhat different than in the previous clause:

$$\phi_v = \phi_{v_0} + C_d a_c$$

$$a_i = a_c \cos \phi_v$$

Where

ϕ_{V_0} is the initial alignment angle
 $C_d a_c$ is the variation in ϕ_V due to a_c

Since $C_d a_c$ is assumed to be a small angle, trigonometric approximations may be used in the expansion of $\cos \phi_V$:

$$\cos \phi_V = \cos \phi_{V_0} \cos(C_d a_c) - \sin \phi_{V_0} \sin(C_d a_c) = \cos \phi_{V_0} - C_d a_c \sin \phi_{V_0} \quad (37)$$

$$a_i = a_c \cos \phi_{V_0} - C_d a_c^2 \sin \phi_{V_0} \quad (38)$$

The first a_i term, which contains only static components of v , shows only a small error in apparent radius due to uncertainties in ϕ_{V_0} , generally within the uncertainty band for radius. The second term adds a quadratic linearity error.

If the test sensor is assumed to have linearity errors of the form

$$a_s = a_i + K_2 a_i^2 + K_{\phi} a_i a_p + K_{pp} a_p^2 + \text{terms containing } a_0, a_0 a_i, \text{ or } a_0 a_p \quad (39)$$

and a_0 is small, then to second order in a_c

$$a_s = a_c \cos \phi_{V_0} + K_2 a_c^2 \cos^2 \phi_{V_0} + K_{\phi} a_c^2 \cos \phi_{V_0} \sin \phi_{V_0} + K_{pp} a_c^2 \sin^2 \phi_{V_0} \quad (40)$$

More than one orientation will be required to separate all the input and output terms that contain a_c^2 . When $\phi_{V_0} = 0^\circ$, there is no input error due to the effects of this clause; therefore, K_2 can be determined unambiguously. When $\phi_{V_0} = 90^\circ$, the C_d input error can be confused with the K_{pp} output error. If the test is repeated with $\phi_{V_0} = -90^\circ$, however, and the fixture design is such that C_d does not change, the input error will change sign, and the two effects will be separated. Similarly, if $\phi_{V_0} = 45^\circ$, even after the K_2 and K_{pp} terms are accounted for, there remains a C_d term that can masquerade as K_{ip} . Once again, if the test is repeated at -135° , the input error will change sign while the output error remains unchanged.

The magnitude of the effect depends, of course, on the accuracy and compliance of the fixture. If the uncertainty in ϕ_{V_0} is 0.001 rad and if $C_d a_c = 0.001$ rad at $a_c = 200 \text{ m/s}^2$, ~~20g~~ then $C_d \equiv \underline{5 \text{ } \mu\text{rad}/(\text{m/s}^2)}$. ~~50- $\mu\text{rad/g}$~~ . The apparent error varies from zero at $\phi_{V_0} = 0^\circ$ to 0.1% at $\phi_{V_0} = 90^\circ$. The K_{pp} error, if uncorrected, is $5 \times 10^{-6} (\text{m/s}^2)/(\text{m/s}^2)^2$, and the K_{ip} error is $7 \times 10^{-6} (\text{m/s}^2)/(\text{m/s}^2)^2$.¹⁶

6.2.4 Misalignment about a radial axis

Misalignment of the test item about a radial centrifuge axis will have no effect on tests conducted with the accelerometer IA along the centrifuge arm. For orientations in which the accelerometer IA is perpendicular to the centrifuge arm, this misalignment will result in a component of gravitational acceleration being sensed by the accelerometer. This misalignment, if not compensated for, results in an apparent bias error but does not affect the determination of accelerometer nonlinearities, the primary purpose of most centrifuge tests.

6.2.5 Instrumentation methods and errors

The measurement of angular deflections of the test item about any of three orthogonal axes is best accomplished by some on-board device, as the nature of the rotating centrifuge environment makes the detection by some stationary off-board device nearly impossible. The most practical on-board devices are the same ones that are used for dynamic radius measurements (see 6.1.8). The same arguments as [found in 6.1.8](#) apply for the optical and proximity detector devices; therefore, the accuracy, readout resolution, slip ring requirements, etc., for the device must be selected commensurate with the test requirements.

The reference with which these measurements are taken must be chosen with care. The measurement should be made with respect to some fixed reference (i.e., gravity). However, since the device is mounted on the centrifuge arm, it can move relative to the reference. Movements of the measurement device itself must either be minimized by careful placement on the centrifuge or be measured and compensated for.

6.3 Angular velocity uncertainty

Measurement of average angular velocity may not define the centrifuge acceleration with sufficient precision in the presence of a periodic variation of angular velocity. This subclause examines the effects of low frequency variation (wow—typically once per revolution), higher frequency variation (flutter—which may or may not be synchronous with rotational angular velocity), and frequency drift during the period of measurement. Earth rotation Coriolis effects are also defined.

6.3.1 Periodic angular velocity variation

The average centripetal acceleration on a centrifuge is given by

$$a_c = \frac{1}{t_1} \int_0^{t_1} \omega(t)^2 R dt$$

where

- t_1 is the period over which the average is taken
- $\omega(t)$ is the centrifuge angular velocity
- R is the centrifuge radius
- t is the time

If periodic variations (wow or flutter) occur, the centrifuge angular velocity can be given by

$$\omega(t) = \omega_0 + \omega_p \sin(2\pi f_w t + \phi) \quad (42)$$

where

- ω_0 is the average angular velocity
- ω_p is the amplitude of periodic variations
- f_w is the frequency of periodic variations
- ϕ is the phase angle of periodic variation at the start of the data sample

When the radius is constant, the average acceleration now becomes

$$a_c = \frac{R}{t_1} \int_0^{t_1} [\omega_0 + \omega_p \sin(2\pi f_w t + \phi)]^2 dt$$

The procedure commonly used to determine the average acceleration is to measure the average centrifuge rate over some time period t_1 . The measured acceleration $\hat{a}_c$ is then given by

$$\hat{a}_c = R \left\{ \frac{1}{t_1} \int_0^{t_1} [\omega_0 + \omega_p \sin(2\pi f_w t + \phi)]^2 dt \right\}$$

The error caused by using the measured average rate is $\hat{a}_c - a_c$.

After multiplying out the squared terms and evaluating the integrals, $\hat{a}_c - a_c$ becomes

$$\begin{aligned} \hat{a}_c - a_c = -R\omega_p^2 \left\{ \frac{1}{2} + \frac{1}{8\pi f_w t_1} [\sin 2\phi - \sin 2(2\pi f_w t_1 + \phi)] \right. \\ \left. - \frac{1}{(2\pi f_w t_1)^2} [\cos(2\pi f_w t_1 + \phi) - \cos \phi]^2 \right\} \end{aligned}$$

[Equation \(45\) consists of constant terms plus time dependent terms. If the sample period consists of a whole number of cycles of periodic variations \(\$f_w\$ \) or of sufficiently long averaging times \(where \$t_1 \gg 1/f_w\$ \), Equation \(45\) becomes](#)

$$\hat{a}_c - a_c = -\frac{R\omega_p^2}{2}$$

or if $\omega_p \ll \omega_0$

$$\frac{\hat{a}_c - a_c}{a_c} = -\frac{1}{2} \left(\frac{\omega_p}{\omega_0} \right)^2$$

A peak variation of 0.1% in angular velocity results in a -0.5 ppm error in the measured acceleration. When ω_p is a fixed percentage of ω_0 , the estimated radius is in error, with negligible effect on derived nonlinearity coefficients. For variable ratios of ω_p/ω_0 , odd coefficients such as K_{oq} and K_3 may be significantly affected. For example, some specially instrumented centrifuges have been observed to produce a variation in ω_p/ω_0 that is approximately proportional to $1/a_c$. In this case, the centrifuge-produced acceleration error is proportional to $1/a_c$, and odd order because it increases the magnitude of a_c for either polarity of input. This error in the centrifuge input will appear to be an error in the accelerometer output and will corrupt the values defined for such odd coefficients as K_{oq} and K_3 .

6.3.2 Angular velocity drift

The average acceleration in the presence of drifts in angular velocity can be determined by expressing the angular velocity as

$$\omega(t) = \omega_s (1 + Ct) \quad (48)$$

where

- C is the rate of drift in angular velocity (1/s)
- ω_s is the centrifuge angular velocity at the start of data sample (rad/s)

The average centripetal acceleration now becomes

$$a_c = \frac{R}{t_1} \int_0^{t_1} \omega_s^2 (1 + Ct)^2 dt = R \omega_s^2 \left(1 + Ct_1 + \frac{C^2 t_1^2}{3} \right) \quad (49)$$

The measured average angular velocity is

$$\omega_0 = \frac{1}{t_1} \int_0^{t_1} \omega_s (1 + Ct) dt = \omega_s \left(1 + \frac{Ct_1}{2} \right) \quad (50)$$

The acceleration as determined from measured average angular velocity is

$$\hat{a}_c = R \omega_0^2 = R \omega_s^2 \left(1 + \frac{Ct_1}{2} \right)^2 \quad (51)$$

The error in determination of acceleration caused by drift in angular velocity can be shown to be

$$\hat{a}_c - a_c = -\frac{R \omega_s^2 C^2 t_1^2}{12} \quad (52)$$

or, if $\omega_p \ll \omega_0$,

$$\frac{\hat{a}_c - a_c}{a_c} = -\frac{C^2 t_1^2}{12} \left(\frac{\omega_p}{\omega_0} \right)^2 = -\frac{C^2 t_1^2}{12} \quad (53)$$

since $(\omega_p/\omega_0)^2 = 1$.

If the angular velocity drifts at a rate of 0.1% per minute, the error in the determination of acceleration is proportional to the square of the sample time at a rate of -0.08 ppm/min^2 .

6.3.3 Earth rotation Coriolis effect

Whenever a body moves in a rotating reference frame, such as the earth's surface, it is subject to the Coriolis acceleration, which is expressed by the vector equation

$$\vec{a} = 2\vec{\Omega} \times \vec{V}$$

where in the case of a centrifuge, V is the tangential velocity of the sensor ECM with respect to the surface of the earth, and $\vec{\Omega}$ is the earth rotation rate.

The horizontal component of this cross product is along the arm and is given by

$$a_b = 2\omega R \Omega_v$$

where Ω_v is the local vertical component of the earth rotation rate, closely approximated by

$$\Omega_v = |\vec{\Omega}| \sin \lambda$$

The Coriolis acceleration consequently has a value proportional to ω and either aids or opposes the centrifuge acceleration depending on rotation sense and latitude λ .

Alternatively, the same algebraic result may be derived by assuming that the vertical component of the earth rotation rate simply adds to or subtracts from the centrifuge rate ω .

Since the centrifuge acceleration is

$$a_c = R \omega^2$$

then

$$\frac{a_b}{a_c} = 2 \frac{\Omega_v}{\omega}$$

As an example, the value of the Coriolis acceleration for a level centrifuge turning counter clockwise at 1 r/s at 40° north latitude is 15 ppm and adds to the centripetal acceleration.

Since the magnitude varies linearly with ω , the Coriolis acceleration varies as the square root of centrifuge acceleration. Most centrifuge tests are made with the same rotation direction for both positive and negative accelerations; therefore, the Coriolis effect is that of an odd function in the data analysis. The effect can be calculated and compensated for by adding it vectorially to the estimated input data.

The vertical component of the cross product is along the rotation axis and is given by

$$a_v = 2 \omega R \Omega_h \cos(\omega t)$$

where

Ω_h is $|\vec{\Omega}| \cos \lambda$, the horizontal component of the earth rotation rate
 ω is the arm angle from north

An accelerometer with its IA mounted vertically will sense this periodic error, which can be eliminated by averaging over complete arm revolutions.

6.3.4 Instrumentation methods and errors

Average centrifuge angular velocity can be determined by measuring the time required for the centrifuge to rotate through a known angle. This step is most conveniently and accurately done by measuring the time required for a number of complete revolutions. The time is measured with a counter that has a precision time source as its input. The counter is triggered by an event marker that occurs once per centrifuge revolution. The accuracy of this measurement is limited only by the accuracy of the time source and the position repeatability of the event marker. A time source good to 1 μ s will produce an angular velocity uncertainty of 1 ppm when measuring a time of 1 s. A 1 variation in the location of the event marker will produce an error of 0.8 ppm in angular velocity in a ~~1-revolution~~ measurement [made over a single revolution](#). A greater concern in determining centrifuge angular velocity is ensuring that the average angular velocity is determined over the same time period that the accelerometer output is measured. This point is especially important if the centrifuge angular velocity is subject to large variations.

An alternate procedure for determining centrifuge angular velocity exists if the centrifuge is equipped with a continuous position readout such as a resolver or encoder. The time for the centrifuge to rotate through a known angle as given by the position readout can be determined in the same manner as was described above using the once-per-revolution event marker. The error sources are the same as when using the marker. However, the accelerometer sample period is no longer required to be a whole number of centrifuge revolutions, but can be any time, provided the position readout can be read at the start and end of the data sample.

6.4 Sensor temperature effects

6.4.1 Sensor self-heating

[Servoed sensors may also experience errors due to thermal gradients induced in the sensor by self-heating.](#) This error source is found primarily in servo force or torque [rebalance](#) accelerometers in which the power dissipated in the sensor varies as a function of input acceleration. Its precise form depends on the nature of the capture loop on the geometry of the sensor, and on its mechanical interfaces. [Certain capture loops, such as pulse torque servo amplifiers, mitigate this error because they use constant current across all accelerations.](#) This error source displays both a transient and a steady-state mode of behavior. [The fact that the error has both modes of behavior](#) is significant because standard centrifuge tests are nearly steady-state in nature, whereas the high acceleration levels that produce significant self-heating tend to be transient in actual applications. The result is that the response measured may not be appropriate to the end-use conditions, even though the applied acceleration and the measurement of the response may be perfectly accurate. Sensor thermal time constants may be on the order of minutes, whereas the duration of high accelerations in actual use may be only seconds. As a result, normal measurement periods, which are typically several minutes, may be inappropriate. The general form of the self-heating equations starts with the following temperature equation: