

An American National Standard

IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data

Sponsor
**Statistical Technical Committee
of the
IEEE Dielectrics and Electrical Insulation Society**

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Foreword

(This Foreword is not a part of ANSI/IEEE Std 930-1987, IEEE Guide for the Statistical Analysis of Electrical Insulation Voltage Endurance Data.)

This document is the first version of a guide to promote standardized methods for the statistical analysis of data from voltage endurance tests on electrical insulation. Simple methods to calculate confidence intervals have received special attention. We strongly encourage users of this guide to calculate confidence intervals for their test data, as this will help interested parties in evaluating the precision of the tests. Much work remains to be done in extrapolating the results of tests from accelerated conditions to normal operating stress (Section 6).

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CLAUSE	PAGE
1. Scope and References	1
1.1 Scope	1
1.2 References	1
2. Introduction	2
2.1 Steps Required to Analyze Voltage-Aging Data	2
3. Data Analysis	3
3.1 Extreme-Value Distributions	3
3.2 The Lognormal Distribution	4
3.3 Censored Data	5
3.4 Probability Graph Papers	5
4. Estimation of Distribution Parameters	9
4.1 Graphical Parameter Estimates	9
4.2 Accurate Parameter Estimates	12
4.3 Confidence Intervals for Parameters	14
4.4 Confidence Intervals for Percentage Failed	21
5. Comparison Tests	23
5.1 Rigorous Hypothesis Tests	23
5.2 Simplified Method to Compare Percentiles of Extreme Value Distributions	23
6. Failure Models	25
6.1 Inverse Power Model	26
6.2 Exponential Model	26
6.3 Fitting Data to Failure Models	26
Annex A (Informative) Program to Estimate Weibull and Gumbel Parameters	29
Annex B (Informative) Examples	31

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1. Scope and References

1.1 Scope

This guide describes, with examples, statistical methods to analyze the data for time-to-failure from *constant-stress* voltage endurance tests or breakdown voltage from *progressive-stress* tests on specimens or systems of electrical insulation. Methods to compare test data are also given. The methods are principally applied to data from tests on solid insulation, but may also apply to the analyses of data from tests on gas, liquid, and composite systems. The statistical methods discussed here do not take into consideration the physical mechanism of voltage aging. The methods assume that the only aging stress is alternating voltage of constant frequency. The methods may not apply if there is more than one aging stress. Methods to ascertain the short time withstand voltage or operating voltage of an insulation system are not presented in this guide. The mathematical techniques contained in this guide may not directly apply to the estimation of equipment life.

1.2 References

This guide shall be used in conjunction with the following publications:

- [1] IEEE Std 101-1972 (R 1980) , IEEE Guide for the Statistical Analysis of Thermal Life Test Data.¹
- [2] BROOKES, A. S. The Weibull Distribution: Effect of Length and Conductor Size of Test Cables. *Electra* no 33, p 49.
- [3] COCHRAN, W. G. and SNEDECOR, G. W. *Statistical Methods*. Iowa State, 1976, 6th ed.
- [4] DAKIN, T. W. and STUDNIARZ, S. A. The Voltage Endurance of Cast Epoxy Resins. 1978 *IEEE International Symposium on Electrical Insulation*. Philadelphia, PA: p 216.
- [5] DISSADO, L. A., FOTHERGILL, J. C., HILL, R. M., and WOLFE, S. V. Weibull Statistics in Dielectric Breakdown, Theoretical Basis, Applications, and Implications. *IEEE Transactions*, EI-19, June 1984, p 227.
- [6] ENDICOTT, H. S. and WEBER, K. H. Area Effect and its Extremal Basis for the Electric Breakdown of Transformer Oil. *AIEE Transactions* pt III, vol 75, June 1956, p 371.

¹ IEEE publications are available from IEEE Service Center, 445 Hoes Lane, PO Box 1331, Piscataway, NJ 08855-1331.

- [7] FURTIG, K. W., MANN, N. R., and SCHEURER, E. W. Confidence Bounds and a New Goodness-of-Fit Test for the Two-Parameter Weibull or Extreme-Value Distributions with Tables for Censored Samples of Size 3(1)25. *Aerospace Research Laboratories Report*, ARL 71-0077, Wright-Patterson AFB, 1971.
- [8] GLADSTEIN, D., NELSON, W., and SCHMEE, J. Confidence Limits for Parameters of a Normal Distribution from Singly Censored Samples Using Maximum Likelihood. *Technometrics*, May 1985.
- [9] HAHN, G. J. and NELSON, W. A Comparison of Methods for Analyzing Censored Life Data to Estimate Relationships Between Stress and Product Life. *IEEE Transactions on Reliability*, R-23, April 1974, p 2.
- [10] KIERSZTYN, S. E. Formal Theoretical Foundation of Electrical Aging of Dielectrics. *IEEE Transactions (Power Apparatus and Systems)*, PAS-100, Nov 1981, p 4333.
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- [13] NATRELLA, M. G. Experimental Statistics. *NBS Handbook*, 91, 1966, pp 2–14.
- [14] NELSON, W. *Applied Life Data Analysis*. New York: John Wiley and Sons, 1982.
- [15] ROSEN, H., STONE, G. C. Some Graphical Techniques for Estimating Weibull Confidence Intervals *IEEE Transactions on Reliability*, R-33, Dec 1984, p 362.
- [16] SIMONI, L. *Voltage Endurance of Electrical Insulation*. Bologna: Technoprint Publ, Dec 1974.

2. Introduction

2.1 Steps Required to Analyze Voltage-Aging Data

2.1.1

Select a probability distribution that adequately represents the test data. Data on solid insulation tests are most often represented by the (two-parameter) Weibull distribution [5]² whereas the Gumbel (smallest extreme-value) distribution is often employed for liquids [6]. The lognormal distribution may also be relevant. The adequacy of the assumed distribution should always be tested, since another distribution could possibly fit better, unless experience or theory suggests a particular distribution.

2.1.2

Estimate the distribution parameters and their confidence intervals. The confidence interval objectively quantifies the uncertainty in a parameter estimate. Generally the more data, the narrower are these confidence intervals.

2.1.3

Perform a hypothesis test if data are to be compared. A hypothesis test is an objective method of ascertaining if the probability distributions of two data sets differ by a convincing amount, or whether such a conclusion is possible due to the inherent variability of the test data.

² The numbers in brackets correspond to those of the references in 1.2.

2.1.4

Estimate the life distribution at normal operating stress by extrapolating the accelerated aging test data. This is particularly prone to error and should be done with care, as the mathematical model, test conditions, etc, may be in error. Any conclusions derived from statistical analysis of test data are rigorously valid only for the test conditions and particular specimens used. In general, voltage endurance data are best used to make comparisons between different groups of specimens at the accelerated test levels.

3. Data Analysis

This section presents the Weibull and Gumbel probability distributions. These are the distributions most often employed to represent breakdown times, or breakdown voltages of insulation, or both. The lognormal probability distribution is also described since it is sometimes used in data analysis, especially in connection with failure models (Section 6). Also described are some of the graphical methods for statistical analysis.

3.1 Extreme-Value Distributions

The Weibull and Gumbel distributions are two types of extreme-value distributions. The extreme-value distributions have been found to represent a wide variety of failure data [2], [11], and [14].

Furthermore, the effect of the size of the test specimens (that is, thickness, area, or volume, or a combination of these) on life or breakdown voltage can easily be modeled with these distributions [2] and [14].

Statistical techniques commonly used for estimating normal distribution quantities do not apply to the extreme-value distributions. In particular, the estimates of the mean standard deviation, and the confidence intervals and hypothesis tests for the normal distribution do not apply to extreme-value distributions.

3.1.1 The Weibull Distribution

The Weibull distribution is most often used to represent both the times-to-breakdown (obtained from a constant-stress test) and the breakdown voltages (obtained from a progressive-stress test) in tests on solid insulation. The Weibull cumulative distribution function for the population fraction below x is

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad x \geq 0 \quad (1)$$

where

- α = scale parameter and is positive
- β = shape parameter and is positive
- x = random variable, usually the time to breakdown or the breakdown voltage
- $F(x)$ = probability of failure at voltage or time (less than or equal to) x

The probability of failure $F(x)$ is zero at $x = 0$. The probability of failure rises continuously as x increases. As the time or voltage increases, the probability of failure approaches certainty, that is, $F(\infty) = 1$.

The scale parameter α represents the time (or voltage) for which the failure probability is 0.632. It is analogous to the mean of the normal distribution.³ The units of α are the same as x , that is, voltage, electric stress, or time.

³ Note that a mean and a standard deviation can be determined for the Weibull distribution. These quantities, however, are seldom used.

The shape parameter β is a measure of the spread of the failure times or voltages. The larger β is, the smaller is the range of breakdown voltages or times.

The two-parameter Weibull distribution of Eq 1 is a special case of the three-parameter Weibull distribution that has the cumulative distribution function

$$\begin{aligned} F(x) &= 1 - e^{-[(x-\gamma)/\alpha]^\beta}, x \geq \gamma \\ &= 0, x < \gamma \end{aligned} \quad (2)$$

The additional term γ is called the location parameter. $F(x) = 0$ for $x < \gamma$, that is, the probability of failure for $x < \gamma$ is zero. The value of γ is usually assumed to be zero in practical applications.

3.1.2 The Gumbel Distribution

The Gumbel distribution is most often used to represent the breakdown voltages of liquid and solid insulations [6]. The cumulative Gumbel (smallest extreme-value) distribution function for the population fraction below y is

$$G(y) = 1 - e^{-e^{-(y-u)/b}}, -\infty \leq y \leq \infty \quad (3)$$

where

- u = location parameter and may have any value
- b = scale parameter and is positive
- y = random variable, usually the breakdown voltage
- $G(y)$ = probability of failure at voltage or time $\leq y$

The Gumbel distribution is asymmetrical and can have a physically impossible finite probability of breakdown for $y < 0$. This distribution is also called the smallest extreme-value (that is, weakest-link) distribution. If y is voltage, then the units of u and b are also voltage.

The Gumbel distribution is closely related to the Weibull distribution. That is, if x has a Weibull distribution then $y = \ln(x)$ has a Gumbel distribution

where

$$\begin{aligned} u &= \ln(\alpha) \\ b &= 1/\beta \end{aligned} \quad (4)$$

Estimation techniques (see Section 4) for one distribution (Gumbel or Weibull) apply to the other if the transformation equation, Eq 4, is utilized.

3.2 The Lognormal Distribution

The lognormal distribution has sometimes been used to represent failure data from insulation systems, but it has not been used nearly as often as the extreme-value distributions in 3.1. However, since this probability distribution is a simple logarithmic transformation of the well-known normal or Gaussian distribution, methods for data analysis are available in all standard statistical references. The probability density function of the lognormal distribution is

$$f(z) = (1/\sqrt{2\pi}\sigma)e^{-(z-u)^2/2\sigma^2} \quad (5)$$

where

z	$= \log x$
x	$= \text{breakdown voltage or time-to-failure}$
μ	$= \text{logarithmic mean}$
σ	$= \text{logarithmic standard deviation}$

The cumulative density function is the integral of the above. There is no closed-form equation for the integral. Values of the distribution are found in [3], [13], or can be obtained from statistical calculators or computer programs.

3.3 Censored Data

Censoring of data occurs when n specimens are started on the test together and the failures of only r ($< n$) are observed.

Censoring is encountered mainly with constant stress tests, where the data are analyzed or the test is terminated before all the specimens fail. Censoring can also occur with progressive stress tests where flashovers or spurious breakdowns occur on specimens that would otherwise survive to high stresses. Censoring can occur by plan or by accident in many insulation tests, and this factor must be taken into account in the data analysis.

There are also other kinds of censoring, such as progressive and multiple censoring. These types of censoring result when failed specimens are disqualified because of specimen failure by spurious mechanisms. Analytical treatment of such data is difficult. See [11] and [14].

3.4 Probability Graph Papers

Special graph papers for the Weibull and Gumbel distributions are shown in Figs 1, 2, 4, and 5 respectively. One axis of the graph paper is a nonlinear cumulative probability scale (This scale is the same for the Weibull and the Gumbel papers). The other axis is the data axis; logarithmic for the Weibull distribution and linear for the Gumbel distribution. This axis is for the failure times or voltages. The axes are scaled so that plotted data from the two-parameter Weibull and Gumbel distributions tend to follow a straight line. Such graph paper is commercially available.⁴

Similar graph paper is also available for the lognormal distribution. The probability scale is the same as for normal distribution graph paper. The other axis however, is logarithmic. If lognormal paper is not available, normal paper can be employed by taking logarithms of the data and plotting them on the linear scale.

The primary uses of the probability plotting paper are to assess the adequacy of the assumed distribution to represent the test data and to obtain rough parameter estimates. Linearity on probability paper is the only easy method to assess which of the Weibull, Gumbel, or lognormal distributions best represents the data. All test data should first be plotted to check that the assumed distribution is adequate.

3.4.1 Weibull Probability Paper

Samples of Weibull probability paper are shown in Figs 1 and 2. To use this paper, order the failure times or voltages from smallest to largest. The most commonly used approximation of the cumulative probability F_i is

$$F_i = \left(\frac{i}{n+1} \right) \cdot 100\% \quad (6)$$

The Weibull example data in Table 1 are plotted in Figs 1 and 2. Data from unfailed specimens should not be plotted, although they should be included in n .

⁴ TEAM, Box 25, Tamworth, NH 03886.

Table 1—Breakdown Data on Epoxy Specimens

Specimen No <i>i</i>	<i>F_i</i> %	Breakdown Time <i>t_i</i> (h)
1	10	15.3
2	20	30.3
3	30	48.5
4	40	89.4
5	50	90.4
6	60	105.7
7	70	144.9
8	80	—
9	90	—

If a reasonably straight line is produced by the plotted data, then it may be reasonable to assume that the data are adequately represented by the Weibull distribution. Some random deviations from a straight line are normally expected. If, however, there is a consistent departure from a straight line (for example, curvature), then another distribution may fit the data better. Figure 3 shows plots on Weibull probability paper of other distribution functions.

3.4.2 Gumbel Probability Paper

A sample of Gumbel probability paper is shown in Figs 4 and 5. The n failure voltages (or times) are ordered from smallest to largest, and the i^{th} smallest voltage y_i can be assigned the plotting position

$$G_i = \left(\frac{i}{n+1} \right) \cdot 100\% \quad (7)$$

The Gumbel data in Table 2 are plotted in Figs 4 and 5. If a reasonably straight line is produced by the plotted data on the Gumbel paper, then the Gumbel distribution adequately represents the failure data. Examples of data from other distributions are plotted on Gumbel paper in Fig 6.

3.4.3 Lognormal Probability Paper

A sample of lognormal probability paper is shown in Fig 7, and contains the data presented in Table 3. The failure data are ordered from smallest to largest, and the i^{th} smallest voltage x_i can be assigned the plotting position

$$F_i = \left(\frac{i}{n+1} \right) \cdot 100\% \quad (8)$$

If a reasonably straight line is produced by the plotted data on the lognormal paper, then this distribution adequately represents the data.

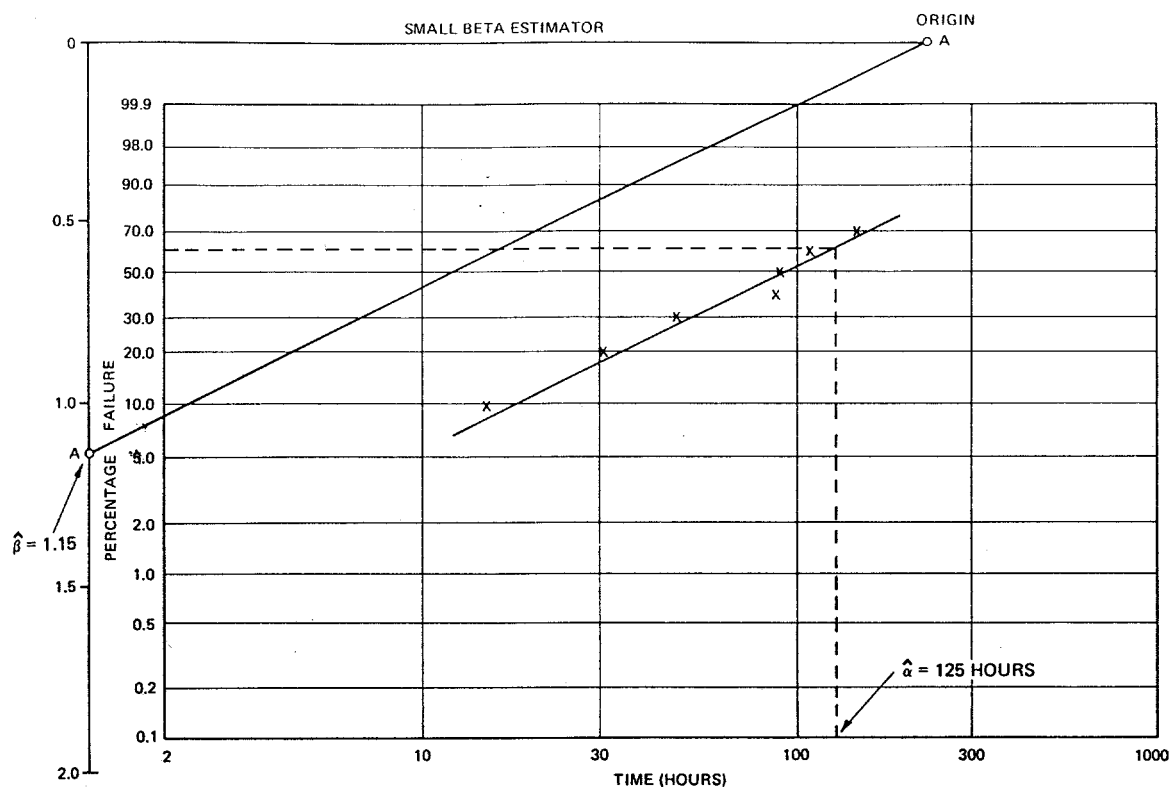


Figure 1—Weibull Plot of Failure Data With Line Fit By Eye