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# **TECHNICAL MEMORANDUM: MEASUREMENT UNCERTAINTY FOR LIGHTING EQUIPMENT CALIBRATION USING INTEGRATING SPHERES**

**AN AMERICAN NATIONAL STANDARD**



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MEASUREMENT UNCERTAINTY FOR  
LIGHTING EQUIPMENT CALIBRATION  
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Publication of this Technical Memorandum  
has been approved by IES.  
Suggestions for revisions  
should be directed to IES.

**Prepared by  
The IES Testing Procedures Committee**



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## 1.0 Introduction and Scope

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### 1.1 Introduction

Uncertainty is inherent in every measurement. To truly understand the value of a measurement, the uncertainty of such should be analyzed. A critical step of this analysis is to know the uncertainty contributed by the instruments used to make the measurement. This Technical Memorandum provides a detailed and step-by-step method for determining uncertainty of certain calibration procedures common to instruments used in lumen measurement laboratories. This document uses the structure proposed in the *Guide to the Expression of Uncertainty in Measurement* (GUM) for its recommendations. For each calibration procedure presented in this document, a mathematical model is developed, estimates of each input quantity are discussed, and guidance is provided for the evaluation of the standard uncertainty of each input estimate. With this information, the combined standard uncertainty is calculated, effective degrees of freedom are calculated, and finally, the expanded uncertainty is reported.

The intent of this document is to provide a common approach to the uncertainty analysis for calibration of lumen measuring equipment (integrating spheres) with standard incandescent lamps, including halogen, that have been assigned values of total luminous flux and/or total spectral radiant flux. In addition, uncertainty budget templates are provided for calibration of incandescent (including halogen) working standard lamps that are virtually identical to the standard lamps they are compared against. These templates are starting points for uncertainty analysis. A laboratory applying these templates is expected to estimate each input quantity and the standard uncertainty of each input quantity, except where industry standard values are specifically noted in this document. Additional input quantities may be required based on specific situations that arise in the uniqueness of each laboratory measurement system. Future documents will cover topics such as uncertainty determination for colorimetric calibrations, and photometric and colorimetric measurements for testing general lighting products.

### 1.2 Scope

This document provides templates for the analysis of measurement uncertainty for the photometric calibration of integrating sphere systems using standard incandescent lamps, as well as the creation of working standard lamps using integrating sphere systems that are effectively identical to the primary standard lamps. It does not provide templates for the analysis of measurement uncertainty for colorimetric calibration of working standard lamps or for the testing of general lighting products.

## 2.0 Normative References

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JCGM 100-2008, *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement* (GUM). Bureau International des Poids et Mesures (BIPM); Sep 2008. Free download: [www.bipm.org/en/publications/guides/gum.html](http://www.bipm.org/en/publications/guides/gum.html).

ANSI/IES LS-1-20, *Nomenclature and Definitions for Illuminating Engineering*. New York: Illuminating Engineering Society; 2017. Free access from [www.ies.org/standards/definitions/](http://www.ies.org/standards/definitions/)

## 3.0 Definitions

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### 3.1 array spectrometer

In this document, an instrument that simultaneously detects and disperses light with respect to wavelength. The CIE S 017/E:2011 (ILV)<sup>1</sup> definition for *array detector* (17-57: “physical arrangement of detector elements that are able to sense the output e.g., of a polychromator. *NOTE:* In some cases, optical components may form a part of the array, as in, for example, a fiber optics faceplate coupled array detector”) is close to the definition used in this document.

### 3.2 combined standard uncertainty

The standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities; it is equal to the positive square root of a sum of terms, the term being the variances or covariances of these quantities weighted according to how the measurement result varies with changes in these quantities. [VIM,<sup>2</sup> Section 2]

### 3.3 coverage factor

A numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty. [2.3.6, VIM<sup>2</sup>]

### 3.4 expanded uncertainty

The quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. [2.3.5, VIM<sup>2</sup>]

*Note 1:* The fraction may be viewed as the coverage probability or level of confidence of the interval.

*Note 2:* To associate a specific level of confidence with the interval defined by the expanded uncertainty requires explicit or implicit assumptions regarding the probability distribution characterized by the measurement result and its combined standard uncertainty. The level of confidence that may be attributed to this interval can be known only to the extent to which such assumptions may be justified.

### 3.5 photometer

An instrument for measuring photometric quantities such as luminance, luminous intensity, luminous flux or illuminance [8.4.1, LS-1-20]. In this document, *photometer* will refer to a physical photometer with reversible changes – an instrument containing a physical receptor and associated filters that is calibrated so as to read photometric quantities directly [8.4.1.3, LS-1-20; see **Section 2 Normative References**]. A *physical receptor* is defined as a device in which incident radiation produces an observable physical change, including but not limited to the generation of electrical current or voltage and the darkening of photographic film. Reversible changes are used in measuring flux and flux-related quantities. Irreversible changes are used in measuring exposure [8.4.1.2, LS-1-20]. The CIE ILV<sup>1</sup> definitions are similar: *photometer* (17-909) – “instrument for measuring photometric quantities”; *detector (of optical radiation)* (17-296) – “device in which incident optical radiation produces a measurable physical effect”; and *output (of a detector of optical radiation)* (17-856) – “physical quantity yielded by a detector in response to an optical input. *NOTE:* This quantity, usually electrical, may for example be current, voltage or a change in resistance; the output may also be chemical, as in a photographic film or an actinometer, or mechanical as in a Golay detector.”

### 3.6 primary standard (lamp)

A light source by which the unit of light is established and from which the values of other standards are derived. This order of standard also is designated as the *national standard* [8.1.1, LS-1-20]. This is very similar to the CIE ILV<sup>1</sup> definition

for *standard lamp* (17-1256): “lamp used as a reference in photometric or spectroradiometric measurements for which the calibration is traceable to a primary photometric or spectroradiometric standard.”

### 3.7 relative uncertainty

The uncertainty of measurement divided by the result of a measurement.

### 3.8 secondary standard (lamp)

A stable light source calibrated directly or indirectly by comparison with a primary standard. This order of standard also is designated as a *reference standard* [8.1.2, LS-1-20]. This is very similar to the CIE ILV<sup>1</sup> definition for *secondary standard lamp* (17-1149): “lamp intended to be used as secondary photometric standard.”

### 3.9 standard deviation (experimental)

For a series of  $n$  measurements of the same measurand, the quantity  $s(q_k)$  characterizing the dispersion of the results and given by the formula:

$$s(q_k) = \sqrt{\frac{\sum_{j=1}^n (q_j - \bar{q})^2}{n-1}},$$

with  $q_k$  being the result of the  $k$ th measurement and  $\bar{q}$  being the arithmetic mean of the  $n$  results considered. [3.8, VIM<sup>2</sup>]

*Note 1:* Considering the series of  $n$  values as a sample of a distribution,  $\bar{q}$  is an unbiased estimate of the mean  $\mu_q$ , and  $s(q_k)^2$  is an unbiased estimate of the variance,  $\sigma^2$ , of that distribution.

*Note 2:* The expression  $s(q_k)/\sqrt{n}$  is an estimate of the standard deviation of the distribution of  $\bar{q}$  and is called the *experimental standard deviation of the mean*.

### 3.10 standard uncertainty

The uncertainty of the result of a measurement expressed as a standard deviation. [2.3.1, VIM<sup>2</sup>]

### 3.11 uncertainty (of measurement)

The parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

*Note 1:* The parameter may be, for example, a standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence.

*Note 2:* Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information.

*Note 3:* It is understood that the result of the measurement is the best estimate of the value of the measurand, and that all components of uncertainty, including those arising from systematic effects, such as components associated with corrections and reference standards, contribute to the dispersion.

### 3.12 working standard (lamp)

A standardized light source for regular use in photometry [8.1.3, LS-1-20]. This is very similar to the CIE ILV<sup>1</sup> definition for *working standard lamp* (17-1444): “lamp intended to be used as a working photometric standard.”

## 4.0 Guide to the Expression of Uncertainty in Measurement (GUM): A Summary

### 4.1 Introduction

The *Guide to the Expression of Uncertainty in Measurement* (GUM; see **Section 2.0 Normative References**) provides a procedural method for the analysis of uncertainty. These concepts will be used in this document for the photometric calibration of integrating spheres using standard incandescent lamps, as well as the calibration using integrating spheres of *working standard lamps* that are effectively identical to the standard lamps. The procedure has eight steps:

1. Define the measurand,  $Y$ , and the model (mathematical equation) to calculate it.
2. Determine an estimate,  $x_i$ , for each input parameter,  $X_i$ .
3. Evaluate the standard uncertainty,  $u(x_i)$ , for each input parameter,  $X_i$ .
4. Calculate an estimate,  $y$ , of the measurand,  $Y$ .
5. Determine the sensitivity coefficient,  $\partial y / \partial x_i$ , for each input parameter,  $X_i$ .
6. Determine the combined standard uncertainty,  $u_c(y)$ .
7. Calculate the coverage factor,  $k$ , based on the degrees of freedom,  $\nu$ , and determine the expanded uncertainty,  $U(y)$ .
8. Report the measurand estimate,  $y$ , the expanded uncertainty,  $U(y)$ , and the coverage factor,  $k$ .

These steps will be contextually expounded on in the following sections.

### 4.2 Description of the Model Equation and Input Parameter Estimation

Per the GUM, a mathematical equation is required to describe the measurement process, considering the various factors that contribute uncertainty to the measurement. The calculated quantity or the result of the mathematical equation is the “measurand” or output variable. For the examples presented in this document, the measurand is the calibration factor required for the various photometric integrating spheres.

The relationship between the measurand,  $Y$ , and the input quantities,  $X_i$ , on which  $Y$  depends, is written mathematically as:  $Y = f(X_1, X_2, \dots, X_N)$ . The function  $f$  should contain every quantity needed to calculate the measurand, including all correction factors that can contribute a significant component of uncertainty to the result of the measurement. An estimated value,  $x_i$ , is determined for each input quantity,  $X_i$ , either based on the statistical analysis of a series of observations or by other means, which may include data from a calibration certificate, manufacturer specification, or published literature, among other sources of information.

### 4.3 Standard Uncertainty Evaluation

For each input estimate,  $x_i$ , an evaluation of the standard uncertainty,  $u(x_i)$ , is required. Per the GUM, the evaluation methods used to determine the magnitude of the standard uncertainty may be grouped into two categories: those that are evaluated by statistical methods, and those that are evaluated by other means. The evaluation of standard uncertainty by the statistical analysis of a series of observations is termed a *Type A* evaluation of uncertainty. The evaluation of uncertainty by means other than the statistical analysis of a series of observations is termed a *Type B* evaluation of uncertainty.

**4.3.1 Type A Uncertainty Evaluation.** A Type A standard uncertainty evaluation is obtained from a probability density function derived from an observed frequency distribution. A Type A analysis involves making repeated direct measurements of a quantity, such as a voltage or a current from a photometer. The best available estimate,  $x_i$ , for an input quantity that varies randomly and where  $n$  independent observations,  $X_{i,k}$ , have been measured under the same conditions, is the arithmetic mean,  $\bar{X}_i$ :

$$x_i = \bar{X}_i = \frac{1}{n} \sum_{k=1}^n X_{i,k} \quad (4.1)$$

The standard uncertainty,  $u(x_i)$ , to be associated with  $x_i$  is the estimated standard deviation of the mean:

$$u(x_i) = s(\bar{X}_i) = \sqrt{\frac{1}{n(n-1)} \sum_{k=1}^n (X_{i,k} - \bar{X}_i)^2} \quad (4.2)$$

A least squares fit to a set of measured data to determine the parameters of the function and the standard deviation of the parameters is a Type A uncertainty. For example, the calibration of a 6.5-digit voltmeter with a calibrated 8.5-digit voltmeter and precision voltage supply involves a linear fit, which is described in **Annex A**. In addition, **Annex A** discusses the uncertainty associated with using the function resulting from the fit.

**4.3.2 Type B Uncertainty Evaluation.** Type B uncertainty evaluations are determined by other than statistical methods. In these cases, the standard uncertainty is often provided in calibration certificates, published literature, and manufacturer specifications or best estimates based on experience. Type B standard uncertainty evaluations may also occur when a Type A uncertainty evaluation cannot be calculated due to insufficient information.

For example, an input quantity where the value is estimated from a rectangular or uniform probability distribution with a lower limit of  $a_-$  and an upper limit of  $a_+$  may be considered. A digital display where the last digit does not vary in time is an example of a rectangular probability distribution for the standard uncertainty. Variation in the measurement exists, but the display does not have enough digits to resolve this variation. In this case, the input quantity is estimated by:

$$x_i = (a_- + a_+)/2, \quad (4.3)$$

and the standard uncertainty,  $u(x_i)$ , associated with  $x_i$  is:

$$u(x_i) = (a_+ - a_-)/(2\sqrt{3}), \quad (4.4)$$

where  $a_+$  is equal to the display plus half the last digit of resolution, and  $a_-$  is equal to the display minus half the last digit of resolution. The GUM provides other examples of Type B evaluations, based on other probability distributions.

Converting the tolerance of an AC-power analyzer to standard uncertainty is another example that uses a uniform distribution. The specifications of a typical AC-power analyzer states that the tolerance for AC voltage measurements at 60 Hz is 0.1% of the reading plus 0.1% of the range plus 10 mV. Measuring a 120-V lamp on a range of 150 V gives a tolerance of  $\pm 0.28$  V for a reading of 120.00 V. The tolerance is modeled by a uniform distribution, where  $a_- = 120.00$  V - 0.28 V = 119.72 V, and  $a_+ = 120.00$  V + 0.28 V = 120.28 V; therefore,  $u(x_i) = 0.16$  V.

#### 4.4 Sensitivity Coefficients and Combined Standard Uncertainty

To determine the contribution of each component to the combined standard uncertainty,  $u_c(y)$ , the standard uncertainty,  $u(x_i)$ , is multiplied by the sensitivity coefficient,  $\frac{\partial y}{\partial x_i}$ . The sensitivity coefficient is described as the influence of the uncertainty component on the measurand,  $y$ ; therefore, if a component changes in value, the calculation of  $y$  is affected. The partial derivative of the model equation for each component of uncertainty describes

this dependence. (**Annex B** provides a brief explanation of the determination of partial derivatives.) Since the partial derivative is the slope of the model equation with a given set of input estimates, another method of determining the sensitivity coefficient is to determine the change in the measurand with respect to the uncertainty component. Using this method, a partial derivative, or sensitivity coefficient, can be calculated by using:

$$\frac{\partial y}{\partial x_i} = \frac{f[x_1, x_2, \dots, x_i + u(x_i), \dots, x_n] - f[x_1, x_2, \dots, x_i - u(x_i), \dots, x_n]}{2[u(x_i)]} \quad (4.5)$$

**Annex C** provides a demonstration of using **Equation 4.5** instead of partial derivatives.

The combined standard uncertainty  $u_c(y)$ , is determined using:

$$u_c(y) = \sqrt{\sum_{i=1}^m \left( \frac{\partial y}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^m \frac{\partial y}{\partial x_i} \frac{\partial y}{\partial x_j} u(x_i) u(x_j) r(x_i, x_j)} \quad (4.6)$$

The combined standard uncertainty may also be expressed as a relative combined standard uncertainty by dividing  $u_c(y)$  by  $y$ . The correlation coefficient in **Equation 4.6**,  $r(x_i, x_j)$ , is the mathematical dependency between two components of uncertainty, and ranges from -1 to +1. As an example, a correlation exists between the increase in luminous flux and the correlated color temperature when the lamp current is increased. The templates provided in this document have been designed to eliminate correlations, such that the contributions due to correlations are zero; i.e.,  $r(x_i, x_j) = 0$  in **Equation 4.6**; therefore, **Equation 4.6** reduces to the square root of the sum of the squares:

$$u_c(y) = \sqrt{\sum_{i=1}^m \left( \frac{\partial y}{\partial x_i} \right)^2 u^2(x_i)} \quad (4.7)$$

Calibrating a sphere's responsivity using four lamps purchased from a national metrology institute is an example of correlation. A laboratory may use four lamps instead of one lamp to calibrate the measurement system because the average result has a tendency to track the scale better. (**Section 5.1.9** shows a more complete example.) The model equation for using four lamps to calibrate an integrating sphere is:

$$y = C_\phi = \frac{C_1 + C_2 + C_3 + C_4}{4}, \quad (4.8)$$

where  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$  are the calibration factors determined for each lamp. Without considering correlation, the combined standard uncertainty might be calculated as:

$$u_c(C_\phi) = \sqrt{\left(\frac{1}{4}\right)^2 u^2(C_1) + \left(\frac{1}{4}\right)^2 u^2(C_2) + \left(\frac{1}{4}\right)^2 u^2(C_3) + \left(\frac{1}{4}\right)^2 u^2(C_4)}, \quad (4.9)$$

where  $u(C_1)$ ,  $u(C_2)$ ,  $u(C_3)$ , and  $u(C_4)$  are the standard uncertainty for each calibration factor determined with each lamp. For example, if,  $u(C_1) = u(C_2) = u(C_3) = u(C_4) = 0.50\%$ , then the combined standard uncertainty would be calculated as  $u_c(C_\phi) = 0.25\%$ , which would be incorrect. The four lamps are highly correlated; therefore,  $r = 1$  between the lamps. The uncertainty budget is predominately Type B uncertainty components, which cannot be averaged or reduced with multiple determinations. The proper combined standard uncertainty calculation for this situation is **Equation 4.10**, for which  $u_c(C_\phi) = 0.50\%$ .

$$u_c(C_\phi) = \sqrt{\begin{aligned} &\left(\frac{1}{4}\right)^2 u^2(C_1) + \left(\frac{1}{4}\right)^2 u^2(C_2) + \left(\frac{1}{4}\right)^2 u^2(C_3) + \left(\frac{1}{4}\right)^2 u^2(C_4) + \\ &2 \frac{\partial C_\phi}{\partial C_1} \frac{\partial C_\phi}{\partial C_2} u(C_1) u(C_2) r(C_1, C_2) + 2 \frac{\partial C_\phi}{\partial C_1} \frac{\partial C_\phi}{\partial C_3} u(C_1) u(C_3) r(C_1, C_3) + \\ &2 \frac{\partial C_\phi}{\partial C_1} \frac{\partial C_\phi}{\partial C_4} u(C_1) u(C_4) r(C_1, C_4) + 2 \frac{\partial C_\phi}{\partial C_2} \frac{\partial C_\phi}{\partial C_3} u(C_2) u(C_3) r(C_2, C_3) + \\ &2 \frac{\partial C_\phi}{\partial C_2} \frac{\partial C_\phi}{\partial C_4} u(C_2) u(C_4) r(C_2, C_4) + 2 \frac{\partial C_\phi}{\partial C_3} \frac{\partial C_\phi}{\partial C_4} u(C_3) u(C_4) r(C_3, C_4) \end{aligned}} \quad (4.10)$$

#### 4.5 Expanded Uncertainty and Degrees of Freedom

For many measurements in photometry, a normal or Gaussian probability distribution characterizes the measurement result,  $y$ , and its combined standard uncertainty,  $u_c(y)$ ; therefore,  $u_c(y)$  defines an interval,  $y - u_c(y)$  to  $y + u_c(y)$ , about the measurement result,  $y$ , where the value of the measurand,  $Y$ , estimated by  $y$ , is believed to lie, with an approximate level of confidence of 68.3%, which is one standard deviation for a normal probability distribution. This is the case when the conditions of the Central Limit Theorem are met. Since the estimate,  $y$ , is not directly measured and is obtained from the estimate of many input quantities that have well-behaved probability distributions, the Central Limit Theorem holds true to an acceptable level.

Traditionally, industry has required a more encompassing measure of uncertainty that defines an interval about the measurement result,  $y$ , where the value of the measurand,  $Y$ , is confidently believed to lie. This coverage probability, or level of confidence, is 95% and used as a normative value in this document. The measure of uncertainty intended to meet this requirement is termed expanded uncertainty,  $U$ . The *expanded uncertainty*,  $U$ , is obtained by multiplying the combined standard uncertainty,  $u_c(y)$ , by a coverage factor,  $k$ .

Conventionally, a coverage factor of  $k = 2$  is used in industry. This coverage factor assumes a large number of degrees of freedom for the final probability distribution associated with the combined standard uncertainty,  $u_c(y)$ . In order to validate that a large number of degrees of freedom exist, the effective degrees of freedom,  $v_{\text{eff}}$ , are calculated using the Welch-Satterthwaite formula:

$$v_{\text{eff}} = \frac{u_c^4(y)}{\sum_{i=1}^N \frac{\left[ \frac{\partial y}{\partial x_i} u(x_i) \right]^4}{v_i}}, \quad (4.11)$$

where  $u_c(y)$  is the combined standard uncertainty,  $\frac{\partial y}{\partial x_i}$  is the sensitivity coefficient for each uncertainty component,  $u(x_i)$  is the standard uncertainty for each uncertainty component, and  $v_i$  is the degrees of freedom for each uncertainty component. For a Type A evaluation, the degrees of freedom are the number of data points minus the parameters used to describe the data. If  $n$  data points are estimated by a mean  $u(x_i)$ , the degrees of freedom is  $v_i = n - 1$ . If  $n$  data points are fit by a curve with  $m$  parameters, the degrees of freedom are  $v_i = n - m$ . The degrees of freedom for uncertainty components obtained using Type B evaluation ranges from  $v_i$  to infinity. The coverage factor,  $k$ , is approximated using the Student's  $t$  distribution,  $t_p(v_{\text{eff}})$ , for the required level of confidence from a table of values of  $t_p(v)$ , as shown in **Table 4.1**.<sup>3</sup> **Table 4.1** only shows the  $t_p(v)$  values for  $p = 95\%$ . The coverage factor may also be approximated using:

$$k_{0.95} = 2 \sqrt{0.95 + \frac{3.3}{(v_{\text{eff}} - 0.92)}}, \quad (4.12)$$

which is a fit to a series of results for the Student's  $t$  distribution with different degrees of freedom at a 95% confidence level, or using the EXCEL® function T.INV.2T(1- $p$ ,  $v$ ).

Using this information and the final uncertainty budget, a laboratory can then determine the ideal number of measurement repetitions for a Type A uncertainty component. One does not need to average any more data than is required to make the uncertainty component very small (on the order of 10% or less) compared to the combined standard uncertainty.

**Table 4.1. Value of  $t_p$  ( $\nu$ ) from the Student's  $t$  Distribution, for Degrees of Freedom  $\nu$  and  $p = 95\%$ .**

| Degrees of Freedom, $\nu$ | $t_{0.95}(\nu)$ | Degrees of Freedom, $\nu$ | $t_{0.95}(\nu)$ | Degrees of Freedom, $\nu$ | $t_{0.95}(\nu)$ |
|---------------------------|-----------------|---------------------------|-----------------|---------------------------|-----------------|
| 1                         | 12.71           |                           | 2.20            |                           | 2.06            |
| 2                         | 4.30            | 12                        | 2.18            | 30                        | 2.04            |
| 3                         | 3.18            | 13                        | 2.16            | 35                        | 2.03            |
| 4                         | 2.78            | 14                        | 2.14            | 40                        | 2.02            |
| 5                         | 2.57            | 15                        | 2.13            | 45                        | 2.01            |
| 6                         | 2.45            | 16                        | 2.12            | 50                        | 2.01            |
| 7                         | 2.36            | 17                        | 2.11            |                           |                 |
| 8                         | 2.31            | 18                        | 2.10            | 100                       | 1.984           |
| 9                         | 2.26            | 19                        | 2.09            |                           |                 |
| 10                        | 2.23            | 20                        | 2.09            | $\infty$                  | 1.960           |

## 5.0 Calibration of Integrating Spheres

### 5.1 Calibration Using a Total Luminous Flux Standard Lamp

**5.1.1 Model for an Integrating Sphere with Photometer.** The scope of this uncertainty budget is the calibration of an integrating sphere measurement system utilizing a photometer as the detection device and an incandescent (including halogen) lamp as the standard lamp calibrated for total luminous flux (lumens) at a particular operating current. The quantity to be determined is the photometer calibration factor,  $C_\phi$ , in units of signal per lumen, and its associated expanded uncertainty,  $U(C_\phi)$ . In the example presented in this section, the photometer calibration factor is referenced to an empty sphere (sphere with the standard lamp removed) and Illuminant A as the source correlated color temperature.

The topic of calibrating secondary or working standard lamps is discussed in **Section 6**.

The model equation describing how to calculate, and the uncertainty components affecting the determination of the sphere calibration factor,  $C_\phi$ , is written as:

$$C_\phi = \frac{y_r - y_{r_d}}{\Phi_r} \cdot \left( \frac{J_r \cdot R}{U_r \cdot c_U} \right)^{m_{J_r}} \cdot \frac{y_E - y_{E_d}}{y_{X_r} - y_{X_{r_d}}} \cdot A_r \cdot C_{f_r}, \quad (5.1)$$

where subscript **r** refers to a standard lamp property, and:

$C_\phi$  **sphere calibration factor:** The measurand, which is in units of signal (volt, amps, or lumens) per lumen.

$\Phi_r$  **total luminous flux of the standard lamp:** The total luminous flux (in lumens) for the standard lamp is provided on the calibration certificate. This is a Type B uncertainty because the source of the standard uncertainty information is a calibration certificate.

In this example, the calibration certificate states 2,800.0 lm, with an expanded uncertainty ( $k = 2$ ) of 16.4 lm. Because the coverage factor is equal to 2, the standard uncertainty is the expanded uncertainty divided by 2, or 8.2 lm. These values are recorded in Line 1 of **Table 5.2**.

$y_r$  **sphere photometer signal:** The photometer signal is a photocurrent, which may be measured using various types of equipment. The photocurrent may be measured with a calibrated ammeter. The repeatability and the calibration uncertainty of the ammeter become the standard uncertainty for this parameter.

A second method uses a transimpedance amplifier to convert the photocurrent to a voltage. The transimpedance amplifier and the voltmeter are calibrated and have certificates with stated uncertainties that need to be included along with the repeatability of the measured signal.

When the user does not have access to the measurement process, a photometer with software reporting a lumen value as the raw signal is an acceptable method. The repeatability of the measurement becomes the standard uncertainty.

To determine the signal and standard uncertainty for this component, 10 measurements are typically made of the illuminated standard lamp. For this example, the signal is the average of the 10 measurements, which is  $y_r = 1.7652$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_r) = 0.0007$  nA. This is a Type A uncertainty, with degrees of freedom equal to  $10 - 1 = 9$ . The signal and standard uncertainty are recorded in Line 2 of **Table 5.2** (see **Section 5.1.4**).

For integrating sphere systems using an array spectrometer as the detection device, the reader is referred to **Section 5.1.9** for a detailed discussion of the uncertainty of the signal component.

$y_{r_d}$  **sphere photometer dark signal:** The photometer signal measured when the standard lamp is mounted in the sphere but not illuminated. To determine the dark signal and standard uncertainty for this component, a user would typically run 10 measurements of the non-illuminated standard lamp. For this example, the signal is the average of the ten measurements, which is  $y_{r_d} = 0.0002$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_{r_d}) = 0.0002$  nA. This is a Type A uncertainty, with degrees of freedom equal to 9. The signal and standard uncertainty are recorded in Line 3 of **Table 5.2**.

$U_r$  **lamp current measurement:** The lamp current measurement for this model is done by measuring the voltage across a calibrated shunt resistor,  $R$ . This is a Type A uncertainty because several measurements are collected. In this example, 30 measurements are made, integrated over 10 power line cycles. The average of the 30 measurements is 0.582743 V and the standard deviation is 0.000015 V, which is the standard uncertainty,  $u(U_r)$ . There are 29 degrees of freedom for this measurement ( $30 - 1$ ). The average, standard uncertainty and degrees of freedom are recorded in Line 4 of **Table 5.2**.

$c_U$  **voltmeter calibration factor:** The voltmeter has a center value close to the nominal reading, with a given uncertainty provided on the calibration certificate. This is a Type B uncertainty because the source of the standard uncertainty information is a calibration certificate.

In this example, the calibration certificate states 1.000032 with an expanded uncertainty ( $k = 2$ ) of 0.000008. The standard uncertainty is the expanded uncertainty divided by 2, or 0.000004. These values are recorded in line 5 of **Table 5.2**.

$J_r$  **reference current for the standard lamp:** The reference current (in amps, A) for the standard lamp is provided on the calibration certificate. This component does not have uncertainty, because it is stated as a given value on the calibration certificate.

In this example, the calibration certificate states 5.825 A. This value is recorded in Line 6 of **Table 5.2**.

$R$  **resistance of the shunt resistor:** The resistor value and uncertainty are listed on the resistor calibration certificate. This is a Type B uncertainty because the source of the standard uncertainty information is a calibration certificate.

In this example, the calibration certificate states 0.100045 ohms ( $\Omega$ ) with an expanded uncertainty ( $k = 2$ ) of 0.000006  $\Omega$ . The standard uncertainty is the expanded uncertainty divided by 2, or 0.000003  $\Omega$ . These values are recorded in Line 7 of **Table 5.2**.

$m_{J_r}$  **sensitivity of luminous flux with respect to current:** For incandescent lamps, including halogen, the change in lumen output with current can be expressed as a ratio of the actual lamp current divided by the desired lamp current, and the ratio then raised to an exponent of 6.50, which has a standard uncertainty of

0.02. These are typical industry standard values. This is a Type B uncertainty because it is based on industry knowledge. These values are recorded in Line 8 of **Table 5.2**.

$y_{x_r}$  **self-absorption signal for the standard lamp:** A lamp mounted in the sphere absorbs some of the light reflected from the sphere wall back onto the lamp, thus reducing the responsivity of the integrating-sphere photometer. The recommended method for correcting this error relies on the use of an auxiliary lamp to determine the magnitude of the self-absorption.

The self-absorption signal for the standard lamp is the photometer signal measured when the auxiliary lamp is on and the standard lamp is mounted in the sphere but not illuminated. For this example, the signal is the average of the 10 measurements, which is  $y_{x_r} = 0.5243$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_{x_r}) = 0.0004$  nA. This is a Type A uncertainty, with degrees of freedom equal to 9. The signal and the standard uncertainty are recorded in Line 9 of **Table 5.2**.

For integrating sphere systems using an array spectrometer as the detection device, the reader is referred to **Section 5.1.9** for a detailed discussion of the uncertainty of the self-absorption signal component.

$y_{x_{rd}}$  **self-absorption dark signal for the standard lamp:** The photometer signal measured when the auxiliary lamp is off and the standard lamp is mounted in the sphere. For this example, the signal is the average of the 10 measurements, which is  $y(y_{x_{rd}}) = 0.0002$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_{x_{rd}}) = 0.0002$  nA. This is a Type A uncertainty, with degrees of freedom equal to 9. The signal and standard uncertainty are recorded in Line 10 of **Table 5.2**.

$y_E$  **self-absorption factor for “empty” sphere:** This measurement is the photometer signal measured when the auxiliary lamp is on and no standard lamp is present in the sphere. A reference condition is required. In this example, it is the sphere with the standard lamp removed; i.e., empty sphere. Otherwise, a self-absorption measurement of the standard lamp would be required with every measurement in order to account for changes in the sphere over time. For this example, the signal is the average of the 10 measurements, which is  $y_E = 0.5437$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_E) = 0.0004$  nA. This is a Type A uncertainty, with degrees of freedom equal to 9. The signal and standard uncertainty are recorded in Line 11 of **Table 5.2**.

$y_{Ed}$  **self-absorption dark signal for “empty” sphere:** The photometer signal measured when the auxiliary lamp is off and no standard lamp is present in the sphere. For this example, the signal is the average of the 10 measurements, which is  $y_{Ed} = 0.0002$  nA. The standard deviation calculated from the 10 measurements is the standard uncertainty,  $u(y_{Ed}) = 0.0002$  nA. This is a Type A uncertainty, with degrees of freedom equal to 9. The signal and standard uncertainty are recorded in Line 12 of **Table 5.2**.

$A_r$  **standard lamp age or burning time:** The operation of an incandescent (including halogen) standard lamp typically reduces the lumen output by 0.4% over a period of 24 hours. Not all lamps meet this condition, and very few are predictable. Many calibration laboratories will provide a statement approximating this dependency on the calibration certificates for the standard lamps they issue. If this information is not available, the decay rate is assumed to be 0.4% per 24 hours. This is a Type B uncertainty because it is based on industry knowledge or data from a calibration certificate.

The value of  $A_r$  is set to 1. It is difficult and risky to apply a correction factor for the aging of the standard lamp without assuming knowledge. Increasing the standard uncertainty with the operation of the standard lamp is a more conservative approach. For this example, assuming the lamp has been used for 2 hours, the standard uncertainty is estimated at 0.4% divided by 24 hours, then multiplied by the 2 hours of use, resulting in a value of 0.00033. These values are recorded in Line 13 of **Table 5.2**.

$C_{fr}$  **additional uncertainty components:** Discussed in **Section 5.1.2**.

**5.1.2 Additional Uncertainty Components.** In order to simplify the presentation of the model equation (**Equation 5.1**), a separate table of uncertainty components is presented, where the components do not change the magnitude of the measurand,  $C_{\phi}$ , but do add uncertainty, and the contribution of which is relatively small (less than 1/5 that of the largest contributor). If the contribution is large, it should be listed in the model equation as a separate factor. **Table 5.1** lists a few examples. The uncertainty contributions are estimated, and the combined standard uncertainty is estimated by the square root of the sum of the squares. These are all Type B uncertainties. The relative standard uncertainty contributions presented in **Table 5.1** are reasonable estimates for industry standard uncertainties. The estimation of  $C_{f_r}$  is equal to 1, and the standard uncertainty for  $C_{f_r}$  is the combined standard uncertainty of the components listed in **Table 5.1**. The estimation of  $C_{f_r}$  and the standard uncertainty for  $C_{f_r}$  are recorded in Line 14 of **Table 5.2**.

**Table 5.1. Additional Uncertainty Components for the Model of an Integrating Sphere with Photometer.**

| Uncertainty Component               | Relative Standard Uncertainty Contribution |
|-------------------------------------|--------------------------------------------|
| Spectral mismatch correction factor | 0.10 %                                     |
| Photometer temperature              | 0.06 %                                     |
| Near-field absorption               | 0.05 %                                     |
| Lamp and sphere ambient temperature | 0.05 %                                     |
| Stray light from the room           | 0.02 %                                     |
| Combined standard uncertainty       | 0.14 %                                     |

**5.1.2.1 Spectral Mismatch Correction Factor.** For a standard lamp, the spectral responsivity of the integrating sphere photometer cannot be perfectly matched to the  $V(\lambda)$  function. The photometer responsivity is referenced to a correlated color temperature (CCT), which is usually (and in this example) 2856 K, the CCT of Illuminant A. When the integrating sphere and photometer are calibrated by standard lamps operating at other correlated color temperatures, a spectral mismatch error occurs. (See ANSI/IES LM-78-20,<sup>4</sup> Section 7.3.2.) Standard lamps typically have correlated color temperatures ranging from 2700 K to 2950 K. A commercially available system should not deviate more than 0.25% over this range of CCTs. For this example, the CCT is close to 2856 K, so the uncertainty due to spectral mismatch correction was approximated to be 0.10%.

**5.1.2.2 Photometer Temperature.** A photometer changes responsivity with temperature. In particular, the filter commonly used to cut off blue wavelength light changes wavelength cutoff with temperature, resulting in a sensitivity of 0.1% per degree Celsius. It is important to note the temperature at which the photometer was calibrated. If the photometer was calibrated at 25 °C and is used in a laboratory maintaining a constant temperature of 25 °C  $\pm$  1 °C, the standard uncertainty would be estimated by a uniform probability function:  $1\text{ °C} - [(-1\text{ °C})/2\sqrt{3}](0.1\%/1\text{ °C}) = 0.06\%$ . Photometers are often temperature controlled, or temperature monitored and corrected, which reduces this contribution of uncertainty to zero.

**5.1.2.3 Near-Field Absorption.** Near-field absorption is described in ANSI/IES LM-78-20 Section 7.3.4. A well-designed system should have a small contribution from near-field absorption. As the sphere diameter decreases, self-absorption of the standard lamp becomes near-field absorption, which is not corrected by the auxiliary lamp measurement. For spheres 1 m in diameter or larger, the near-field absorption should be small. For this example, the standard uncertainty due to near-field absorption is approximated as 0.05%. For spheres smaller than 1 m, the near-field absorption error can be large and difficult to assess. A series of calibrated lamps differing in size (specifically, surface area) may be used to approximate the near-field absorption for smaller spheres, with respect to the surface area of the standard lamp.

**5.1.2.4 Lamp and Sphere Ambient Temperature.** For an incandescent lamp (including halogen lamps), the ambient temperature does not normally affect the operation unless the sphere is unusually small. If the sphere is small enough to affect the operation of the lamp, then the sphere coating reflectance most likely will be affected by the ambient temperature. (Refer to ANSI/IES LM-78-20 Section 5.3 and Section 7.3.5 for more details concerning sphere coating reflectivity and ambient temperature.) For typical measurement systems (and this example), the standard uncertainty due to lamp and sphere ambient temperature is approximately 0.05%.

**5.1.2.5 Stray Light from the Room.** Stray light from the room is typically very small or non-existent. Possible sources of room light include open ports in the sphere, the seam where the sphere halves close, and gaps around detectors or a wire feed-through. If the measurements are normally taken with the room lights on, then the stray light contribution can be approximated by measuring a dark signal with the lights on and off. For typical measurement systems and this example, the standard uncertainty due to room stray light is approximately 0.02%.

**5.1.3 Integrating Sphere with Photometer Sensitivity Coefficients.** As stated in **Section 4.4**, the partial derivative of the model equation for each component of uncertainty describes the dependence of the measurand on the uncertainty components. In this case, the sensitivity coefficients are determined from the partial derivatives of **Equation 5.1 (Section 5.1.1)** with respect to each uncertainty component. A brief description of the techniques used to determine the partial derivatives is described in **Annex B**. The equations for the partial derivatives follow, along with the respective line number in **Table 5.2** where the result is recorded. It should be noted that if the ratio of currents is exactly 1, Line 8 goes to zero; therefore, a small factor that is approximately one-tenth of the resolution should be added to this ratio.

|                                                                                                             |         |                                                                                    |         |                                                                                       |         |
|-------------------------------------------------------------------------------------------------------------|---------|------------------------------------------------------------------------------------|---------|---------------------------------------------------------------------------------------|---------|
| $\frac{\partial C_\phi}{\partial \Phi_r} = \frac{-C_\phi}{\Phi_r}$                                          | Line 1  | $\frac{\partial C_\phi}{\partial y_r} = \frac{C_\phi}{y_r - y_{r_d}}$              | Line 2  | $\frac{\partial C}{\partial y_{r_d}} = \frac{-C}{y_r - y_{r_d}}$                      | Line 3  |
| $\frac{\partial C_\phi}{\partial U_r} = \frac{-m_{J_r} \cdot C_\phi}{U_r}$                                  | Line 4  | $\frac{\partial C_\phi}{\partial c_U} = \frac{-m_{J_r} \cdot C_\phi}{c_U}$         | Line 5  | $\frac{\partial C_\phi}{\partial r} = \frac{m_{J_r} \cdot C_\phi}{R}$                 | Line 7  |
| $\frac{\partial C_\phi}{\partial m_{J_r}} = C_\phi \cdot \ln\left(\frac{J_r \cdot R}{U_r \cdot c_U}\right)$ | Line 8  | $\frac{\partial C_\phi}{\partial y_{x_r}} = \frac{-C_\phi}{y_{x_r} - y_{x_{r_d}}}$ | Line 9  | $\frac{\partial C_\phi}{\partial y_{x_{r_d}}} = \frac{C_\phi}{y_{x_r} - y_{x_{r_d}}}$ | Line 10 |
| $\frac{\partial C_\phi}{\partial y_E} = \frac{C_\phi}{y_E - y_{E_d}}$                                       | Line 11 | $\frac{\partial C_\phi}{\partial y_{E_d}} = \frac{-C_\phi}{y_E - y_{E_d}}$         | Line 12 | $\frac{\partial C_\phi}{\partial A_r} = \frac{C_\phi}{A_r}$                           | Line 13 |
| $\frac{\partial C_\phi}{\partial C_{f_r}} = \frac{C_\phi}{C_{f_r}}$                                         | Line 14 |                                                                                    |         |                                                                                       |         |

**5.1.4 Example Uncertainty Budget.** Provided in **Table 5.2** is an uncertainty budget using **Equation 5.1** as the model and typical values (see **Section 5.1.1**) for a high-quality sphere system with a photometer detector using a high-quality luminous flux standard lamp.

**5.1.5 Measurand and Combined Standard Uncertainty.** Using **Equation 5.1** and the estimated values for  $x_i$  provided in **Table 5.2**, the measurand is calculated:  $C_\phi = 6.537 \times 10^{-4}$  nA/lm.

The combined standard uncertainty is the square root of the sum of squares,  $u_c(C_\phi) = 2.3 \times 10^{-6}$  nA/lm, or 0.35% as a relative combined standard uncertainty. The combined standard uncertainty in this example is dominated by the standard uncertainty of the standard lamp. An increase in the number of measurements for the Type A components would not significantly reduce the combined standard uncertainty.

**Table 5.2. Uncertainty Budget for an Integrating Sphere Using a Photometer Detector and a Total Luminous Flux Standard Lamp.**

|    | Symbol    | Value<br>$x_i$          | Units    | Standard<br>Uncertainty<br>$u(x_i)$ | Type | Deg. of<br>Freedom<br>$\nu$ | Sensitivity<br>$c_i$    | Contribution<br>$u_i(y)$ |
|----|-----------|-------------------------|----------|-------------------------------------|------|-----------------------------|-------------------------|--------------------------|
| 1  | $\Phi_r$  | 2,800.0                 | lm       | 8.2                                 | B    | $\infty$                    | $-2.335 \times 10^{-7}$ | $-1.914 \times 10^{-6}$  |
| 2  | $y_r$     | 1.7652                  | nA       | 0.0007                              | A    | 9                           | $3.704 \times 10^{-4}$  | $2.593 \times 10^{-7}$   |
| 3  | $y_{rd}$  | 0.0002                  | nA       | 0.0002                              | A    | 9                           | $-3.704 \times 10^{-4}$ | $-7.407 \times 10^{-8}$  |
| 4  | $U_r$     | 0.582743                | V        | 0.000015                            | A    | 29                          | $-7.291 \times 10^{-3}$ | $-1.094 \times 10^{-7}$  |
| 5  | $c_U$     | 1.000032                |          | 0.000004                            | B    | $\infty$                    | $-4.249 \times 10^{-3}$ | $-1.700 \times 10^{-8}$  |
| 6  | $J_r$     | 5.8250                  | A        |                                     |      |                             |                         |                          |
| 7  | $R$       | 0.100045                | $\Omega$ | 0.000003                            | B    | $\infty$                    | $4.247 \times 10^{-2}$  | $1.274 \times 10^{-7}$   |
| 8  | $m_{Jr}$  | 6.50                    |          | 0.02                                | B    | $\infty$                    | $5.353 \times 10^{-10}$ | $1.071 \times 10^{-11}$  |
| 9  | $y_{xr}$  | 0.5243                  | nA       | 0.0004                              | A    | 9                           | $-1.247 \times 10^{-3}$ | $-4.989 \times 10^{-7}$  |
| 10 | $y_{xrd}$ | 0.0002                  | nA       | 0.0002                              | A    | 9                           | $1.247 \times 10^{-3}$  | $2.495 \times 10^{-7}$   |
| 11 | $y_E$     | 0.5437                  | nA       | 0.0004                              | A    | 9                           | $1.203 \times 10^{-3}$  | $4.811 \times 10^{-7}$   |
| 12 | $y_{Ed}$  | 0.0002                  | nA       | 0.0002                              | A    | 9                           | $-1.203 \times 10^{-3}$ | $-2.405 \times 10^{-7}$  |
| 13 | $A_r$     | 1.0000                  |          | 0.00033                             | B    | $\infty$                    | $-6.537 \times 10^{-4}$ | $2.157 \times 10^{-7}$   |
| 14 | $C_{fr}$  | 1.0000                  |          | 0.0014                              | B    | $\infty$                    | $6.537 \times 10^{-4}$  | $9.152 \times 10^{-7}$   |
|    | $C_\Phi$  | $6.5369 \times 10^{-4}$ | nA/lm    |                                     |      |                             | $u_c(C_\Phi)$           | $2.29 \times 10^{-6}$    |

**5.1.6 Degrees of Freedom and Expanded Uncertainty.** Using the Welch-Satterthwaite formula (**Equation 4.11**, presented in **Section 4.5**), the effective degrees of freedom for this uncertainty budget,  $\nu_{eff}$ , are 1,950. Using an effective degrees of freedom of 1,950 and the Student's  $t$  approximation for coverage factor (**Equation 4.12**, presented in **Section 4.5**), the coverage factor is approximately 2, resulting in an expanded uncertainty of  $u(C_\Phi) = 4.6 \times 10^{-6}$  nA/lm, or 0.70% as a relative expanded uncertainty. The uncertainty community typically rounds the coverage factor to 2.00 if the calculated value is within 0.10 of 2.00. The final statement of uncertainty would be:

*The luminous flux responsivity of the integrating sphere system on [date] was determined, traceable to the candela SI unit, with a magnitude of  $6.537 \times 10^{-4}$  nA/lm and an associated expanded uncertainty of  $4.6 \times 10^{-6}$  nA/lm, or a relative expanded uncertainty of 0.70%. The stated uncertainty was calculated according to the GUM by a multiplication of the combined standard uncertainty with a coverage factor of  $k = 2$ .*

**5.1.7 Model Modifications.** The model **Equation 5.1** is a starting point. Based on the equipment used by the laboratory, the model equation should be modified to represent the laboratory. For example, if the laboratory has a multimeter or power supply measuring the current directly, the model equation would be altered to:

$$C_\Phi = \frac{(y_r - y_{rd})}{\Phi_r} \cdot \left( \frac{J_r}{J \cdot c_J} \right)^{m_{Jr}} \cdot \frac{(y_E - y_{Ed})}{(y_{Ar} - y_{Ar_d})} \cdot A_r \cdot C_{fr} \quad (5.2)$$

where  $J$ , the measured current, and  $c_J$ , the ammeter calibration, replace the voltmeter measurement,  $U$ , voltmeter calibration,  $c_U$ , and shunt resistor calibration,  $R$ .

The changes to the uncertainty analysis based on **Equation 5.2**, where the current reading comes from the power supply display instead of measured across a shunt, are straightforward. For this example, the current reading,  $J$ , is 5.825 A. The

standard uncertainty of the current reading is a Type B uncertainty determined using **Equation 4.4**. Since the last digit in the display does not vary, the actual current is within the range 5.8245 to 5.8255 amps. The standard uncertainty is  $0.001/2\sqrt{3} = 0.00029$  A. The display has a calibration factor  $c_J = 1.000$  for this example, with a standard uncertainty of 0.1% of the signal:  $u(c_J) = 0.00017$ .

In **Table 5.2**, Lines 4 and 5 are replaced with the lines shown in **Table 5.3**.

**Table 5.3. Replacement Lines in Table 5.2 for Modified Model Equation.**

|   |       |         |   |         |   |          |                         |                         |
|---|-------|---------|---|---------|---|----------|-------------------------|-------------------------|
| 4 | $J$   | 5.825   | A | 0.00029 | B | $\infty$ | $-7.294 \times 10^{-4}$ | $-2.115 \times 10^{-7}$ |
| 5 | $c_J$ | 1.00001 |   | 0.00017 | B | $\infty$ | $-4.249 \times 10^{-3}$ | $-7.223 \times 10^{-7}$ |

The sensitivity coefficients for these two parameters are:

$$\frac{\partial C_\phi}{\partial J} = \frac{-m_{J_r} \cdot C_\phi}{J} \quad \text{and} \quad \frac{\partial C_\phi}{\partial c_J} = \frac{-m_{J_r} \cdot C_\phi}{c_J}$$

The measurand,  $C_\phi$ , is  $6.536 \times 10^{-4}$  nA/lm, and the combined standard uncertainty is  $2.4 \times 10^{-6}$  nA/lm. The degrees of freedom are 2,370, so the coverage factor is approximately 2. The expanded uncertainty is  $4.8 \times 10^{-6}$  nA/lm.

Another possible modification is where the laboratory does not reference to an “empty” sphere and would like to use a working reference lamp every day before starting measurements; therefore, the model equation for the photometer calibration factor would be:

$$C_\phi = \frac{(y_r - y_{r,d})}{\Phi_r} \cdot \left( \frac{J_r \cdot R}{U_r \cdot c_U} \right)^{m_{J_r}} \cdot A_r \cdot C_{f_r} \quad (5.3)$$

**5.1.8 Multiple Standard Lamps.** Many laboratories will not rely on a single standard lamp to calibrate their integrating sphere measurement system. This section describes the uncertainty analysis when three standard lamps are used to calibrate the system. Lamps age differently and tend to operate slightly differently on different days; therefore, the average of three standard lamps tends to produce a more consistent calibration factor over time. However, as discussed in **Section 4.4**, standard lamps are highly correlated, whether they come from the same or different calibration laboratories. The only factor common between the three standard lamp measurements is the determination of the empty sphere self-absorption factor; therefore, the measurement equation is:

$$C_\phi = \left( \frac{C_1 + C_2 + C_3}{3} \right) (y_E - y_{E,d}) \quad (5.4)$$

where

$$C_i = \frac{(y_{r,i} - y_{r,i,d})}{\Phi_{r,i}} \cdot \left( \frac{J_{r,i} \cdot R}{U_{r,i} \cdot c_U} \right)^{m_{J_r}} \cdot \frac{1}{(y_{X_{r,i}} - y_{X_{r,i,d}})} \cdot A_{r,i} \cdot C_{f_r} \quad (5.5)$$

The standard lamp described in **Table 5.2** would now have an uncertainty budget for  $C_1$ , and the combined standard uncertainty,  $u_c(C_1)$ , shown in **Table 5.4**. **Table 5.5** is the uncertainty budget for a second standard lamp, with a higher wattage. **Table 5.6** is the uncertainty budget for a third standard lamp, of lower wattage. All three lamps produce a correlated color temperature near 2856 K and have very similar luminous intensity angular distributions. It is important to notice that the relative standard uncertainty of the total luminous flux for each of the standard lamps is slightly different:  $u(\Phi_{r1}) = 0.29\%$ ,  $u(\Phi_{r2}) = 0.41\%$ , and  $u(\Phi_{r3}) = 0.36\%$ .

**Table 5.4. Uncertainty Budget for Sphere Responsivity Using Standard Lamp 1**

|    | Symbol        | Value<br>$x_i$          | Units    | Standard<br>Uncertainty<br>$u(x_i)$ | Type | Deg. of<br>Freedom<br>$\nu$ | Sensitivity<br>$"c_i$   | Contribution<br>$u_i(y)$ |
|----|---------------|-------------------------|----------|-------------------------------------|------|-----------------------------|-------------------------|--------------------------|
| 1  | $\Phi_{r2}$   | 2,800.0                 | lm       | 8.2                                 | B    | $\infty$                    | $-4.296 \times 10^{-7}$ | $-3.522 \times 10^{-6}$  |
| 2  | $y_{r2}$      | 1.7652                  | nA       | 0.0007                              | A    | 9                           | $6.814 \times 10^{-4}$  | $4.770 \times 10^{-7}$   |
| 3  | $y_{r2d}$     | 0.0002                  | nA       | 0.0002                              | A    | 9                           | $-6.814 \times 10^{-4}$ | $-1.363 \times 10^{-7}$  |
| 4  | $U_{r2}$      | 0.582743                | V        | 0.000015                            | A    | 29                          | $-1.342 \times 10^{-2}$ | $-2.012 \times 10^{-7}$  |
| 5  | $c_U$         | 1.000032                |          | 0.000004                            | B    | $\infty$                    | $-7.818 \times 10^{-3}$ | $-3.127 \times 10^{-8}$  |
| 6  | $J_{r2}$      | 5.825                   | A        |                                     |      |                             |                         |                          |
| 7  | $R$           | 0.100045                | $\Omega$ | 0.000003                            | B    | $\infty$                    | $7.814 \times 10^{-2}$  | $2.344 \times 10^{-7}$   |
| 8  | $m_{Jr}$      | 6.5                     |          | 0.2                                 | B    | $\infty$                    | $9.849 \times 10^{-10}$ | $1.970 \times 10^{-10}$  |
| 9  | $y_{x_{r2}}$  | 0.5243                  | nA       | 0.0004                              | A    | 9                           | $-2.295 \times 10^{-3}$ | $-9.180 \times 10^{-7}$  |
| 10 | $y_{x_{r2d}}$ | 0.0002                  | nA       | 0.0002                              | A    | 9                           | $2.295 \times 10^{-3}$  | $4.590 \times 10^{-7}$   |
| 13 | $y_E$         | 1.0000                  |          | 0.00033                             | B    | $\infty$                    | $1.203 \times 10^{-3}$  | $3.969 \times 10^{-7}$   |
| 14 | $y_{Ed}$      | 1.0000                  |          | 0.0014                              | B    | $\infty$                    | $1.203 \times 10^{-3}$  | $1.684 \times 10^{-6}$   |
|    | $C_{\phi 1}$  | $1.2027 \times 10^{-3}$ | nA/lm    |                                     |      | 36,052                      | $u_c(C_{\phi 1})$       | $4.10 \times 10^{-6}$    |

Since standard lamp 3 is a lower wattage, the gain was changed on the transimpedance amplifier. A silicon-based photometer is inherently very linear (orders of magnitude). The non-linearity of the signal comes from the methods used to determine the photocurrent. Transimpedance amplifiers may be non-linear and, more important, can have large differences as gain ranges are changed. For this example, a calibrated transimpedance amplifier is used, which has a small standard uncertainty,  $U(G)$ , equal to 0.05%. Uncalibrated transimpedance amplifiers may have gain range differences as large as 1%, which can be the most significant uncertainty component. If lamps of different electrical wattages are measured, then the gain factors need to be addressed in the uncertainty estimation, which is accounted for in Line 13 of **Table 5.6**.

The combined standard uncertainty for the average sphere responsivity,  $C_\phi$ , is calculated using **Equation 5.6**, which is based on **Equation 4.6**. All of the correlation coefficients are taken to be equal to 1. **Table 5.7** summarizes the uncertainty budget for the average sphere responsivity,  $C_\phi$ , using three standard lamps.

$$u_c(C_\phi) = \sqrt{\left(\frac{\partial C_\phi}{\partial C_{\phi 1}}\right)^2 u^2(C_{\phi 1}) + \left(\frac{\partial C_\phi}{\partial C_{\phi 2}}\right)^2 u^2(C_{\phi 2}) + \left(\frac{\partial C_\phi}{\partial C_{\phi 3}}\right)^2 u^2(C_{\phi 3}) + \left(\frac{\partial C_\phi}{\partial y_E}\right)^2 u^2(y_E) + \left(\frac{\partial C_\phi}{\partial y_{Ed}}\right)^2 u^2(y_{Ed}) + 2 \frac{\partial C_\phi}{\partial C_{\phi 1}} \frac{\partial C_\phi}{\partial C_{\phi 2}} u(C_{\phi 1}) u(C_{\phi 2}) r(C_{\phi 1}, C_{\phi 2}) + 2 \frac{\partial C_\phi}{\partial C_{\phi 1}} \frac{\partial C_\phi}{\partial C_{\phi 3}} u(C_{\phi 1}) u(C_{\phi 3}) r(C_{\phi 1}, C_{\phi 3}) + 2 \frac{\partial C_\phi}{\partial C_{\phi 2}} \frac{\partial C_\phi}{\partial C_{\phi 3}} u(C_{\phi 2}) u(C_{\phi 3}) r(C_{\phi 2}, C_{\phi 3})} \quad (5.6)$$

Using the Welch-Satterthwaite formula (**Equation 4.11**, presented in **Section 4.5**), the effective degrees of freedom,  $\nu_{eff}$ , for this uncertainty budget are 7,431, which leads to a coverage factor approximately equal to 2, resulting in an expanded uncertainty of  $U(C_\phi) = 5.3 \times 10^{-6}$  nA/lm, or 0.81% as a relative expanded uncertainty. The final statement of uncertainty would be: