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Reference Document 46.1

Terms Commonly Used in the Digital Analysis of Dynamic Data

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1 SCOPE AND LIMITATIONS

The purpose of this Reference Document (RD) is to define, clarify, and disseminate terminology appropriate for digital analysis of dynamic data as used in environmental testing.

This list is in no manner constrained by any standards publications, technical organizations, or societies. However a reasonable effort has been made to keep the definitions consistent with these standards. This document is intended for use by those responsible for procuring, manufacturing, and using computer-controlled environmental test systems, as well as those responsible for educating others in the field of environmental test control.

2 INTRODUCTION

This Reference Document (RD) contains certain terms commonly used in digital analysis of dynamic data generated when measuring in-service conditions of mechanical and electrical apparatus. This RD also covers certain conditions encountered during qualification testing of mechanical systems and components. Recognizing that there are many ways in which dynamic data signals are generated, recorded, processed, and displayed, wherever a choice exists we have emphasized processing of recorded data by digital means.

This document is intended to assist those involved in recording, analyzing, displaying, or interpreting data relating to the dynamic properties of mechanical systems. Such data may include strain, vibration, sound pressure variations, and system electrical characteristics.

In running text, boldface type is used to indicate terminology that is defined in this RD.

3 TERMS AND DEFINITIONS

accelerance
See **inertance, mechanical impedance**.

acceleration SRS
See **shock response spectrum**.

accuracy
The degree of conformity of a measure to a model, standard, or true value. Usually expressed as a percentage of specified value. Should consider other dependent factors such as temperature, frequency, relative value (full scale vs absolute), drift rates, and range.

aliasing
When the sample rate f is less than 2 times the highest frequency in the data, the frequency is ambiguously represented. The frequencies above f/2 will be folded back into the lower frequencies to produce erroneous results.

amplitude density function
See **probability density function**.

amplitude distribution function
See **probability distribution function**.

analog-to-digital converter (ADC)
An electronic device used for converting analog signals to digital form, either binary code or binary-coded-decimal. An ADC is usually used in conjunction with a multiplexer that selects a particular analog signal and with a sample-and-hold amplifier that stores an instantaneous analog voltage. The ADC digitizes (see **digitize**) that voltage into N discrete levels, where $N = 2$ and n is the number of bits of resolution in the device. For dynamic waveforms, the sampling frequency must be high enough to avoid aliasing errors.

analysis range/bandwidth
See **frequency range**.

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anti-aliasing filter
See **filter, anti-aliasing**.

apparent weight/apparent mass
A term sometimes used for the preferred term, **effective mass** (see **mechanical impedance**).

aperture error
See **error, aperture.**

Argand diagram
See **Nyquist diagram.**

array
An ordered set of numbers; in a sampled data system discussed in this Glossary (see **sampling**) the numbers are equally spaced in time or frequency.

autocorrelation function
A measure of the similarity that a function has with a time-displaced version of itself—as a function of the time displacement or lag. When the lag is zero, the value of the autocorrelation is equal to the mean square value of the function. The sample autocorrelation function

$$\hat{R}_x(j)$$

of the sequence x_i is usually computed

$$\hat{R}_x(j) = \frac{1}{N-j}\sum_{i=1}^{N-j} x_i x_{i+j} \qquad j = 0,1,\ldots,m$$

where

x_i = the function $x(t)$ sampled at the time $i\Delta t$

Δt = the sample interval

$j\Delta t$ = the lag

N = the number of samples of x

$m\Delta t$ = the maximum lag

For a discrete **stationary** random quantity where N must be finite, the formula gives an estimate only with a statistical uncertainty that increases as N decreases. It is used to identify periodic components in the sample function x_i.

autospectral density function
See **power spectral density function**.

average, arithmetic
An averaging technique that indicates the average value after N summations:

$$x = \frac{1}{N}\sum_{N-1}^{0} x_i$$

Note that Σx_i steadily grows until $i = N$, at which point it is divided by N.

average, ensemble
A technique for averaging a set of data records. Each record (a member of the ensemble) is sampled at the same instant of time; the sampled values are then averaged. For an ergodic random process (see **ergodic process**) an ensemble average is equivalent to the time average along any record of the ensemble.

average, exponential
An averaging technique that weights newer information more heavily than old. Computed as:

$$\hat{x}_{i+1} = \hat{x}_i + \frac{x_N - \hat{x}_i}{K}$$

where

$\hat{x}_i$ = old average

x_N = latest value

K = averaging weighting factor

average, stable
An averaging technique which indicates the estimated value at each measurement as opposed to an arithmetic average which gives the values only after division by the number of averages. Computed as:

$$\hat{x}_{i+1} = \hat{x}_i + \frac{x_N - \hat{x}_i}{i+1}$$

where

$\hat{x}_i$ = old average

x_N = latest value (N = 1 to M)

i = current count index (O to M)

M = total number of averages

bandpass filter
See **filter, bandpass**.

bandreject filter
See **filter, bandreject**.

bandwidth, 3-db (b3db)
The interval between the upper and lower frequencies at which the filter attenuates an applied signal by 3 dB (see **decibel**) with respect to the gain at the center frequency of the filter.

bandwidth, equivalent noise
See **effective (noise) filter bandwidth**.

bandwidth, real time
The range of frequencies over which a real time analysis can be performed (see **real time analysis**).

baseband analysis
Analysis over a frequency range from zero to a specified upper frequency (see **zoom analysis**.)

Bessel filter
See **filter, Bessel**.

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bias
A tendency of a statistical estimate to deviate in one direction away from a true value.

bias error
See **error, bias**.

block
See **frame, memory length**.

block, synchronous
A data sampling or processing technique in which the beginning or ending of a data block is synchronized with an external event.

blocked constraints
See **mechanical impedance/mobility**.

boxcar function
See **window**.

Butterworth filter
See **filter, Butterworth**.

calibration factor (CF; also called scale factor)
Data acquired by an **analog-to-digital converter (ADC)** is in terms of binary counts representing some fraction of a specified full-scale voltage input. The calibration factor (CF) is the multiplier that must be applied to the digitized (see **digitize**) value to convert it to engineering units. For example, an acceleration signal of 5 g/V is to be digitized by a 12-bit (11 bits + sign) ADC having a full input of 10 V. The calibration factor is

$$\mathrm{CF} = 5\,\frac{\mathrm{g}}{\mathrm{V}} \times \frac{10\,\mathrm{V}}{2^{11}\mathrm{counts}} = 24.414 \times 10^{-3}\,\mathrm{g/count}$$

An ADC count of 1562 would then be equivalent to

$$\mathrm{g} = (1562) \times (24.\,414 \times 10^{-3}) = 38.135\ \mathrm{g}$$

center frequency
A characteristic of a bandpass or band reject filter or a constant percentage filter of these types, the center frequency is the geometric mean of the upper (f_U) and the lower (f_L) cutoff frequencies:

$$\mathrm{f_c} = \sqrt{\mathrm{f_L f_U}}$$

For a constant bandwidth filter, the center frequency is the arithmetic mean (see **average, arithmetic**) of the upper and lower frequencies:

$$\mathrm{f_c} = \frac{1}{2(\mathrm{f_L} + \mathrm{f_U})}$$

Chebyshev filter
See **filter, Chebyshev**.

chi square
See **goodness of fit**.

circular convolution (periodic convolution)
The periodic convolution z_i of two time sequences x_i and y_j (i = 0, 1, 2, … N-1) is defined as

$$z_i = \sum_{m=0}^{N-1} \bar{x}_m y_{i-m}$$

where $\bar{x}_m$ is a periodic extension of the sample set x_m, i, e., $\bar{x}_{i+kn} = x_i$ (n = 0, 1, 2, … N-1) (k = 0, ±1, ±2 …). The inverse discrete Fourier transform (DFT^{-1}) of the product of two DFTs is a periodic convolution of the original time sequences (see **DFT, wrap-around error**).

circular correlation
A correlation function computed in terms of a circular convolution.

clipping
The term applied to the generally undesirable (but sometimes intentional) circumstance when an output signal is limited in some sense by the full-scale range of an amplifier, ADC, or other device. Clipping may be hard, that is, when the signal is strictly limited at some level; or it may be soft, in which case the clipped signal continues to follow the input at some reduced gain (see **gain**).

co (coincident, real)
The real part of a complex function, that is, the component that is in phase with the input excitation. In frequency domain analysis, the coincident terms are the cosine terms of the Fourier transform (see **frequency domain, Fourier transform**).

coherence function
A frequency domain function generally computed to show the degree of a linear, noise-free relationship between a system input and output. Values of coherence satisfy the relationship

$$O < \gamma 2(\mathrm{f}) < 1,$$

where a value of 0.0 indicates no causal relationship between an input and the output, and 1. O indicates the existence of linear noise-free frequency response function between input and output (see **frequency domain**).

coherence, multiple
For a system having multiple inputs $x_i(t)$ and one output $y(t)$, the multiple coherence represents the fraction of power in the output accounted for by simultaneous linear filter relationships with all the inputs. This coherence function obeys the usual inequality, and will be unity under noise-free ideal conditions when there is a true linear relationship occurring in a multiple-input/single-output system.

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coherence, ordinary
For a system having a single input $x(t)$ and single output $y(t)$, the ordinary coherence is defined as

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_x(f)G_y(f)}$$

where

$G_x(f)$, $G_y(f)$ = power spectral density functions of $x(t)$, $y(t)$

$G_{xy}(f)$ = cross spectral density function between $x(t)$ and $y(t)$

For a linear system, $\gamma^2_{xy}(f)$ can be interpreted as the fractional portion of the power output $y(t)$, which is contributed by the input $x(t)$ at frequency f. The coherence function is a measure of the statistical validity of the transfer function estimate. A value of $\gamma_{xy}^2 < 1.0$ indicates the existence of a nonlinear system, the presence of extraneous noise, or the existence of other uncorrelated inputs. Note that the coherence function is independently normalized at each frequency and is therefore independent of the shape of the frequency response function between measurement points (see **frequency response function**).

coherence, partial
For a system having multiple inputs $x_i(t)$ and one output $y(t)$, the partial coherence is the coherence computed between any individual input and the output when the effect of all other inputs is removed from the output by a linear least squares prediction. This coherence obeys the usual inequality, and will reveal the existence of a linear relationship between a particular residual input and the output even when the relationship is not apparent from the ordinary coherence function (see **least squares prediction**).

complex conjugate
The result of multiplying the imaginary part of a complex quantity by -1 (see **quad** and **complex function**).

complex demodulation
See **zoom analysis**.

complex function
A complex function is any mathematically defined relationship of the form

$$y(x) = a(x) + ib(x),$$

where

x = real variable,

$a(x)$ = the real part of $y(x)$, $a = \mathrm{Re}\{y(x)\}$,

$b(x)$ = the imaginary part of $y(x)$, $b = \mathrm{Im}\{y(x)\}$,

$i = \sqrt{-1}$

Complex functions are often represented in terms of their amplitude

$$|y(x)| = [a^2(x) + b^2(x)]^{1/2}$$

and phase

$$\varphi y(x) = \tan^{-1}\frac{b(x)}{a(x)}$$

A form of the arc tangent function that returns angles in all four quadrants should be used.

confidence bands
See **confidence interval**.

confidence interval
The range, with a specified value of uncertainty, within which the true value of a measured quantity will lie. For example, consider a random variable X with a true mean of x. Note that the true mean is never known, but can only be estimated. An estimate of the mean, x–, is then made with a 90% confidence interval of $\pm a$. The confidence statement means that if we made 100 such independent estimates of x, about 90 would be in the range of $x \pm a$, and about 10 would lie outside this range. Note that the confidence statement does not state how close a particular estimate x is to the true value x (a is a function of the numbered samples and the variance of the samples).

constraints, blocked and free
See **mechanical impedance/mobility**.

continuous spectrum
The type of spectrum produced from nonperiodic data. The spectrum is continuous in the frequency domain (see **line spectrum, frequency domain**).

convolution
A mathematical concept. The convolution of continuous functions $x(t)$ and $y(t)$ is defined as

$$x(t) \times y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$$

Let $W(f)$, $X(f)$, $Y(f)$, and $Z(f)$ be the Fourier transforms of $w(t)$, $x(t)$, $y(t)$, and $z(t)$, respectively. It can be shown that if $z(t) = x(t) \times y(t)$, then

$$Z(f) = X(f) \times Y(f)$$

and if $W(f) = X(f) \times Y(f)$, then

$$x(t) \times y(t) = w(t).$$

If $x(t)$ is the input to a linear system whose impulse response is $h(t)$, then the output of the system is the convolution $x(t) \times h(t)$ (see **circular convolution**).

correlation function
See **autocorrelation function; cross correlation function.**

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covariance
A statistical concept. The covariance of two random variables, x and y, is defined as

$$\begin{aligned}\text{Cov}(x, y) &= E[(x - E[x])(y - E[y])] \\ &= E[x, y] - E[x]E[y]\end{aligned}$$

where

$E[\ ]$ is the expected value (mean) of the quantity in brackets.

The covariance is a measure of the correlation of the two variables. If

$$\text{Cov}(x,y) = \sigma_x\sigma_y$$

where $\sigma_x\sigma_y$ are the standard deviations of x and y, the signals are fully correlated. If

$$\text{Cov}(x,y) = O$$

the signals are uncorrelated. A normalized quantity, ρ_{xy}, where

$$\rho_{xy} = \frac{\text{Cov}(x,y)}{\sigma_x\sigma_y}$$

is called the correlation coefficient.

critical damping coefficient
Critical damping is the smallest amount of damping at which a system will respond to a step function without overshoot (see **damping**). The critical damping coefficient for a linear, viscously damped, single-degree-of-freedom mechanical system is defined as

$$C_c = 2\sqrt{km} = 4\pi m f_n$$

where

k = spring constant

m = mass

f_n = undamped natural frequency.

(see **degrees of freedom: viscously damped**).

cross correlation function
A measure of the similarity of two functions with the time displacement (lag) between them used as an independent variable. The sample cross correlation function $R(j)$ between two sequences x_i and y_i is usually computed as:

$$\hat{R}_{xy}(j) = \frac{1}{N-j}\sum_{i=N-j-1}^{0} x_i y_{i+j}$$

$$j = 0,1,\ldots, m$$

where

x_i, y_i, $\hat{R}_{xy}(j)$ = the respective functions sampled at the time $i\Delta t$

Δt = the sample interval

$j\Delta t$ = the lag

N = the number of samples of x and y

$m\Delta t$ = the maximum lag.

For discrete **stationary** random quantities where N must be finite, the formula gives an estimate only with a statistical uncertainty, which increases as N decreases.

cross spectral density function
A measure in the frequency domain of the similarity of two functions. It is usually computed from the Fourier transforms of two discrete functions x_i and y_i to

$$\tilde{G}_{xy}(k) = \frac{2}{\ } \tilde{G}_{xy} = \frac{2\Delta t}{N}[X_k^* Y_k]$$

where

Δt = sample interval,

N = the frame size of x_i and y_i,

X_k^* = complex conjugate of X_k, and

X_k, X_k = the discrete Fourier transform of x_i, y

$\tilde{G}_{xy}$ is a **raw** cross spectral estimate, The cross spectral density is then estimated by averaging M frames of $\tilde{G}_{xy}$. It can also be computed from the **cross-correlation function** as

$$G_{xy}(k) = 2F[R_{xy}(j)]$$

where

$F[\ ]$ = Fourier transform of quantity within brackets

$R_{xy}(j)$ = cross-correlation of x_i and y_i

For discrete **stationary** random quantities, the formulas give an estimate only with a statistical uncertainty (error) which increases as M decreases. Typically, the data is also multiplied by a window and this will affect the effective resolution of the estimate. (See **Fourier transform window**).

cumulative probability distribution function
See probability distribution function.

curve fitting
The process whereby coefficients of an arbitrary function (usually a polynomial) are computed such that the function approximates the values in a given data set. A mathematical function, such as the minimum mean squared error, is used to judge the goodness of fit (see mean squared error, test).

cutoff frequency
The frequency at which the rolloff skirt of the filter shape is down from the nominal unity gain passband level by a specified amount.

damping
Dissipation of energy.

damping factor
See damping ratio.

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damping ratio
For a linear, viscously damped, single-degree-of-freedom, mechanical system, the ratio of the actual damping coefficient, C, to the critical damping coefficient, C_c, is called the damping ratio ζ,

where

$$\zeta = \frac{C}{C_c} = \frac{1}{2Q} \approx \frac{1}{2}\frac{B_{3dB}}{f_n}$$

$$C_c = 2\sqrt{km} = 4\pi\ mf_n$$

k = spring constant,

m = mass,

f_n = undamped natural frequency

Q = Quality factor,

B_{3dB} = half power frequency bandwidth,

(See **viscously damped; degrees of freedom; damping**).

decibel (dB)
A measurement unit that denotes the ratio of the magnitude squared of a quantity with respect to an arbitrarily established reference value of the quantity, expressed as 10 times the logarithm to the base 10 of the ratio of the quantities, e. g.,

$$\mathrm{dB} = 10\log_{10}\frac{\mathrm{V}^2}{\mathrm{V}_{\mathrm{ref}}^2} = 20\log_{10}\frac{\mathrm{V}}{\mathrm{V}_{\mathrm{ref}}}.$$

decimation (of sampled data)
A process used to decrease the amount of data for analysis. jth order decimation consists of keeping only every jth sample, discarding the remaining samples.

degrees of freedom
The minimum number of independent generalized coordinates required to define completely the positions of all parts of the system at any instant of time.

delay
In reference to filtering, refers to the time lag between the filter input and output. Delay shows up as a frequency-dependent phase shift between output and input, and depends on the type and complexity of the filter.

delay time
See **time lag**.

delta f
See **resolution**.

delta frequency
See **frequency increment**.

delta function
See **impulse function**.

delta time
See **sampling interval**.

deterministic function
A function whose future values can be predicted on the basis of some known rules and of its past values.

deterministic system
Systems whose behavior can be described by an explicit mathematical relationship; that is, given the present state of a system, its exact state at any instant of time in the future can be calculated.

Discrete Fourier Transform (DFT)
See **Fourier transform**.

discrete inverse Fourier transform (DFT-1)
See **Fourier transform**.

difference spectrum
See **error spectrum**.

Digital-to-analog converter (DAC)
The process of producing a continuous analog signal from discrete quantized levels. The result is a continuous waveform designed to match as closely as possible a previously sampled signal or a synthesized result. Usually, it is followed by a low pass filter (see **quantization; low pass filter**).

digitize
The process of converting an analog measurement of a quantity into a numerical value.

displacement SRS
See **shock response spectrum**.

distribution function
See **probability distribution function**.

domain off
See **function**.

double length transform
A $2N$ point DFT of sample set x_i (i = 0, 1, ... N-l) extended with N zeros. This transform does not add any new information in the frequency domain, but is sometimes required to prevent circular convolution errors. Sometimes refers to a technique for obtaining the Fourier transform of a time series of length $2N$ by utilizing an N point fast Fourier transform algorithm (see **frequency domain; circular convolution; Fourier transform**).

dynamic mass
See **mechanical impedance**.

dynamic range
For spectrum measurements, the difference, in dB, between the overload level and the minimum detectable signal level (above the noise) within a measure-

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