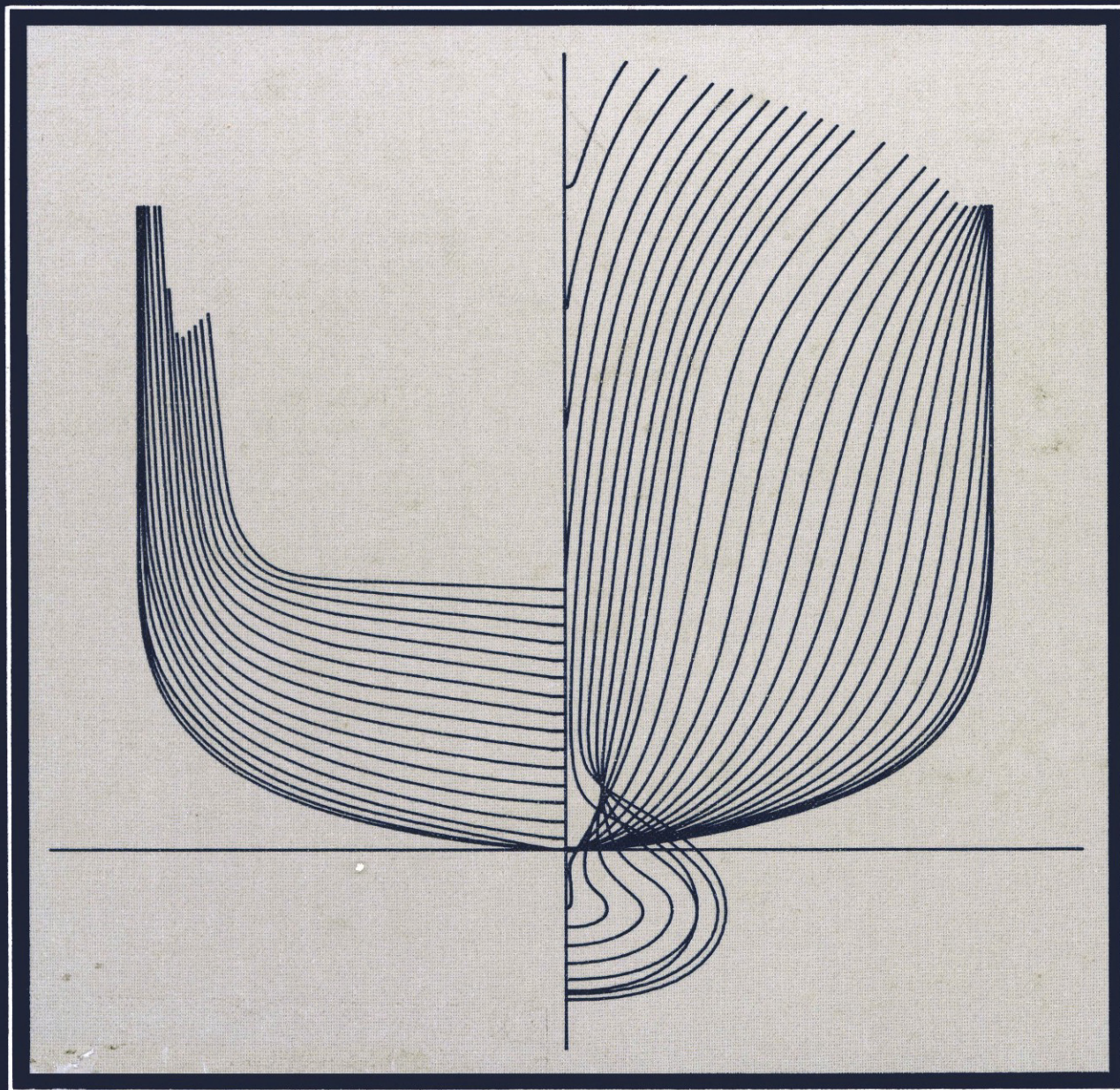


Introduction to NAVAL ARCHITECTURE



By Thomas C. Gillmer and Bruce Johnson

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BY THOMAS C. GILLMER AND BRUCE JOHNSON



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Preface

This textbook has been prepared to satisfy the educational requirements of the Naval Systems Engineering Department at the U.S. Naval Academy. The department offers two third-class (sophomore-level) courses that teach engineering fundamentals of naval architecture, especially those connected with naval ship design. A four-semester-hour course, Naval Engineering I, is taught to most of the non-engineering majors at the Naval Academy whose background includes chemistry, physics, and mathematics through differential equations. Because these students have not had the engineering fundamentals courses, this textbook contains a brief summary of engineering statics (chapter 2), engineering materials (chapter 4), strength of materials (chapter 5), and fluid mechanics (chapter 11).

A three-semester-hour course, Introduction to Naval Systems Engineering, is taught to majors in ocean engineering and marine engineering to give them a background in naval architecture and to acquaint them with aspects of engineering design early in the curriculum. It is hoped that this text will satisfy the requirements of other institutions that cover this subject matter at the sophomore level. Engineering majors can skip sections of chapters 2, 4, and 5, if they have already had courses covering this material, in order to complete most of the book during a three-semester-hour course.

The text does not attempt to exhaust the subject of naval architecture. The general design of merchant ships, cargo-handling equipment, and habitability systems, as well as costing and contracting, computer-aided ship design, ship construction, launching, trials preparation, and delivery are omitted from the text. These subjects are extensively covered in a SNAME publication, *Ship Design and Construction*, to which this book frequently refers.

The text makes extensive use of material from *Modern Ship Design* by Professor Thomas Gillmer. Professor Bruce Johnson has contributed a considerable amount of new material, including example problems.

Because of the desired 1982 publication date, there was insufficient time to convert existing material to dual English-metric units. This will be accomplished for the second edition of the book. The symbols generally conform to those adopted by the International Towing Tank Conference (ITTC) and the International Ship Structures Conference (ISSC), except in those cases where symbols have not yet been standardized or where certain symbols have been chosen for pedagogical reasons. Professor Johnson is currently responsible for updating the international standard symbol list as a member of the Information Committee for the Nineteenth ITTC, which will meet in 1984.

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Critical reviews of portions of the manuscript were supplied by Dr. John Ince (chapters 1–10), Dr. Robert Pond (chapter 4), Mr. Robert Peach (chapters 4 and

5), Mr. Robert Batman of the Naval Sea Systems Command (chapter 9), Mr. John Hill (chapters 11 and 12), and Mr. Howard Chatterton (chapters 13 and 14). For his assistance in the selection of homework problems and in checking the accuracy of answers, special thanks are due to Lieutenant Commander Dennis Jones.

The authors acknowledge the support and encouragement of the Naval Institute staff, especially Mr. Richard Hobbs, who guided the project, Ms. Constance Buchanan, who carefully edited the manuscript, and Ms. Beverly Baum and Ms. Cynthia Taylor, who coordinated the layout and artwork. The authors also acknowledge the assistance of Mrs. Dorothy Johnson throughout the project.

Much of the basic material in this text is borrowed from the source book *Modern Ship Design* by Thomas C. Gillmer. The authors are indebted to those who cooperated with him in the two editions of that volume, which is still in print today.

Symbols and Abbreviations

The following symbols are considered the most applicable and convenient notations for use by a student of naval architecture. In most cases, they conform to the standard usages approved by the Society of Naval Architects and Marine Engineers and the International Towing Tank Conferences. In certain instances, minor deviations to avoid duplication are considered more advantageous for use in this text. Numerical subscripts following a symbol denote successive locations, progressions, or values of the basic symbol.

A —area, in general	b —span of a control surface (perpendicular to direction of flow)
A_M —area, midships section	$\bar{b}$ —mean span of a control surface
A_O —area of propeller disk	C_A —model-ship correlation allowance
A_P —projected area of a foil or propeller disk	C_B —block coefficient, ∇/LBT
A_W —area, waterplane	C_D —drag coefficient, $D/1/2\rho Av^2$
A_X —area, maximum transverse section	C_F —frictional-resistance coefficient, $R_F/1/2\rho Sv^2$
AP —after perpendicular	C_{IL} —longitudinal-waterplane inertia coefficient, $12I_L/L^3B$
AR —aspect ratio, $\bar{b}/\bar{c}$	C_{IT} —transverse-waterplane inertia coefficient, $12I_T/LB^3$
$\overline{AF}$ —distance from after perpendicular to center of flotation	C_L —lift coefficient, $L/1/2\rho Av^2$
$\overline{AG}$ —distance from assumed center of gravity to actual center of gravity	C_M —midship section coefficient, A_M/BT
$\overline{AZ}$ —horizontal distance from assumed center of gravity to Z	C_P —prismatic coefficient, ∇/A_XL
a —area (generally a small area within a system)	C_R —residual-resistance coefficient, $R_R/1/2\rho Sv^2$
a —linear acceleration	C_S —wetted-surface coefficient, $S/\pi BL$
B —beam or breadth, molded, of a ship	C_T —total-resistance coefficient, $R_T/1/2\rho Sv^2$
B —position of center of buoyancy	C_V —viscous-resistance coefficient, $R_V/1/2\rho Sv^2$
B_1 (etc.) —changed position of the center of buoyancy	C_{VP} —vertical prismatic coefficient, ∇/A_WT
BHP —brake horsepower	C_W —wavemaking-resistance coefficient, $R_W/1/2\rho Sv^2$
$\overline{BM}$ —transverse metacenter above center of buoyancy (metacentric radius)	C_{WP} —waterplane-area coefficient, A_W/LB
$\overline{BM}_L$ —longitudinal metacenter above center of buoyancy (longitudinal metacentric radius)	C_X —maximum-transverse-section coefficient, $A_X/(BT)_X$
b —width of a compartment or tank	c —chord of a control surface (parallel to direction of flow)
b —center of buoyancy of an added buoyant layer	$\bar{c}$ —mean chord of a control surface
	c —damping coefficient
	c —distance from neutral axis to extreme fiber
	c_w —celerity or phase velocity of a wave
	$CL(\text{C})$ —centerline
	$\overline{D}$ —drag force
	D —diameter, generally
	D —molded depth of a ship hull
	DWL —designed load waterline
	DWT —deadweight tons
	dx —increment of length along an axis (also dy , dz , dA , etc.)



E —modulus of elasticity or Young's modulus	$\overline{KB}$ —distance from the keel (baseline) to the center of buoyancy
E —energy, generally	$\overline{KG}$ —distance from the keel (baseline) to the center of gravity
EHP —effective horsepower	$\overline{KM}$ —distance from the keel (baseline) to the transverse metacenter
e —elongation	$\overline{KM}_L$ —distance from the keel (baseline) to the longitudinal metacenter
F —position of center of flotation (centroid of waterplane)	$\overline{Kg}$ —distance from the keel (baseline) to the center of gravity of a component weight
F_1 —changed position of the center of flotation	k —radius of gyration about an axis, i.e. k_x, k_y
F —force, generally	k —form factor for hull forms, $(C_V - C_F)/C_F$
F_B —buoyancy force	k —spring constant
F_n —Froude number, $v/\sqrt{gL}$	L —lift force
F_R —resultant force	L —length, generally
$\overline{FB}$ —distance from forward perpendicular to center of buoyancy	L —reference length of a ship
$\overline{FF}$ —distance from forward perpendicular to center of flotation	L_{OA} —length of a ship, overall
$\overline{FG}$ —distance from forward perpendicular to center of gravity	L_{OS} —length of a ship, overall, submerged
FP —forward perpendicular	L_{PP} —length of a ship between perpendiculars
FW —fresh water (usually taken as 36 cu ft per ton)	L_{WL} —length of a ship on the waterline
f —coefficient of friction	L_w —length of a wave from crest to crest
f —frequency, cycles per unit time	LCB —position of longitudinal center of buoyancy
f_E —frequency of encounter	LCF —position of longitudinal center of flotation
G —position of ship's center of gravity	LCG —position of longitudinal center of gravity
G_1 (etc.) —changed position of ship's center of gravity	LWL —load, or design, waterline
G —shear modulus, $E/2(1 + \mu)$	l —length of an internal compartment or tank
$\overline{GG}_1$ —distance through which the ship's center of gravity moves	l —longitudinal distance through which a weight is moved
$\overline{GM}$ —transverse metacentric height, height of M above G	M —moment, generally
$\overline{GM}_L$ —longitudinal metacentric height, height of M_L above G	M —position of transverse metacenter
$\overline{GZ}$ —righting arm; horizontal distance from G to Z	M_L —position of longitudinal metacenter
g —acceleration of gravity	M_x —bending moment at any section in a ship's length
g —position of center of gravity of a component weight	MTI'' —moment to change trim one inch
g_1 —changed position of center of gravity of a component weight	m —mass, generally, (W/g) or (w/g)
H —wave height, trough to crest	N —total number of increments in a summation
h —depth of water or of submergence	NA —neutral axis
I —moment of inertia, generally	n —revolutions per unit time (generally per second)
I_L —longitudinal moment of inertia of waterplane	n —any integer
I_T —transverse moment of inertia of waterplane	O —origin of coordinates
I_x —moment of inertia of hull girder section about neutral axis	OX —longitudinal axis of coordinates
i —moment of inertia of free surface in a compartment	OY —transverse axis of coordinates
i —incrementing integer in a summation	OZ —vertical axis of coordinates
J —advance coefficient, v_A/nD	P —power, generally
K —any point in a horizontal plane through the baseline (keel)	P —pitch of a propeller
K_Q —torque coefficient, $Q/\rho n^2 D^5$	P —load force
K_T —thrust coefficient, $T/\rho n^2 D^4$	$P.C.$ —propulsive coefficient
	PHP —propeller horsepower
	p —pressure, generally
	p_o —static pressure
	Q —torque, generally
	Q_D —torque delivered to the propeller
	Q_o —open-water propeller torque
	Q_R —rudder torque



- Q_S —torque measured on the shafting just inside the stern tube
 q —dynamic pressure, $\frac{1}{2}\rho v^2$
 R —resultant force
 R —radius of a propeller
 R —resistance, in general
 R_{AP} —appendage resistance
 R_F —frictional resistance
 R_R —residual (residuary) resistance
 R_T —total resistance
 R_V —viscous resistance
 R_W —wave-making resistance
 R_n —Reynolds number, vL/ν
 r —position vector
 rpm —revolutions per minute
 rps —revolutions per second
 S —wetted-surface area
 SHP —shaft horsepower
 SW —salt water (usually taken as 35 cu ft per ton)
 s_A —apparent slip ratio of a propeller, $1 - v/Pn$
 s_R —real slip ratio of a propeller, $1 - v_A/Pn$
 T —draft
 T_A —draft, aft
 T_F —draft, forward
 T_M —mean draft, $(T_A + T_F)/2$
 T —thrust (force)
 T —period of oscillation, generally
 T —wave period
 T_z —natural period for heaving
 T_θ —natural period for pitching
 T_ϕ —natural period for rolling
 TPI —tons-per-inch immersion
 THP —thrust horsepower
 t —time, generally
 t —transverse distance through which a weight is moved
 t —thickness of a foil section
 t —thrust-deduction fraction, $(T - R_T)/T$
 U —velocity of incident flow
 u —deformation in the x direction
 V —volume, in general
 V —linear velocity in general; speed of a ship
 V_A —speed of advance of a propeller
 V_M —model speed in knots
 V_S —ship speed in knots
 V_x —shearing force across a section at point x along the ship's length
 v —vertical distance through which a weight is moved
 v —specific volume, $1/\rho g$
 v —particle velocity in a gravity wave
 v_M —model velocity in ft/sec
 v_S —ship velocity in ft/sec
 VCB —vertical position of center of buoyancy
 VCG —vertical position of center of gravity
 W —weight in general; weight of a ship
 WL —any waterline parallel to the baseline
 WL_1 —changed position of WL
 w —weight of an individual item
 w —Taylor wake fraction, $1 - V_A/V_S$
 w_x —weight per unit length
 w —specific weight, ρg
 x_A —added-mass coefficient
 x —distance from origin along X -axis
 y —distance from origin along Y -axis
 y —distance from neutral axis to a fiber
 y —transverse distance from ship's centerline to center of gravity of flooding water in free communication with the sea
 y —distance from the water surface along an inclined axis
 Z —a point vertically over B , opposite G
 Z —section modulus, I_x/c
 z —distance from origin along Z -axis
 z —heave-motion displacement
- ### Special Naval Architectural Symbols
- (Φ) —centerline or centerplane axis
 $(\mathbb{B})$ —baseline
 $\overline{\otimes}$ —midship-section designation
 $\overline{\otimes}_{pp}$ —midlength between perpendiculars
 $\overline{\otimes}B$ —longitudinal distance from amidships to center of buoyancy, B
 $\overline{\otimes}G$ —longitudinal distance from amidships to center of gravity, G
 $\overline{\otimes}F$ —longitudinal distance from amidships to center of flotation, F
 $\overline{\otimes}b$ —longitudinal distance from amidships to the center of the added buoyant layer (mean center of flotation)
 $\overline{\otimes}g$ —longitudinal distance from amidships to the center of gravity of an added or removed weight, w
 ∇ —volume of displacement ("vol"), 35 Δ
- ### Greek Symbols
- α (alpha) —angle of incidence; angle of attack
 β (beta) —angle of attack in yaw on the hull (or drift angle), measured from the resultant ship velocity V to the xz_0 -plane
 γ (gamma) —specific gravity
 γ —shearing strain
 Δ (Delta) —displacement in tons salt water
 δ (delta) —small increment
 δ_R —rudder angle
 δT_M —parallel sinkage
 $\delta trim$ —change in trim
 ϵ (epsilon) —normal strain
 η (eta) —efficiency, generally

η_B —propeller efficiency behind hull
 η_D —quasi-propulsive coefficient
 η_H —hull efficiency, $(1 - t)/(1 - w)$
 η_O —propeller efficiency in open
 η_P —propulsive efficiency, $\eta_D \times \eta_S$
 η_R —relative rotative efficiency
 η_S —shaft transmission efficiency
 Θ —angle, in general
 θ (theta) —angle of pitch or of trim (about OY -axis)
 Λ (Lambda) —tuning factor, relative to synchronism
 λ (lambda) —linear scale ratio, ship to model
 μ (mu) —Poisson's ratio
 μ (mu) —coefficient of dynamic viscosity
 μ —volume permeability factor
 μ_s —surface permeability factor
 ν (nu) —coefficient of kinematic viscosity; μ/ρ
 ρ (rho) —mass density, weight per unit volume/g
 σ (sigma) —normal stress
 τ (tau) —shear stress
 ϕ (phi) —angle of heel or roll (about OX -axis)
 ψ (psi) —angle of yaw (about OZ -axis)
 ω (omega) —angular velocity
 ω (omega) —circular frequency, $2\pi/T$

Mathematical Symbols

∂ —partial derivative sign
 i — $\sqrt{-1}$
 $\approx$ —is approximately equal to
 $<$ —is less than

$>$ —is greater than
 $\dot{x}$ (one dot over a variable) —the first derivative of the variable
 $\ddot{x}$ (two dots over a variable) —the second derivative of the variable
 $\propto$ —proportional to
 ∞ —infinity
 δ (delta) —a finite increment
 Σ (Sigma) —summation of
 π (pi) —ratio of circumference of circle to diameter
 $\int$ —integral of

Abbreviations for References

ABS —American Bureau of Shipping
 ASNE —American Society of Naval Engineers
 ASCE —American Society of Civil Engineers
 ATTC —American Towing Tank Conference
 BSRA —British Shipbuilding Research Association
 DTNSRDC —David Taylor Naval Ship Research and Development Center
 IMCO —International Maritime Consultative Organization
 ITTC —International Towing Tank Conference
 (R) INA —(Royal) Institution of Naval Architects
 SNAME —Society of Naval Architects and Marine Engineers
 USNA —United States Naval Academy

Introduction to Naval Architecture

CHAPTER 1

Ships Categorized

1-1 Introduction

The forms a ship can take are innumerable. A vessel might appear to be a sleek seagoing hotel carrying passengers along to some exotic destination; a floating fortress bristling with missile launchers; or an elongated box transporting tanks of crude oil and topped with complex pipe connections. None of these descriptions of external appearance, however, does justice to the *ship system* as a whole and integrated unit—self-sufficient, seaworthy, and adequately stable in its function as a secure habitat for crew and cargo. This is the concept that the naval architect keeps in mind when designing the ship and that provides the basis for subsequent discussions, not only in this chapter but throughout the entire book.

In order to discuss naval architecture, it is helpful to place ships in certain categories. For purposes of this text, ships are classified according to their means of physical support and their designed purposes.

1-2 Ships Typed According to Means of Physical Support

The mode of physical support by which vessels can be categorized assumes that the vessel is operating under designed conditions. Ships are designed to operate above, on, or below the surface of the sea, so the air-sea interface will be used as the reference datum. Because the nature of the physical environment is quite different for the three regions just mentioned, the physical characteristics of ships designed to operate in those regions can be diverse.

AEROSTATIC SUPPORT

There are two categories of vessels that are supported above the surface of the sea on a self-induced cushion of air. These relatively lightweight vehicles are capable

of high speeds, since air resistance is considerably less than water resistance, and the absence of contact with small waves combined with flexible seals reduces the effects of wave impact at high speed. Such vessels depend on *lift fans* to create a cushion of low-pressure air in an underbody chamber. This cushion of air must be sufficient to support the weight of the vehicle above the water surface.

The first type of vessel has flexible “skirts” that entirely surround the air cushion and enable the ship to rise completely above the sea surface. This is called an *air cushion vehicle* (ACV), and in a limited sense it is amphibious (figure 1-1).

The other type of air-cushion craft has rigid side walls or thin hulls that extend below the surface of the water to reduce the amount of air flow required to maintain the cushion pressure. This type is called a *captured-air-bubble vehicle* (CAB). It requires less lift-fan power than an ACV, is more directionally stable, and can be propelled by water jets or supercavitating propellers (figure 1-2). It is not amphibious, however, and has not yet achieved the popularity of the ACVs, which include passenger ferries, cross-channel automobile ferries, polar-exploration craft, landing craft, and riverine warfare vessels.

HYDRODYNAMIC SUPPORT

There are also two types of vessels that depend on *dynamic support* generated by relatively rapid forward motion of specially designed hydrodynamic shapes either on or beneath the surface of the water. A principle of physics states that *any moving object that can produce an unsymmetrical flow pattern generates a lift force perpendicular to the direction of motion*. Just as an airplane wing (airfoil) produces lift when moving through the air, a *hydrofoil*, located beneath the surface and attached by means of a surface piercing strut, can dy-



Figure 1-1. A one-hundred-ton surface effect ship, an air-cushion vehicle (ACV), making a test run. These vessels are supported on a cushion of relatively low-pressure air supplied by ducted internal fans. They are propelled at potential speeds up to 80 knots by turbine-driven water jets. (Courtesy of Bell Aero Space Co.)

namically support a vessel's hull above the water (figure 1-3).

Planing hulls are hull forms characterized by relatively flat bottoms and shallow V-sections (especially forward of amidships) that produce partial to nearly full dynamic support for light displacement vessels and small craft at higher speeds (figure 1-4). Planing craft are generally restricted in size and displacement because of the required power-to-weight ratio and the structural

stresses associated with traveling at high speed in waves. Most planing craft are also restricted to operations in reasonably calm water, although some “deep V” hull forms are capable of operation in rough water.

HYDROSTATIC SUPPORT

Finally, there is the oldest and most reliable type of support, *hydrostatic support*. All ships, boats, and primitive watercraft up to the twentieth century have de-



Figure 1-2. Another type of surface effect ship, a captured-air-bubble vehicle (CAB), is supported on a cushion of air that is screened at the sides by the rigid side structure of the vessel.



Figure Figure 1-3. The hydrofoil-supported vessel. These craft have the best seakeeping potential of the various hydrodynamically supported vessels.

pendent upon the easily attained buoyant force of water for their operation.

This hydrostatic support, commonly recognized as *flotation*, can be explained by a fundamental physical law that the ancient philosopher-mathematician Archimedes defined in the second century B.C. *Archimedes' Principle* states that a body immersed in a liquid is buoyed up (or acted upon) by a force equal to the weight of the liquid displaced. This principle applies to all vessels that float (or submerge) in water—salt or fresh. And from this statement the name of the ships in the category are derived; they are generally called *displacement hulls*.

Although this ship type is very familiar, its subcategories warrant special discussion. For example, in some vessels reasonably high speed must be combined with the ability to carry light cargo or to move more comfortably in rough water than a planing hull. High-speed planing-hull characteristics can be modified to produce a semidisplacement hull or semiplaning hull (figure 1-5). These compromise craft, of course not as fast as full-planing hulls but faster than conventional displacement hulls, must have more power and less weight than the latter. Such types are obviously the result of “trade-offs.”

The example cited above lies between clear-cut physically defined categories—it is not a good example of a variation of a true displacement-type ship. The latter must be recognized primarily as a displacement vessel, and its variations depend primarily on the distribution of buoyant volume—the extent of the depth and breadth of the hull below the water.

The most ubiquitous type of displacement ship can be generally classified as the *common carrier*, a seagoing vessel. It may be employed for passenger service, light cargo-carrying, fishing by trawling, or for hundreds of other tasks that do not require exceptional capacity, speed, submergence, or other special performance (figure 1-6). It is the most common and easily recognizable type of ship, with moderate displacement, moderate speeds, moderate to large lengths, and moderate capacities. It usually embodies the maximum in cruising range and seaworthiness. It is the “ship for all seasons.” It is the standard to which all other ship classifications in the displacement category may be referred (figure 1-7).



Figure 1-4. HMS *Tenacity*, a fast fisheries patrol vessel, is essentially a planing hull. A large example of this type, she is 142 feet long, powered by gas turbines, and capable of 40-kt speeds. She was originally intended as a fast surface-to-surface missile carrier. (Courtesy of Vosper-Thornycroft, Ltd.)



Figure 1-5. The USS *Welsh*, a fast navy gunboat, is capable of planing speeds but has a combination power plant more economical for cruising at lower speeds. She thus operates as a semiplaning hull.



Figure 1-6. The handsome and conventional displacement ship is an excellent example of a modern luxury ship designed as a passenger cruise ship, one of the last examples of a disappearing type, the seagoing passenger ship. (Courtesy of Swan-Hunter Shipbuilders, Ltd.)



Figure 1-7. The hard-working and valuable displacement ship of the SL 7 type is currently the fastest cargo ship in the world, capable of sustained cruising speeds of 33 knots. (Courtesy of Sea-Land Industries, Inc.)

The closest relative to this standard vessel, which plays a crucial role not only in world commerce but in the survival of the industrial world as well, is the bulk oil carrier, the tanker, or supertanker (figure 1-8). These terminologies are common but unspecific, and in this discussion they are inadequate, for what was called a supertanker several years ago is today not a supertanker. The industry itself has created a far more explicit nomenclature. Based upon the index of 100,000 tons oil cargo capacity, the size categories are LCC (Large Crude Carrier), VLCC (Very Large Crude Carrier), and ULCC (Ultra Large Crude Carrier). Any tanker greater than 100,000 tons but less than 200,000 is a LCC, those between 200,000 and 400,000 are VLCCs, and those over 400,000 are ULCCs. The current necessity for these designations becomes clear when we realize that before 1956 there were no tankers larger than 50,000 tons, and not until the early sixties were any ships built larger than 100,000 tons. In 1968 the first ship over 300,000 tons was built. With their bulk and enormous capacity (four football fields can be placed end to end on one of their decks), these ships are designed and built to be profit-makers, enormously long,

wide, and deep, carrying thousands of tons of crude oil per voyage at the least cost. Few of these elephantine tankers have more than one propeller shaft or rudder. Their navigation bridges are nearly one quarter of a mile from their bows. Their top service speed is so low that a voyage from an Arabian oil port to a European destination normally takes two months.

Such vessels belong to a category of displacement ship that has a great range of buoyant support. They have a very large and disproportionate hull volume below the surface when fully loaded. Indeed, the cargo weight far exceeds the weight of the ship itself. The draft or depth of water required for a fully loaded VLCC runs to 50 or 60 feet and the ULCC may be 80 feet. Such ships belong in the exclusive category of displacement vessels called *deep displacement* ships.

There exists another type of displacement hull with extreme draft. However, its similarity to the crude-oil carrier of the preceding discussion goes no further than that. This type of vessel is called the SWATH (Small Waterplane Area Twin Hull). Briefly, this rather rare breed of ship is designed for relatively high speed and stable platform in moderately rough water. Its future

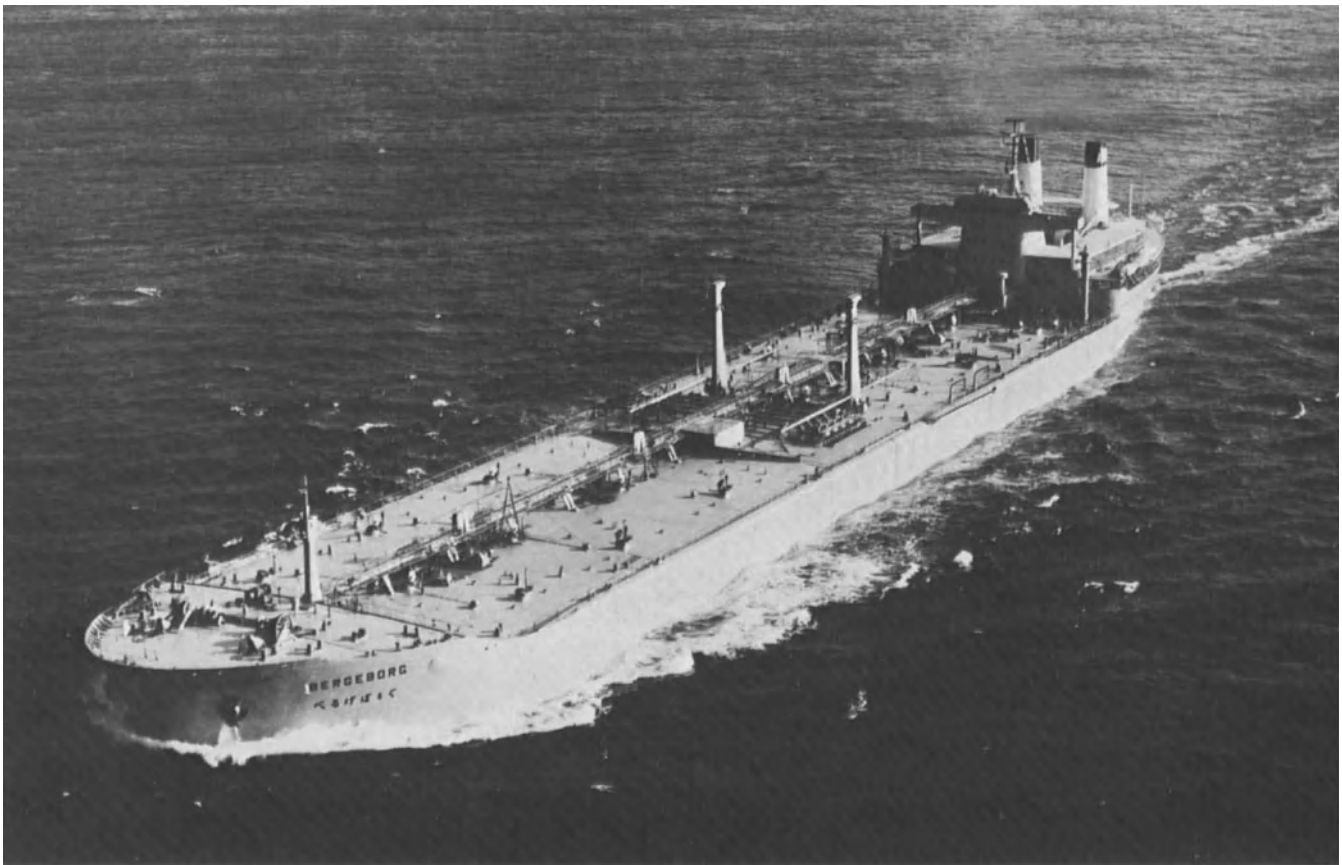


Figure 1-8. The heaviest and largest of all displacement ships, the crude oil tanker. Sometimes referred to as Large Crude Carriers and Very Large Crude Carriers, such ships are built as large as one-half-million tons capacity.

is problematical, but the theory of placing the bulk of the displacement well below the surface and extending the support to the above-water platform or deck through the narrow waterline fins or struts is sound. Twin hulls connected by an upper platform provide the necessary operating stability (figure 1-9).

The most significant class of displacement hull for special application is the submarine, a vessel for completely submerged operation. The nature of the submarine and a description of her various operational attitudes, both static and dynamic, is covered in subsequent chapters. It is only necessary here to emphasize that submersible vessels are specifically displacement vessels applying the theory of Archimedes' Principle and all that it implies (figure 1-10).

MULTIHULL VESSELS

There is one other type of hull in common use that has not yet been mentioned, primarily because it fits into none of the categories described but rather can exist comfortably in any. This craft is the so-called *multihull*

vessel—the catamaran and the trimaran. These vessels are most frequently displacement hulls in their larger sizes, such as the SWATH mentioned above, or more conventionally, ocean research vessels requiring stable platforms and protected areas for launching equipment (figure 1-11). There are also the twin-hulled CAB vessels mentioned earlier and high-speed planing catamarans. Actually, the multihull ship is an adaptation of any of the basic hull categories to a special application that requires exceptional transverse stability and/or the interhull working area.

Figure 1-12 indicates the body profiles (with no relative scale) that have just been described and relates them to their means of physical support. They are arranged from high to low speed, except for the multihull types, which may be either speed, depending upon their purpose.

1-3 Other Criteria

There are other criteria that justify the widely varied configurations of ship design. They are the result of

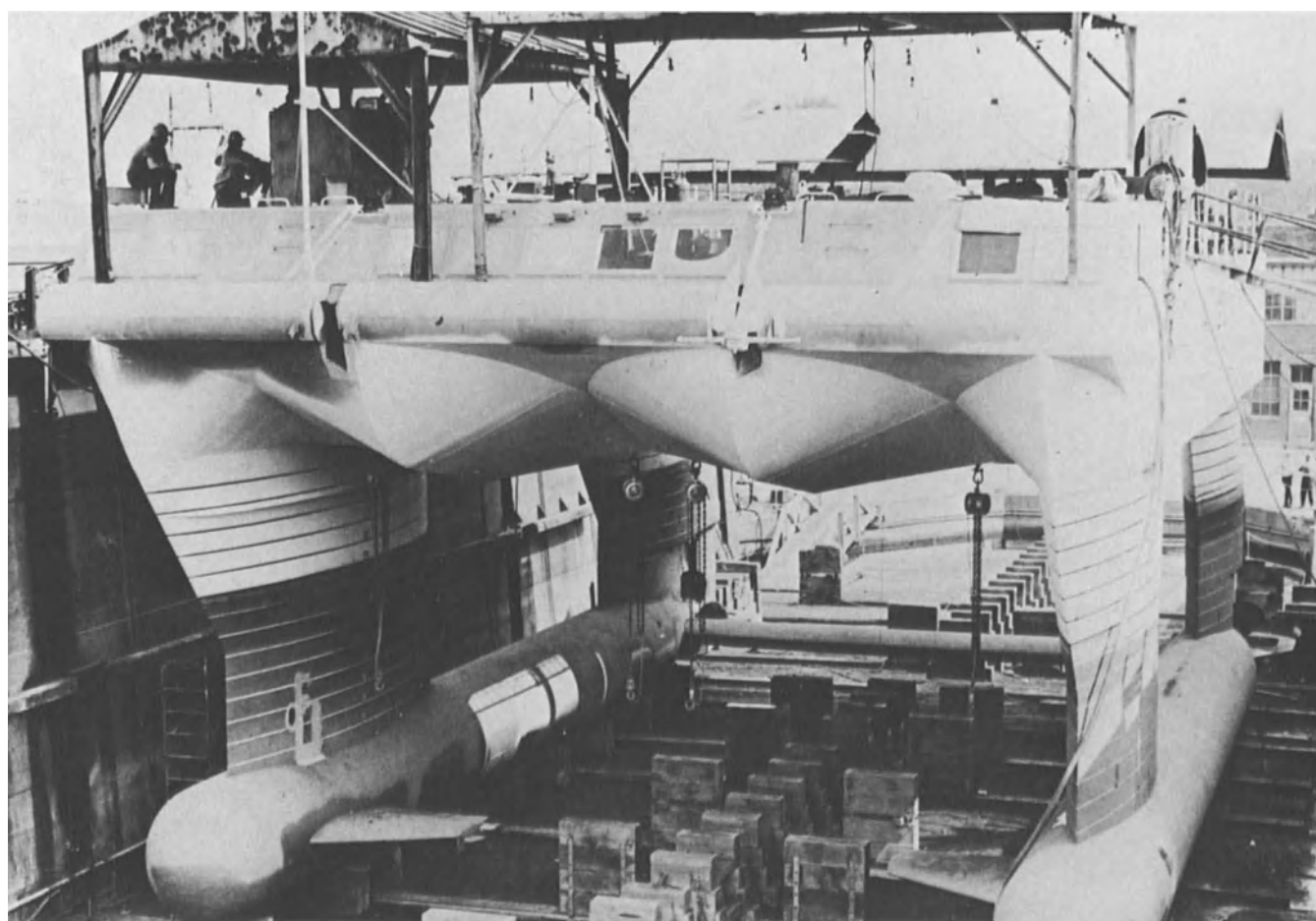


Figure 1-9. The deep-draft types are still largely experimental concepts. They are categorized as small waterplane area twin-hull designation, or SWATH. Potentially fast for displacement ships, they are very stable with minimum motion in a seaway.

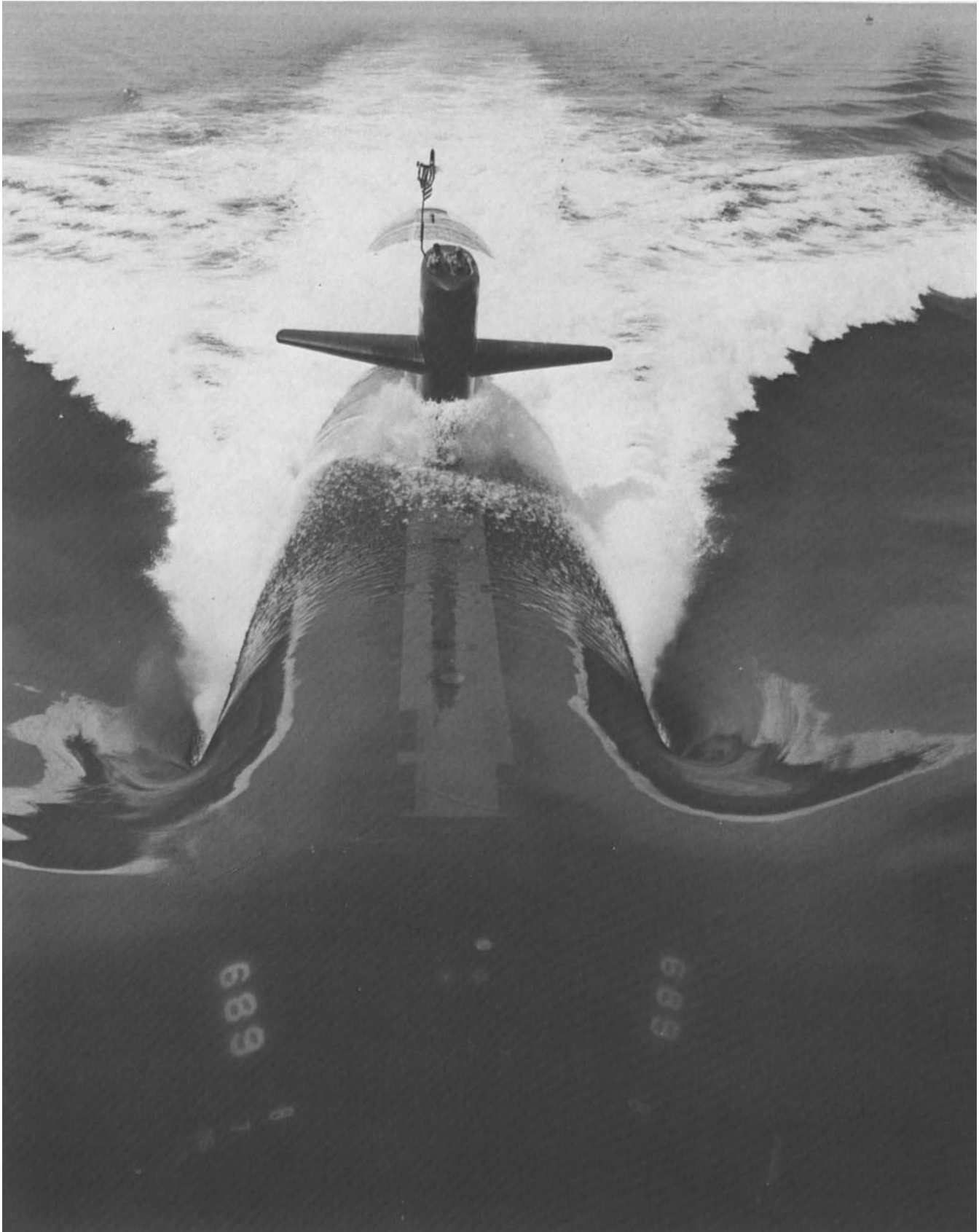


Figure 1–10. The modern navy attack submarine, the best example of an advanced submersible vessel. Nuclear powered, they are designed for optimum performance entirely below the surface.

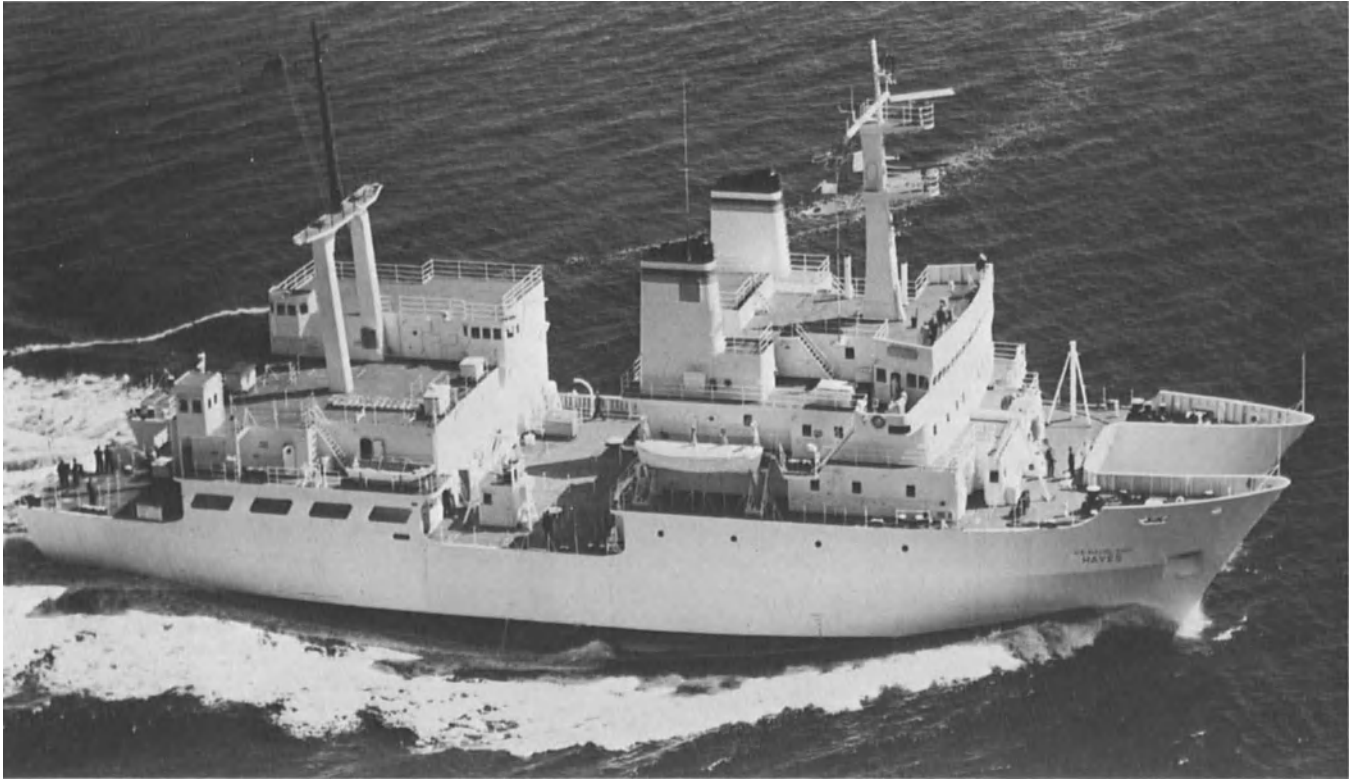


Figure 1–11. The oceanographic research vessel USNS *Hayes*, one of the best examples of a multi-hull vessel, generally a costly, controversial type

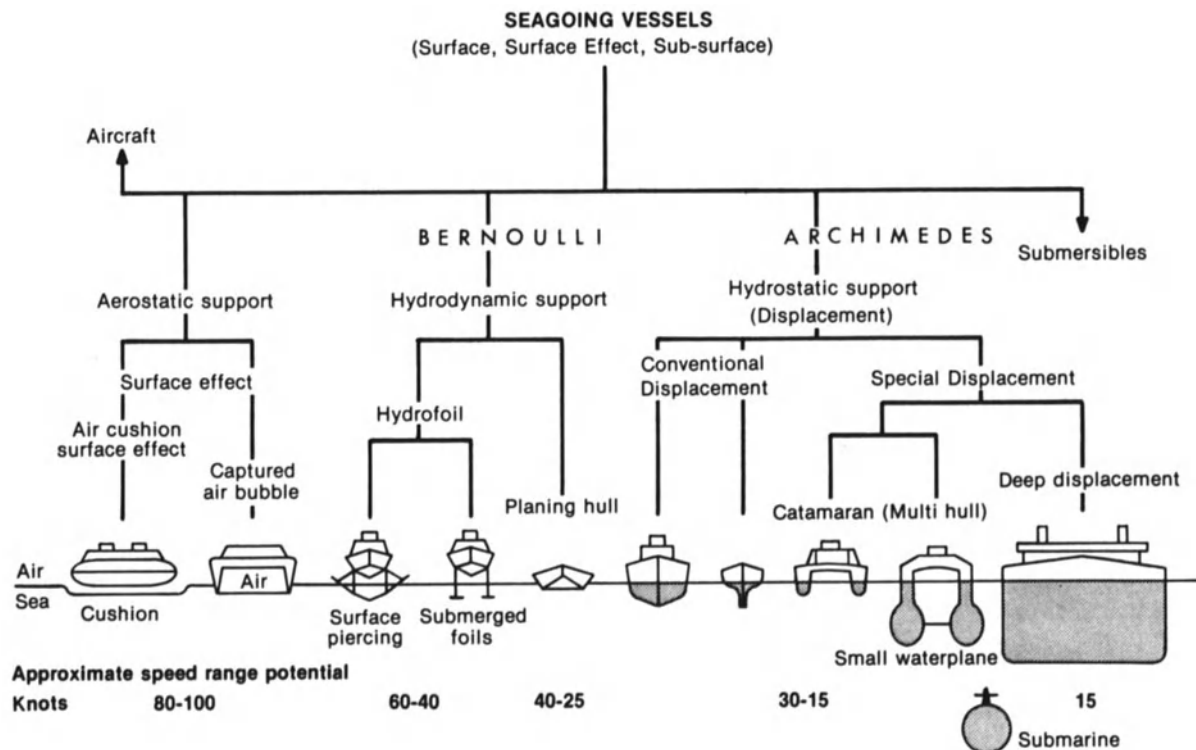


Figure 1–12. Categories of seagoing vessels arranged according to their mode of support on or in the sea

trade-offs concerning cost, mission, speed, endurance, payload (cargo or weapons capacity), operating environment (stability, survivability, and port requirements), reliability, appearance, personal comfort and habitability, and political considerations. The relative importance of the various factors is dictated by the purpose of the vessel, which is set by the commercial firm, government, or individual who purchases the vessel. A useful classification based on purpose includes the following categories: merchant and commercial ships, naval vessels, and pleasure craft.

MERCHANT AND COMMERCIAL SHIPS

Merchant and commercial ships are generally bought to earn a profit. The previously discussed cargo ships are designed for the minimum (or at least competitive) “required freight rate,” which involves predicting the “life-cycle cost” of the ship, including the acquisition costs, the operating and maintenance costs, and any salvage value remaining when the ship is sold. A “cash-flow analysis” is made to establish what rate of return can be expected on the owner’s investment.

New designs of all commercial vessels, including cargo ships, passenger liners, fishing boats, offshore supply vessels, and tugboats, must compete economically with similar vessels available from the many worldwide shipbuilders. Government subsidies protecting the national shipbuilding industry from foreign competition can result in lower costs to the purchaser, even when the actual ship construction costs are higher. Thus, political

considerations can play an important role in the economics of commercial ship design and construction.

Appearance, personal comfort, and reliability are necessary for a luxury passenger liner to attract customers, whereas payload, endurance, and ability to survive a hostile sea environment are important considerations in the design of fishing vessels. Offshore supply vessels are concerned with speed for oil-rig crew transport or emergency services, but slower speeds may be acceptable when payloads such as drill pipe and drilling mud are the principal cargo. Operating environment includes both wind and wave conditions at sea and port and harbor capabilities ashore. Thus, deep-draft vessels may be excluded from certain geographic areas. Special-purpose cargo-handling devices such as the unloading ramps on roll-on/roll-off (Ro/Ro) ships (figure 1–13) may be necessary for quick turnaround both at principal worldwide ports and those of underdeveloped countries. The latter ports impose other cargo-handling restrictions on the ship designer.

NAVAL AND COAST GUARD VESSELS

Naval vessels are generally classified as combatants or auxiliaries, although there are special-purpose craft that do not fit easily into either category. For large combatant warships such as aircraft carriers, guided-missile cruisers, destroyers, and nuclear submarines, all of the previously mentioned factors become important—hence the enormous cost of such ships. Their military mission is of prime importance, but carrying out the mission



Figure 1–13. The *Lipscomb Lykes*, a Ro/Ro ship. (Courtesy of Melvin Freden.)

depends on speed, endurance (possibly aided by at-sea replenishment for surface ships), weapons payload, and ability to operate and survive in hostile environments. Reliability under combat conditions, the appearance of military power, crew habitability that influences reenlistments, and the political importance of who becomes the prime contractor and principal weapons-system subcontractors: all these are factors that must be taken into consideration, making the construction and operation of warships very expensive for taxpayers.

Naval auxiliaries are more closely related to commercial ships in appearance, but their mission may involve operating with warships, which requires compatibility in terms of speed, endurance, required payload, and the ability to conduct replenishment operations during poor sea conditions. Thus, one can expect the cost of such ships to be greater than that of their commercial counterparts.

Oceanographic research vessels, Coast Guard cutters, and ice-breakers all have missions in which endurance, reliability, ability to operate in difficult environments, and habitability are important. Since smaller vessels have limited fuel capacity, there is a trade-off between speed and endurance; hence two types of power plant are frequently used to optimize both speed and endurance. The more exotic craft discussed in the previous sections generally sacrifice payload and endurance for speed.

PLEASURE CRAFT

Pleasure craft, both motor powered and sail powered, come in a wide variety of sizes and shapes to suit individual requirements and tastes. The economic trade-offs are based on what the potential buyer can afford or thinks he can afford. Appearance, speed, personal comfort and habitability, and stability are the major criteria for designs that satisfy the purpose of the craft, which is the enjoyment of leisure time.

GENERAL CONSIDERATIONS

In such general comparisons and categorizations it is necessary to return to a practical perspective. It is all very well to arrange type categories with respect to the common denominator of supporting force or mission, but the question of relative significance in harder terms must ultimately be answered. The graphic comparisons in figure 1-14 convey the great variation in relative performance and capability as a function of ship category based on type of support.

How many ships of each of these categories can justify themselves in terms of economic support and environmental capabilities? How many are purely experimental? What can be expected of their future? Where these questions apply to the problems faced by the ship designer, an attempt has been made in the following chapters to provide the background for adequate evaluation.

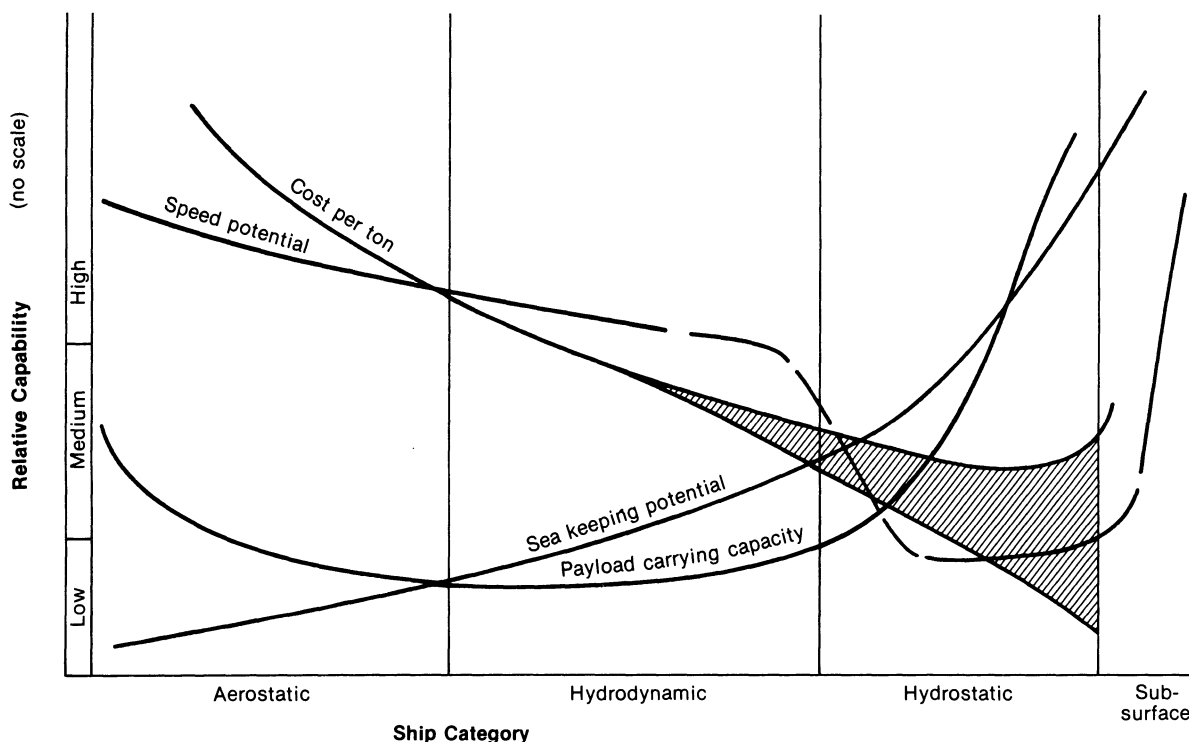


Figure 1-14. The relative performance and associated capabilities of the ship categories of figure 1-12

A more detailed discussion of these comparative factors can only be made after the technical presentations of this book have been absorbed. But it must be emphasized here that most of this book will deal with the physical nature of *displacement* ships, simply because almost all of the ships on the world's oceans are and probably will be of this type. They carry the raw materials of world commerce and a nation's military strength to most parts of the globe. Without them the civilized industrialized world would quickly collapse.

The ships of recent years have noticeably progressive features in their external configuration. The old stacks or funnels have been replaced by raked, streamlined stacks or transverse pairs of slim diesel funnels. Superstructures have become crisp and uncluttered. Hulls of tankers and bulk carriers have become monstrous in volume. Fast cargo carriers and naval vessels have acquired new grace in their sheer and flare. Below the waterline, improved hydrodynamic knowledge has resulted in bulbous forefoot extensions and improved rudder configurations. There are a multitude of internal developments provided by modern technology, including the less visible changes in strength and performance allowed by improved metals and other materials.

1-4 The Systems Approach

The greatest change in new ships, however, is not very evident in their structure. This is because designers, planners, and operators recognize that a ship is an extremely complex but integrated total *system* (figure 1-15).

It is increasingly difficult to design and build a ship without regard to the systems-engineering approach. Because of the rapidly mushrooming technology of this century, there has been growing specialization within

the engineering professions. This has led to the need for a way to deal with complex assemblies made up of many specialized components. If they are to be capable of optimum performance, such complex assemblies as the Trident submarine or the nuclear aircraft carrier must be designed in an orderly manner. This integrated approach is ordinarily referred to as *systems engineering*.

Systems engineering is employed in the design of all naval vessels and most commercial craft today, and the student of ship design should become familiar with it early in his engineering education. We might define the approach as a process for *achieving significant objectives, allocating resources, and organizing information* so that all major aspects of a problem can be precisely determined and coordinated according to a plan. Systems engineering supplies the bridge between what is needed and what is technically feasible.

SYSTEMS IN SHIPS

Systems engineering, whether it is applied to a large ocean transport ship, a warship, or a very small vessel, implies total integration of all subsystems to provide a functional unit that achieves the basic mission of the ship. This means that ship control must function through the internal and external communications systems, and the machinery and propulsion systems must react to control, signaling their responses on display instruments at the central control station. The weapons systems of a warship must function on order with simultaneous execution and respond to all safety and protective systems. Systems engineering includes all automatic control systems as well as a multitude of engineering and electronic subsystems that maintain order and perform daily living and emergency functions. In the last century



Figure 1-15. The USS *Enterprise* (CVAN 65), the largest of all ship systems

or more of successful mechanical propulsion, the ship has undergone fundamental changes; no longer is she merely a large floating vessel with a relatively isolated power plant, isolated cargo holds and living quarters, and a lonely navigation bridge with its crude mechanical or sound-signaling device to the engine room. In a sense, the ship of a century ago was a system too, but her design lacked the systematic, integrated approach demanded for the successful modern ship.

In the design of a modern warship, shipbuilders and naval architects recognize the following major subsystems of functional groupings:

1. Hull
2. Machinery and propulsion plant
3. Electric plant
4. Command and surveillance
5. Auxiliary systems
6. Outfit and furnishings
7. Armament

The interfaces and interplay among these subsystems must be identified and defined. The secondary subsystems in each must then be defined by more detailed functional blocks. Components for emergency operation and casualty procedures must be provided, and in the design stages particularly, prime consideration must be given to weight and buoyancy, speed, power and endurance, volume relationships, payload capacity, general arrangements and habitability, ship-motion effects, and acoustic influences. Integration of these multiple considerations is generally accomplished before the actual preliminary design is begun.

It has been suggested that an integrated ship has a tendency to become an inflexible black box, with no capacity for growth. If this were true, the ship would contain components that might be incompatible with the logistics plans supporting other ships, and the defects in such resulting ships could be located and eliminated in the design process. Actually, the integrated ship can be made *more* rather than less compatible with logistics by using the maximum of standardized components and adapters for special applications in the subsystem design. The command control subsystem, for example, requires multipurpose, interchangeable displays tied together at the central computer subsystem and properly programmed to achieve the required compatibility and flexibility. It must also be recognized that in the “whole” ship design, proper systems engineering provides for standardization, interchangeability, and central procurement of unitized components.

THE EFFECTIVE SHIP'S SYSTEM

From the preceding section, one might get the impression that nowadays all modern ships are designed and

built as completely integrated systems. This is not completely true; rather, systems integration is a goal to be achieved insofar as economics allow, demands require, and skills of the designers and builders enable. In other words, all ships are systems, but the integration of their subsystems varies. Their usefulness in fulfilling their missions depends upon the degree of integration. The measure of this degree is found in the science of *systems effectiveness*. While it is not the purpose of this text to discuss this science in detail (most engineering students encounter the study of systems effectiveness early in their engineering curriculum), it is helpful to mention that the designer should expose his ship's system design to this orderly evaluation procedure.

Because of the great number of ship types, the varying complexities of their designs, and their nearly infinite range requirements, it is not possible to standardize an approach to effective system design. Simply stated, systems effectiveness is defined as the probability that the system will operate successfully under specific conditions throughout a given time period. This definition is general and can be applied to any system, including the ship itself.

While actual operational systems effectiveness extends beyond the designer's control and contains at least two other factors—human decisions and responses—external to design, the designer is the initial creator of the system and its effectiveness.

Some examples of engineering integration may illustrate what is *not* an effective system. A whole system comprised of determinate subsystems cannot be effective where there are such things as overlapping requirements, incompatibility, mechanical deficiencies, and lack of environmental adaptability. For example, the propulsion unit of a high-speed planing hull, if it is a marine propeller, must have comparatively small diameter, high pitch, and rapid revolutions per minute to be compatible with the hull system. To be compatible with its environment, the propeller must be made of a metal that will not form a terminal of an electrolytic system and that will therefore protect against electrolysis. The designer must ask himself such questions as: Does subsystem *A* have a material reliability of 50 percent when subsystem *B*—which is dependent on subsystem *A*—has a reliability of 95 percent and the total system requirement is to be 90 percent reliable? Further, are associated subsystems compatible? Are there gasoline auxiliary generators in a system where the primary fuel is diesel oil?

From these simple examples of systems-effectiveness evaluation, it is but a short step to realizing the need for full evaluation of the many systems existing in a modern, complex ship. Such responsibilities for the U.S. Navy lie with the Naval Sea Systems Command.

CHAPTER 2

Engineering Fundamentals in Ship Design

2-1 Introduction to Mechanics

Newtonian mechanics is the study of the motion of objects and bodies, both solid and fluid. That portion of mechanics applicable to ship design views solid bodies and fluids as continuous media. Although many of the analytical laws of motion were originally developed for particles and rigid bodies, they can be considered relevant to continuous media when applied to the center of mass or center of gravity. Isaac Newton (1642–1727) formulated the basic laws of motion when he postulated that there exists a frame of reference for which

1. Every body continues in its state of rest or of uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it.
2. The change of motion [i.e., mechanical momentum] is proportional to the motive force impressed; and is made in a direction of the right [straight] line in which that force is impressed.
3. For every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

The postulated frame of reference is called an *inertial frame*, and the commonly accepted inertial frame of reference is the fixed stars. For most engineering work at moderate speeds near the earth's surface, the earth itself can be considered a satisfactory frame of reference.

Newton's second law is commonly formulated as

$$\Sigma \mathbf{F} = \frac{d}{dt} (m\mathbf{v}) \quad (2-1)$$

where $\mathbf{F}$ = force, m = mass, $\mathbf{v}$ = velocity.

(Note the use of bold-faced type to indicate a vector quantity.)

In words, the second law can be formulated as follows: *The summation of forces on an object equals the time rate of change of momentum of the object.* For rigid bodies, this reduces to

$$\Sigma \mathbf{F} = m\mathbf{a} \quad (2-2)$$

where $\mathbf{a}$ = acceleration.

These are *vector* equations, since force systems and momentum changes involve both magnitude and direction. More about the nature of vector quantities will be stated in the next section.

Mechanics can be further subdivided into the following categories:

1. Statics—The study of situations for which the summation of both forces and moments is equal to zero. This special case includes bodies at rest and in uniform, unaccelerated motion.
2. Kinematics—The study of the geometry of motion.
3. Dynamics—The study of the forces and moments causing and resulting from motion.

Engineering students generally take separate courses covering various aspects of mechanics. These courses may include

1. Statics—The study of forces and moments on rigid, unaccelerated objects. A brief introduction is included in this chapter.
2. Dynamics—The study of the effects of forces and moments on the kinematics of rigid body motion. Applications involving ship motions and maneuverability are covered in chapters 13 and 14.
3. Material science—The study of the properties of materials. This is briefly summarized for ship applications in chapter 4.

4. Solid mechanics or strength of materials—The study of the external and internal forces associated with the

deformation of solid structures. This is summarized for ship applications in chapter 5.

5. Fluid mechanics (both hydrostatics and fluid dynamics)—The study of the forces and motions associated with substances that cannot sustain shear forces when in static equilibrium. (Fluids deform continuously when subjected to shear stresses.) The principles of the conservation of mass and energy are added to the basic laws of motion for such studies. The subject of hydrostatics is summarized in this chapter. Applications of hydrostatics to ship design are covered in chapter 3 and in chapters 6 to 9. Hydrodynamics is covered in chapters 11 to 14.

2-2 Introduction to Statics: Vectors and Scalars

As stated in the previous section, statics is the study of situations for which the summation of both forces and moments is equal to zero. Since both forces and moments represent vector quantities, those aspects of vector algebra that apply to summations will be reviewed. But first it is essential that two quantities be defined.

A *scalar* is a quantity, completely specified by a number on an appropriate scale, such as mass, length, time, or temperature. It is independent of the choice of a coordinate system.

A *vector* is a quantity completely specified by a magnitude and a direction, such as force, velocity, or acceleration. It is generally represented by a directed line segment or arrow located somewhere in a coordinate

system and is identified in this text by bold-faced letters. If the coordinate system is based on an inertial frame of reference, the magnitude and direction are fixed quantities, but if the coordinate system is moving relative to an inertial frame of reference, the magnitude and direction of the vector may depend upon the motion of the coordinate system, which is itself a vector quantity. A typical example of this phenomenon is the observation of relative motion from a moving vehicle. Velocities perpendicular to the direction of motion appear the same to observers in both frames of reference. However, velocities parallel to the direction of motion appear as *relative* velocities to the moving observer and must be corrected for the motion of the moving frame of reference if *absolute* magnitudes and directions are desired.

Although this distinction will become important in later chapters that treat the dynamic nature of ship motions, the study of statics does not involve moving frames of reference.

MAGNITUDE AND DIRECTION

The magnitude of a vector is always a positive quantity and corresponds to the length of vector represented graphically. For example, local wind speed corresponds to the length of a wind vector on a diagram of a storm system, or the particle velocities in a breaking wave are represented by an array of vectors as illustrated in figure 2-1, in which the length of the vector represents the

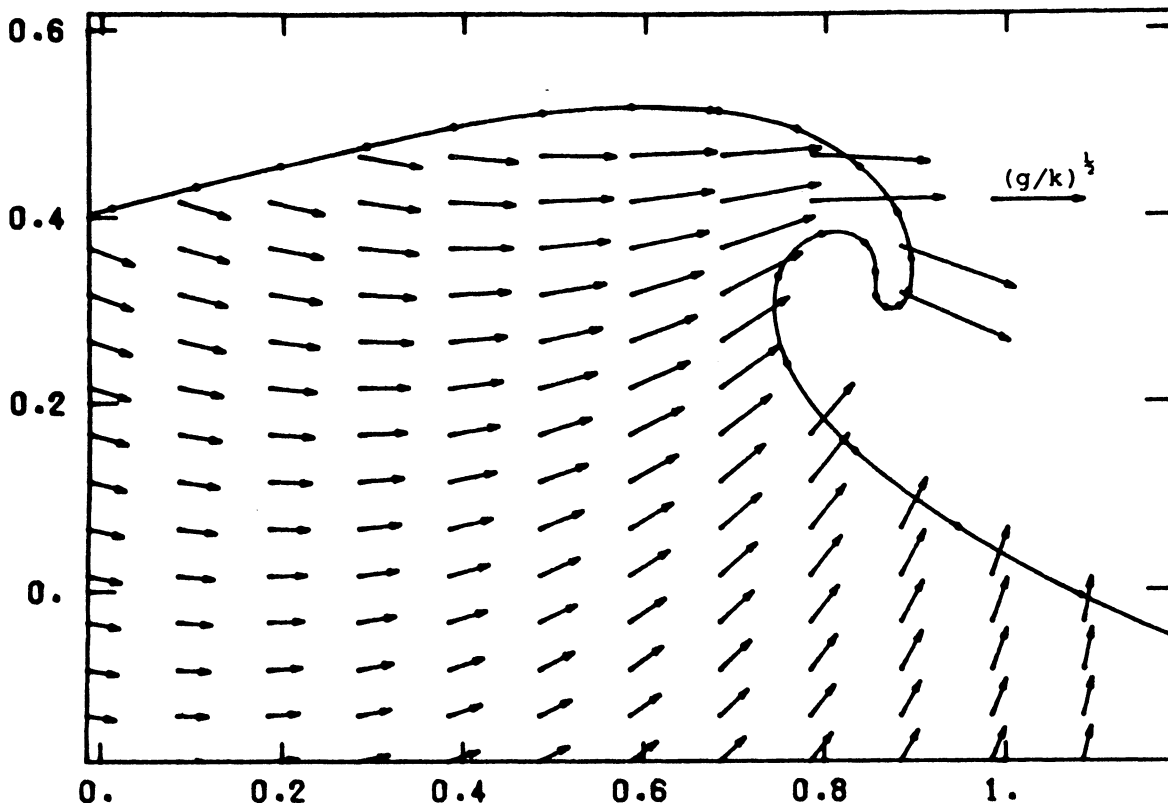


Figure 2-1. Particle velocities for a breaking wave showing the plunging jet. (From Cokelet 1978.)

speed of the particles at a particular location in the breaking wave (Cokelet 1978).

The magnitude of a vector is usually indicated by using an absolute value sign, but it may also be indicated by eliminating the boldface.

$$\text{magnitude of vector } \mathbf{A} = |\mathbf{A}| = A \quad (2-3)$$

When a vector quantity $\mathbf{a}$ is multiplied by a scalar quantity m , as in equation 2-2, the direction of both $\mathbf{a}$ and the resultant scalar product $\mathbf{F} = m\mathbf{a}$ are the same. The magnitude of $|\mathbf{F}|$ equals the magnitude of $|\mathbf{a}|$ multiplied by m . The vector $-\mathbf{F}$ is defined as a vector with magnitude equal to $\mathbf{F}$ but having an opposite sense to $\mathbf{F}$. The direction of a vector is indicated by the angle its representing arrow makes with a coordinate-system axis, or graphically by the arrow itself. It may be defined analytically by the coordinates of the head of the arrow when the tail is placed at the origin of the set of coordinates.

ADDITION AND SUBTRACTION OF VECTORS

The summations of forces and moments involve the addition and subtraction of vectors and may be accomplished either graphically or analytically. The graphical method involves the use of the parallelogram law based on the commutative property of vectors.

To illustrate vector addition and subtraction, let $\mathbf{A}$ and $\mathbf{B}$ be two vectors represented by the two arrows $\mathbf{A}$ and $\mathbf{B}$ in figure 2-2, which in turn represent the directed

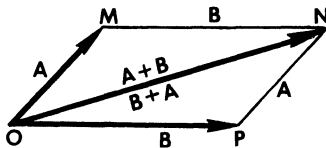


Figure 2-2. Vector addition

line segments $\mathbf{OM}$ and $\mathbf{OP}$. The sum $\mathbf{A} + \mathbf{B}$ is represented by the diagonal of the parallelogram $\mathbf{OMNP}$, which also demonstrates the commutative law for vector addition

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \quad (2-4)$$

Figure 2-3 illustrates the head-to-tail addition method

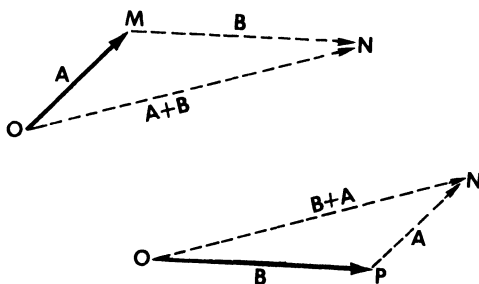


Figure 2-3. Commutative law (triangle rule)

(triangle rule) that may be employed to add several vectors using the commutative law.

The associative law of vector addition,

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C}) \quad (2-5)$$

is demonstrated in figure 2-4 using the head-to-tail addition method. If the $\mathbf{A} + \mathbf{C}$ parallelogram were added to the diagram, a total of six different sequences for the addition of the three vectors could be followed, all giving the same result.

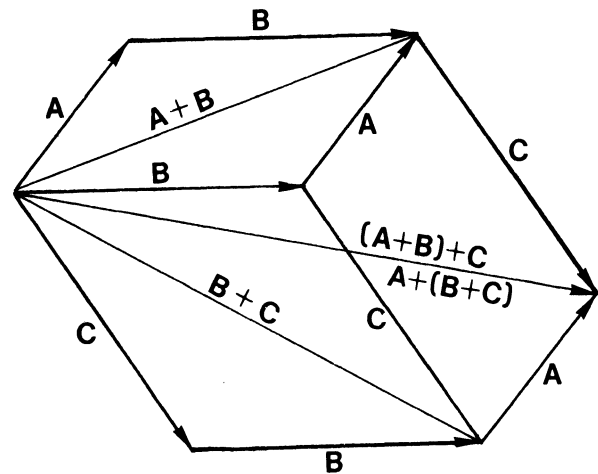


Figure 2-4. Associative law

Vector subtraction is accomplished by adding a negative vector using head-to-tail addition or by constructing the parallelogram using the appropriate negative vector as illustrated in figure 2-5. As can be seen from

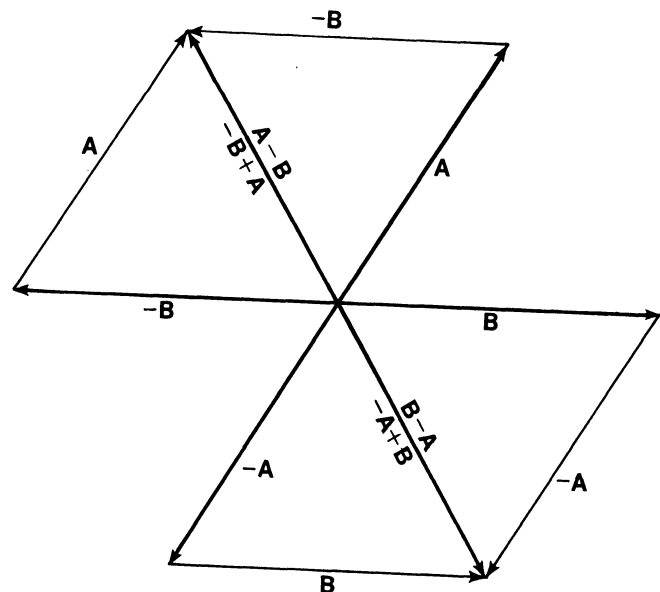


Figure 2-5. Vector subtraction

the diagram,

$$\mathbf{A} - \mathbf{B} = -(\mathbf{B} - \mathbf{A}) \quad (2-6)$$

equations 2-4, 2-5, and 2-6 have a meaning that is slightly different from the customary one because they are vector equations and require the magnitude as well as the direction of the left-hand side to be equal to that of the right-hand side. Two vectors can be said to be equal only if their directions as well as their magnitudes are equal to each other.

The analytical method for vector summations involves the resolution of the vectors into components along coordinate axes. This method has a distinct advantage over the graphical method for problems in three (or more) dimensions. Arbitrary vectors could be used for the development of the analytical method for vector summations, but since much of the text involves the analysis of forces, their resolution into components will be used as the example.

2-3 Resolution of Forces into Components

The most common method for the resolution of forces into components involves the use of an orthogonal coordinate system with mutually perpendicular axes, such as the Cartesian coordinate system illustrated in figure 2-6. In this case

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$$

and

$$|\mathbf{F}| = \sqrt{|\mathbf{F}_x|^2 + |\mathbf{F}_y|^2 + |\mathbf{F}_z|^2} \quad (2-7)$$

where $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ are the rectangular components of $\mathbf{F}$ obtained by projecting $\mathbf{F}$ onto the x -, y -, and z -axes.

It is useful at this point to introduce the concept of the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ of magnitude 1, directed

respectively along the positive x -, y -, and z -axes. This will enable the rectangular components F_x , F_y , F_z to be treated as scalars multiplied by the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$$\mathbf{F}_x = F_x \mathbf{i} \quad \mathbf{F}_y = F_y \mathbf{j} \quad \text{and} \quad \mathbf{F}_z = F_z \mathbf{k}$$

Thus,

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \quad (2-8)$$

as shown in figure 2-7. The vector components of $\mathbf{F}$, $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ can now be treated as scalar components F_x , F_y , and F_z , which may be positive or negative, depending on the sense of the vector components.

The actual computation of the magnitude of the force components is much simpler in two dimensions than in three dimensions, because only one angle is involved. Since most of the problems to follow will involve forces in a two-dimensional plane, the remaining discussion of vector forces will be limited to the resolution of $\mathbf{F}$ into $\mathbf{F}_x$ and $\mathbf{F}_y$ components.

The placement of coordinate axes is always arbitrary, and one can change their orientation to a more convenient one at the beginning of any problem. Consider a force $\mathbf{F}$ that lies in the plane of the paper as illustrated in figure 2-8. By changing the x - and y -axes from their arbitrary orientation to another orientation, such as the x' - and y' -axes, $\mathbf{F}$ can be made equal to the addition of many different sets of component vectors $\mathbf{F}_x'$ and $\mathbf{F}_y'$.

If Θ represents the angle between the positive x -axis and $\mathbf{F}$ measured counterclockwise, then

$$F_x = |\mathbf{F}| \cos \Theta \quad \text{and} \quad F_y = |\mathbf{F}| \sin \Theta \quad (2-9)$$

Note that the scalar components F_x and F_y take their sign from the angle Θ as the quadrant is established relative to the positive x -axis. Thus,

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} = F \cos \Theta \mathbf{i} + F \sin \Theta \mathbf{j} \quad (2-10)$$

and

$$\tan \Theta = \frac{F_y}{F_x}$$

Forces can be added in terms of their components. Consider two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, acting at a particular location as illustrated in figure 2-9. Determine the resultant force $\mathbf{F}_3$.

$$\begin{aligned} \mathbf{F}_3 &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= F_{1x} \mathbf{i} + F_{1y} \mathbf{j} + F_{2x} \mathbf{i} + F_{2y} \mathbf{j} \\ &= (F_{1x} + F_{2x}) \mathbf{i} + (F_{1y} + F_{2y}) \mathbf{j} \end{aligned} \quad (2-11)$$

Example 2-1: If $\mathbf{F}_1$ equals 50.0 Newtons at 30° and $\mathbf{F}_2$ equals 30.0 Newtons at 130° as illustrated in figure 2-9, calculate the magnitude and direction of $\mathbf{F}_3$.

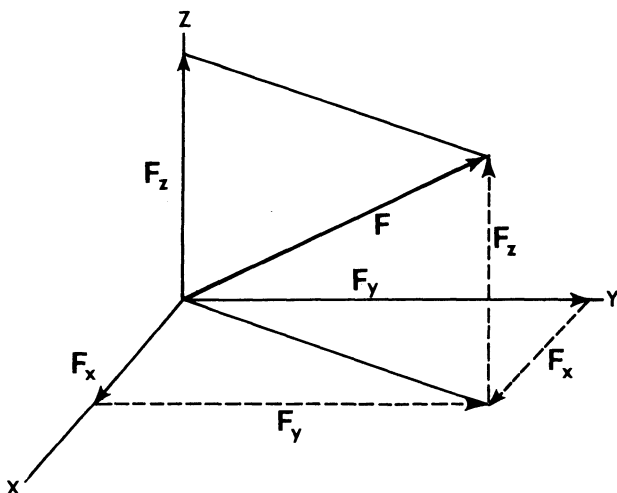


Figure 2-6. Resolving a vector into its components

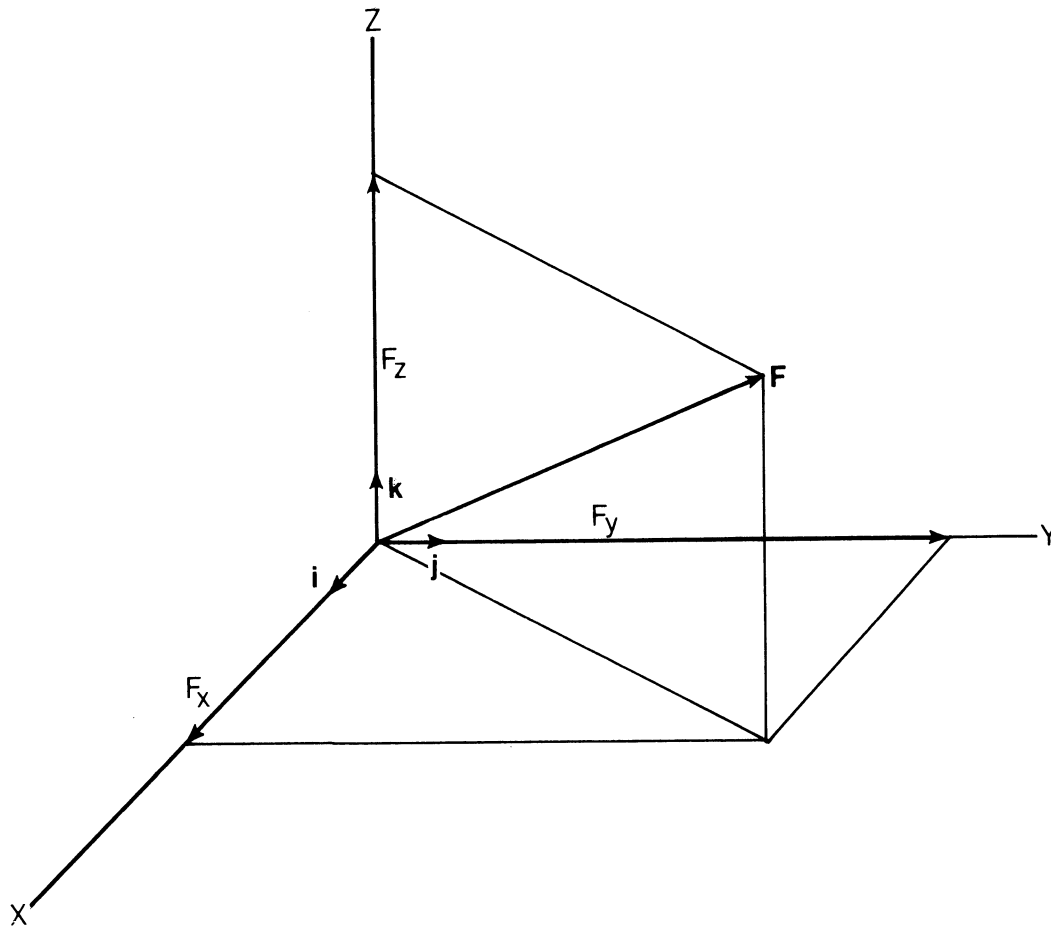


Figure 2-7. Concept of unit vectors

Solution:

$$F_{1x} = 50 \cos 30^\circ = 43.3 \text{ N}$$

$$F_{2x} = 30 \cos 130^\circ = -19.3 \text{ N}$$

$$F_{1y} = 50 \sin 30^\circ = 25.0 \text{ N}$$

$$F_{2y} = 30 \sin 130^\circ = 23.0 \text{ N}$$

$$F_{3x} = 43.3 - 19.3 = 24.0 \text{ N}$$

$$F_{3y} = 25 + 23 = 48.0 \text{ N}$$

$$F_3 = \sqrt{24^2 + 48^2} = 53.7 \text{ Newtons}$$

$$\tan \Theta_3 = \frac{F_{3y}}{F_{3x}} = \frac{48}{24}$$

$$\Theta_3 = 63.4^\circ$$

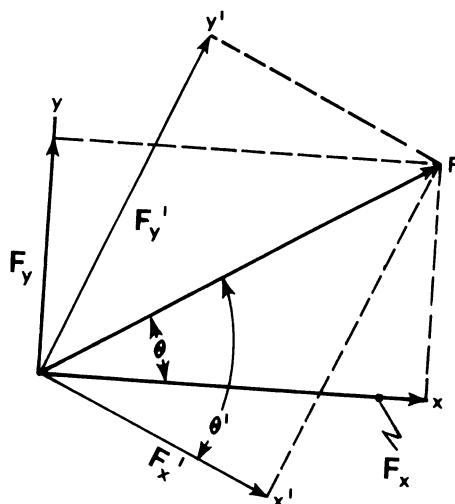


Figure 2-8. Change of axis orientation

2-4 Forces, Moments, and Couples

At this point in the development of basic equations governing problems in statics a distinction should be

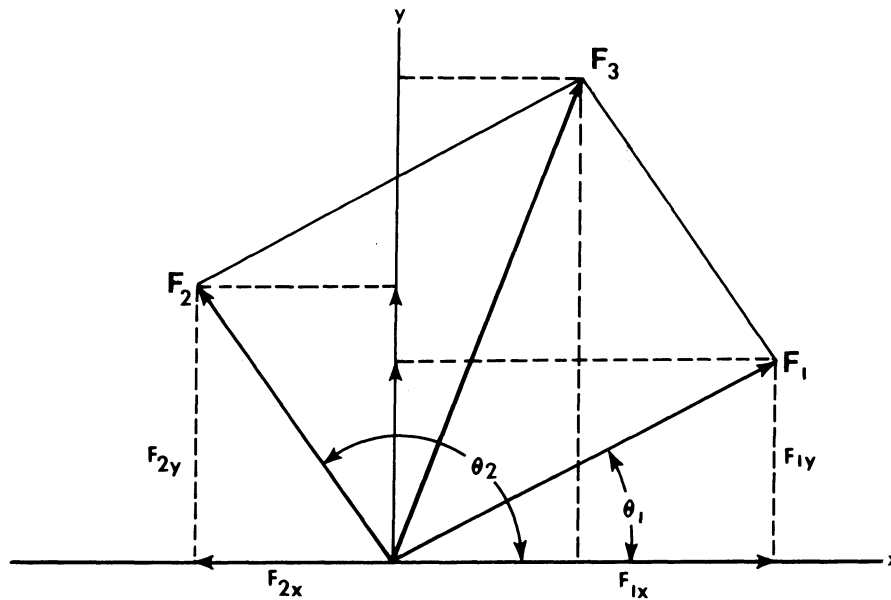


Figure 2-9. Addition of forces

made between the types of vectors used to represent various physical quantities.

Free vectors may be freely translated to any position parallel to the original line of action without affecting the analysis of the problem. Only the magnitude and direction must be preserved. Examples of free vectors are the displacement and velocity of a nonrotating rigid body and the concept of a couple as discussed in this section.

Sliding vectors may be positioned anywhere along the original line of action without affecting the analysis of the problem. Examples of sliding vectors are the forces that act on a rigid body and the tensile force in a “weightless” cable that supports some known weight. Moments caused by such vectors are not affected by changing the position of a vector along its line of action.

Fixed vectors are those whose effect depends on the local point of application. Examples of fixed vectors include forces acting on individual particles and forces acting on elastic or deformable bodies. The deformation of a rubber ball depends on the point of application of the force.

FORCE SYSTEMS

There are several special categories of force systems that should be distinguished.

Concurrent forces all have lines of action that pass through the same point. The analysis of forces on a particle or system of particles falls into this category, and therefore rotations caused by unbalanced moments are not considered in particle mechanics.

Colinear forces all have the same direction and the same line of action. This generally simplifies the analysis to that for a one-dimensional problem.

Coplanar forces all lie in the same plane, and therefore such force systems can be analyzed as two-dimensional problems.

MOMENTS

In the analysis of the effects of external and internal forces on rigid bodies, the concept of the moment of a vector about a line or axis must be introduced. Since the forces on a rigid body can be considered to be sliding vectors, the moment of such a force (torque) is defined as the product of the force times the perpendicular distance to the line of action of the force about some point in space.

Consider the force $\mathbf{F}$ acting on a rigid rod, both in the x - y plane as illustrated in figure 2-10. The rod is located with one end at the origin, and the point of application of the force is characterized by the position vector $\mathbf{r}$. The moment of $\mathbf{F}$ about the z -axis in scalar form is

$$M = Fd = F(r \sin \Theta) \quad (2-12)$$

since $\sin \Theta$ equals $\sin (180 - \Theta)$. In vector form, the equation is

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \quad (2-13)$$

since equation 2-12 gives the magnitude of the vector product, called a *cross product*, as indicated in equation 2-13. It should also be noted that $F \sin \Theta$ represents

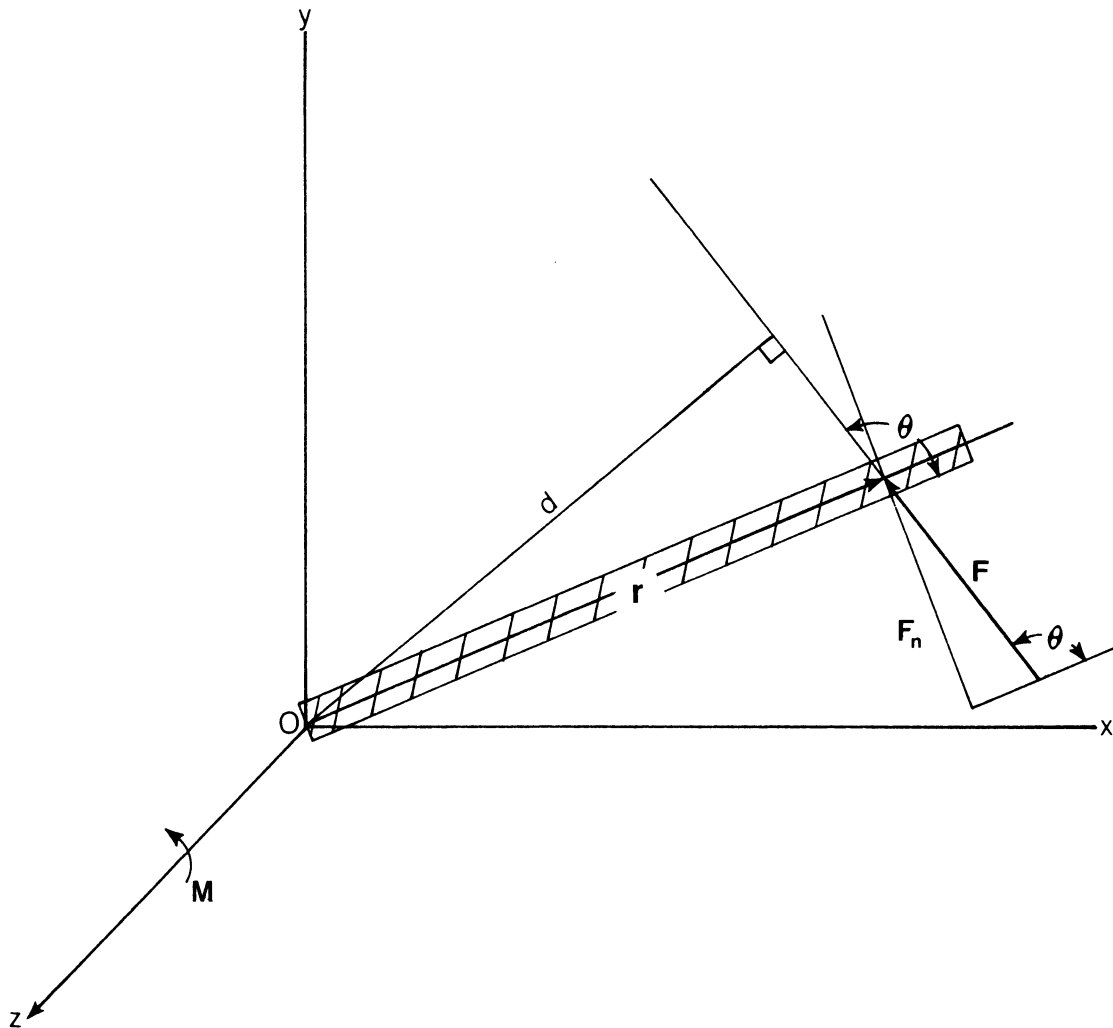


Figure 2-10. Moment generated by a force action on a rigid rod

the normal component of the force $\mathbf{F}$ with respect to the position vector $\mathbf{r}$, such that the magnitude of $\mathbf{M}$ can be found from

$$M = F_n r \quad (2-14)$$

where

$$F_n = F \sin \theta$$

Thus, the moment of a vector about a line may also be found from the product of the distance to the point of application times the component of the force normal to the position vector $\mathbf{r}$.

The sign of the moment M is established by the *right-hand rule*, which is positive counterclockwise when θ is between 0° and 180° .

If several forces are all applied at the same point, Varignon's theorem, or the "principle of moments," makes it possible to determine force

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \dots$$

and find the resultant moment

$$\mathbf{M}_R = \mathbf{r} \times \mathbf{F}_R \quad (2-15)$$

since

$$\begin{aligned} \mathbf{M}_R &= \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \dots \\ &= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \dots) \end{aligned}$$

This theorem can also be used to resolve the resultant force into x - and y -components and calculate the moment as indicated in figure 2-11.

$$M_R = -(r_x F_{Ry} + r_y F_{Rx})$$

where the minus sign results from $180^\circ < \theta < 360^\circ$.

COUPLES

A *couple* is a pair of coplanar forces of equal magnitude acting in parallel but opposite directions, capable of causing rotation but not translation (i.e., a pure moment). Consider two equal and opposite forces, $\mathbf{F}_1$ and

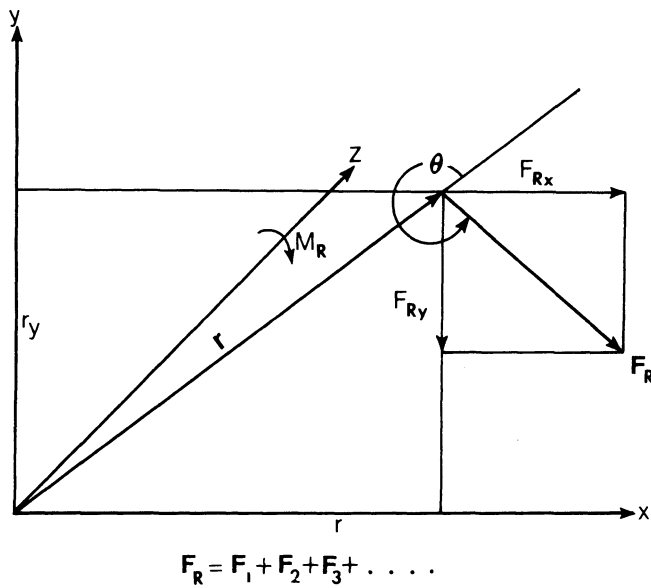


Figure 2-11. Moment of a resultant force

F_2 , acting in the x - y plane in figure 2-12. The moment of each force about the z -axis is

$$M_1 = \mathbf{r}_1 \times \mathbf{F}_1$$

$$M_1 = -r_1 \sin \theta F_1$$

$$M_2 = \mathbf{r}_2 \times \mathbf{F}_2$$

$$M_2 = r_2 \sin \theta F_2$$

The summation of moments about the origin is

$$M_0 = (r_2 \sin \theta - r_1 \sin \theta) F_1$$

since F_1 and $F_2 = F$

$$M_0 = (r_2 - r_1) \sin \theta F$$

$$= dF \quad (2-16)$$

This demonstrates that the moment of a couple is independent of the distance from the axis and is only a function of the magnitude of the forces and the perpendicular distance between the lines of action. Thus, the moment of a couple is a free vector that may be located anywhere in the plane of the couple.

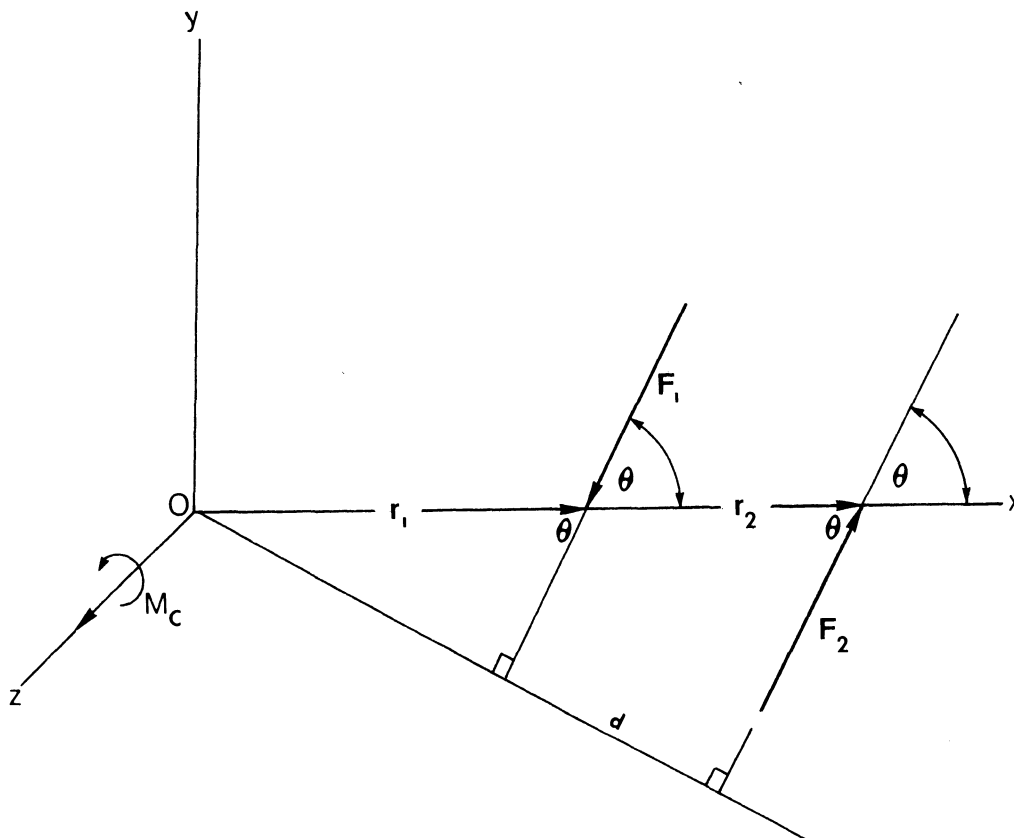


Figure 2-12. A couple

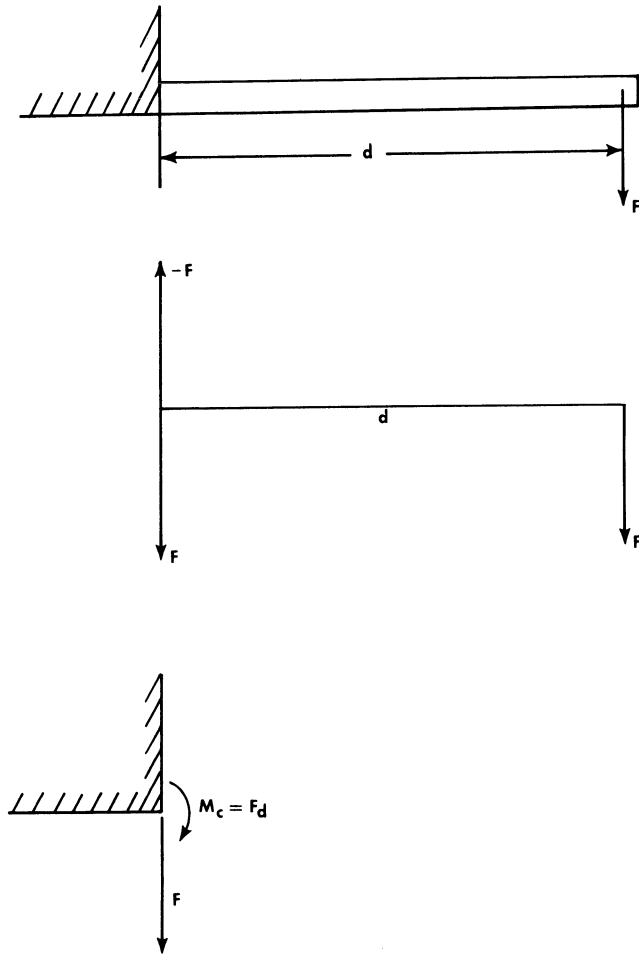


Figure 2-13. Replacement of a moment by a force and a couple

It is often useful to replace the moment of a force located at some distance from a particular location with a force acting at that location plus a couple about the axis passing through the location. Consider the force F acting on a rod attached rigidly to the wall, as in figure 2-13. The force acting on the wall may be found by replacing the force acting at perpendicular distance d from the wall with a force F acting at the wall and a couple moment $M_c = Fd$ also acting at the wall.

A system of forces acting at different locations on a rigid body may also be reduced to a single resultant force acting at the origin plus a couple that is the sum of all the couples and moments about the origin.

$$\mathbf{F}_R = \Sigma \mathbf{F} \quad (2-17)$$

$$\mathbf{M}_{R0} = \Sigma \mathbf{M}_0 \quad (2-18)$$

2-5 Static Equilibrium

The necessary and sufficient conditions for a rigid body to be in a state of static equilibrium are that the summation of forces and the summation of moments, as indicated in equations 2-17 and 2-18, be equal to zero.

$$\Sigma \mathbf{F} = 0 \quad (2-19)$$

$$\Sigma \mathbf{M}_0 = 0 \quad (2-20)$$

This will ensure that the body will experience neither translational (linear) motion nor rotational (angular) motion. Note that equations 2-19 and 2-20 provide a total of six equations that may be solved for six unknowns in a three-dimensional problem. Note also that if the forces are concurrent and are applied at the center of gravity of the body (taken as the origin), equation 2-20 is automatically satisfied, and the analysis becomes a problem involving the principles of particle mechanics (i.e., $\Sigma \mathbf{F}$ equals zero).

Most of the analyses in this text will involve coplanar force systems, however. In the case of real three-dimensional problems in ship stability, the analysis will be divided into longitudinal and transverse stability portions, so that coplanar force systems can be assumed in order to simplify the problem. For two-dimensional force systems, the equilibrium conditions reduce to a set of three scalar equations,

$$\Sigma F_x = 0 \quad (2-21a)$$

$$\Sigma F_y = 0 \quad (2-21b)$$

$$\Sigma M_0 = 0 \quad (2-21c)$$

which may be solved for a maximum of three unknowns. In certain types of problems, it is necessary to sum moments about more than one axis and eliminate one of the force summations. The resulting restrictions on linear independence of the equations are beyond the scope of this text, and the reader is referred to one of the standard texts concerning engineering statics (Beer and Johnston 1977; Hibbeler 1978).

FREE BODY DIAGRAMS

In order to apply the equations of equilibrium to a particular problem in statics, it is necessary to construct a *free body diagram*. This is done to account for all the forces and moments present, including the reaction forces, where the body comes in contact with another body, a supporting or restraining cable, or the ground plane. These reactions are generally classified in terms of the number of unknown forces and moments that may be present at the reaction.

1. One unknown force with a known line of action—

These reactions include cables, links, and situations where only a normal force to the surface can be supported. The latter includes assumed *frictionless surfaces* approximated by rollers, rockers, ball-bearing collars, and pins constrained in a particular direction such that the tangential force is always equal to zero (figure 2-14).

2. One unknown resultant force with an unknown line of action—

This type of reaction is found at pinned or hinged joints and rough surfaces that can support tangential as well as normal forces, but cannot support a couple or moment. If coplanar forces are used,

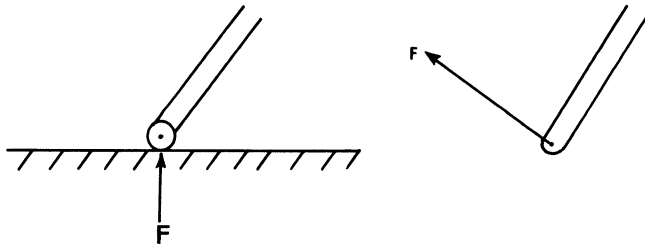


Figure 2-14. Frictionless support: one unknown force of known direction

there are still two unknown quantities involved (figure 2-15).

3. *One unknown resultant force with an unknown line of action plus an unknown couple*—This type of reaction is found at a fixed support that can support both forces and moments and can constrain the rigid body completely (figure 2-16).

Steps in the Construction of a Free Body Diagram

1. Sketch the contour or outline of the body detached (free) from other bodies or the ground. Dimension the sketch as required to calculate moments of the forces involved.

2. Indicate on the sketch all the external forces, both known and unknown, and the couples that may act on the body. This will include all reaction forces and couples as well as applied external loads and the weight of the body acting at the center of gravity.

3. Label the known forces and couples with the numerical values of their magnitude and direction in appropriate units for the required calculations. Label all the unknown forces and couples with letters appropriate to the equations of equilibrium. Remember that the sense of the direction of the external forces is determined by the effect of the surroundings *on the body*, not vice versa. The assumed direction for the unknown

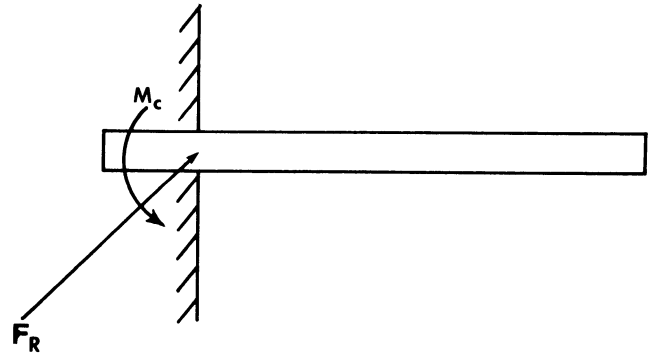


Figure 2-16. Fixed support: one unknown force of unknown direction and an unknown couple

forces is correct if the solution yields a positive number. A negative value for an unknown force indicates that the assumed direction of the force was incorrect.

The problem can now be solved, providing there are not more than three unknowns for two-dimensional problems involving the equations of static equilibrium.

Example 2-2: A rotating cargo crane on a merchant ship lifts a 10,000-lb load, as illustrated in figure 2-17(a). The boom is 60 feet long and will be considered weightless for purposes of the problem. The crane weighs 7,000 pounds, and its center of gravity can be assumed to be located 2 feet from the axis of rotation towards the boom (thus compensating for the weightless boom assumption). For the loading condition and angles indicated in the sketch, determine the compression load in the crane boom and the tension in the cable between A and B. Also determine the reaction forces and moment at the base of the crane (point D).

Solution: To determine the compression load in the crane boom $\overline{BC}$ and the cable tension in $\overline{AB}$, sketch a free body diagram of the boom, cable, and load, as illustrated in figure 2-17(b). Note that by neglecting the

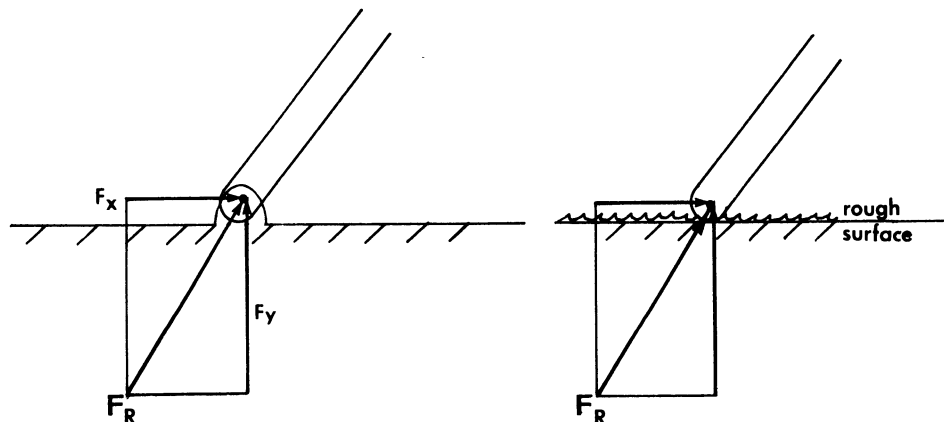


Figure 2-15. Pinned support: one unknown force of unknown direction

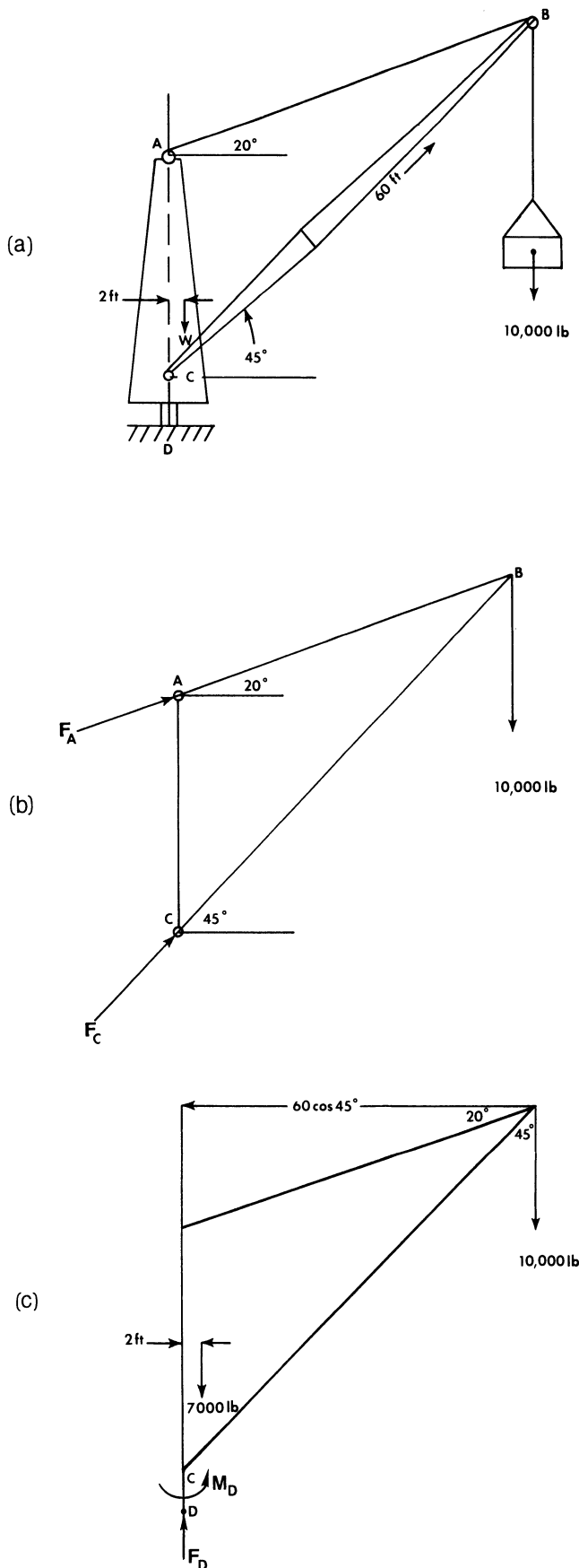


Figure 2-17(a),(b),(c). Cargo crane

weight of the boom, the remaining forces are concurrent through point B . Thus, the condition that ΣF equals zero at point B is sufficient to determine the unknown reaction forces F_A and F_C .

$$\Sigma F_x = 0$$

$$F_A = \cos 20^\circ + F_C \cos 45 = 0$$

$$\Sigma F_y = 0$$

$$F_A \sin 20^\circ + F_C \sin 45 - 10,000 = 0$$

$$F_A = -F_C \frac{\cos 45}{\cos 20} \text{ from } \Sigma F_x = 0$$

$$-F_C \cos 45 \frac{\sin 20}{\cos 20} + F_C \sin 45 = 10,000$$

$$F_C (\sin 45 - \cos 45 \tan 20) = 10,000 \text{ lb}$$

$$F_C = \frac{10,000}{0.45} = \boxed{22,235 \text{ lb}}$$

$$F_A = -F_C \frac{\cos 45}{\cos 20} = \boxed{-16,730 \text{ lb}}$$

Thus, the assumed direction of F_A on the free body diagram was incorrect; i.e., the cable is in tension.

To determine the reaction at the base of the crane, point D , sketch a free body diagram of the entire crane as illustrated in figure 2-17(c).

By inspection,

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$F_D - 7,000 \text{ lb} - 10,000 \text{ lb} = 0$$

$$F_D = \boxed{17,000 \text{ lb}}$$

$$M_0 = 0 \text{ about } D$$

$$M_D - 7,000 (2) - 10,000 (60 \cos 45) = 0$$

$$M_D = 14,000 + 424,300$$

$$= \boxed{438,300 \text{ ft-lb}}$$

2-6 Analysis of Distributed Quantities

In the preceding discussion, the weight of an object was assumed to be concentrated at a single point so that the force and moment equations would be easy to apply. For simple geometrical shapes, such as rectangular or circular bars, the position of the center of gravity (where the sum of all the gravitational forces is assumed to be concentrated) is quite easy to calculate. For more complex shapes such as ships, integrals and more complex

summations must be used. For situations where the weight (density) of the component parts is uniform or can be described by an equation, an integral formulation of the properties of distributed quantities can be made. When the component weights have discrete values, a numerical summation must be used. When using a digital computer for the calculations, numerical integration methods are used, which are discussed in appendix B.

THE FIRST MOMENT OF AREAS AND VOLUMES: THE CENTROID

The center of gravity or centroid of an object with a uniform weight per unit area or volume may be found by using the first moment of area or volume. Since the weight density is uniform, it may be eliminated from the calculation, making the problem one of geometric analysis.

Two-dimensional Shapes

Consider a flat horizontal plate of homogeneous density and thickness aligned with the x - y plane, as in figure 2-18. The centroid of an arbitrary section of the plate may be found by using the first moment of area about the x - and y -axes. The first moment of area about the x -axis is defined as

$$M_x = \int y dA \quad (2-22)$$

And the first moment of area about the y -axis is defined as

$$M_y = \int x dA \quad (2-23)$$

Plane shapes having at least one side coinciding with an axis are frequently found in naval architecture. These may include situations where the ordinate values are known functions of the abscissa, i.e., y equals $f(x)$ at discrete values of the abscissa and at spacings δx_i . For

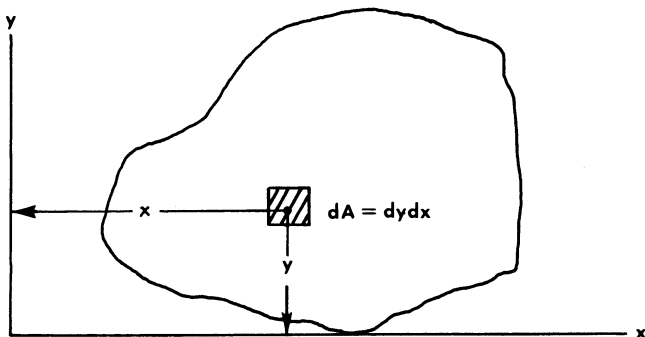


Figure 2-18

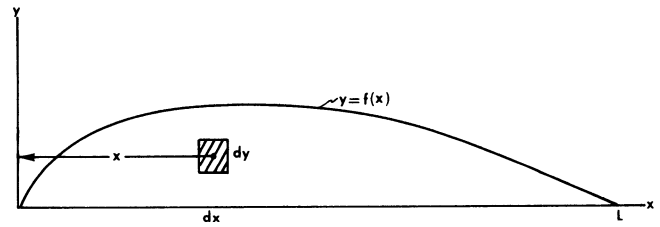


Figure 2-19

this situation, illustrated in figure 2-19,

$$\begin{aligned} M_x &= \int_0^L \int_0^y y \, dy \, dx \\ M_x &= \int_0^L \frac{y^2}{2} \, dx \end{aligned} \quad (2-24)$$

which may be approximated numerically by using N rectangles of height y and width δx_i

where

$$i = 1, 2, \dots, N$$

$$M_x = \sum_{i=1}^N \frac{y^2}{2} \delta x_i \quad (2-24a)$$

Similarly the first moment of area about the y -axis is

$$\begin{aligned} M_y &= \int_0^L \int_0^y x \, dy \, dx \\ M_y &= \int_0^L xy \, dx \end{aligned} \quad (2-25)$$

which may be approximated numerically by

$$M_y = \sum_{i=1}^N xy \delta x_i$$

The centroid of the plane shapes discussed above may be found from

$$\bar{x} = \frac{M_y}{A} = \frac{\int x dA}{\int dA} \quad (2-26)$$

$$\bar{y} = \frac{M_x}{A} = \frac{\int y dA}{\int dA} \quad (2-27)$$

where A = total area.

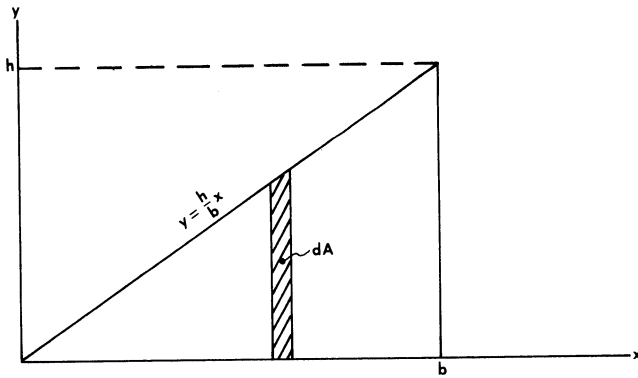


Figure 2-20

Example 2-3: Compute the centroid of the right triangle shown in figure 2-20.

Solution: Since the shape of the triangle may be expressed in equation form, and the base of the triangle corresponds to the x -axis, the integral method using equations 2-24 and 2-25 may be used.

$$M_x = \int_0^b \frac{y^2}{2} dx$$

$$= \int_0^b \frac{1}{2} \left(\frac{h}{b} x \right)^2 dx = \frac{1}{2} \frac{h^2}{b^2} \frac{x^3}{3} \Big|_0^b$$

$$M_x = \frac{bh^2}{6}$$

$$M_y = \int_0^b xy \, dx$$

$$= \int_0^b x \frac{h}{b} x \, dx = \frac{h}{b} \frac{x^3}{3} \Big|_0^b$$

$$M_y = \frac{b^2h}{3}$$

Thus,

$$\bar{x} = \frac{M_y}{A} = \frac{b^2h}{3} \bigg/ \frac{bh}{2} = \frac{2}{3}b$$

$$\bar{y} = \frac{M_x}{A} = \frac{bh^2}{6} \bigg/ \frac{bh}{2} = \frac{h}{3}$$

as expected.

Composite Shapes

In many instances, the centroid of two-dimensional shapes consisting of rectangles, triangles, and other common shapes may be calculated by summing the moments

about a common axis for the individual components and setting the sum equal to the total area of the shape multiplied by the centroid of the composite area.

$$M_x = \sum_{i=1}^N y_i A_i$$

$$M_y = \sum_{i=1}^N x_i A_i$$

$$\bar{x} = \frac{M_y}{A} = \frac{\sum x_i A_i}{\sum A_i} \quad (2-28)$$

$$\bar{y} = \frac{M_x}{A} = \frac{\sum y_i A_i}{\sum A_i} \quad (2-29)$$

where N = number of shapes.

Example 2-4: Locate the centroid of the V-block shape shown in figure 2-21(a).

Solution: The origin of the coordinate system may be placed anywhere on or near the section and would normally be chosen on a line of symmetry to simplify the calculation. In order to demonstrate the method of composite sections, however, the origin will be chosen at the lower left-hand side of the shape, as illustrated in figure 2-21(b). The section is then divided into one

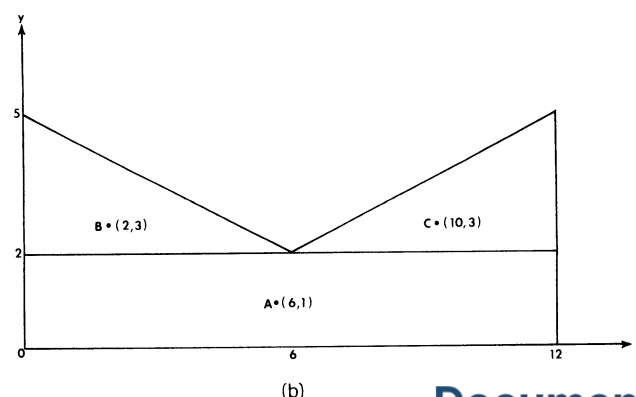
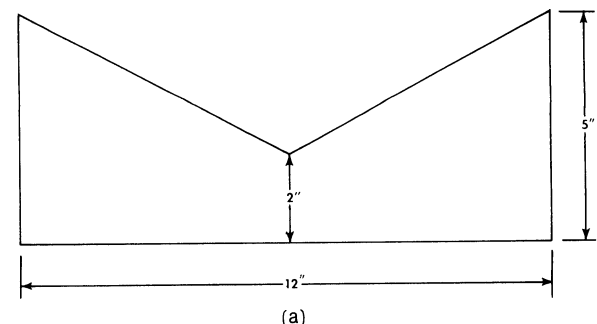


Figure 2-21(a),(b). V-block



rectangular area A and two triangular areas B and C , with centroids as indicated. Thus,

$$A_A = 2 \times 12 = 24 \text{ in}^2$$

$$A_B = A_C = \frac{6 \times 3}{2} = 9 \text{ in}^2$$

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} = \frac{6(24) + 2(9) + 10(9)}{24 + 9 + 9}$$

$$= \frac{252}{42} = \boxed{6 \text{ in}} \text{ as expected}$$

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{1(24) + 3(9) + 3(9)}{24 + 9 + 9}$$

$$= \frac{78}{42} = \boxed{1.86 \text{ in}}$$

An alternate solution for $\bar{y}$ could involve subtracting the V-notch area from the enclosing rectangle.

$$\bar{y} = \frac{2.5(5 \times 12) - 4\left(\frac{3 \times 12}{2}\right)}{5 \times 12 - \frac{3 \times 12}{2}} = \frac{78}{42} = \boxed{1.86 \text{ in}} \text{ as before.}$$

The centroids of other standard shapes are given in handbooks and in textbooks on statics and strength of materials.

Three-dimensional shapes. The centroid of a three-dimensional shape may be found from the first moment of volume about the three principal axes. Thus,

$$\bar{x} = \frac{\int x dV}{V} = \frac{\sum_{i=1}^N \bar{x}_{ci} A_{yzi} \delta x_i}{\sum_{i=1}^N A_{yzi} \delta x_i} \quad (2-30a)$$

$$\bar{y} = \frac{\int y dV}{V} = \frac{\sum_{i=1}^N \bar{y}_{ci} (A_{xz} \delta y)_i}{\sum_{i=1}^N (A_{xz} \delta y)_i} \quad (2-30b)$$

$$\bar{z} = \frac{\int z dV}{V} = \frac{\sum_{i=1}^N \bar{z}_{ci} (A_{xy} \delta z)_i}{\sum_{i=1}^N (A_{xy} \delta z)_i} \quad (2-30c)$$

where $\bar{x}_{ci}$ = centroid of individual components

A_{yzi} = component area in a yz plane

δx_i = thickness of layer A_{yz}

N = number of components.

Numerical integration techniques for making these calculations are found in appendix B.

For composite three-dimensional shapes consisting of known component weights W_i and corresponding centers of gravity $\bar{x}_i$, $\bar{y}_i$, and $\bar{z}_i$, of this inhomogeneous combination, the centroid may be found by summing moments of the weights or masses about the three principal axes.

$$\bar{x} = \frac{\sum \bar{x}_i W_i}{\sum W_i} \quad (2-31a)$$

$$\bar{y} = \frac{\sum \bar{y}_i W_i}{\sum W_i} \quad (2-31b)$$

$$\bar{z} = \frac{\sum \bar{z}_i W_i}{\sum W_i} \quad (2-31c)$$

This formulation will be used in chapter 6 to determine the changes in the center of gravity of a ship.

2-7 Second Moment of Area: Moment of Inertia

The second moment of area, which derives its name from the power on the variable in the defining equation, is generally called the *moment of inertia*, regardless of whether or not the equation includes the mass density that relates the quantity to “inertia.” It is defined with respect to a given axis as follows:

$$I_x = \int y^2 dA \quad (2-32)$$

$$I_y = \int x^2 dA \quad (2-33)$$

The moment of inertia is always a positive quantity and is used in the basic equations for strength of materials (chapter 5), ship hydrostatics (chapters 6 and 7), ship motions (chapter 11), and in section 2-9 of this chapter. Moments of inertia are tabulated for many structural shapes in handbooks on civil and mechanical engineering. Statics and strength of materials textbooks also contain tables of moments of inertia for standard shapes (Beer and Johnston 1977; Hibbeler 1978). Unfortunately, many ship applications involve calculating the moment of inertia of the “waterplane area” for various cross sections of the ship, and these are rarely standard shapes. However, they do have one side coinciding with an axis when calculations about the cen-

terline are involved. Thus, the equations can be set up for numerical integration in a fashion similar to that for the centroid calculation illustrated in figure 2-17 of the previous section.

$$I_x = \int_0^L \int_0^y y^2 dy dx = \int_0^L \frac{y^3}{3} dx \quad (2-34)$$

$$= \sum_{i=1}^N \frac{y_i^3}{3} \delta x_i$$

$$I_y = \int_0^L \int_0^y x^2 dy dx = \int_0^L x^2 y dx \quad (2-35)$$

$$= \sum_{i=1}^N x_i^2 y_i \delta x_i$$

Since most ships are symmetrical about the centerline, hull-form offsets (y -values for each x_i) are based on one-half of a ship or "half breadth," as explained in chapter 3. Therefore, the moment of inertia of the whole ship about the centerline is two times the moment of inertia for one side about the centerline.

Example 2-5: Calculate the moment of inertia of the triangle in example 2-3 about the x -axis.

Solution:

$$\begin{aligned} I_x &= \int y^2 dA = \int_0^b \frac{y^3}{3} dx \\ &= \frac{1}{3} \int_0^b \left(\frac{hx}{b} \right)^3 dx \\ &= \frac{1}{3} \left(\frac{h}{b} \right)^3 \frac{x^4}{4} \bigg|_0^b \\ &= \boxed{\frac{bh^3}{12}} \end{aligned}$$

This can be shown to be true for all triangles about the base.

PARALLEL AXIS THEOREM

It is frequently necessary to calculate the moment of inertia of an area about some other axis parallel to the axis through the centroid of the area for which the moment of inertia is known. This will be required in the calculation of the moment of inertia of composite areas. The theorem can be developed by calculating the moment of inertia of an arbitrary shape about the x -axis using y -values related to the distance d from the axis passing through the centroid c to the new axis, as illustrated in figure 2-22.

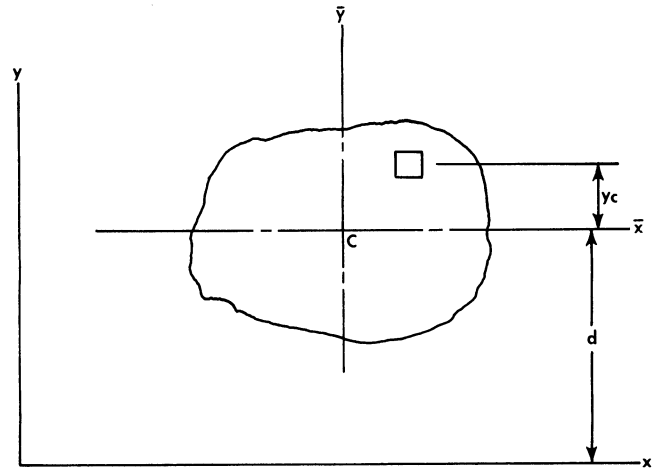


Figure 2-22. Parallel axis theorem

$$\begin{aligned} I_x &= \int y^2 dA \quad y = y_c + d \\ &= \int (y_c + d)^2 dA \\ &= \int y_c^2 dA + d \int y_c dA + d^2 \int dA \\ &= I_x + dM_c + d^2 A \end{aligned}$$

where I_x = moment of inertia about the centroid

d = parallel distance the axis is shifted

M_c = moment of area about the centroid

= 0 by definition

y_c = distance of elemental area from the centroidal axis.

Thus,

$$I_x = I_x + d^2 A \quad (2-36)$$

Similarly, letting d equal the distance from the y axis to the vertical centroidal axis,

$$I_y = I_y + d^2 A \quad (2-37)$$

Example 2-6: Determine the moment of inertia of a rectangle with respect to an axis parallel to its base and passing (a) through its centroid, and (b) about the base, as in figure 2-23.

Solution: Using equation 2-32 and noticing the symmetry about the x -axis passing through the centroid,

$$\begin{aligned} I_x &= 2 \int_0^{h/2} \int_0^b y^2 dy dx \\ &= 2 \frac{y^3}{3} \bigg|_0^{h/2} x \bigg|_0^b = \frac{2}{3} \frac{h^3}{8} b \\ &= \boxed{\frac{bh^3}{12}} \end{aligned}$$



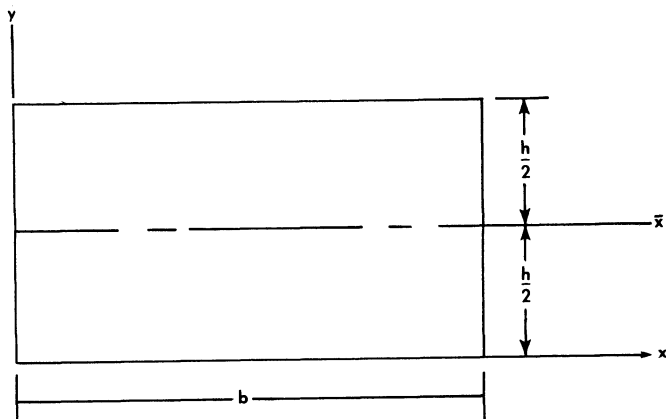


Figure 2-23

To calculate the moment of inertia about the base, the parallel axis theorem can be used.

$$\begin{aligned}
 I_x &= I_{\bar{x}} + d^2 A \\
 &= \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 bh \\
 &= \boxed{\frac{bh^3}{3}}
 \end{aligned}$$

As a check, return to equation 2-32 and integrate about the base.

$$\begin{aligned}
 I_x &= \int_0^b \int_0^h y^2 dy dx \\
 &= \frac{y^3}{3} \bigg|_0^h x \bigg|_0^b \\
 &= \frac{bh^3}{3} \text{ as before.}
 \end{aligned}$$

Moment of Inertia of Composite Areas

The moment of inertia of composite areas consisting of combinations of simple shapes may be calculated by adding or subtracting the moments of inertia of the shapes making up the composite area, using the parallel axis theorem for any shapes whose centroids are located off the centroidal axis of the composite area.

Example 2-7: Calculate the moment of inertia of an I-beam that is made from three sections of 2"-by-8" steel plate welded together as illustrated in figure 2-24.

Solution: Both an additive method that requires the use of the parallel axis theorem and a subtractive method will be used for comparison.

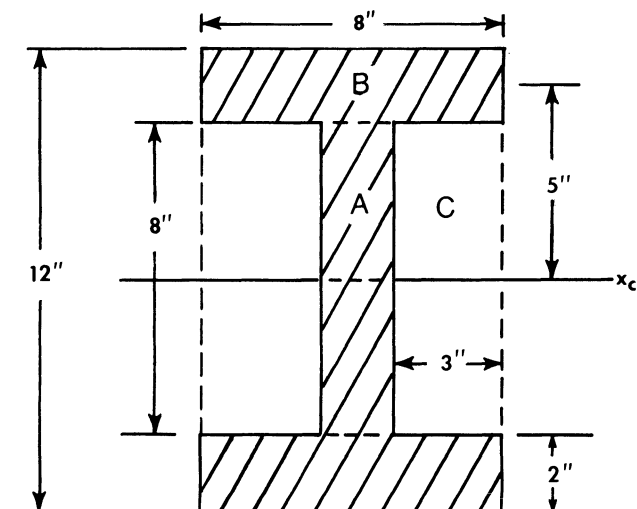


Figure 2-24. I-beam

Additive method:

$$\begin{aligned}
 I_x &= I_A + 2[I_B + d^2 A_B] \\
 &= \frac{2(8)^3}{12} + 2 \left[\frac{8(2)^3}{12} + 5^2(8 \times 2) \right] \\
 &= \boxed{896 \text{ in}^4}
 \end{aligned}$$

Subtractive method:

$$\begin{aligned}
 I_x &= I_{8 \times 12} - 2I_c \\
 &= \frac{8(12)^3}{12} - 2 \left(\frac{3(8)^3}{12} \right) \\
 &= \boxed{896 \text{ in}^4}
 \end{aligned}$$

RADIUS OF GYRATION

Occasionally it is desirable to relate a plane area to its moment of inertia by defining a radius of gyration k_x or k_y .

$$I_x = k_x^2 A \quad \text{or} \quad k_x = \sqrt{\frac{I_x}{A}} \quad (2-38)$$

$$I_y = k_y^2 A \quad \text{or} \quad k_y = \sqrt{\frac{I_y}{A}} \quad (2-39)$$

This is equivalent to concentrating the area into a thin strip of negligible inertia about its own centroidal axis and locating it at a distance from the appropriate axis equal to the radius of gyration (see parallel axis theorem). This concept is used extensively in structural mechanics, dynamics, and ship-motion studies.

COEFFICIENT OF INERTIA: C_I

In ship design it is convenient to relate the moment of inertia of a waterplane area (the area of the ship's section)

of the ship's hull at a particular draft with the surface of the ocean) with that of a circumscribing rectangle having the same waterline length and beam as the ship.

$$I_y = I_L = C_{IL}BL^3/12 \quad (2-40a)$$

$$I_x = I_T = C_{IT}LB^3/12 \quad (2-40b)$$

where C_{IL} = longitudinal waterplane inertia coefficient

C_{IT} = transverse waterplane inertia coefficient.

This concept will be discussed further in chapters 3 and 6.

2-8 Hydrostatics

FLUID PROPERTIES

Hydrostatics is the study of liquids at rest. Liquids and gases are fluids (i.e., substances that cannot sustain shear forces when in static equilibrium and that deform continuously when subjected to shear stresses while in motion). The following definitions are needed to describe the basic hydrostatic properties of fluids:

1. *Density and specific weight*—The density ρ of a fluid is its mass per unit volume, while the specific weight w is its weight per unit volume. The system of units relating the two concepts can be obtained from Newton's second law applied to the acceleration caused by gravity g ; i.e.,

$$F = ma \quad \text{becomes} \quad W = mg \quad (2-41)$$

where W = weight of body of mass m .

Thus, in the English gravitational system of units, an lbf is equivalent to a slug-ft/sec², and in the metric system, a Newton is equivalent to a kilogram-meter/sec². If both sides of the expression are divided by the volume of the substance, one obtains

$$\frac{W}{V} = \frac{m}{V} g$$

Thus the specific weight w and the density ρ are related by

$$w = \rho g \quad (2-42)$$

2. *Specific volume*—The specific volume of a fluid is its volume per unit weight, which is the reciprocal of the specific weight. Since w will also be used to indicate weight changes on ships, ρg will generally be used to indicate specific weight. Thus,

$$v = \frac{1}{w} = \frac{1}{\rho g} \quad (2-43)$$

3. *Specific gravity*—The specific gravity γ of a liquid is the ratio of its density to that of pure water at a

standard temperature. Physicists use 39.2°F (4°C) as the standard, but engineers and naval architects ordinarily use 59°F (15°C). In the metric system, the density of water at 4°C is 1.00 g/cm³, hence the specific gravity (which is dimensionless) has the same numerical value for a liquid as its density in that system. Since the density of a fluid varies with temperature, specific gravities must be determined and specified at particular temperatures as shown in appendix A.

For purposes of making approximate engineering calculations, the following table gives commonly accepted values of the properties of fresh water and salt water in both English and metric units. These values should be used in solving homework problems unless otherwise specified or implied by some specific temperature being given.

	<i>English</i>	<i>Metric</i>
<i>Specific weight of fresh water</i>	62.4 lbf/ft ³	.0098MN/m ³
<i>Specific weight of salt water</i>	64.0 lbf/ft ³	.01MN/m ³
<i>Specific volume of fresh water</i>	36 ft ³ /ton	102 m ³ /MN
<i>Specific volume of salt water</i>	35 ft ³ /ton	99.5 m ³ /MN
<i>Mass density of fresh water</i>	1.94 slugs/ft ³	1,000 kg/m ³
<i>Mass density of salt water</i>	1.99 slugs/ft ³	1,025 kg/m ³
<i>Specific gravity of salt water</i>	1.025	1.025
<i>Acceleration of gravity</i>	32.17 ft/sec ²	9.807 m/sec ²

Note that 1,000 kg equals a metric ton in mass units and that MN stands for meganewton (10⁶ Newtons) in force units. The English long ton (2,240 lbf) is equivalent to 1.01605 metric tons.

The specific weight of a liquid varies only slightly with pressure; it also depends on temperature, and the variation may be considerable. Since specific weight w is equal to ρg , it depends on the local value of the acceleration of gravity in addition to the variations with temperature and pressure. The text deals specifically with ships at sea level, however, where g can be considered constant, so this variation can be neglected. The variation of the specific weight of water with temperature under normal conditions, where g equals 32.17 fps/sec, is shown in appendix A.

PRESSURE

There are no shear stresses in fluids at rest; hence, only normal stresses are present. The average pressure intensity is defined as the normal force exerted on a unit area. If F represents the normal force on some finite

area A , while dF represents the force on an infinitesimal area dA , the pressure is defined as follows:

$$p = \lim_{\delta A \rightarrow 0} \frac{\delta F}{\delta A} = \frac{dF}{dA} \quad (2-44)$$

If the pressure is uniform over the total area, then p equals F/A . In engineering units, pressure is generally expressed in pounds per square inch (psi) or pounds per square foot (psf). In a solid, because of the possibility of tangential stresses between adjacent particles, the stresses at a given point may be different in different directions. But in a fluid at rest, no tangential stresses can exist, and the only forces between adjacent surfaces are pressure forces normal to the surfaces. Therefore, the pressure at any point in a fluid at rest is the same in every direction and is a scalar quantity.

PRESSURE-DEPTH RELATIONSHIP

The pressure in a fluid at rest changes only in the direction of gravitational acceleration. Thus, the pressure in a horizontal plane is everywhere the same for fluids of the same density. To determine the variation of pressure in a vertical direction, consider a small volume of fluid at rest, as in figure 2-25. The weight of the fluid in the volume equals $\rho g A \delta z$. Since the summation of forces equals zero,

$$\begin{aligned} (p + \delta p)A - pA - \rho g A \delta z &= 0 \\ \delta p &= \rho g \delta z \\ \text{or } \rho g &= \lim_{\delta z \rightarrow 0} \frac{\delta p}{\delta z} = \frac{dp}{dz} \end{aligned} \quad (2-45)$$

If the fluid can be considered incompressible (i.e., the specific weight is nearly constant for the range of height or depth variation in a particular problem), the derivative form can be replaced by the integration of equa-

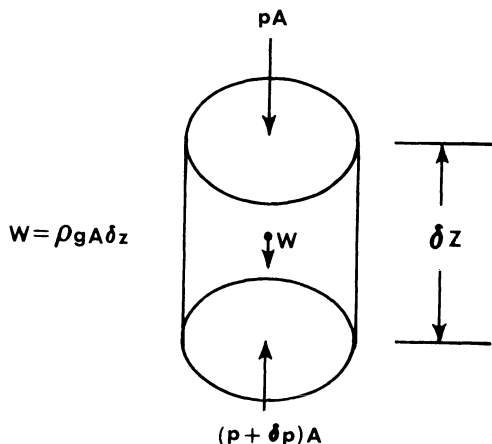


Figure 2-25

tion 2-45, and one obtains

$$\delta p = \rho g h \quad (2-46)$$

where h = change in depth.

ABSOLUTE AND GAGE PRESSURE

A relationship for calculating the change in pressure with depth was obtained in the previous section. A reference pressure is thus required to establish the value of the pressure at any point. Pressures are generally measured relative to zero normal stress (absolute pressure) or to the zero reading on a pressure gage (gage pressure). The relationship between these two reference pressures is illustrated in figure 2-26, where it is assumed that atmospheric pressure exists on the exterior of the gage-pressure measuring device. This relationship is summarized by the following equation:

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gage}} \quad (2-47)$$

where P_{abs} = absolute pressure

P_{atm} = atmospheric pressure

P_{gage} = gage pressure.

The variation of pressure in a liquid at rest is thus:

$$p = p_0 + \rho g h \quad (2-48)$$

where p_0 is the reference pressure at the liquid surface. If the reference pressure is taken to be zero gage pressure, then the stated pressure p is a gage pressure. If the reference pressure is taken to be the atmospheric pressure, then the pressure will be an absolute pressure. The standard atmospheric pressure is taken to be 14.7 psi, which is equivalent to 29.92 inches of mercury (760 mm of mercury) with a specific gravity of 13.6.

2-9 Hydrostatic Forces on Submerged Surfaces

As can be seen from equation 2-48, the pressure varies linearly with changes in depth, provided that gage pressure is used and that the changes in depth do not involve changes in the density of water. When a fluid is at rest no tangential forces can be sustained, so the hydrostatic forces are always normal to the surface. Consider the following situations:

1. Forces on a horizontal plane surface.
 - a) The magnitude of the force is equal to the integral of the pressure over the area.

$$\begin{aligned} F &= \int p dA = \int \rho g h dA = \rho g h \int dA \\ &= \rho g h A \end{aligned} \quad (2-49)$$

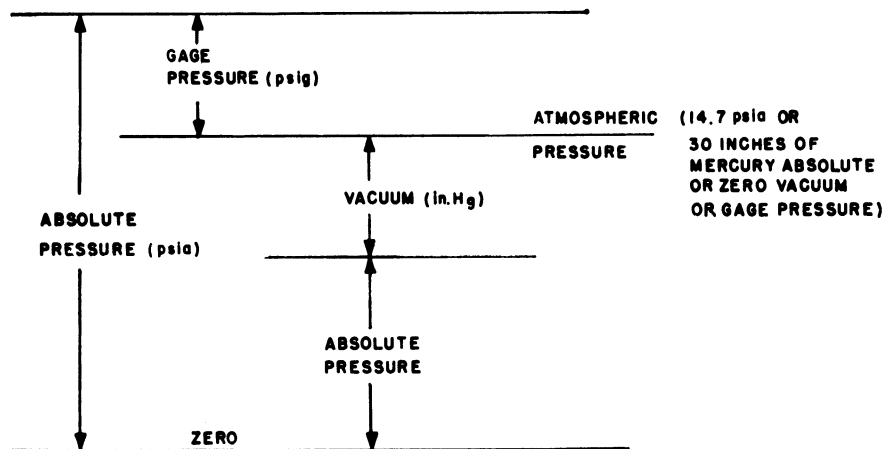


Figure 2-26. Relationships among gage pressure, atmospheric pressure, vacuum, and absolute pressure

- b) The point of application of the force is at the centroid of the horizontal area.
2. Forces on an inclined plane surface.
- a) The magnitude of the force is equal to the integral of the pressure over the area such as AB illustrated in figure 2-27.

$$F = \int_A^B p dA = \rho g \int_A^B h dA$$

Now

$$h = y \sin \theta$$

$$F = \rho g \sin \theta \int_A^B y dA$$

but

$$\int_A^B y dA = \bar{y}A = \frac{\bar{h}A}{\sin \theta}$$

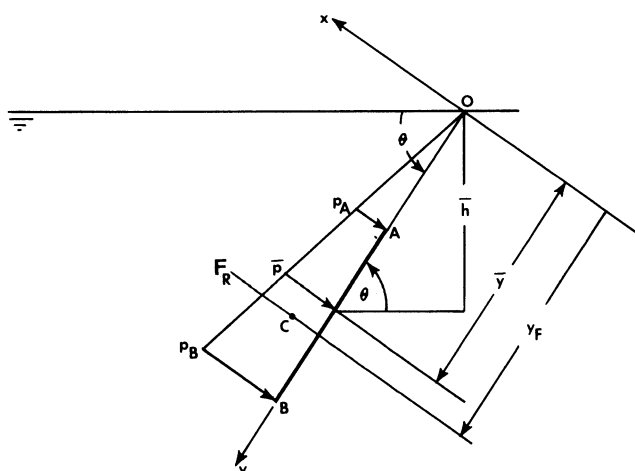


Figure 2-27. Force on an inclined surface

Thus,

$$F = \rho g \bar{h}A = \bar{p}A \quad (2-50)$$

where $\bar{h}$ = depth of the centroid of the area
 $\bar{p}$ = mean pressure at the centroid of the area.

- b) The point of application of the resultant force is such that the moment of the resultant force about a horizontal axis at the surface is equal to the moment of the distributed force about the same axis.

$$\begin{aligned} y_F F_R &= \int y p dA = \int y (\rho g h) dA \\ &= \int y (\rho g y \sin \theta) dA \\ &= \rho g \sin \theta \int_{y_A}^{y_B} y^2 dA \end{aligned}$$

But

$$\int_{y_A}^{y_B} y^2 dA = I_0 = \begin{array}{l} \text{moment of inertia of} \\ \text{surface } AB \text{ about the axis} \\ \text{at the watersurface} \end{array}$$

$$\begin{aligned} \therefore y_F F_R &= \rho g \sin \theta I_0 \\ &= \rho g \sin \theta (I_c + \bar{y}^2 A) \end{aligned}$$

where I_c = moment of inertia of surface AB about its centroid

A = area of surface AB .

Now $F_R = \rho g \sin \theta \bar{y}A$ from equation 2-50

$$y_F = \frac{\rho g \sin \theta (I_c + \bar{y}^2 A)}{\rho g \sin \theta \bar{y}A}$$

$$y_F = \frac{I_c}{\bar{y}A} + \bar{y}$$



or

$$y_F - \bar{y} = \frac{I_c}{\bar{y}A} \quad (2-51)$$

Thus, the resultant force on an inclined surface acts at a distance below the centroid of the surface equal to the moment of inertia of the surface about the centroid divided by $\bar{y}A$. Note that for a given surface area A , the distance the resultant force acts below the centroid diminishes with increasing depth.

Example 2-8: Consider the hydrostatic forces acting on the side of a ship hull as illustrated in figure 2-28. Compare the resultant force acting on a 6-ft by 6-ft section located on the surface ($\bar{h}_1 = 3$ ft), and (b) the section located with the centroid 15 feet below the surface. The specific weight for sea water is $\rho g = 64 \text{ lb/ft}^3$.

Solution: For vertical surfaces, it is customary to replace y with the depth h as indicated in the sketch.

$$F_1 = \bar{p}_1 A = \rho g \bar{h}_1 A$$

$$= 64(3)36$$

$$= \boxed{6,912 \text{ lb}}$$

$$h_{F_1} = \bar{h}_1 + \frac{I_c}{\bar{h}_1 A}$$

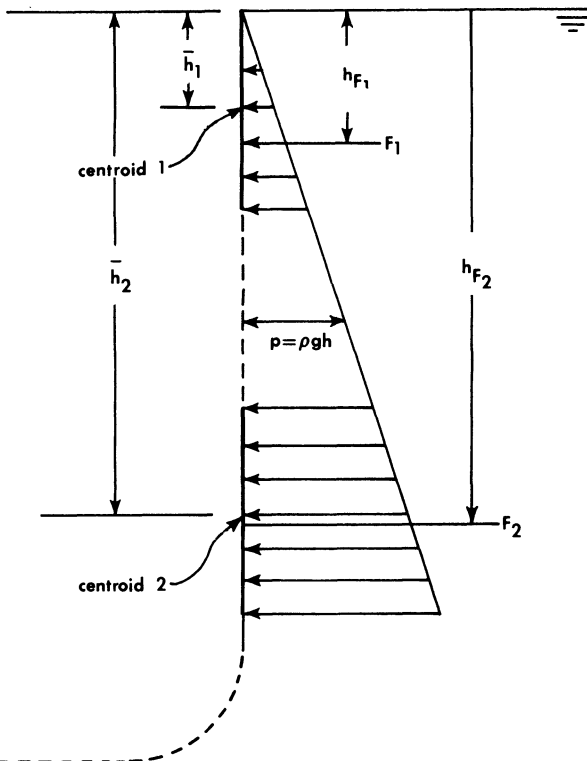


Figure 2-28. Forces on a ship hull

$$I_c = \frac{bh^3}{12} = \frac{6(6)^3}{12} = 108 \text{ ft}^4$$

$$h_{F_1} = 3 + \frac{108}{3(36)} = \boxed{4 \text{ ft}}$$

which coincides with the depth of the centroid of the pressure-versus-depth triangle in figure 2-26.

$$F_2 = \bar{p}_2 A = \rho g \bar{h}_2 A$$

$$= 64(15)36$$

$$= \boxed{34,560 \text{ lb}}$$

$$h_{F_2} = \bar{h}_2 + \frac{I_c}{\bar{h}_2 A}$$

$$= 15 + \frac{108}{15(36)}$$

$$h_{F_2} = \boxed{15.2 \text{ ft}}$$

It can be shown that this depth also coincides with the centroid of the pressure-versus-depth trapezoid for the lower surface as illustrated in figure 2-26.

3. Forces on submerged curved surfaces.

Since the pressure forces always act normal to the surface it is possible to find the resultant forces by finding the horizontal and vertical components of the resultant force.

- The horizontal component of the hydrostatic force on a curved surface is equal in magnitude and in point of application to the resultant force on a vertical plane projection of the curved surface as illustrated in figure 2-29.
- The vertical component of the hydrostatic force on a curved surface is equal in magnitude to the weight of the liquid extending to the free surface vertically above the curved surface. It acts on a vertical line of action through the centroid of the volume of liquid above the surface. Although the horizontal component of the hydrostatic force can be easily computed, it is seen that the vertical components will require integration techniques, both to find the magnitude of the force and to locate the centroid of the volume.

ARCHIMEDES' PRINCIPLE

The techniques just developed can be used to demonstrate Archimedes' Principle, which can be stated as

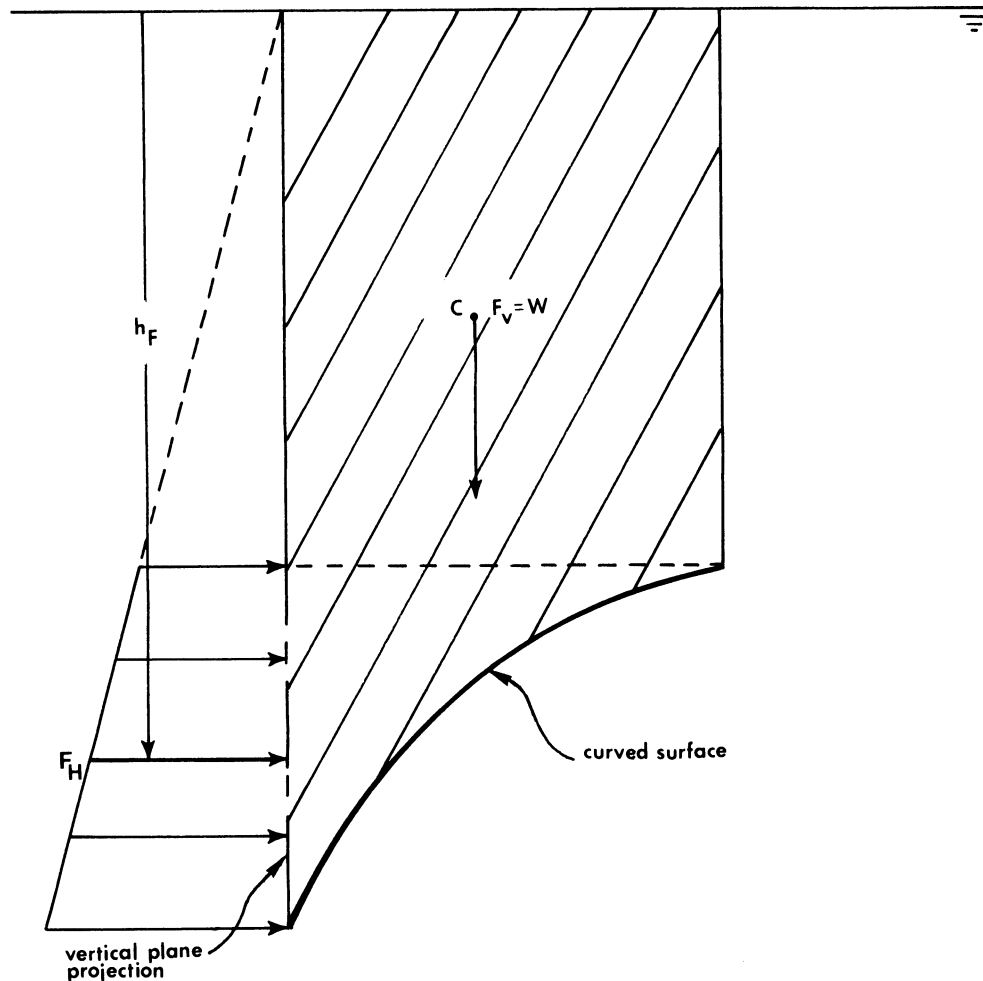


Figure 2-29. Forces on a curved surface

follows: *A body partially or completely immersed in a fluid is buoyed up, or sustained, by a force equal to the weight of fluid displaced.*

Consider the ellipsoidal body immersed in a liquid as illustrated in figure 2-30. It is seen from the figure that the horizontal pressure forces cancel each other since they both act on identical projected areas. The vertical force acting downward on surface ABC is equal to the weight of liquid directly above ABC or

$$F_{ABC} = \rho g (\text{volume } EABCF)$$

where ρ = density of the fluid.

The vertical force acting upward on surface ADC is equal to the weight of liquid directly above ADC or

$$F_{ADC} = \rho g (\text{volume } EADCF)$$

The net force or buoyant force acting on the surface $ABCD$ is thus

$$F_{ADC} - F_{ABC} = \rho g (\text{volume } EADCF - \text{volume } EABCF)$$

or

$$\begin{aligned} \text{buoyant force} &= \rho g (\text{volume } ABCD) \\ &= \text{weight of liquid displaced.} \end{aligned} \quad (2-52)$$

Volume $ABCD$ is called the displaced volume of the body, and equation 2-52 holds for both submerged and floating bodies. The buoyant force acts through the centroid of the displaced volume.

Problems

2-1. Determine the tension in the cable in figure 2-a. Consider the beam to be weightless.

2-2. Determine the tension in each of the cables in figure 2-b.

2-3. Determine the reactions at A and B for the

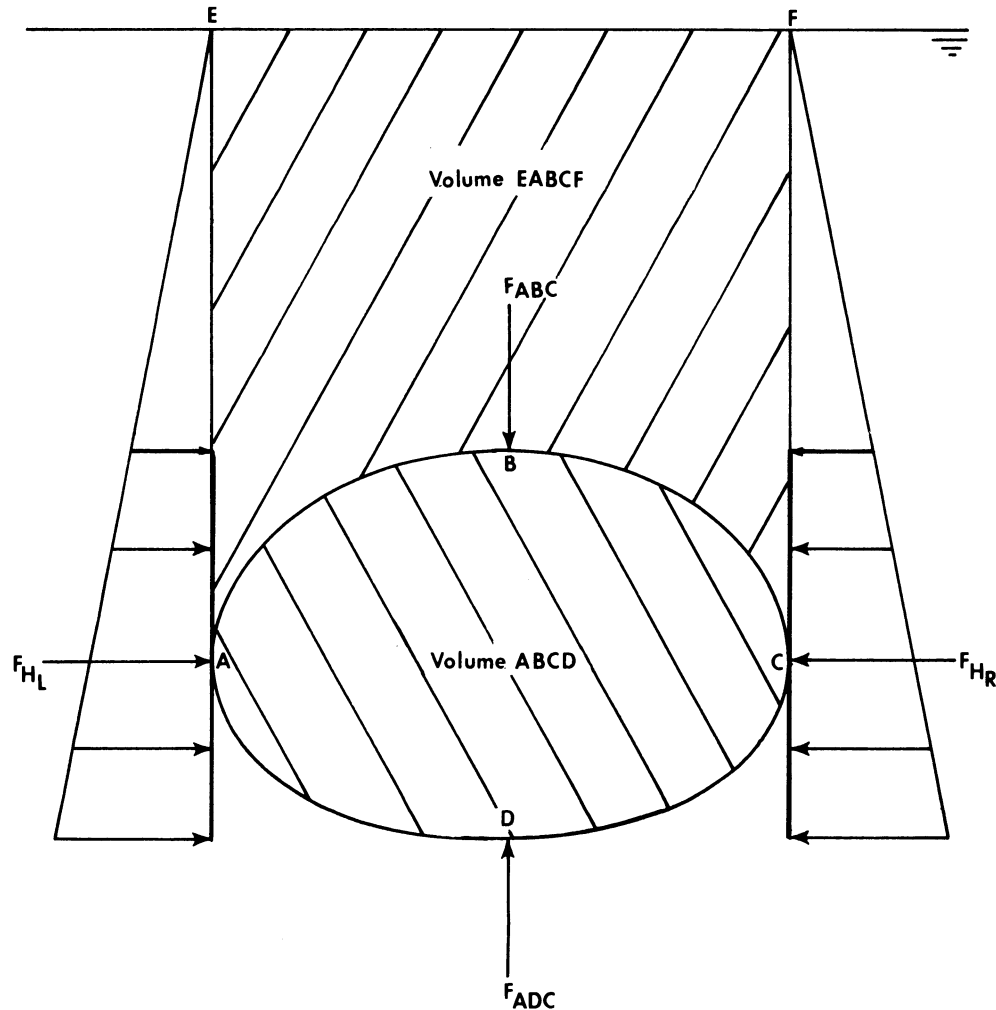


Figure 2-30. Forces on a submerged body

simply supported beam in figure 2-c. Consider the beam to be weightless.

2-4. Determine the reactions at point A for the cantilevered weightless beam shown in figure 2-d.

2-5. A ship is moored to a pier and can be boarded via a 30-ft-long brow weighing 500 pounds connected to the quarterdeck by a pinned joint at point A in figure 2-e. The other end of the brow rests on rollers on the pier at point B, ten feet below the quarterdeck. Two sailors are standing on the brow at the locations shown in the sketch. Determine the reactions at A and B.

2-6. Calculate the moment of inertia of a triangle about its centroid.

2-7. Calculate the moment of inertia of the V-block in example 2-4 about its base.

2-8. Calculate the centroid of the triangle in example 2-3 by using a horizontal strip for dA and integrating with respect to y .

2-9. Develop a general expression for the moment of inertia of an I-beam of width w , thickness t , and height h about the vertical and horizontal centroidal

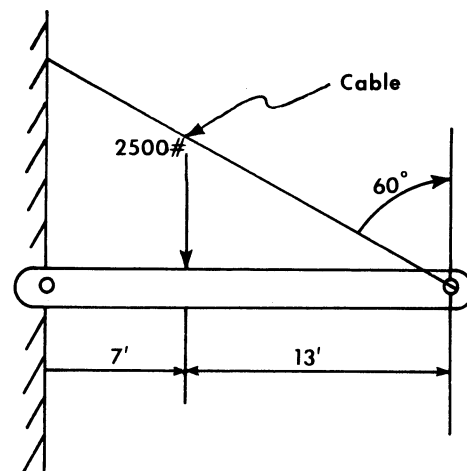


Figure 2-a.

axes. Compare these expressions with those for a channel section of the same dimensions.

2-10. Calculate the radius of gyration of the I-beam in example 2-7.

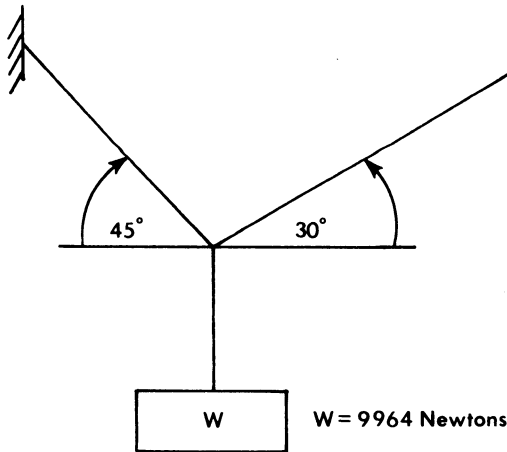


Figure 2-b

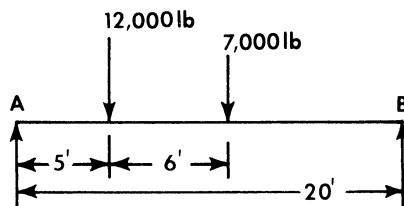


Figure 2-c

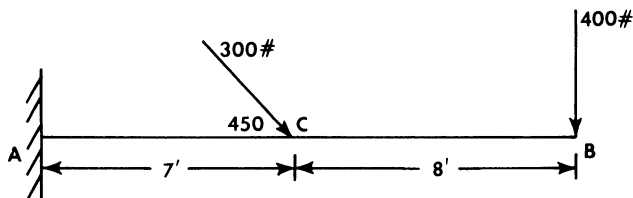


Figure 2-d.

2-11. If the pressure gauge on a steam condenser indicates a reading of 24 inches of mercury vacuum, what is the absolute pressure in psi if the barometer reads 750 mm of mercury?

2-12. Prove that the resultant force F_2 in example 2-8 acts at the centroid of the pressure-vs-depth trapezoid for the lower surface in figure 2-26.

2-13. A rectangular gate 6 feet high and 4 feet wide is hinged as shown in figure 2-f and is used to control the height of water in the reservoir. A spring is provided to hold the gate closed against the stops if the water is below a certain level. By adjusting the spring tension, the gate can be made to open at a given water depth. Find a) the horizontal force on the gate

b) the location of the resultant horizontal force on the gate

c) the spring force required to just keep the gate closed if the water is at elevation 9 feet but that will let the gate open for greater water elevations.

2-14. An irregularly shaped volume of a new ma-

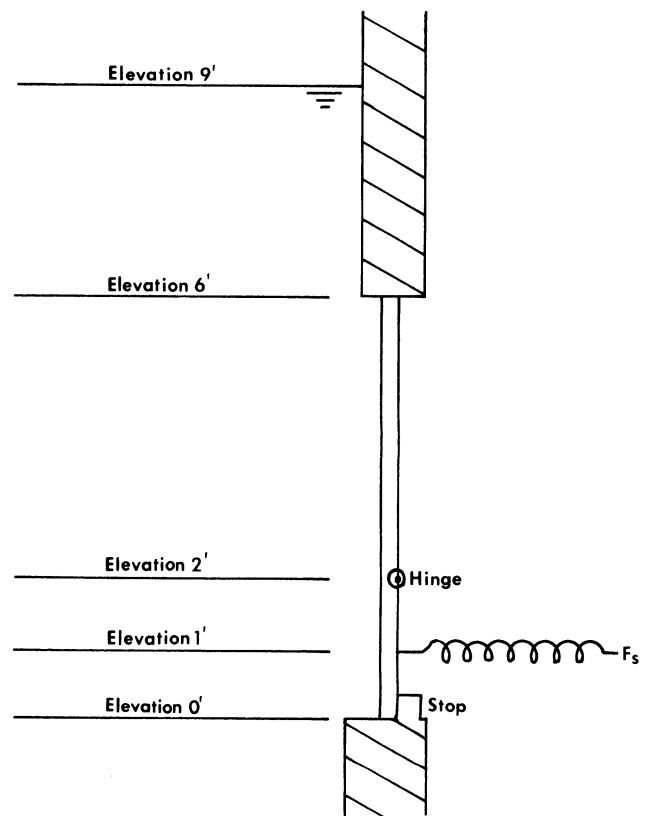


Figure 2-f

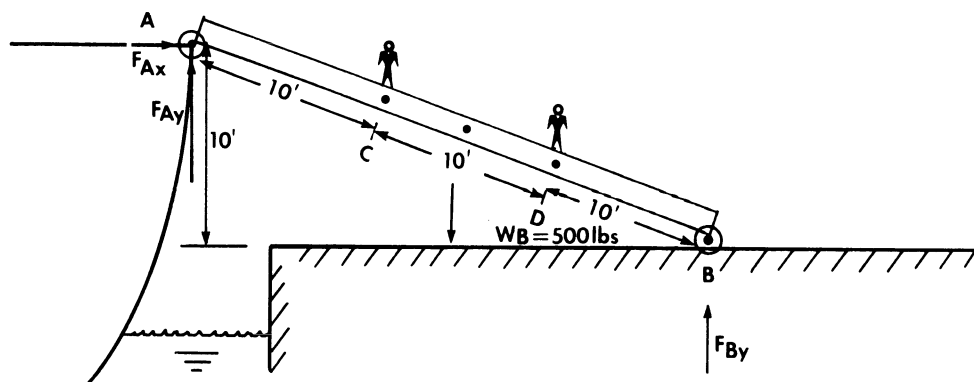


Figure 2-e

terial is attached to a scale and lowered into a tank of fresh water, as illustrated in figure 2-g. When the body is completely submerged, the scale reads 35 pounds and the level of the water in the 1.0-ft-diameter tank has risen 1.0 inch. Determine the specific gravity and the density of the material.

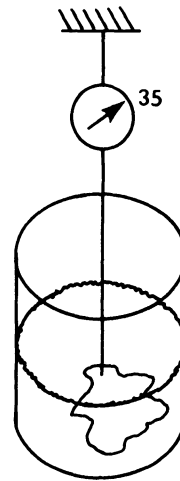


Figure 2-g.