

# INTERNATIONAL STANDARD

# ISO 10725

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## Acceptance sampling plans and procedures for the inspection of bulk materials

*Plans et procédures d'échantillonnage pour acceptation pour le contrôle de  
matériaux en vrac*



Reference number  
ISO 10725:2000(E)

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 10725 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 3, *Application of statistical methods in standardization*.

Annexes A and B form a normative part of this International Standard. Annexes C and D are for information only.

## Introduction

The application of statistical methods in the field of sampling of bulk materials has been developed since the late 1940s, principally for large quantities of raw materials, such as coals or iron ores, where major interest was to obtain an accurate estimate of the lot mean with reasonable cost, so as to adjust the price and process duly when necessary.

Recently, the need for acceptance sampling of bulk materials has increased especially for industrial products, such as powder chemicals or plastic beads, where the determination of acceptability of a lot is more important than to acquire an accurate estimate of the lot mean. This International Standard has been developed for the former purpose.

The subject of this International Standard is situated on the border line between ISO/TC69/SC 3 dealing with bulk sampling and ISO/TC 69/SC 5 dealing with acceptance sampling, and some SC 5 experts have assisted in the drafting.



# Acceptance sampling plans and procedures for the inspection of bulk materials

## 1 Scope

This International Standard specifies acceptance sampling plans by the determination of variables and use of acceptance inspection procedures for bulk materials. These sampling plans comply with specific operating characteristic curves at reasonable cost.

This International Standard is applicable to the inspection where the lot mean of a single quality characteristic is the principal factor in the determination of lot acceptability, but it also gives special procedures for multiple quality characteristics. This International Standard is applicable to the cases where the values of standard deviations at individual stages of sampling are known or are imprecise.

This International Standard is applicable to various kinds of bulk materials, but is not always applicable to minerals such as iron ores, coals, crude petroleum, etc., where accurate estimation of the lot mean is more important than the determination of lot acceptability.

For special cases when standard procedures are not always adequate and the measurement standard deviation is dominant, this International Standard specifies special acceptance sampling plans and procedures, such as in the case for liquids.

## 2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 2859-1:1999, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*.

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control*.

ISO 5725-1:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions*.

ISO 11648-1:—<sup>1)</sup>, *Statistical aspects of sampling from bulk materials — Part 1: General principles*.

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1) To be published.

### 3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 2859-1, ISO 3534-1, ISO 3534-2, ISO 5725-1 and the following apply.

#### 3.1

##### **acceptance sampling**

sampling inspection in which decisions are made to accept or not to accept a lot based on the results of a sample or samples selected from that lot

#### 3.2

##### **acceptance inspection**

inspection to determine whether an item or lot delivered or offered for delivery is acceptable

#### 3.3

##### **sampling system**

collection of sampling plans, together with criteria by which appropriate sampling plans may be chosen

#### 3.4

##### **sampling plan**

combination of sample size and associated acceptability criteria

#### 3.5

##### **sample size**

total number of tests or measurements and elements thereof

NOTE 1 In this International Standard, the sample size is, for example, the number of sampling increments in a composite sample, the number of composite samples per lot, the number of test samples prepared from a composite sample, the number of measurements per test sample. The number of measurements is the same as the number of test portions.

NOTE 2 In this International Standard, this term should not be used for sample amount such as the volume or mass of a sampling increment.

#### 3.6

##### **acceptability criteria**

criteria or element of the criteria (for instance an acceptance value) for the determination of lot acceptability, i.e. to accept or not to accept a lot

#### 3.7

##### **acceptance quality limit**

when a continuing series of lots is considered, a level of the lot mean which for the purposes of sampling inspection is the limit of the satisfactory process average

#### 3.8

##### **non-acceptance quality limit**

when a continuing series of lots is considered, a level of the lot mean which for the purposes of sampling inspection is the limit of the unsatisfactory process average

#### 3.9

##### **one-sided specification limit**

specification limit of either a lower or an upper limit for the lot mean

#### 3.10

##### **two-sided specification limits**

specification limits of both lower and upper limits for the lot mean

#### 3.11

##### **bulk material**

amount of material within which component parts are not initially readily distinguishable on the macroscopic level

NOTE This International Standard excludes paper rolls, wire coils, iron scrap or similar materials, because it is difficult to apply the specified sampling procedures.

### 3.12

#### **sampling increment**

amount of bulk material taken in one action by a sampling device

### 3.13

#### **composite sample**

aggregation of two or more sampling increments taken from a lot for inspection of the lot

### 3.14

#### **test sample**

sample, as prepared for testing or analysis, the whole amount or a part of it being used for testing or analysis at one time

### 3.15

#### **test portion**

part of a test sample which is used for testing or for analysis at one time

### 3.16

#### **acceptance value**

limiting value of sample average that permits lot acceptance

### 3.17

#### **discrimination interval**

interval between the acceptance quality limit and the non-acceptance quality limit

### 3.18

#### **limiting interval**

minimum interval between upper and lower acceptance quality limits, when two-sided specification limits are specified

### 3.19

#### **relative standard deviation**

ratio of a standard deviation relative to the discrimination interval

### 3.20

#### **repeatability**

precision under repeatability conditions, i.e. where independent test results are obtained with the same method on identical test items in the same laboratory, by the same operator using the same equipment within short intervals of time

### 3.21

#### **intermediate precision measurement**

precision under intermediate precision conditions, i.e. where test results are obtained with the same method on identical test items in the same laboratory, under some different operating conditions (time, calibration, operator and equipment)

## 4 Symbols and abbreviated terms

The symbol and the abbreviated terms used in this International Standard are as follows:

|       |                                                                  |
|-------|------------------------------------------------------------------|
| $C$   | varying cost per lot                                             |
| $C_I$ | sum of costs proportional to total number of sampling increments |
| $C_M$ | sum of costs proportional to total number of measurements        |

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|          |                                                                                                                                                                                                                          |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $C_T$    | sum of costs proportional to the total number of test samples                                                                                                                                                            |
| $c_I$    | cost of drawing a sampling increment                                                                                                                                                                                     |
| $c_M$    | cost of a measurement                                                                                                                                                                                                    |
| $c_T$    | cost of preparing a test sample                                                                                                                                                                                          |
| $c_{TM}$ | cost of treating a test sample ( = $c_T + n_M c_M$ )                                                                                                                                                                     |
| $D$      | discrimination interval                                                                                                                                                                                                  |
| $D_N$    | narrow discrimination interval for multiple characteristics                                                                                                                                                              |
| $d_I$    | relative standard deviation between sampling increments ( = $\sigma_I/D$ )                                                                                                                                               |
| $d_T$    | relative test sample standard deviation ( = $\sigma_T/D$ )                                                                                                                                                               |
| $d_O$    | relative overall standard deviation ( = $\sigma_O/D$ )                                                                                                                                                                   |
| $f_D$    | correction factor for multiple characteristics                                                                                                                                                                           |
| $f_U$    | factor for obtaining upper control limit                                                                                                                                                                                 |
| $G$      | number of lots used for re-estimation of standard deviations                                                                                                                                                             |
| $J$      | number of quality characteristics                                                                                                                                                                                        |
| $K_p$    | the upper $p$ -fractile of the standardized normal distribution<br>(Examples of $p$ are $\alpha$ , $\beta$ and $P_a$ . For $\alpha = 0,05$ , $K_\alpha = 1,644\ 85$ . For $\beta = 0,10$ , $K_\beta = 1,281\ 55$ , etc.) |
| $L_{CL}$ | lower control limit                                                                                                                                                                                                      |
| $L_{SL}$ | lower specification limit for the lot mean                                                                                                                                                                               |
| $m$      | lot mean                                                                                                                                                                                                                 |
| $m_A$    | acceptance quality limit for the lot mean                                                                                                                                                                                |
| $m_R$    | non-acceptance quality limit for the lot mean                                                                                                                                                                            |
| $n_I$    | number of sampling increments per composite sample                                                                                                                                                                       |
| $n_M$    | number of measurements per test sample                                                                                                                                                                                   |
| $n_T$    | number of test samples per composite sample                                                                                                                                                                              |
| $P_a$    | probability of acceptance                                                                                                                                                                                                |
| $Q_{CR}$ | consumer's risk quality                                                                                                                                                                                                  |
| $Q_{PR}$ | producer's risk quality                                                                                                                                                                                                  |
| $R_C$    | cost ratio ( = $c_{TM}/c_I$ )                                                                                                                                                                                            |
| $s_c$    | composite sample standard deviation                                                                                                                                                                                      |
| $s_{cT}$ | combined sample standard deviation                                                                                                                                                                                       |

|                 |                                                                                                        |
|-----------------|--------------------------------------------------------------------------------------------------------|
| $s_M$           | measurement standard deviation                                                                         |
| $s_T$           | test sample standard deviation                                                                         |
| $t_p(\nu)$      | the lower $p$ -fractile of the $t$ -distribution with $\nu$ degrees of freedom                         |
| $U_{SL}$        | upper specification limit for the lot mean                                                             |
| $U_{CL}$        | upper control limit                                                                                    |
| $x_{ijk}$       | measured value for the $k$ -th test portion from $j$ -th test sample from the $i$ -th composite sample |
| $\bar{X}_{...}$ | sample grand average                                                                                   |
| $\bar{X}_L$     | lower acceptance value                                                                                 |
| $\bar{X}_U$     | upper acceptance value                                                                                 |
| $\alpha$        | producer's risk                                                                                        |
| $\alpha^*$      | individual producer's risk                                                                             |
| $\beta$         | consumer's risk                                                                                        |
| $\beta^*$       | individual consumer's risk                                                                             |
| $\gamma$        | constant for obtaining the acceptance value                                                            |
| $\Delta$        | interval between the upper and lower acceptance quality limits                                         |
| $\delta$        | constant for obtaining the limiting interval                                                           |
| $\nu$           | degrees of freedom of a standard deviation                                                             |
| $\nu_E$         | degrees of freedom of an estimate standard deviation                                                   |
| $\sigma_C$      | composite sample standard deviation                                                                    |
| $\sigma_E$      | estimate standard deviation for a lot mean                                                             |
| $\sigma_M$      | measurement standard deviation                                                                         |
| $\sigma_O$      | overall standard deviation                                                                             |
| $\sigma_T$      | test sample standard deviation ( $\sigma_T^2 = \sigma_P^2 + \sigma_M^2 / n_M$ )                        |
| $\sigma_I^2$    | variance component between sampling increments                                                         |
| $\sigma_M^2$    | variance component between measurements                                                                |
| $\sigma_P^2$    | variance component between test samples (variance for test sample preparation).                        |

NOTE 1 The symbols accompanied by a subscript, "L" or "U", denote that they are for the lower or upper specification limit, respectively.

NOTE 2 The symbol  $\sigma$  is used for a population standard deviation, while the symbol  $s$  is used for a sample value.

## 5 Sampling plans

### 5.1 General

At the beginning of the acceptance sampling, the following items should be established for satisfactory inspection of a lot of bulk material.

### 5.2 Applicability

#### 5.2.1 Lot mean

This International Standard is applicable when the lot mean of a single quality characteristic is the principal factor in the determination of lot acceptability.

When the material is homogenized through further processing in the consumer's plant, the consumer may be principally interested in the lot mean.

If two or more quality characteristics are specified for a material, then the procedures given in annex A shall be applied. Annex A also provides optional procedures for multiple characteristics to prevent an increase in both the producer's risk and the consumer's risk.

This International Standard is based on the assumption that the lot mean is kept unchanged during acceptance sampling for the lot, or that the expected values of the physical average and the arithmetic mean are equal. Special care is necessary for some unstable characteristics, such as moisture of particulate material. There may be some exceptional cases where this assumption is not true, such as shown in the following example.

**EXAMPLE** CMC (carboxymethyl cellulose) powder is used as an additive to cement, and in this application one of its most important characteristics is the viscosity of the aqueous solution. If two samples, of equal mass, one having a high value of viscosity and the other a low value, are blended, the viscosity of the blended sample will always be lower than the arithmetic mean of the original two sample values. This International Standard is not applicable to such cases.

#### 5.2.2 Standard deviations

This International Standard is based on the assumption that the values of the individual standard deviation of the specified quality characteristic is known and stable. Guidelines to judge the stability of the individual standard deviation are as follows:

- a) in the standard procedure, if both  $s_c$  and  $s_T$  control charts have no out-of-control point, and if no other evidence gives doubt about the stability, one can deem that all standard deviations are stable. If  $\sigma_M$  is large and unstable, then this fact will probably be detected by the  $s_T$  control chart. If  $\sigma_M$  is sufficiently small, its instability can be neglected, because its precise estimate is unnecessary;
- b) in the special procedure in annex B, if the  $s_T$  control chart has no out-of-control point, and if no other evidence gives doubt about the stability, all standard deviations can be deemed to be stable. In this case, the instability of  $\sigma_I$  and  $\sigma_T$  can be neglected, because their precise estimates are unnecessary.

However, at the start of acceptance sampling, the precise value and/or the stability of the individual standard deviation may not be sufficiently known. Furthermore, minor and temporary deviation from the stability guidelines given above may occur during application of this acceptance sampling system. In such cases, the procedures for imprecise standard deviations are applicable, where assumed values of standard deviations of the specified quality characteristic are used.

If relevant values of standard deviations are not available at all, this International Standard is not applicable.

#### 5.2.3 Inspection lots

These sampling plans are intended to be used primarily for a continuing series of lots. However, if the requirements for standard deviations are satisfied, these plans may also be used for isolated lots.

### 5.3 Standardized sampling procedures

#### 5.3.1 General

This International Standard contains the following procedures for inspection of an individual lot:

- a) increment sampling;
- b) constitution of composite samples;
- c) preparation of test samples; and
- d) measurements.

Figure 1 illustrates the schematic flow of the above procedures. In order to avoid overcrowding Figure 1, the numbers of unused test samples and test portions drawn are far smaller than the usual values, respectively (see C.2.7).

Representative sampling shall be used throughout the above-mentioned procedures. For example, it is required that individual composite sample can represent the whole lot. In order to obtain reliable results, it is important to specify instructions or standardized procedures. It is recommended that reference be made to ISO 11648-1 beforehand, so that reasonable sampling procedures may be specified.

#### 5.3.2 Increment sampling (see Figure 1)

Take sampling increments of  $2n_I$  from a lot. It is recommended that dynamic sampling be used, where sampling increments are taken from a moving lot. However, the use of static sampling is allowed, where the lot stands still.

It is also recommended that an appropriate sampling device be used. When the material contains coarse lumps, the volume of individual sampling increments should be sufficiently large that representative samples may be obtained.

#### 5.3.3 Constitution of composite samples (see Figure 1)

Pool sampling increments of  $n_I$  together and form two composite samples. In this International Standard, two composite samples have been adopted. Each composite sample shall be representative of the whole lot. This requirement may be attained by carrying out systematic duplicate sampling, described as follows:

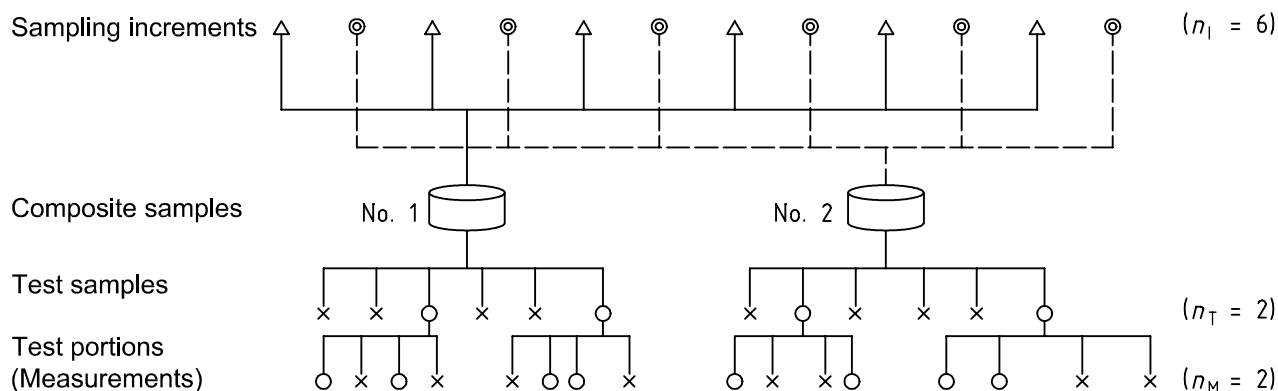
Among  $2n_I$  sampling increments numbered in order, pool those with odd numbers (1, 3, ...,  $2n_I - 1$ ) to form composite sample No. 1, and those with even numbers (2, 4, ...,  $2n_I$ ) to form composite sample No. 2.

#### 5.3.4 Preparation of test samples (see Figure 1)

Prepare  $n_T$  test samples from each of the two composite samples. Establish the procedure for test sample preparation beforehand, taking into account the nature of the material to be inspected.

When the material contains coarse lumps, make sure the procedure of the test sample preparation includes one or more stages of particle size reduction (such as crushing and grinding), homogenization (such as mixing) and sample division. The procedure should specify the mass of the test sample and, if necessary, the particle size of the test sample. When the material is liquid, test samples may be taken directly from the composite sample, after sufficient stirring.

**NOTE** If an adequate procedure for test sample preparation is chosen, then a variance component between test samples,  $\sigma_T^2$ , can be far smaller than the variance component between sampling increments,  $\sigma_I^2$ . On the other hand, economical considerations are also important. For example, grinding of lumps is effective in reducing  $\sigma_T^2$ , but fine grinding of the total amount of composite sample is frequently too expensive.



### Key

- $\triangle$  odd numbered sampling increments
- $\odot$  even numbered sampling increments
- $\circ$  used
- $\times$  not used

**Figure 1 — Schematic model of bulk acceptance sampling procedures**

### 5.3.5 Measurements (see Figure 1)

Draw test portions of  $n_M$  from  $2n_T$  test samples respectively, and carry out  $2n_T n_M$  measurements per lot. It is necessary to specify the measurement conditions in detail.

## 5.4 Standard deviations

### 5.4.1 General

In the case of imprecise standard deviations, determine the sample standard deviations at various stages in accordance with 6.2.9. In the case of known standard deviations, determine the sample standard deviations at various stages in accordance with 6.2.3. Necessary information common to both cases is given in 5.4.2 to 5.4.4.

If the results of past inspection are available, the estimates of standard deviations can be obtained from them. If knowledge of the standard deviations at their respective stages is insufficient, then it is necessary to obtain relevant values. It is recommended to use one of experimental techniques described in ISO 11648-1 and ISO 11648-2<sup>[1]</sup>. In many cases a nested experiment is the most suitable.

### 5.4.2 Standard deviation between sampling increments

The values of the quality characteristics of sampling increments taken from a lot may vary due to differing sources of variation. The standard deviation between sampling increments,  $\sigma_I$ , is the positive square root of the variance component between the sampling increments,  $\sigma_I^2$ .

**NOTE** If the material is a gas or a low viscosity liquid in one container, the sampling increment standard deviation,  $\sigma_I$ , can be assumed to be zero. However, if the material is a viscous liquid or solid, and if it is in two or more containers, the sampling increment variance,  $\sigma_I^2$ , is usually composed of two components, i.e. the sampling increment variances within the container and between containers.

### 5.4.3 Standard deviation between test samples

Test samples prepared from a composite sample, in accordance with the specified test sample preparation procedure, may contain variation due to division of the composite sample. The standard deviation between test samples,  $\sigma_T$ , is the positive square root of the variance component between test samples,  $\sigma_T^2$ .

NOTE The standard deviation between test samples,  $\sigma_T$ , composed of solids is not always negligible except when the particle size is sufficiently small. In contrast, in the case of a low viscosity liquid or a gas,  $\sigma_T$  is frequently negligible.

#### 5.4.4 Measurement standard deviation

The results of measurements may vary due to differing sources of variation, including the measurement procedure and the test portion variation within a test sample.

When the total number of measurements is small, the measurement standard deviation usually corresponds to the repeatability. When the total number of measurements is large, it is rather difficult to maintain repeatability conditions, then the appropriate intermediate precision measures shall be used. For more information, see ISO 5725-1, ISO 5725-2<sup>[2]</sup> and ISO 5725-3<sup>[3]</sup>.

NOTE In this International Standard, the approximate ratio of  $\sigma_M/\sigma_T$  is more important than  $\sigma_M$  itself. If  $\sigma_M/\sigma_T$  is sufficiently small (for example, less than 0,2), it is not necessary to know the exact value of  $\sigma_M$ .

### 5.5 Costs

#### 5.5.1 General

This International Standard uses the following cost values for obtaining the economical sampling plan. When the knowledge of these cost values is insufficient, it is also possible to obtain an applicable sampling plan (see 5.5.6).

#### 5.5.2 Components of costs

The total varying cost per lot,  $C$ , consists of the sum of the costs proportional to the total number of sampling increments, to the total number of test samples and to the total number of measurements, as follows:

$$\begin{aligned} C &= C_I + C_T + C_M \\ &= 2n_I c_I + 2n_T c_T + 2n_T n_M c_M \end{aligned}$$

The unit cost values,  $c_I$ ,  $c_T$  and  $c_M$ , are used to obtain economical sampling plans.

#### 5.5.3 Cost of taking a sampling increment

The sum of costs proportional to the total number of sampling increments,  $C_I$ , contains the following elements:

- a) the cost of taking the sampling increments;
- b) the cost of pooling to form a composite sample.

The cost of taking one sampling increment,  $c_I$ , is given by the following equation:

$$c_I = \frac{C_I}{2n_I}$$

#### 5.5.4 Cost of preparing a test sample

The sum of the cost proportional to the total number of test samples,  $C_T$ , contains the following elements:

- a) the cost of size reduction and sample division;
- b) the cost of preparing test samples.

The cost of preparing a test sample,  $c_T$ , is given by the following equation:

$$c_T = \frac{C_T}{2n_T}$$

### 5.5.5 Cost of a measurement

The cost of a measurement,  $c_M$ , is given by the following equation:

$$c_M = \frac{C_M}{2n_T n_M}$$

where the sum of the costs,  $C_M$ , is proportional to the total number of measurements.

### 5.5.6 Procedures for cases when cost values are insufficiently known

At the start of the contract, the knowledge of the above cost values may be insufficient. In such cases, the following procedures should be used in order to obtain an applicable sampling plan.

- a) When knowledge of the above cost values is insufficient, assume the approximate ratio,  $c_I:c_T:c_M$ , and use each term of the ratio in place of the respective cost value.

EXAMPLE At the start of the contract, the approximate cost ratio was assumed as follows:

$$c_I:c_T:c_M = 3:1:0,5$$

Putting  $c_I = 3$ ,  $c_T = 1$  and  $c_M = 0,5$ , the sampling plan was obtained in accordance with the standard procedures.

After running five lots, the cost ratio was revised as follows:

$$c_I:c_T:c_M = 3,5:1:0,4$$

Using the new cost values ( $c_I = 3,5$ ,  $c_T = 1$  and  $c_M = 0,4$ ), the applicable sampling plan was obtained again, but both the cost ratio level and the sampling plan remained unchanged.

- b) If it is difficult to assume the approximate cost ratio, then use the following ratio:

$$c_I:c_T:c_M = 1:1:1$$

## 5.6 Acceptance quality limit and non-acceptance quality limit

### 5.6.1 General

The quality measures, the acceptance quality limit,  $m_A$ , and the non-acceptance quality limit,  $m_R$ , should be specified in accordance with the following procedures.

### 5.6.2 Interval between $m_R$ and specification limit

It is recommended that the interval between the non-acceptance quality limit and the specification limit ( $m_{R,L} - L_{SL}$  or  $U_{SL} - m_{R,U}$ ) be specified taking account of the actual use of an accepted lot. For example, if an accepted lot is divided into sub-lots in actual use, then variation between sub-lots should be considered when determining the interval.

When two-sided specification limits are specified, the two intervals ( $m_{R,L} - L_{SL}$  or  $U_{SL} - m_{R,U}$ ) may be different.

This interval can be adjusted to the quality limit of the supplied material. If the quality limit is far from satisfactory, this interval can be increased so that the consumer's risk at the specification limit can be reduced. For this purpose annex D gives useful information. On the contrary, if the quality limit is satisfactory, this interval can be reduced to zero or even can be a negative value.

### 5.6.3 Discrimination interval

The discrimination interval,  $D$ , is the interval between the acceptance quality limit and the non-acceptance quality limit. It is recommended that the value of  $D$  be specified, taking account of the values of the standard deviations,  $\sigma_L$ ,  $\sigma_T$  and  $\sigma_M$ . If the value of discrimination interval is too small, then this International Standard may not give any applicable sampling plan, and the choice of the acceptance quality limit and/or the non-acceptance quality limit will need to be reconsidered.

When two-sided specification limits,  $L_{SL}$  and  $U_{SL}$ , are specified, the two discrimination intervals ( $m_{A,L} - m_{R,L}$  and  $m_{R,U} - m_{A,U}$ ) shall be equal.

The discrimination interval can be adjusted to the quality limit of the supplied material. If the quality limit is satisfactory, the discrimination interval can be increased to achieve a reduction in costs.

NOTE This interval should be determined mainly from the technical aspects. ISO 10576<sup>[4]</sup> may give useful information when determining this interval.

### 5.6.4 Interval between acceptance quality limits

When two-sided specification limits are specified, the interval between the upper and lower acceptance quality limits,  $\Delta$ , should be equal to or greater than the limiting interval,  $\delta \times D$ . That is:

$$\Delta = m_{A,U} - m_{A,L} \geq \delta \times D$$

For the standard procedures of known standard deviations,  $\delta = 0,636$ , and for the optional procedures,  $\delta = 0,566$ .

For the procedures of imprecise standard deviations, the value of  $\delta$  can be obtained from Table 1, which is indexed by  $\nu_E$ . The value of  $\nu_E$  will be given together with the sample sizes. At the preliminary stage, it is convenient to assume the following interim values:

$$\nu_E = 8 \quad \text{and} \quad \delta = 0,566$$

**Table 1 — Values of  $\delta$  for two-sided specification limits (imprecise standard deviations)**

| $\nu_E$                                                                                                     | $\delta$ |
|-------------------------------------------------------------------------------------------------------------|----------|
| 3,0 to 3,9                                                                                                  | 0,929    |
| 4,0 to 4,9                                                                                                  | 0,758    |
| 5,0 to 5,9                                                                                                  | 0,670    |
| 6,0 to 6,9                                                                                                  | 0,617    |
| 7,0 to 7,9                                                                                                  | 0,582    |
| $\geq 8,0$                                                                                                  | 0,566    |
| NOTE The value of $\delta$ is used for determining the applicability of the two-sided specification limits. |          |

## 5.7 Responsible authority

### 5.7.1 Functions

The responsible authority has various functions such as:

- a) to approve the values of standard deviations;

- b) to judge stability of standard deviations;
- c) to select between imprecise and known standard deviations;
- d) to approve the values of  $m_A$  and  $m_R$ ;
- e) to decide whether or not to use optional procedures;
- f) other specified or implied functions.

It is desirable that the responsible authority have sufficient knowledge and ability so as to maintain the neutrality of this acceptance sampling system and to smoothly perform the acceptance sampling procedures.

### 5.7.2 Contractual relationship

The responsible authority may be either of the following:

- a) the first party;
- b) the second party;
- c) the third party; or
- d) any of the first, the second or the third party, differing according to function.

The responsible authority should be specified in advance of the acceptance sampling, in the contract or any other relevant document. See also 3.1.12 of ISO 2859-1:1999.

## 6 Inspection procedures

### 6.1 General

This clause gives the following procedures:

- a) assessment of the standard deviations;
- b) determination of sample sizes;
- c) selection and preparation of samples;
- d) determination of the acceptance value;
- e) determination of lot acceptability.

### 6.2 Assessment of the standard deviations

#### 6.2.1 General

If the results of past inspection are available, the necessary values of standard deviation,  $\sigma_I$ ,  $\sigma_T$  and  $\sigma_M$ , can be obtained through the procedures for confirmation and re-estimation given below. If not, see 6.2.9. It is permitted to use the values of standard deviations specified by an agreement between the supplier and the purchaser.

It is especially advantageous to use the values of standard deviations specified by an agreement in the following cases:

- a) at the start of the contract, when knowledge of  $\sigma$  values is not always sufficient; or
- b) the producer has far more knowledge about the standard deviations than the consumer; or
- c) during inspection, when either the  $s_c$  or the  $s_T$  control chart has given an out-of-control point, but the process has been improved thereafter, and it is believed that the standard deviations have been decreased.

### 6.2.2 Confirmation procedures

When the procedures are applied to a continuing series of lots, the stability of the standard deviations should be checked lot by lot. For this purpose, it is convenient to use control charts as shown in 6.2.4. If the check results indicate a lack of stability of the standard deviations, then appropriate action should be taken.

On the other hand, values of the population standard deviation should be re-estimated periodically. Unless otherwise specified, the standard deviations should be re-estimated from the results of the immediately preceding 10 lots, and after each five subsequent lots, regardless of lot acceptability. If the standard deviation is re-estimated, then sample sizes,  $n_M$ ,  $n_I$  and  $n_T$ , should be re-evaluated. If the re-estimation leads to sample sizes other than the preset ones, then the new sampling plan should be used for the next lot, because the appropriate sample sizes have changed. For additional information, see C.2.3.

### 6.2.3 Standard deviations

The composite sample standard deviation (sample value,  $s_c$ ) is obtained from the composite sample averages using the following simpler equation

$$s_c = \sqrt{\frac{(\bar{x}_{1..} - \bar{x}_{2..})^2}{2}}$$

The test sample standard deviation (sample value,  $s_T$ ) is obtained from the test sample averages using the following equation:

$$s_T = \sqrt{\frac{1}{\nu_T} \sum_{i=1}^2 \sum_{j=1}^{n_T} (\bar{x}_{ij.} - \bar{x}_{i..})^2}$$

where  $\nu_T = 2(n_T - 1)$ .

If  $n_T = 2$ , then the following simpler equation is applicable.

$$s_T = \sqrt{\frac{1}{2} \sum_{i=1}^2 \frac{(\bar{x}_{i1.} - \bar{x}_{i2.})^2}{2}}$$

If  $n_M > 1$ , then the measurement standard deviation (sample value,  $s_M$ ) can be obtained similarly.

$$s_M = \sqrt{\frac{1}{\nu_M} \sum_{i=1}^2 \sum_{j=1}^{n_T} \sum_{k=1}^{n_M} (\bar{x}_{ijk} - \bar{x}_{ij.})^2}$$

where  $\nu_M = 2n_T(n_M - 1)$ .

If  $n_M = 2$ , then the following simpler equation is applicable.

$$s_M = \sqrt{\frac{1}{\nu_M} \sum_{i=1}^2 \sum_{j=1}^{n_T} \frac{(\bar{x}_{ij1} - \bar{x}_{ij2})^2}{2}}$$

where  $\nu_M = 2n_T$ .

If the total number of measurements is sufficiently small, then a pocket calculator can be used for the calculation of the above values, otherwise the use of appropriate software is recommended.

## 6.2.4 Control charts

### 6.2.4.1 General

Control charts used in this International Standard are a of specific type. They have an upper control limit,  $U_{CL}$ , but they do not have a lower control limit,  $L_{CL}$  (for more information see C.2.3).

#### 6.2.4.2 $s_C$ control chart

The upper control limit  $U_{CL,C}$  of an  $s_C$  control chart should be obtained, for each of the 10 lots (or other number of lots used for the check), regardless of lot acceptability, using the following equation:

$$U_{CL,C} = f_U \times \sigma_C$$

where

$f_U$  is a factor given in Table 2, as a function of degrees of freedom ( $\nu_C = 1$ );

$\sigma_C$  is a population standard deviation between composite samples.

If none of the standard deviations (sample value,  $s_C$ ) exceeds the corresponding  $U_{CL,C}$ , then the  $s_C$  control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE 1 Because the number of composite samples is kept to two, the  $U_{CL,C}$  is common to all the lots.

NOTE 2 If a sufficient number of preceding inspection results are not available, then  $\sigma_C$  is given by the following equation:

$$\sigma_C = \sqrt{\frac{\sigma_I^2}{n_I} + \frac{\sigma_T^2}{n_T}}$$

#### 6.2.4.3 $s_T$ control chart

The upper control limit,  $U_{CL,T}$ , of an  $s_T$  control chart should be obtained, for each of the 10 lots (or the other number of lots used for the check), regardless of lot acceptability, using the following equation:

$$U_{CL,T} = f_U \times \sigma_T$$

where

$f_U$  is a factor given in Table 2, as a function of degrees of freedom for the test sample,  $\nu_T$ ;

$\sigma_T$  is a population standard deviation between test samples.

If none of the standard deviations (sample value,  $s_T$ ) exceeds the corresponding  $U_{CL,T}$ , then the  $s_T$  control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE 1 If  $n_T$  is kept constant, then the  $U_{CL,T}$  is common to all the lots.

NOTE 2 If a sufficient number of preceding inspection results are not available, then  $\sigma_T$  is given by the following equation:

$$\sigma_T = \sqrt{\sigma_P^2 + \frac{\sigma_M^2}{n_M}}$$

**Table 2 — Values of  $f_U$  for  $U_{CL}$**

| $\nu$ | $f_U$ | $\nu$ | $f_U$ | $\nu$ | $f_U$ | $\nu$ | $f_U$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 2,800 | 31    | 1,331 | 61    | 1,235 | 104   | 1,180 |
| 2     | 2,297 | 32    | 1,326 | 62    | 1,233 | 108   | 1,176 |
| 3     | 2,065 | 33    | 1,321 | 63    | 1,231 | 112   | 1,173 |
| 4     | 1,924 | 34    | 1,316 | 64    | 1,230 | 116   | 1,170 |
| 5     | 1,827 | 35    | 1,311 | 65    | 1,228 | 120   | 1,167 |
| 6     | 1,755 | 36    | 1,307 | 66    | 1,226 | 124   | 1,165 |
| 7     | 1,700 | 37    | 1,303 | 67    | 1,224 | 128   | 1,162 |
| 8     | 1,654 | 38    | 1,299 | 68    | 1,223 | 132   | 1,159 |
| 9     | 1,617 | 39    | 1,295 | 69    | 1,221 | 136   | 1,157 |
| 10    | 1,585 | 40    | 1,291 | 70    | 1,219 | 140   | 1,155 |
| 11    | 1,558 | 41    | 1,287 | 71    | 1,218 | 144   | 1,153 |
| 12    | 1,534 | 42    | 1,284 | 72    | 1,216 | 148   | 1,150 |
| 13    | 1,513 | 43    | 1,281 | 73    | 1,215 | 152   | 1,148 |
| 14    | 1,494 | 44    | 1,277 | 74    | 1,213 | 156   | 1,147 |
| 15    | 1,477 | 45    | 1,274 | 75    | 1,212 | 160   | 1,145 |
| 16    | 1,462 | 46    | 1,271 | 76    | 1,211 | 164   | 1,143 |
| 17    | 1,448 | 47    | 1,268 | 77    | 1,209 | 168   | 1,141 |
| 18    | 1,435 | 48    | 1,265 | 78    | 1,208 | 172   | 1,140 |
| 19    | 1,424 | 49    | 1,263 | 79    | 1,206 | 176   | 1,138 |
| 20    | 1,413 | 50    | 1,260 | 80    | 1,205 | 180   | 1,136 |
| 21    | 1,403 | 51    | 1,257 | 82    | 1,203 | 184   | 1,135 |
| 22    | 1,393 | 52    | 1,255 | 84    | 1,200 | 188   | 1,133 |
| 23    | 1,385 | 53    | 1,252 | 86    | 1,198 | 192   | 1,132 |
| 24    | 1,377 | 54    | 1,250 | 88    | 1,196 | 196   | 1,131 |
| 25    | 1,369 | 55    | 1,248 | 90    | 1,193 | 200   | 1,129 |
| 26    | 1,362 | 56    | 1,246 | 92    | 1,191 | 220   | 1,123 |
| 27    | 1,355 | 57    | 1,243 | 94    | 1,189 | 240   | 1,118 |
| 28    | 1,348 | 58    | 1,241 | 96    | 1,187 | 260   | 1,113 |
| 29    | 1,342 | 59    | 1,239 | 98    | 1,185 | 280   | 1,109 |
| 30    | 1,336 | 60    | 1,237 | 100   | 1,183 | 300   | 1,105 |

NOTE Where  $\nu$  is not given, linear interpolation may be used.

#### 6.2.4.4 $s_M$ control chart

If  $n_M > 1$ , then the upper control limit,  $U_{CL,M}$ , for an  $s_M$  control chart can be calculated similarly.

$$U_{CL,M} = f_U \times \sigma_M$$

where

$f_U$  is a factor given in Table 2, as a function of degrees of freedom for the measurement,  $\nu_M$ ;

$\sigma_M$  is a population standard deviation of measurement.

If none of the standard deviations (sample values,  $s_M$ ) exceeds the corresponding  $U_{CL,M}$ , then the  $s_M$  control chart may be considered to show a state of control, otherwise it should be considered as showing an out-of-control state.

NOTE If both of the sample sizes,  $n_T$  and  $n_M$ , are kept constant, then the  $U_{CL,M}$  is common to all the lots.

### 6.2.5 Re-estimation of population standard deviations

#### 6.2.5.1 General

Re-estimation of population standard deviations shall be made for the immediately preceding  $G$  lots. Unless otherwise specified by the responsible authority,  $G$  shall be 10. It is recommended that re-estimation be carried out after every five subsequent lots.

#### 6.2.5.2 Constant sample sizes

When sample sizes are constant for  $i$  lots, the population standard deviations,  $\sigma_C$  and  $\sigma_T$ , are estimated from the standard deviations (sample values,  $s_C$  and  $s_T$ ) using the following formulae:

$$\sigma_C = \sqrt{\frac{\sum_{i=1}^G s_{C,i}^2}{G}}$$

and

$$\sigma_T = \sqrt{\frac{\sum_{i=1}^G s_{T,i}^2}{G}}$$

where  $G$  is the number of lots used for the re-estimation.

If  $n_M > 1$ , then the measurement standard deviation,  $\sigma_M$ , can be estimated similarly:

$$\sigma_M = \sqrt{\frac{\sum_{i=1}^G s_{M,i}^2}{G}}$$

#### 6.2.5.3 Varying sample sizes

When sample sizes are not constant for  $G$  lots, population standard deviations,  $\sigma_C$  and  $\sigma_T$ , are estimated from the standard deviations (sample values,  $s_C$  and  $s_T$ ) using the following equations:

$$\sigma_c = \sqrt{\frac{\sum_{i=1}^G v_{c,i} \times s_{c,i}^2}{\sum_{i=1}^G v_{c,i}}}$$

and

$$\sigma_T = \sqrt{\frac{\sum_{i=1}^G v_{T,i} \times s_{T,i}^2}{\sum_{i=1}^G v_{T,i}}}$$

where  $G$  is the number of lots used for the re-estimation;

If  $n_M > 1$ , then the measurement standard deviation,  $\sigma_M$ , can be estimated similarly:

$$\sigma_M = \sqrt{\frac{\sum_{i=1}^G v_{M,i} \times s_{M,i}^2}{\sum_{i=1}^G v_{M,i}}}$$

## 6.2.6 Re-estimation of variance components

### 6.2.6.1 General

Population standard deviations,  $\sigma_c$  and  $\sigma_T$ , vary if the sample sizes,  $n_M$ ,  $n_T$  or  $n_I$ , change, whereas  $\sigma_I$  and  $\sigma_P$  are not affected by the sample size. The variance component between sampling increments,  $\sigma_I^2$ , and the variance component between test samples,  $\sigma_P^2$ , are somewhat different from ordinary variances. It is impossible to obtain a variance component directly, but it can be obtained as differences of other variances.

### 6.2.6.2 Variance component between sampling increments

The variance component between sampling increments,  $\sigma_I^2$ , is given by the following equation:

$$\sigma_I^2 = n_I \left( \sigma_c^2 - \frac{\sigma_T^2}{n_T} \right)$$

If  $\sigma_I^2 < 0$ , then assume that  $\sigma_I^2 = 0$ .

### 6.2.6.3 Variance component between test samples

If  $n_M > 1$ , then the variance component between test samples,  $\sigma_P^2$ , is given by the following equation:

$$\sigma_P^2 = \sigma_T^2 - \frac{\sigma_M^2}{n_M}$$

If  $\sigma_P^2 < 0$ , then assume that  $\sigma_P^2 = 0$ ;

If  $n_M = 1$ , then it is not necessary to separate  $\sigma_P^2$  and  $\sigma_M^2$ .