
Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

*Engrenages — Roues et engrenages cylindriques à développante —
Concepts et géométrie*



PDF disclaimer

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.



COPYRIGHT PROTECTED DOCUMENT

© ISO 2007

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword.....	iv
1 Scope	1
2 Normative references	1
3 Symbols, subscripts and units.....	2
3.1 Symbols	2
3.2 Subscripts	8
3.3 Units	9
4 Individual cylindrical gears.....	10
4.1 Concepts for an individual gear	10
4.2 Reference surfaces, datum lines and reference quantities.....	12
4.3 Involute helicoids.....	17
4.4 Angular pitch and pitches.....	21
4.5 Diameters of gear teeth	24
4.6 Gear tooth height	24
4.7 Tooth thickness, space width.....	25
5 Cylindrical gear pairs	27
5.1 Concepts for a gear pair	27
5.2 Mating quantities	28
5.3 Calculation of the sum of the profile shift coefficients.....	31
5.4 Tooth engagement.....	32
5.5 Backlash	40
5.6 Sliding conditions at the tooth flanks.....	41
6 Tooth flank modifications	44
6.1 Tooth flank modifications which restrict the usable flank	44
6.2 Transverse profile modifications	45
6.3 Flank line (helix) modifications	48
6.4 Flank face modifications	49
6.5 Descriptions of modifications by functions.....	51
7 Geometrical limits.....	52
7.1 Counterpart rack tooth profile	53
7.2 Machining allowance	54
7.3 Deviations in tooth thickness	55
7.4 Generating profile shift, generating profile shift coefficient.....	56
7.5 Generated root diameter	57
7.6 Usable area of the tooth flank, tip and root form diameter	57
7.7 Undercut	59
7.8 Overcut	59
7.9 Minimum tooth thickness at the tip circle of a gear	59
Annex A (informative) Calculations related to tooth thickness.....	60
Bibliography	83

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 21771 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771 cancels and replaces ISO/TR 4467:1982, of which it constitutes a technical revision.

Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

1 Scope

This International Standard specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included.

It also covers the concepts and parameters for cylindrical gear pairs with parallel axes and a constant gear ratio, which consist of cylindrical gears according to it. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

The equations given are not restricted to the pressure angle, $\alpha_p = 20^\circ$.

The standard is structured as follows.

- Listing of symbols and nomenclature for a unique description of gears and gear pairs (see Clause 3).
- Equations and explanations of the relevant values for defining a cylindrical gear and its tooth system. The equations for determination of the nominal values for zero-deviation gear description parameters are stated for radial tooth dimensions (gear tooth heights), the distance between flanks of the same hand, the distance between flanks of opposite hand, as well as the tooth flank characterizing parameters (see Clause 4).
- Equations and explanations of the relevant values for defining cylindrical gear pairs. The equations for the essential parameters characterizing the engagement conditions of the unloaded gear pair are listed (see Clause 5).
- Equations and suggestions for desired flank modifications (see Clause 6).
- Concepts and recommendations needed for a unique geometrical definition of the intended results from manufacture (Clause 7).
- Equations for determination of the nominal values or the limiting values for the most used inspection methods for tooth thickness (see Annex A).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*

ISO 1328-2:1997, *Cylindrical gears — ISO system of accuracy — Part 2: Definitions and allowable values of deviations relevant to radial composite deviations and runout information*

3 Symbols, subscripts and units

3.1 Symbols

Symbol	Description	Used in
a_w	centre distance of a cylindrical gear pair	5.2.3
a_0	centre distance in the generating gear unit	7.5
a_L	centre distance for tooth flank engagement	A.8
b	facewidth	4.2.8
b_F	usable facewidth	4.2.8
b_M	contact line overlap (for measuring base tangent length)	A.2.1
b_w	active facewidth (the facewidth used)	5.4.7.2
c	tip clearance	5.2.7
c_F	form over dimension	5.4.4
d	reference diameter	4.2.4
d_a	tip diameter	4.5.3
d_{a0}	tip diameter of tool	7.5
d_{aM}	tip diameter of overcut cylindrical gears	A.9
d_b	base diameter	4.3.10
d_{b0}	base diameter of the pinion-type cutter	7.6
d_f	root diameter (nominal dimension)	4.5.4
d_{fE}	root diameter produced	7.5
d_{f0}	root diameter of the pinion-type cutter	A.9
d_v	V-circle diameter	4.5.1
d_w	working pitch diameter	5.2.5
d_y	Y-circle diameter	4.3.3
d_{Fa}	tip form diameter	7.6
d_{Fa0}	tip form diameter of the pinion-type cutter	7.6
d_{Ff}	root form diameter	7.6
d_K	diameter of circle through centre of ball	A.5
d_M	diameter of a measuring circle	A.2.1
d_{Na}	active tip diameter	5.4.1
d_{Nr}	start of active profile diameter (SAP diameter, active root diameter)	5.4.1
e_t	space width on the reference cylinder	4.7.3
e_{yt}	space width on the Y-cylinder	4.7.3

Symbol	Description	Used in
e_p	space width of the standard basic rack tooth profile	4.2.3
g_a	length of addendum path of contact	5.4.5.2
g_f	length of dedendum path of contact	5.4.5.2
g_α	length of path of contact	5.4.5.2
$g_{\alpha y}$	distance of a point Y from pitch point C	5.6.1
g_β	arc of contact	5.4.7.4
h	tooth depth (between tip line and root line)	4.6.1
h_a	addendum	4.6.2
h_{aP}	addendum of the standard basic rack tooth profile	Figure 4
h_{aP0}	addendum of the tool standard basic rack tooth profile	7.5
h_f	dedendum	4.6.2
h_{fP}	dedendum of the standard basic rack tooth profile	Figure 4
h_{fP0}	dedendum of the tool standard basic rack tooth profile	A.9
h_w	working depth of teeth in a gear pair	5.2.6
h_{FFP}	depth of dedendum form of the standard basic rack tooth profile	Figure 4
h_K	radius of the tip corner chamfering or tip corner rounding	6.1.2
h_P	tooth depth of standard basic rack tooth profile	Figure 4
inv	involute function	4.3.9
j_{bn}	contact backlash	5.5
j_r	radial backlash	5.5
j_t	circumferential backlash at the reference circle	5.5.2
j_{wt}	circumferential backlash at the pitch circle	5.5
k	number of teeth, spaces or pitches in a span (e.g. number of teeth spanned)	A.2.1
k	addendum modification coefficient	4.5.2
$l_{\max}$	path of engagement	5.4.8
Σl	sum of path of contact	5.4.8
m_n	normal module	4.2.7
m_t	transverse module	4.2.7
m_x	axial module	4.2.7
n_a	number of revolutions of driving gear (rpm)	5.2.2
n_b	number of revolutions of driven gear (rpm)	5.2.2
p	pitch, pitch on the reference cylinder	Figure 4

Symbol	Description	Used in
p_{bn}	normal pitch on the base cylinder	4.4.5
p_{bt}	transverse pitch on the base cylinder	4.4.5.1
p_{en}	normal base pitch on the path of contact	4.4.5.2
p_{et}	transverse base pitch on the path of contact	4.4.5.1
p_n	normal pitch	4.4.2.2
p_t	transverse pitch	4.4.2.1
p_x	axial pitch	4.4.4
p_{yn}	normal pitch on the Y-cylinder	4.4.3
p_{yt}	transverse pitch on the Y-cylinder	4.4.3
p_z	lead	4.3.2
q	machining allowance on tooth flank	7.2
q_{Fs}	undercut	Figure 24
s_{aK}	residual tooth thickness at tip with tip corner chamfering or tip corner rounding	6.1.2
s_{bn}	normal tooth thickness on the base circle	A.2.2
s_n	normal tooth thickness on the reference circle	4.7.5
s_{ni}	minimum normal tooth thickness on the reference circle	7.3
s_{ns}	maximum normal tooth thickness on the reference circle	7.3
s_t	transverse tooth thickness on the reference circle	4.7.1
s_{yn}	normal tooth thickness on the Y-cylinder	4.7.5
s_{yt}	transverse tooth thickness on the Y-cylinder	4.7.1
s_p	tooth thickness of the standard basic rack tooth profile	4.2.3
u	gear ratio	5.2.1
v_g	sliding speed	5.6.1
v_{ga}	sliding speed at the addendum	5.6.1
v_{gf}	sliding speed at the dedendum	5.6.1
v_n	normal speed	5.6.1
x	profile shift coefficient	4.2.9
x_E	generating profile shift coefficient	7.4
x_{Emin}	generating profile shift coefficient at undercut limit	7.7
x_L	profile shift coefficient of master gear	A.8
z	number of teeth	4.1.5
z_a	number of teeth of driving gear	5.2.2

Symbol	Description	Used in
z_b	number of teeth of driven gear	5.2.2
z_L	number of teeth of master gear	A.8
z_0	number of teeth of pinion-type cutter	7.6
A	starting point of meshing	5.4.3
B	starting point of single tooth contact on driving gear	5.4.5.1
C	pitch point, depth of relief for modifications	5.4.3
C_{ay}	modification of the profile	6.5
$C_{\beta y}$	modification of the flank line	6.5
$C_{\Sigma y}$	modification of the flank surface	6.5
$C_{\alpha a}$	amount of tip relief	6.2.1
$C_{\alpha f}$	amount of root relief	6.2.1
C_{Ea}	tip amount of triangular end relief modification	6.4.2
C_{Ef}	root amount of triangular end relief modification	6.4.2
$C_{i,j}$	amount of modification at point (i,j)	6.4.1
$C_{H\alpha}$	amount of transverse profile slope modification	6.2.2
C_{α}	amount of profile crowning (barrelling)	6.2.3
$C_{\beta I}, C_{\beta II}$	amount of end relief	6.3.1
C_{β}	amount of flank line crowning	6.3.3
$C_{H\beta}$	amount of flank line slope modification	6.3.2
D_M	measuring ball or measuring cylinder diameter	A.5
D	end point of single tooth contact point on driving gear	5.4.5.1
E	end point of meshing	5.4.3
E_{sn}	normal tooth thickness deviation limit (or allowance)	A.9
E_{sni}	lower deviation limit for tooth thickness	7.3
E_{sns}	upper deviation limit for tooth thickness	7.3
K_g	sliding factor	5.6.2
K_{ga}	sliding factor at tooth tip	5.6.2
K_{gf}	sliding factor at tooth root	5.6.2
L_{AE}	roll length	6.2
L_{Ca}	tip relief roll length	6.2.1
L_{Cf}	root relief roll length	6.2.1
L_{CI}, L_{CII}	length of end relief	6.3.1
L_{Ea}	tip roll length of triangular end relief modification	6.4.2

Symbol	Description	Used in
L_{Ef}	root roll length of triangular end relief modification	6.4.2
M_{dK}	dimension over balls	A.7
M_{dZ}	dimension over cylinders	A.7.1
M_{rK}	radial single-ball dimension	A.5
M_{rZ}	radial single-cylinder dimension	A.6
N	number of tooth or pitch	4.1.6
O	centre of a circle	Figure 10
S_{α}	twist of the transverse profile	6.4.3
S_{β}	twist of the flank line	6.4.3
T_{sn}	tooth thickness tolerance	Figure 37
T	contact point of tangent (lines of engagement) at base circle	Figure 10
U	involute point of origin	4.3.7
W_k	base tangent length over k measured teeth or measured spaces	A.2.1
Y	any point on a tooth flank or involute	4.3.5
α_n	normal pressure angle	4.3.6
α_t	transverse pressure angle	4.3.5
α_{wt}	working transverse pressure angle of gear pair	5.2.4
α_{wt0}	working transverse pressure angle in the generating gear unit	7.6
α_{yn}	normal pressure angle at the Y-cylinder	4.3.6
α_{yt}	transverse pressure angle at the Y-cylinder	4.3.5
α_{Ff}	pressure angle at root form circle	7.6
α_K	pressure angle at circle through centre of ball	A.5
α_{Kt}	transverse pressure angle at a point at circle through centre of ball	A.5
α_{Mt}	transverse pressure angle at a point at measuring circle	A.5
α_p	pressure angle of the standard basic rack tooth profile	4.3.6
α_{p0}	pressure angle of the tool basic rack tooth profile	7
α_L	working transverse pressure angle for double-flank engagement	A.8
α_{vt}	transverse pressure angle at the V-cylinder	A.5
β	helix angle	4.3.3
β_b	base helix angle	4.3.3
β_y	helix angle at Y-cylinder	4.3.3
δ_w	angle of rocking for span measurement	A.2.1

Symbol	Description	Used in
γ	lead angle at reference cylinder	4.3.3
γ_y	lead angle at Y-cylinder	4.3.3
ε_α	transverse contact ratio	5.4.7.1
ε_β	overlap ratio	5.4.7.3
ε_γ	total contact ratio	5.4.7.5
ζ	specific sliding	5.6.3
ζ_f	specific sliding at end points of path of contact	5.6.3
η	space width half angle at reference circle	4.7.4
η_b	base space width half angle	4.7.4
η_y	space width half angle at Y-circle	4.7.4
ξ_y	rolling angle of the involute at point Y	4.3.7
ξ_{Fa0}	rolling angle at tip form circle of pinion-type cutter	7.6
ξ_{Ff}	rolling angle at root form circle	7.6
ξ_{Na}	rolling angle at active tip circle	5.4.1
ξ_{Nf}	rolling angle at active root circle	5.4.1
ρ_{fP}	root radius on the standard basic rack tooth profile	Figure 4
ρ_y	radius of curvature of the involute at point Y	4.3.8
τ	angular pitch	4.4.2
φ_j	backlash angle	5.5.2
φ_α	transverse angle of transmission	5.4.7.1
φ_β	overlap angle	5.4.7.3
φ_γ	total angle of transmission	5.4.7.5
ψ	tooth thickness half angle at reference circle	4.7.2
ψ_b	base tooth thickness half angle	4.7.2
ψ_y	tooth thickness half angle at Y-circle	4.7.2
ω_a	angular velocity of driving gear	5.2.2
ω_b	angular velocity of driven gear	5.2.2
Σx	sum of profile shift coefficients	5.3
Σx_E	sum of profile shift coefficient, non-zero backlash	5.3

3.2 Subscripts

Subscript	Description	Used in ^b
—	a	
a	for quantities associated with the tip of a tooth or for the driving gear	5.2.2
b	for quantities associated with the base cylinder	4.3.10
b	for quantities associated with the driven gear	5.2.2
e	for quantities associated with the plane of action	
f	for quantities associated with the root	
g	for “sliding”	
i	for the lower limit in the case of deviations	
k	for a number of teeth, spaces, pitches or spans	
l	for “left-hand”	
m	for a mean value	
max	for a maximum value	
min	for a minimum value	
n	for quantities in a normal section	4.2.6.2
r	for “right-hand”	
s	relating to “tooth thickness”, for the upper limit in the case of deviations	
t	for quantities in a transverse section	4.2.6.1
v	for quantities associated with the V-cylinder	4.5.1
w	for quantities associated with the pitch cylinder and working values of a gear pair	
x	for quantities in an axial section	4.2.6.3
y	for values at a point Y (on the Y-cylinder)	
E	relating to “generating” (e.g. quantities generated on the cylindrical gear) or “generator”	
F	for quantities determining form circles and maximum usable flank area	
K	for quantities resulting from corner chamfering or for ball dimensions	
L	for designating a master gear	
L	for designating left flanks	4.1.8.2
M	for designating a measured value	
N	for active circles	
P	for quantities of the standard basic rack tooth profile	
P0	for quantities of the tool standard basic rack tooth profile	
R	for designating right flanks	4.1.8.2
V	for working side, for rough gear cutting	
W	for measuring base tangent length	

Subscript	Description	Used in ^b
Z	for quantities associated with cylinder dimensions	
α	for quantities associated with contact	
β	for quantities associated with a tooth trace	
γ	for total contact ratio	
Σ	for "sum"	
0	for quantities associated with the generating tool or the generating gear unit	7
1	for quantities associated with the pinion (smaller gear) of a gear pair	5.1.3
2	for quantities associated with the wheel (larger gear) or internal gear, used for designating a coefficient relating to the module	5.1.3
I	for locating face	4.2.1
II	for the face opposite the locating face	4.2.1
^a No subscript designates quantities associated with the reference cylinder. ^b Used with the symbols listed in 3.1 or as additions.		

3.3 Units

The quantities dealt with in this International Standard are to be stated in the following units:

- modules, lengths and linear dimensions in millimetres (mm);
- angles which are to be used in equations in radians (rad);
- angles which can be used for entries or to display results in degrees (°);
- angular velocity in radians per second (rad/s).

NOTE The notation $|z|$, denotes the absolute value, which is always positive, e.g. $|-50| = +50$. The expression $\frac{z}{|z|}$ is used to extract the sign of the tooth number and is convenient for programming. In particular, it is used often to determine the appropriate sign for an element of an expression; the result is 1 for external gears and -1 for internal gears.

4 Individual cylindrical gears

In this clause, the geometry of gear teeth is described using a generation process based on zero backlash engagement with a basic rack. The relationships are valid for any basic rack, but the standard basic rack (see ISO 53) is used for illustration. The standard basic rack tooth profile of the tooth system has straight flanks. Its datum line is the straight line on which the nominal dimensions of tooth thickness and space width are defined as equal to half the pitch. The standard basic rack tooth profile has the same pressure angles for the left and right flanks and the addendum plus bottom clearance equal to the dedendum. The helix angles for all the tooth flanks of a gear have the same nominal value.

4.1 Concepts for an individual gear

4.1.1 Gear, cylindrical gear, external gear, internal gear

A gear is a rotationally symmetrical object (gear blank) with a tooth system worked into the rim. A cylindrical gear is a gear with a cylindrical reference surface. A distinction is made between external and internal gears according to the radial arrangement of the teeth in each case. The tips of the teeth point outwards in an external gear and inwards in an internal gear.

4.1.2 Tooth system, external teeth, and internal teeth

The tooth system refers to all the teeth and space widths around the rim of a gear. As in 4.1.1, a distinction is made between internal and external gear teeth.

4.1.3 Tooth and space

A tooth is a geometrical element on the gearwheel body that enables the transmission of force and motion. The form and dimensions of the teeth and the distance between consecutive teeth are defined by the tooth system parameters. The space is the gap between two consecutive teeth.

4.1.4 Tooth system parameters

The nominal dimensions of involute cylindrical gear teeth are uniquely determined by the diameter of the reference cylinder, the associated basic rack and its position in relation to the reference circle. The nominal dimensions are defined by the following parameters, which are independent of each other:

- number of teeth, z ;
- standard basic rack tooth profile;
- normal module, m_n ;
- helix angle, β , and flank direction;
- profile shift coefficient, x ;
- tip diameter, d_a ;
- facewidth, b .

4.1.5 Number of teeth and sign of number of teeth

The number of teeth around the rim of the gearwheel is denoted by z .

The number of teeth, z , of an external cylindrical gear must be taken as a positive value in the following equations while the number of teeth, z , in an internal cylindrical gear is to be taken as a negative value.

In the case of segments, the number of teeth, z , used in calculations is the number that there would be on the whole circumference.

4.1.6 Tooth number

When numbering teeth, the designations tooth 1, tooth 2, etc. are to be defined on a transverse surface (datum face) viewed in an agreed direction so that the teeth are numbered in ascending order (moving in a clockwise direction). If the letter N is used to denote a reference tooth, the next tooth in the direction of counting is denoted by $N + 1$ and the previous tooth going in the opposite direction by $N - 1$. Tooth No. z is followed by tooth 1 in the direction of counting, see Figure 1.

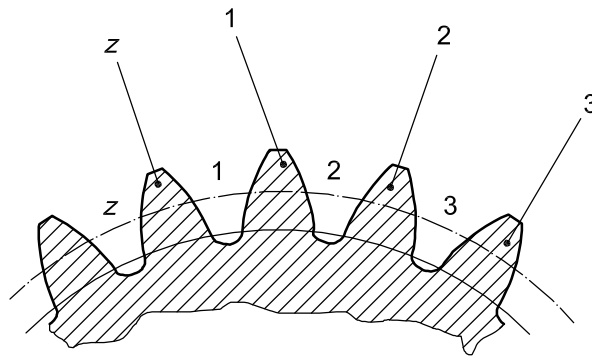


Figure 1 — Numbering of teeth and spaces on datum face

4.1.7 Top land and bottom land

4.1.7.1 Top land

The top land of a tooth is the outermost (innermost in the case of internal gears) periphery of the tooth concentric to the reference cylinder, see Figure 2.

4.1.7.2 Bottom land

The bottom land is the innermost (outermost in the case of internal gears) periphery of the space width concentric to the reference cylinder, see Figure 2.

4.1.8 Tooth flanks and flank sections

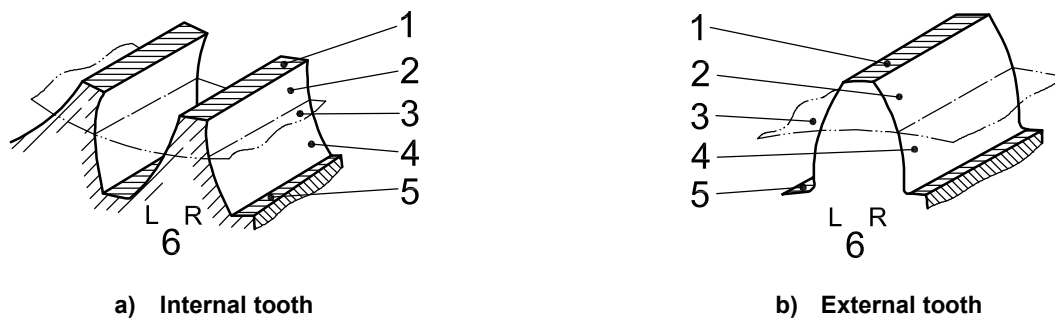
4.1.8.1 Tooth flank

Tooth flanks are those parts of the surface of a tooth that are located between the top land and the bottom land, see Figure 2.

4.1.8.2 Right flank, left flank

The right flank (or left flank) is the tooth flank that an observer sees on the right-hand (or left-hand) side when viewing the datum face of a tooth when it is pointing upwards. This definition applies to both external and internal gears, see Figure 2.

Right flank parameters are indicated by the subscript R and left flank parameters by the subscript L.

**Key**

- 1 top land
- 2 addendum flank
- 3 reference cylinder
- 4 dedendum flank
- 5 bottom land
- 6 datum face

Figure 2 — Top land, bottom land and tooth flank with division (internal and external teeth)

4.1.8.3 Addendum flank, dedendum flank

The addendum flank (or dedendum flank) is that part of a tooth flank that is located between the reference cylinder and the top land (or the bottom land), see Figure 2.

4.1.8.4 Usable flank

The usable flank is that part of a tooth flank that can be used to engage with a mating flank. On a cylindrical gear, it is part of the involute helicoid including any flank modifications.

4.2 Reference surfaces, datum lines and reference quantities

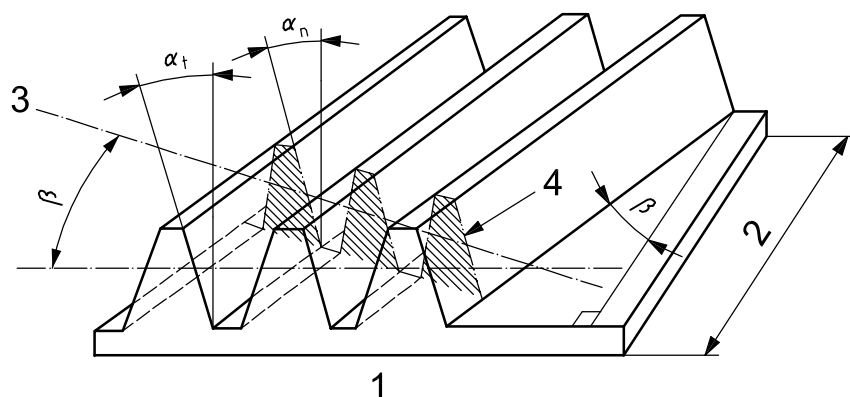
4.2.1 Reference surface, datum surface, datum face

The reference surface of the teeth is an imaginary surface to which the geometrical parameters relate. In the case of cylindrical gears, the reference surface is termed the *reference cylinder*.

The agreed front of the gear (usually used for text or suitably marked) is used as the datum face. Parameters which relate to the datum face are denoted by the subscript I while parameters which relate to the opposite face are denoted by the subscript II.

4.2.2 Reference rack

The reference rack is the rack that can be produced using the same gear-cutting tool, gear-cutting method and pitch point (pitch axis) as the actual cylindrical gear. It is characterized by its profile, the direction of its teeth in relation to the pitch axis of the generating gear unit, tip plane, root plane and facewidth, see Figure 3.

**Key**

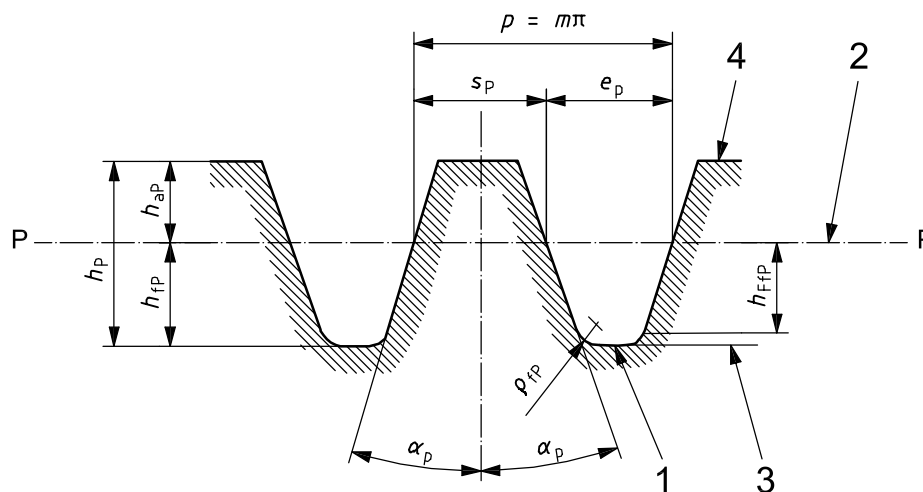
- 1 transverse section
- 2 facewidth
- 3 normal section
- 4 standard basic rack tooth profile

Figure 3 — Concepts and parameters relating to reference rack

4.2.3 Basic rack tooth profile for involute gear teeth

The basic rack tooth profile is defined in a normal section. The flanks of the basic rack tooth profile of involute teeth are straight lines. Tooth thickness, s_p , and space width, e_p , on the datum line of the basic rack (P-P) in the reference plane are equal, see Figure 4.

The standard basic rack tooth profile for involute teeth is standardized in ISO 53.

**Key**

- 1 basic rack profile
- 2 datum line
- 3 root line
- 4 tip line

Figure 4 — Terms and parameters relating to basic rack tooth profile in normal section

4.2.4 Reference cylinder, reference circle, reference diameter

The reference cylinder is the reference surface for the cylindrical gear teeth. Its axis coincides with the axis of the gear (gear axis). The reference circle is the intersection of the reference cylinder with a transverse plane section. The reference diameter, d , is determined by

$$d = |z| m_t = \frac{|z| m_n}{\cos \beta} \quad (1)$$

4.2.5 Gear axis

The axis of a gear (gear axis) is the axis that is defined by the geometrical axis of the support surfaces.

4.2.6 Sections through a cylindrical gear

4.2.6.1 Transverse section, transverse profile

The sectioning of cylindrical gear teeth by a plane perpendicular to the gear axis yields a transverse section. For helical gears, quantities in the transverse section are denoted by the subscript t . The intersection of a tooth with a transverse plane is termed the transverse profile.

4.2.6.2 Normal section, normal profile

The sectioning of involute helical gear teeth by a surface perpendicular to the flank lines of the involute helicoid yields a normal surface. The normal surface is curved three-dimensionally.

Quantities on the normal surface are denoted by the subscript n . The intersection of a tooth with a normal surface is termed the normal profile.

4.2.6.3 Axial section, axial profile

The sectioning of cylindrical gear teeth by a plane containing the gear axis yields an axial section.

Quantities in the axial section are denoted by the subscript x . The intersection of a tooth with an axial plane is termed the axial profile.

4.2.6.4 Cylindrical section, flank lines

The flank lines are lines of intersection of the right and left flanks with a cylinder that has an axis which coincides with the gear axis. Hence, right and left flank lines are to be distinguished.

The reference flank line (tooth trace) is the line of intersection of the flank with the reference cylinder. The base flank line is the line of intersection of the involute flank — possibly imagined as extended — with the base cylinder. The base flank line is a helix on the base cylinder. The origin of the involute helicoid is a base flank line. The tip flank line is the line of intersection of the involute flank — possibly imagined as extended — with the tip cylinder.

The flank lines are helices in the case of helical gear teeth and straight lines in the case of spur gear teeth.

4.2.7 Module

The module of a basic rack is found as the pitch of the rack divided by the number π (see Figure 4). The normal module, m_n , of the cylindrical gear is found as the module of the standard basic rack tooth profile (module series ISO 54).

For a helical gear, the transverse module, m_t , is found as

$$m_t = \frac{m_n}{\cos \beta} \quad (2)$$

and the axial module, m_x , as

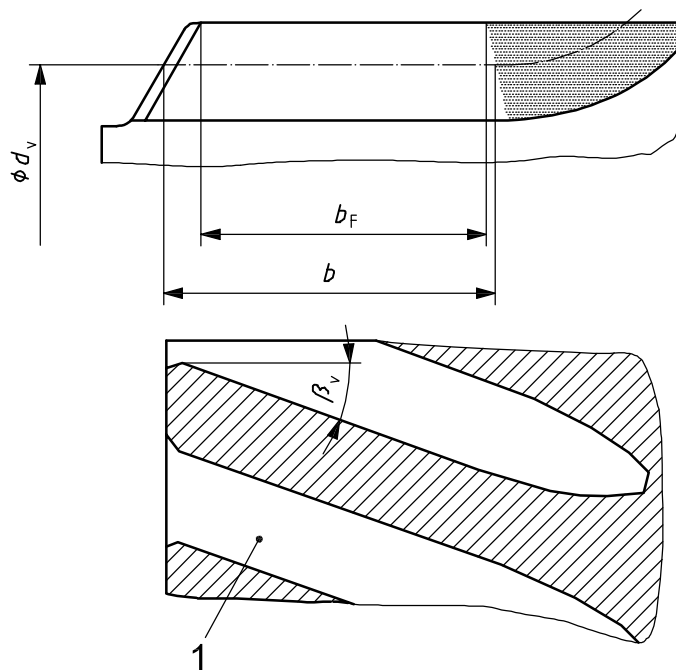
$$m_x = \frac{m_n}{\sin \beta} = \frac{m_n}{\cos \gamma} = \frac{m_t}{\tan \beta} \quad (3)$$

For a spur gear, the module is $m = m_t = m_n$.

4.2.8 Facewidth

The facewidth, b , is the length of the toothed part of the cylindrical gear measured in the axial direction on the V-cylinder. (See 4.5.1.)

The usable facewidth, b_F , is the distance between two transverse sections that contain the fully developed height of the tooth flank. (See Figure 5.)



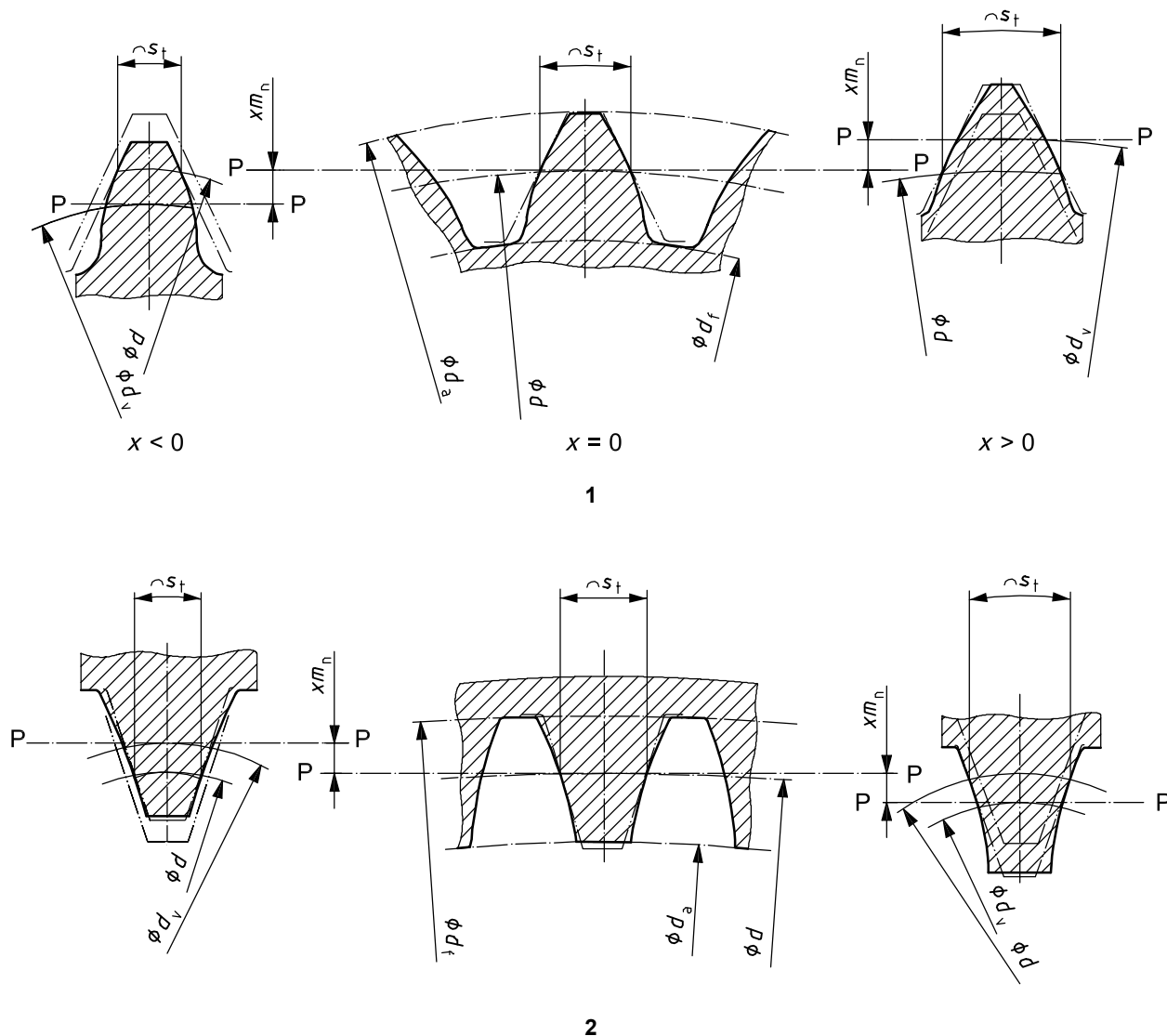
Key

- 1 developed view of V-cylinder

Figure 5 — Facewidth b , usable facewidth b_F

4.2.9 Profile shift, profile shift coefficient and sign of profile shift

The profile shift, xm_n , for involute gear teeth is the displacement of the basic rack datum line from the reference cylinder. The magnitude of the profile shift can be made non-dimensional by dividing by the normal module, and it is then expressed by the profile shift coefficient, x . Positive profile shift increases the tooth thickness on the reference cylinder. (See Figure 6.)



Key

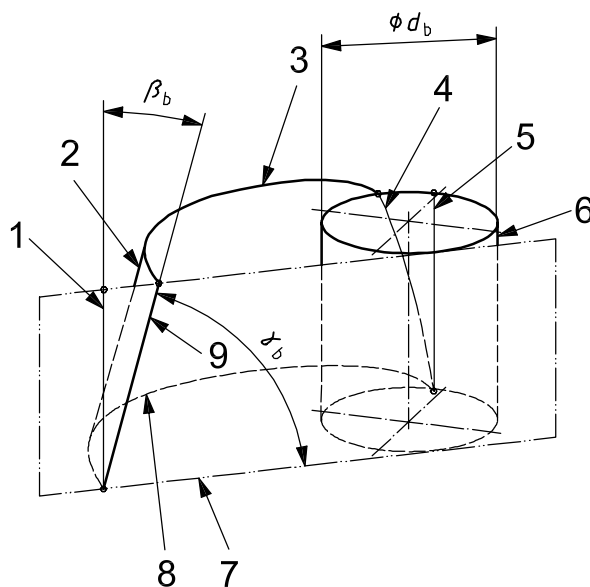
- $\sim$ measurement along an arc
- P-P datum line of basic rack
- 1 external gear
- 2 internal gear

Figure 6 — Profile shift for external and internal gear teeth

4.3 Involute helicoids

4.3.1 Generator of involute helicoids

In developing the base cylinder surface as a plane, a flank line on the base cylinder describes an involute helicoid. The straight line inclined to the axial line in the developed surface (base cylinder tangential plane) is the generator of the involute helicoid, see Figure 7.



Key

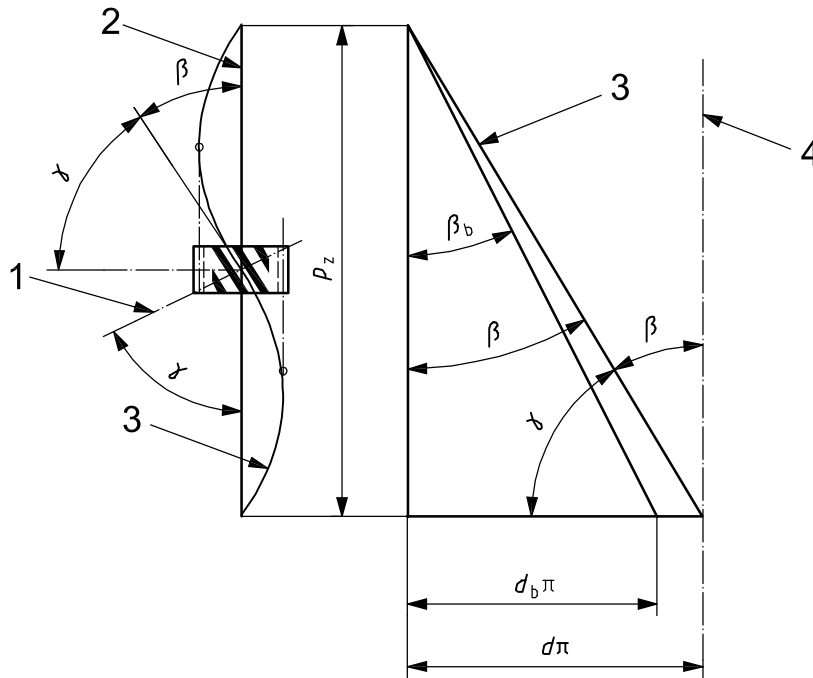
- 1 developed axial line
- 2 involute helicoid
- 3 involute
- 4 base helix
- 5 base cylinder axial line
- 6 base cylinder
- 7 developed base cylinder envelope
- 8 involute
- 9 straight line generator

Figure 7 — Base cylinder with generator and involute helicoids

4.3.2 Lead

The lead, p_z , is the distance between successive intersections of an axial line with the involute helicoid, see Figure 8. The lead is independent of the diameter of the cylinder.

$$p_z = \frac{|z| m_n \pi}{\sin \beta} = \frac{|z| m_t \pi}{\tan \beta} = |z| p_x \quad (4)$$



$$\beta + \gamma = 90^\circ$$

Key

- 1 normal plane
- 2 reference cylinder envelope line, gear axis
- 3 reference trace
- 4 projection of the gear axis

Figure 8 — Lead triangle, lead, helix angle, lead angle

4.3.3 Helix angle, lead angle

The helix angle, β , is the angle between a tangent to a reference helix and the reference cylinder envelope line through the tangent contact point. In special cases, the helix angle, β_R , of right flanks may differ from the helix angle, β_L , of left flanks; however, all equations are based on equal helix angles.

The relationship between β and the base helix angle, β_b (helix angle on the base cylinder), is found from Equations (5) to (7):

$$\tan \beta_b = \tan \beta \cos \alpha_t \quad (5)$$

$$\sin \beta_b = \sin \beta \cos \alpha_n \quad (6)$$

$$\cos \beta_b = \cos \beta \frac{\cos \alpha_n}{\cos \alpha_t} = \frac{\sin \alpha_n}{\sin \alpha_t} = \frac{\sin \alpha_{yn}}{\sin \alpha_{yt}} = \cos \alpha_n \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \quad (7)$$

On a cylinder with arbitrary diameter, d_y , the helix angle, β_y , is found from Equations (8) to (10):

$$\tan \beta_y = \tan \beta \frac{d_y}{d} = \tan \beta \frac{\cos \alpha_t}{\cos \alpha_{yt}} = \tan \beta_b \frac{d_y}{d_b} = \frac{\tan \beta_b}{\cos \alpha_{yt}} \quad (8)$$

$$\sin \beta_y = \sin \beta \frac{\cos \alpha_n}{\cos \alpha_{yn}} = \frac{\sin \beta_b}{\cos \alpha_{yn}} \quad (9)$$

$$\cos \beta_y = \frac{\tan \alpha_{yn}}{\tan \alpha_{yt}} = \frac{\cos \alpha_{yt} \cos \beta_b}{\cos \alpha_{yn}} \quad (10)$$

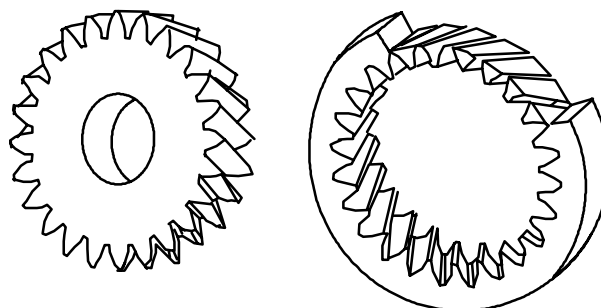
The lead angle, γ , is the angle at which the normal plane crosses the gear axis, see Figure 8. It is also the angle between a tangent to a reference helix (reference flank line) and the transverse section through the tangent contact point:

$$\gamma_y = 90^\circ - \beta_y \quad (11)$$

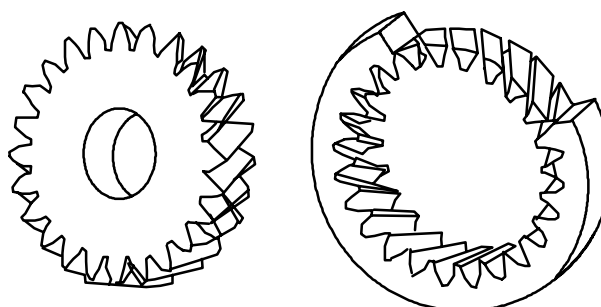
For spur gears, $\beta = 0^\circ$ and $\gamma = 90^\circ$.

4.3.4 Flank direction

The flank direction is right-handed if the flank line describes a right-hand helix and left-handed if the flank line describes a left-hand helix. (See Figure 9.)



a) Right-hand teeth



b) Left-hand teeth

Figure 9 — Direction of helix

4.3.5 Transverse pressure angle at a point, transverse pressure angle

In a transverse section, the tangent to the involute at the arbitrary point Y is inclined to the radius to that point by the transverse pressure angle, α_{yt} :

$$\cos \alpha_{yt} = \frac{d_b}{d_y} = \frac{d}{d_y} \cos \alpha_t \quad (12)$$

(See Figure 10.)

The transverse pressure angle, α_t , is the acute angle between the tangent to the involutes at their point of intersection with the reference circle and the radius through this point of intersection. It is expressed by

$$\cos \alpha_t = \frac{d_b}{d} \quad (13)$$

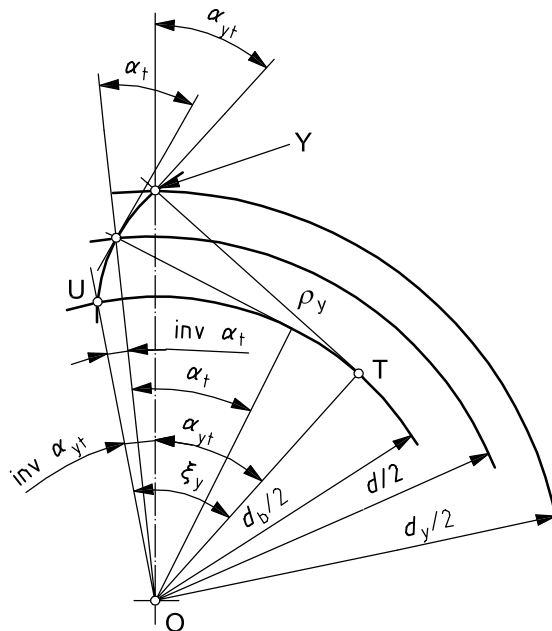


Figure 10 — Parameters relating to involute

4.3.6 Normal pressure angle at a point, normal pressure angle

In the normal section of the involute helicoid, the tangent to this section at an arbitrary point Y is inclined to the radius through Y by the normal pressure angle at that point, α_{yn} . The corresponding angle of inclination at the reference cylinder is the normal pressure angle, α_n ; this is equal to the pressure angle, α_P , of the standard basic rack tooth profile.

$$\tan \alpha_n = \tan \alpha_t \cos \beta \quad (14)$$

$$\tan \alpha_{yn} = \tan \alpha_{yt} \cos \beta_y \quad (15)$$

For a spur gear, $\alpha_n = \alpha_t$ and $\alpha_{yn} = \alpha_{yt}$.

4.3.7 Roll angle of the involute

The angle at the centre over the base circle arc from the origin, U, of the involute to the contact point, T, of the tangent from point Y to the base circle is the roll angle, ξ_y , of the involute, see Figure 10. The base circle arc, UT, is equal to the tangent portion, YT, hence

$$\xi_y = \tan \alpha_{yt} \quad (16)$$

4.3.8 Radius of curvature of the involute, length of roll

The tangent portion, YT, is the radius of curvature, ρ_y , of the involute at point Y and at the same time the length of roll, L_y , belonging to point Y, i.e. the developed base circle arc from the involute origin, U. In the triangle OTY it is the side opposite the transverse pressure angle, α_{yt} , at the centre of the circle O:

$$L_y = \rho_y = \frac{z}{|z|} \frac{d_b}{2} \xi_y = \frac{z}{|z|} \frac{d_b}{2} \tan \alpha_{yt} = \frac{z}{|z|} \frac{\sqrt{d_y^2 - d_b^2}}{2} \quad (17)$$

(See Figure 10.)

4.3.9 Involute function

The angular difference, $\xi_y - \alpha_{yt}$, is termed the involute function of angle α_{yt} and is denoted by $\text{inv } \alpha_{yt}$ (to be read as “involute α_{yt} ”):

$$\text{inv } \alpha_{yt} = \xi_y - \alpha_{yt} = \tan \alpha_{yt} - \alpha_{yt} \quad (18)$$

(See Figure 10.)

4.3.10 Base cylinder, base circle, base diameter

The base cylinder is that cylinder coaxial with the gear axis that is determinative for the generation of the involute helicoids, see Figure 10. Quantities associated with the base cylinder are denoted by the subscript b.

The base circle is the intersection of the base cylinder with a plane of transverse section. The involutes from the base circle form the transverse profiles of the gearing. The base diameter, d_b , is given by

$$d_b = d \cos \alpha_t = |z| m_t \cos \alpha_t = \frac{|z| m_n \cos \alpha_t}{\cos \beta} = \frac{|z| m_n}{\sqrt{\tan^2 \alpha_n + \cos^2 \beta}} \quad (19)$$

$$d_b = |z| m_n \frac{\cos \alpha_n}{\cos \beta_b} \quad (20)$$

4.4 Angular pitch and pitches

4.4.1 Angular pitch

The angular pitch, τ , is that angle laying in transverse sections that result from the dividing of the complete periphery of a circle into z equal parts.

$$\tau = \frac{2\pi}{|z|} = \frac{2p_{yt}}{d_y} \text{ in radians} \quad (21)$$

$$\tau = \frac{360}{|z|} \text{ in degrees} \quad (22)$$

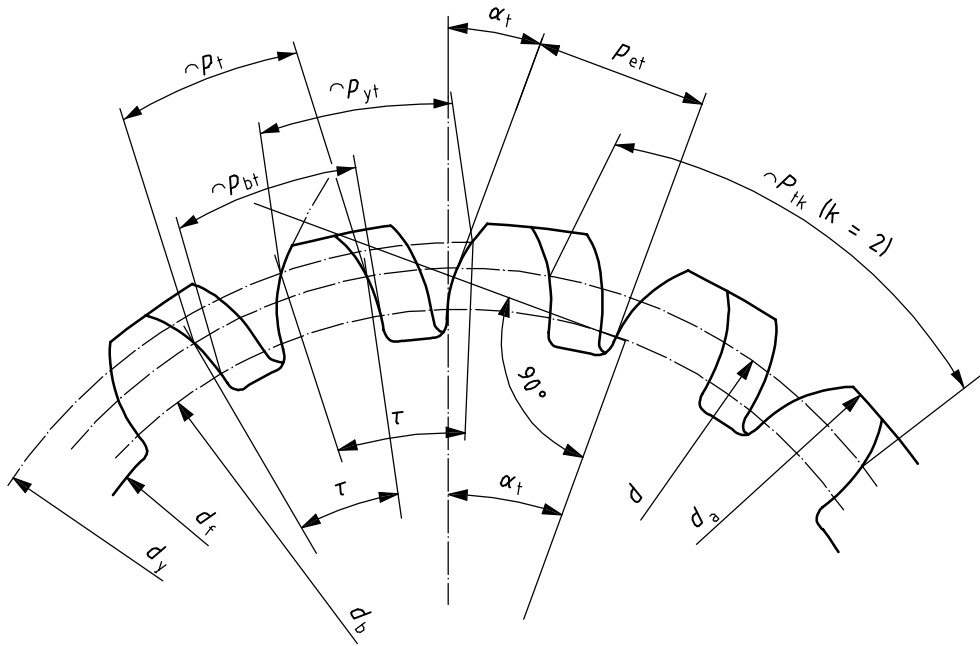
4.4.2 Pitches on the reference cylinder

4.4.2.1 Transverse pitch

The (reference cylinder) transverse pitch, p_t , is the length of the reference circle arc between two successive equal-handed tooth flanks (right or left flanks):

$$p_t = \frac{\pi m_n}{\cos \beta} = \frac{d}{2} \tau = \frac{\pi d}{|z|} = \pi m_t \quad (23)$$

(See Figure 11.)



Key

⌒ measurement along an arc

Figure 11 — Diameter, angular pitch, transverse pitches on helical cylindrical gear

4.4.2.2 Normal pitch

The (reference cylinder) normal pitch, p_n , is the length of the helix arc between two successive equal-handed tooth flanks (right or left flanks) on the reference cylinder in the normal section of the gear:

$$p_n = \pi m_n = p_t \cos \beta \quad (24)$$

(See Figure 12.)

4.4.3 Pitches on any cylinder

It is necessary to distinguish between the transverse pitch, p_{yt} , and the normal pitch, p_{yn} , on a cylinder of any diameter, d_y , (Y-cylinder):

$$p_{yt} = \frac{d_y}{2} \tau = \frac{\pi d_y}{|z|} = \frac{d_y}{d} p_t \quad (25)$$

$$p_{yn} = p_{yt} \cos \beta_y \quad (26)$$

4.4.4 Axial pitch

The axial pitch, p_x , of a helical gear is the portion of a generation line of a cylinder concentric with the gear axis between two successive equal-handed tooth flanks (right or left flanks), see Figure 12. The axial pitch is independent of the diameter of the cylinder. Axial pitch does not apply to spur gears. It is expressed by

$$p_x = \frac{\pi m_n}{\sin \beta} = \pi m_x = \frac{p_z}{|z|} = \frac{\pi m_t}{\tan \beta} = \frac{p_{yt}}{\tan \beta_y} = \frac{p_{yn}}{\sin \beta_y} \quad (27)$$

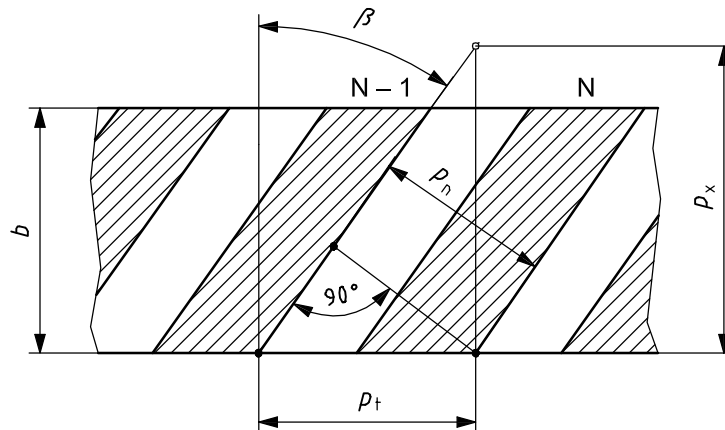


Figure 12 — Geometrical relations between transverse, normal and axial pitch in a developed view of a reference cylinder

4.4.5 Base pitch

The distance between successive equal-handed tooth flanks (right or left flanks) on the developed base cylinder tangential plane is the base pitch.

— Transverse base pitch:

$$p_{bt} = \frac{d_b}{2} \tau = p_t \cos \alpha_t = p_{yt} \cos \alpha_{yt} = \frac{\pi d_b}{|z|} = \frac{d_b}{d} p_t \quad (28)$$

— Normal base pitch:

$$p_{bn} = p_n \cos \alpha_n = p_{bt} \cos \beta_b \quad (29)$$

4.4.5.1 Transverse base pitch on the path of contact

The transverse base pitch on path of contact, p_{et} , is the distance between two parallel tangents in a transverse section which contact two successive equal-handed tooth flanks:

$$p_{et} = p_{bt} \quad (30)$$

(See Figure 11.)

4.4.5.2 Normal base pitch on the plane of contact

The normal base pitch, p_{en} , is the distance between two parallel tangential planes which contact two successive equal-handed tooth flanks:

$$p_{en} = p_{bn} \quad (31)$$