
**Sampling procedures for inspection
by variables —**

**Part 3:
Double sampling schemes indexed
by acceptance quality limit (AQL)
for lot-by-lot inspection**

Règles d'échantillonnage pour les contrôles par mesures —

*Partie 3: Plans d'échantillonnage doubles pour le contrôle lot par lot,
indexés d'après le niveau de qualité acceptable (NQA)*



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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-3 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

- *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*
- *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*
- *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*
- *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

The following part is under preparation:

- *Part 4: Procedures for assessment of declared quality levels*

Introduction

Inspection by variables for percentage nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation which may appear quite complicated to the user:

- a) procedures for unknown process standard deviation (the “ s ” method), or procedures for where the process standard deviation is originally unknown then estimated with fair precision, or known since the start of inspection (the “ σ ” method);
- b) a single specification limit, or double specification limits with separate, combined or complex control;
- c) normal inspection, tightened inspection or reduced inspection;
- d) Form k plans and Form p^* plans;
- e) a single quality characteristic (the univariate case) or a number of unrelated quality characteristics (the multivariate independent case).

The text has been arranged so that the simpler procedures may be implemented without necessarily understanding the more complicated procedures. The main text of this part of ISO 3951 is confined to the univariate case. The multivariate independent cases are treated separately in Annex A for the “ s ” method, in Annex B for the “ σ ” method and in Annex C for combined “ s ” method and “ σ ” method procedures. Annex D facilitates the use of the main text of the standard by directing the user to the clauses and tables concerning any univariate situation with which he might be confronted; it only deals with Clauses 16, 17, 21, 22 and 23 and, in every case, it is necessary to have read Clauses 1 to 15 and Clauses 18 to 20 first.

This part of ISO 3951 is complementary to the double sampling plans and procedures of ISO 2859-1. When specified by the responsible authority, it would be valid to reference both ISO 3951-3 and ISO 2859-1 in a product specification, a contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The “responsible authority” can then be designated in one of these documents.

In all parts of ISO 3951:

- the acronym AQL stands for “acceptance quality limit” rather than “acceptable quality level”, in order to more accurately reflect its function;
- procedures are given for the case where the process standard is unknown (the “ s ” method) and for the case where it may be presumed to be known (the “ σ ” method);
- the sampling plans have been chosen so that their operating characteristic curves closely match those of the corresponding single sampling plans in ISO 2859-1;
- minimal statistical theory has been given (it being planned ultimately to provide this in a guidance document to sampling procedures for inspection by variables);
- text, charts and tables that are only informative have been consigned to annexes wherever practicable.

In none of the parts have methods been given based on the sample range, now that the availability of computers and calculators with a standard deviation function key is so widespread. Data for acceptance sampling by variables is often substantially more expensive to acquire than data for sampling by attributes, and the “ s ” method makes more efficient use of these data.

The coverage of ISO 3951-1 is constrained to the case of a single, normally distributed, quality characteristic with a single class of nonconformity, and includes the case of combined control of double specification limits.

ISO 3951-2 provides a more comprehensive treatment of single sampling plans by variables, including procedures for separate and complex control of double specification limits. Procedures are also given for multiple independent quality characteristics and/or multiple AQLs.

ISO 3951-3 provides plans for double sampling by variables, which on average provide substantial savings of inspection effort in comparison with plans for single sampling by variables. The savings are achieved by first selecting from the lot and inspecting a random sample that is typically nearly 40 % smaller than that of the corresponding single sampling plan. If these inspection results satisfy an acceptance criterion, an immediate decision is made to accept the lot without further inspection. Alternatively, if the inspection results satisfy a non-acceptance criterion, an immediate decision not to accept the lot is made without further inspection. Thus, when quality is very good or very poor, the saving in inspection effort can amount to nearly 40 %. Only when the inspection results from the first sample are equivocal is a second random sample, of the same size as the first, selected; the acceptability of the lot is then resolved by combining the results of the first and second samples and determining whether they satisfy a second acceptance criterion.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

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Sampling procedures for inspection by variables —

Part 3:

Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection

1 Scope

This part of ISO 3951 specifies an acceptance sampling system of double sampling schemes for inspection by variables for percent nonconforming. It is indexed in terms of the acceptance quality limit (AQL).

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of non-accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide

- automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected, and
- an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly or explicitly determined from an estimate of the percentage of nonconforming items in the process, based on either one or two random samples of items from the lot.

This part of ISO 3951 is primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process; if there are different producers or production processes, apply this part of ISO 3951 to each one separately;
- b) where the items of product have a single quality characteristic (for multiple quality characteristics, see informative Annexes A, B and C);
- c) where the quality characteristic is measurable on a continuous scale;
- d) where the measurement error is negligible (i.e. with a standard deviation of no more than 10 % of the corresponding process standard deviation);
- e) where production is stable (under statistical control) and the quality characteristic is distributed, at least to a close approximation, according to a normal distribution;

CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

- f) where the possibility of having to select and inspect a second sample is administratively acceptable;

g) where a contract or standard defines an upper specification limit U , a lower specification limit L or both on the quality characteristic. An item is deemed to conform if its measured quality characteristic x satisfies the appropriate one of the following inequalities:

- 1) $x \geq L$ (i.e. the lower specification limit is not violated);
- 2) $x \leq U$ (i.e. the upper specification limit is not violated);
- 3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

NOTE Inequalities 1) and 2) are called cases with a “single specification limit”, and 3) is the case with “double specification limits”. For double specification limits, a further distinction is made between combined control, separate control and complex control, as follows:

- combined control is where a single AQL applies to nonconformity beyond both limits;
- separate control is where separate AQLs apply to nonconformity beyond each of the limits;
- complex control is where one AQL applies to nonconformity beyond the limit that is of greater seriousness, and a larger AQL applies to the total nonconformity beyond both limits.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this part of ISO 3951, the definitions given in ISO 3534-1, ISO 3534-2 and the following apply. References are given in square brackets for definitions that have been re-expressed in the vocabulary of this part of ISO 3951 for the user's convenience.

3.1

inspection by variables

inspection by measuring the magnitude(s) of the characteristic(s) of an item

[ISO 3534-2:2006, definition 4.1.4]

3.2

sampling inspection

inspection of selected items in the group under consideration

[ISO 3534-2:2006, definition 4.1.6]

3.3

acceptance sampling inspection

acceptance inspection where the acceptability is determined by means of sampling inspection

[ISO 3534-2:2006, definition 4.1.8]

3.4**double sampling inspection, double sampling**

acceptance sampling inspection based initially on a first sample, of size n_1 , which leads to a decision to accept, non-accept, or to inspect a second sample, of size n_2 , before taking the decision whether or not to accept

NOTE 1 The decisions are made according to defined rules.

NOTE 2 In this part of ISO 3951, both sample sizes are equal and denoted by n , i.e. $n_1 = n_2 = n$.

3.5**acceptance sampling inspection by variables**

acceptance sampling inspection in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[ISO 3534-2:2006, definition 4.2.11]

NOTE An acceptable process is a process that generates nonconforming items at a rate no worse than the AQL.

3.6**process average**

rate at which nonconforming items are generated by a process

3.7**acceptance quality limit****AQL**

worst tolerable product quality level

NOTE See Clause 5.

3.8**quality level**

quality expressed as a rate of nonconforming units

3.9**limiting quality****LQ**

quality level, when a lot is considered in isolation, which, for the purposes of acceptance sampling inspection, is limited to a low probability of acceptance (in this part of ISO 3951: 10 %)

NOTE See Clause 8.

3.10**nonconformity**

non-fulfilment of a requirement

[ISO 9000:2005, definition 3.6.2]

NOTE 1 Nonconformity will generally be classified by its degree of seriousness, such as:

Class A. Nonconformity of a type considered to be of the highest concern for the product or service. Such types of nonconformity will typically be assigned very small AQL values.

Class B. Nonconformity of a type considered to have the next lower degree of concern; this is typically assigned a larger AQL value than that in class A and smaller than that in class C if a third class exists, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

NOTE 2 The main text of this part of ISO 3951 deals with the univariate case, for which there will be either one or two classes of nonconformity.

3.11

nonconforming unit

unit with one or more nonconformities

[ISO 3534-2:2006, definition 1.2.15]

3.12

“s” method acceptance sampling plan

acceptance sampling plan by variables using the sample mean(s) and sample standard deviation(s)

NOTE See Clause 16.

3.13

“σ” method acceptance sampling plan

acceptance sampling plan by variables using the sample mean(s) and the presumed value(s) of the process standard deviation(s)

NOTE See Clause 17.

3.14

specification limit

limiting value stated for a characteristic

[ISO 3534-2:2006, definition 3.1.3]

3.15

lower specification limit

L

specification limit that defines the lower limiting value

[ISO 3534-2:2006, definition 3.1.5]

3.16

upper specification limit

U

specification limit that defines the upper limiting value

[ISO 3534-2:2006, definition 3.1.4]

3.17

combined control

requirement when nonconformity beyond both the upper and the lower specification limits of a quality characteristic belongs to the same class to which a single AQL is applied

NOTE 1 See 5.3, 16.4 and 17.4.

NOTE 2 The use of a combined AQL requirement implies that nonconformities beyond either specification limit are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18

separate control

requirement when nonconformity beyond the upper and the lower specification limits of a quality characteristic belongs to different classes, to which separate AQLs are applied

NOTE See 5.3, 16.3 and 17.3.

3.19**complex control**

requirement when nonconformity beyond both the upper and the lower specification limits of a quality characteristic belongs to one class and nonconformity beyond either the upper or the lower specification limit belongs to a different class, with separate AQLs being applied to the two classes

NOTE See 5.3, 16.5 and 17.5.

3.20**acceptability constants**

k, p^*

constants depending on the specified value of the acceptance quality limit, sample size and inspection severity, used in the criteria for accepting the lot in an acceptance sampling plan by variables

NOTE 1 See Clauses 16 and 17.

NOTE 2 For double sampling, there will be three such pairs of acceptability constants, one for acceptance at the first sample, one for non-acceptance at the first sample and one for acceptance with the combined first and second samples.

3.21**quality statistic**

Q

(acceptance sampling) function of the specification limit, the sample mean, and the sample or process standard deviation, used in assessing the acceptability of a lot

[ISO 3534-2:2006, definition 4.4.9]

NOTE See also 3.22 and 3.23.

3.22**lower quality statistic**

Q_L

function of the lower specification limit, the sample mean, and the sample or process standard deviation

NOTE See Clause 4 for further details.

3.23**upper quality statistic**

Q_U

function of the upper specification limit, the sample mean, and the sample or process standard deviation

NOTE See Clause 4 for further details.

3.24**maximum sample standard deviation****MSSD**

$s_{\max}$

largest sample standard deviation for a given sample size code letter and acceptance quality limit for which it is possible to satisfy an acceptance criterion for double specification limits when the process variability is unknown

NOTE 1 The MSSD depends on whether the double specification limits are under combined, separate or complex control and on the inspection severity (i.e. normal, tightened or reduced).

NOTE 2 See 16.4.2 and Table 16, 17 or 18.

NOTE 3 For double sampling plans, there are two MSSDs under each combination of inspection severity and type of control, one for the first sample and one for the combined first and second samples.

3.25

maximum process standard deviation

MPSD

$\sigma_{\max}$

largest process standard deviation for a given sample size code letter and acceptance quality limit for which it is possible to satisfy an acceptance criterion for double specification limits under all inspection severities (i.e. normal, tightened or reduced) when the process variability is known

NOTE 1 An MPSD depends on whether the double specification limits are under combined, separate or complex control, but does not depend on the inspection severity or on whether the sample is the first or second.

NOTE 2 See 17.3, 17.4 and 17.5 and Tables 19, 20 and 21.

3.26

switching rule

instruction within an acceptance sampling scheme for changing from one acceptance sampling plan to another of greater or lesser severity based on demonstrated quality history

NOTE 1 Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of severity of sampling.

[ISO 3534-2:2006, definition 4.3.4]

NOTE 2 See Clause 21.

3.27

measurement

set of operations having the object of determining the value of a quantity

[ISO 3534-2:2006, definition 3.2.1]

3.28

average sample size

ASSI

average number of units in a sample inspected per lot in reaching decisions to accept or not to accept when using a given acceptance sampling scheme

NOTE ASSI is dependent on the actual quality level of the submitted lots.

[ISO 3534-2:2006, definition 4.7.3]

3.29

responsible authority

concept used to maintain the neutrality of this part of ISO 3951, irrespective of whether it is being invoked or applied by the first, second or third party

NOTE 1 The responsible authority may be:

- a) the quality department within a supplier's organization (first party);
- b) the purchaser or procurement organization (second party);
- c) an independent verification or certification authority (third party);
- d) any of a), b) or c), differing according to function (see Note 2) as described in a written agreement between two of the parties, for example a document between supplier and purchaser.

NOTE 2 The duties and functions of a responsible authority are outlined in this part of ISO 3951 (see 5.3, 6, 10, 11, 16.4.3.2.1, 17.1, 19.1, 20.2, 21.4, 23.1, 23.2 and 23.3).

4 Symbols and abbreviations

4.1 Symbols

For the purposes of this part of ISO 3951, the following symbols apply.

A_c	acceptance number
c_u	factor given in Table 29 for determining the upper control limit for sample standard deviations (see 23.2)
f_s	factor given in Tables 16, 17 and 18 for combined control of double specification limits, relating the maximum sample standard deviation (MSSD) to the difference between U and L , for normal, tightened and reduced inspection respectively (see 16.4.2 and 16.4.3.1)
	NOTE 1 $f_{s,1}$ and $f_{s,c}$ represent respectively the factors applicable to the standard deviation of the first sample and to the combined standard deviation of the first and second samples.
f_σ	factor given in Tables 19, 20 and 21 that relates the maximum process standard deviation (MPSD) to the difference between U and L , for combined, separate and complex control respectively (see 17.4 and 17.5)
k	Form k acceptability constant
	NOTE 2 k_a , k_r and k_c represent respectively the Form k acceptability and non-acceptability constants at the first sample and the acceptability constant for the combined first and second samples.
L	lower specification limit (as a suffix to a variable, denotes its value at L).
μ	process mean
N	lot size (number of items in a lot)
n	sample size (number of items in each sample)
p	process fraction nonconforming
$\hat{p}$	estimate of the process fraction nonconforming
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit
p^*	Form p^* acceptability constant, the largest acceptable value of the estimate of the process fraction nonconforming
	NOTE 3 p_a^* , p_r^* and p_c^* represent respectively the Form p^* acceptability and non-acceptability constants at the first sample and the acceptability constant for the combined first and second samples.
P_a	probability of acceptance
Q	quality statistic
Q_L	lower quality statistic
	NOTE 4 Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and $(\bar{x} - L)/\sigma$ when it is presumed to be known.
	NOTE 5 $Q_{L,1}$ is defined as $(\bar{x}_1 - L)/s_1$ or $(\bar{x}_1 - L)/\sigma$; $Q_{L,c}$ is defined as $(\bar{x}_c - L)/s_c$ or $(\bar{x}_c - L)/\sigma$.
Q_U	upper quality statistic
	NOTE 6 Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and $(U - \bar{x})/\sigma$ when it is presumed to be known.
	NOTE 7 $Q_{U,1}$ is defined as $(U - \bar{x}_1)/s_1$ or $(U - \bar{x}_1)/\sigma$; $Q_{U,c}$ is defined as $(U - \bar{x}_c)/s_c$ or $(U - \bar{x}_c)/\sigma$.

s sample standard deviation of the measured values of the quality characteristic, also an estimate of the process standard deviation, i.e.

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}} = \sqrt{\frac{n \sum_{j=1}^n x_j^2 - \left(\sum_{j=1}^n x_j \right)^2}{n(n-1)}}$$

$s_{\max}$ maximum allowable sample standard deviation (see 3.24)

NOTE 8 $s_{1,\max}$ and $s_{c,\max}$ represent, respectively, the maximum standard deviation for the first sample and the maximum standard deviation for the combined first and second samples (see also f_s).

σ standard deviation of a process whose inherent variability is under statistical control

NOTE 9 σ^2 , the square of the standard deviation, is known as the process variance.

$\sigma_{\max}$ maximum allowable process standard deviation (see 3.25 and also f_σ)

U upper specification limit (as a suffix to a variable, denotes its value at U)

x_j measured value of the quality characteristic for the j th item of a sample

$\bar{x}$ arithmetic mean of the measured values of the quality characteristic in a sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

NOTE 10 $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_c$ represent, respectively, the mean of the first sample, the mean of the second sample and the mean of the first and second samples combined. As the first and second sample sizes are equal, the combined sample mean takes the simple form $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$.

$\bar{x}_L$ lower acceptance value for $\bar{x}$

$\bar{x}_U$ upper acceptance value for $\bar{x}$

4.2 Abbreviations

AQL	acceptance quality limit
ASSI	average sample size
LQ	limiting quality
MSSD	maximum sample standard deviation
MPSD	maximum process standard deviation
MVUE	minimum variance unbiased estimator
OC	operating characteristic

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process average when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the acceptance quality limit might be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to have process averages consistently better than the AQL, thereby protecting the consumer from the situation where the long run process average is worse than the AQL. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, in the contract or by the responsible authority. In the case

- a) where an upper specification limit for the quality characteristic is given, or
- b) where a lower specification limit is given,

then a single AQL applies to the indicated limit.

Where both upper and lower specification limits are given for the quality characteristic, three further cases can be identified:

- c) combined control of double specification limits, where a single AQL applies to the total percentage nonconforming beyond both limits;
- d) separate control, where separate AQLs apply to the percent nonconforming beyond each limit;
- e) complex control, where one AQL applies to the percent nonconforming beyond the limit that is of greater seriousness, while a larger AQL applies to the total percent nonconforming beyond both limits.

Acceptance tests shall be carried out according to the provisions of this part of ISO 3951 for each AQL. The lot shall only be accepted if the AQL requirement is satisfied in cases a), b) or c), or if both AQL requirements are satisfied in cases d) or e).

5.4 Preferred AQLs

The sixteen AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this part of ISO 3951 is not applicable. (See 14.2.)

5.5 Caution

From the above description of the AQL concept, it follows that the desired protection can only be assured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL does not imply that the supplier has the right to knowingly supply any nonconforming items of product.

6 Switching rules for normal, tightened and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the process average is not satisfactory. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level significantly better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see 20.1) that the variability is in statistical control, consideration should be given to switching to the “ σ ” method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see 23.1).

When it has been necessary to discontinue sampling inspection, inspection should not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses 21, 22 and 23.

7 Relation to ISO 2859-1

7.1 Similarities to ISO 2859-1

- a) The double sampling plans by variables in this part of ISO 3951 are complementary to the double sampling plans by attributes provided in ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.
- b) Both standards use the AQL to index the sampling plans and the preferred values used in this document are identical to those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).
- c) In this part of ISO 3951, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. Then, general tables give the sample sizes to be taken and the acceptability criteria, indexed by the sample size code letter and the AQL. Separate tables are given for the “ s ” and “ σ ” methods, and for normal, tightened and reduced inspection.
- d) In this part of ISO 3951, the first and second sample sizes of the double sampling plans are the same.
- e) The switching rules are essentially equivalent.
- f) The operating characteristic (OC) curves of the variables plans in this part of ISO 3951 are closely matched to those of the corresponding single sampling attributes plans in ISO 2859-1 (see Annex M).
- g) The classification of nonconformities by degree of seriousness into class A, class B, etc., remains unchanged.

7.2 Differences from ISO 2859-1

- a) **Determination of acceptability.** Acceptability for an ISO 2859-1 attributes double sampling plan for percent nonconforming is determined by the numbers of nonconforming items found in the samples. Acceptability for a plan for inspection by variables is based on the distances of the estimated process means from the specification limit(s) in terms of the estimated or known process standard deviations or, by implication, the estimated process fractions nonconforming. In this part of ISO 3951, two methods are considered: the “*s*” method for use when the process standard deviation σ is unknown and the “ σ ” method for use when σ is presumed to be known. In the case of a single specification limit, or for separate control of double specification limits, acceptability is determined most easily by comparing quality statistics with “Form *k*” acceptability constants (see 16.2, 16.3, 17.2 and 17.3). For combined or complex control of double specification limits, acceptability is determined by comparing estimates of the process percent nonconforming for each class of nonconformity with “Form *p*” constants (see 16.4, 16.5, 17.4 and 17.5). (Annexes A, B and C provide procedures for two or more unrelated quality characteristics.)
- b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements should be distributed according to a normal distribution (or at least a close approximation to a normal distribution), either originally or after a known transformation.
- c) **Sample sizes.** Unlike the sample sizes in ISO 2859-1, the sample sizes along rows of the master tables in this part of ISO 3951 are not constant. This was necessary to obtain closely matched OC curves [see 7.1 f)].
- d) **Producer’s risk.** For process quality precisely at the AQL, the producer’s risk (see 8.3) that a lot will not be accepted is similar, but not identical, to the corresponding producer’s risk in ISO 2859-1 (see Annex I).
- e) **Average sample sizes (ASSIs).** The ASSIs of double sampling schemes by variables are generally much smaller than the ASSIs for corresponding schemes by attributes at any given process quality level (see Annex L).
- f) **Multiple sampling plans.** No multiple sampling plans are given in this part of ISO 3951.
- g) **Average outgoing quality limit (AOQL).** The AOQL concept applies when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

8 Limiting quality protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Sometimes specific individual plans are selected from this part of ISO 3951 and used without the switching rules. For example, a purchaser might be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as “inspection in compliance with ISO 3951-3”. When used in such a way, ISO 3951-3 simply represents a collection of individual double sampling plans indexed by AQL. The operating characteristic curves and other measures of a plan so chosen should be assessed individually from the tables provided.

8.2 Consumer's risk quality tables

If the series of lots is not long enough to allow the switching rules to be applied, it might be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give consumer's risk quality not more than the specified limiting quality protection. Sampling plans for this purpose may be selected by choosing a consumer's risk quality (CRQ) and a consumer's risk to be associated with it, and then referring to the tables on Charts C to R (see Figures 2 to 15).

8.3 Producer's risk tables

The producer's risk is the probability of non-acceptance under either the " s " or " σ " methods for lots produced when the process average equals the AQL. The producer's risks of the double sampling plans of this part of ISO 3951 are given for the " s " method in Annex I. The corresponding producer's risks for the " σ " method are broadly similar.

8.4 Operating characteristic curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the OC curves. However, the degree of consumer protection provided by an individual sampling plan at any process quality may be judged from its OC curve. OC curves for the " s " method sampling plans of this part of ISO 3951 are given in Charts C to R, which should be consulted when choosing a sampling plan. Also given on these charts are respectively tables of process qualities at nine standard probabilities of acceptance for all the " s " method sampling plans in this part of ISO 3951.

Some of the OC curves in Charts C to R apply to tightened or reduced inspection as well as, or rather than, normal inspection. The numerical legend on the charts refers to normal inspection. To locate an OC curve for tightened or reduced inspection, examine the appropriate column of the corresponding table at the bottom of the page; if the relevant plan is not also used for normal inspection, it will be identified by the letter T (for tightened) or R (for reduced) followed by a number, e.g. R1, for cross-referencing to the corresponding OC curve.

EXAMPLE For tightened inspection with sample size code letter G, turn to Chart G. The OC curve for tightened inspection when the AQL is 2,5 % is the same as the one on the uppermost graph corresponding to an AQL of 1,5 % under normal inspection. For reduced inspection, it is necessary to turn back to the chart that is two pages earlier. For example, the OC curve for code letter G for an AQL of 0,65 % under reduced inspection is the one identified as R1 on the uppermost graph of Chart E.

These OC curves and tables apply to a single specification limit under the " s " method. Most of them also provide a good approximation to the " σ " method and to the cases of combined, separate or complex control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the " σ " method, refer to Annex J.

9 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge both of statistics and the product to be inspected. Clauses 10 to 14 of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable, and the choices to be made when selecting an appropriate standard plan.

10 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.

- b) In terms of the knowledge gained, the advantage lies with inspection by variables, as the information obtained indicates more precisely how good the product is. Earlier warning will therefore tend to be given if quality begins to deteriorate.
- c) An attributes scheme can be more readily understood and accepted; for example, it might at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See Example 2 in 16.2.)
- d) Enormous improvements in average sample size (ASSI) can be achieved by replacing double sampling schemes by attributes by double sampling schemes by variables. Annex L illustrates this for normal, tightened and reduced inspection, showing the maximum and minimum values of the quotients of the ASSI for double sampling by variables and the ASSI for the corresponding schemes by attributes. The tables in Annex L show that the advantage of using “s” method double sampling plans tends to become more marked with increases in lot size and decreased in the AQL.

NOTE 1 For normal and tightened inspection, the corresponding schemes by attributes on and below the fourth diagonal of the master tables are double sampling schemes. For reduced inspection, the corresponding schemes by attributes on and below the fifth diagonal of the master tables are double sampling schemes. For normal, tightened and reduced inspection, the attributes schemes corresponding to the first diagonal are single sampling schemes with accept number zero. For all other diagonals on the master tables, the corresponding attributes scheme is a single sampling plan with a fractional accept number.

- e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.
- f) Variables sampling has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.
- g) The use of this part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of each quality characteristic is normal, or normal after a known transformation, and that the quality characteristics are independent. In the case of doubt, the responsible authority should be consulted.

NOTE 2 ISO 5479 gives detailed procedures for tests for departure from normality.

NOTE 3 Departure from normality is also dealt with in ISO 2854, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

11 Choice of method

If it is desired to apply inspection by variables, the next question is whether to use the “s” method or the “ σ ” method. As already indicated in 10 d) above, the “ σ ” method is the most economical in terms of sample size but, before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the “s” method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the “ σ ” method? The sample sizes will generally be smaller and the acceptability criteria simpler to implement under the “ σ ” method. On the other hand, it will still be necessary to calculate the sample standard deviations s for record purposes and to keep the control charts up to date. (See Clause 20.)

12 Choice between single and double sampling plans

Another question to consider is whether it is preferable to use single sampling or double sampling plans.

The advantage of using double sampling plans is that the average amount of sampling is reduced, the reduction depending on the process quality level. The maximum and minimum percentage reductions in the average sample sizes (ASSIs) for double sampling schemes in comparison with the sample size for the

corresponding single sampling plans by variables are shown for normal inspection in Annex K. Table K.1 shows the reductions for the “s” method (used when the process standard deviation is unknown) and Table K.2 for the “σ” method (used when the process standard deviation is presumed to be known).

NOTE Annexes F and G provide the corresponding “s” method and “σ” method single sampling plans by variables used in these comparisons.

However, double sampling plans can also have a number of disadvantages. When items take a long time to test, but can be inspected or tested simultaneously, replacing a single sampling plan by a double sampling plan can double the time needed to produce an accept or non-accept decision. This problem is exacerbated if time has to be booked in advance at the inspection facility.

Even worse is the case where items need to be transported a considerable distance to be tested. This raises a number of questions. Should both samples be transported to the inspection facility at the same time? Should time for one or for both samples to be inspected be booked in advance, i.e. what are the costs of booking time that is subsequently not used? If the second sample is transported but not required, can it be transported back again and returned to the lot from which it was drawn, i.e. can it be assumed to be not adversely affected by its long journey? Will any delay caused by the use of double sampling cause a storage problem for the lots that are awaiting a disposition decision? Are the savings from the use of double sampling more than cancelled out by extra administrative and logistical costs?

The decision whether or not to replace single sampling plans by double sampling plans therefore depends on whether the potential savings from the reduction in the average amount of sampling and inspection outweighs the negative aspects of double sampling.

13 Choice of inspection level and AQL

For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate OC curve or table from Charts C to R shows the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

14 Choice of sampling scheme

14.1 Standard schemes

The standard procedure may be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the “s” method, has been found in practice to produce workable sampling plans, but it rests on the assumption that the order of priority is first the AQL, second the sample size and last, probabilities of acceptance at poorer quality levels such as the indifference and limiting qualities.

NOTE The indifference and limiting qualities are the quality levels which, if offered for inspection, would have a 50 % or 10 % probability of acceptance, respectively. However, the actual risk taken by the consumer varies according to the probability that goods at such low quality levels are offered for inspection.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 21, 22 and 23), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

However, if, in certain circumstances, these lower quality levels have a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable scheme from this part of ISO 3951 may be selected by using Chart A (see Figure 1). Construct a vertical line through the acceptable value for the indifference quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a curve indexed by the sample size code letter of a standard normal inspection double sampling scheme, which meets the specified requirements. This should be verified by inspecting the OC curve from among Charts C to R relating to this code letter and AQL.

EXAMPLE

Suppose that an acceptable value for the indifference quality is 3,0 % nonconforming and that the desired quality with a 95 % probability of acceptance is 1,0 % nonconforming. A vertical line on Chart A (see Figure 1) at 3,0 % nonconforming and a horizontal line at 1,0 % nonconforming intersect just below the sloping line indexed by the letter K. Examining Chart K, it is seen that a plan with sample size code letter K and AQL 1,0 % meets the requirements.

If the lines intersect at a point above the line marked R in Chart A, this indicates that the specification cannot be met by any of the plans in this part of ISO 3951.

14.2 Special schemes

If none of the standard schemes are acceptable, it will be necessary to devise a special scheme. It then has to be decided which combination of AQL, limiting quality, and sample sizes is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

NOTE This choice is not completely unfettered; the fact that the sizes of the samples are necessarily whole numbers, and that for pragmatic reasons they are constrained to be equal, imposes some limitations. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

15 Preliminary operations

Before starting inspection by variables,

- a) check that production is considered to be continuing and that the distribution of the quality characteristic can be considered to be normal or may be transformed to a normal distribution;

NOTE If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

- b) check whether the “s” method is to be used initially or whether the process standard deviation is stable and known, in which case, the “ σ ” method should be used;
- c) check that the inspection level to be used has been designated. If none has been given, use inspection level II;
- d) check, when the quality characteristic has double specification limits, whether the limits are under combined, separate or complex control. For combined control, check that nonconformity beyond each limit is of equal importance; for separate or complex control, check to which class of nonconformity each limit has been assigned;
- e) check that an AQL has been designated for each class of nonconformity, and that it is one of the preferred AQLs used in this part of ISO 3951. If it is not, then the tables are not applicable.

16 Standard univariate “s” method procedures

16.1 Obtaining a plan, sampling and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

- With the inspection level given (normally this will be II) and with the lot size, obtain the sample size code letter from Table 9.
- For a single specification limit, enter Table 10, 11 or 12 (for normal, tightened or reduced inspection, respectively) with this code letter and the AQL, and obtain the sample sizes n and the Form k acceptability constants k_a , k_r and k_c . For separate control of double specification limits, do this for both limits. For combined control of double specification limits, enter Table 23, 24 or 25, as appropriate, and obtain the sample sizes n and the Form p^* acceptability constants p_a^* , p_r^* and p_c^* . For complex control of double specification limits, enter Table 23, 24 or 25, as appropriate, twice, once with the AQL applying to the combined control part of the specification and once with the smaller AQL that applies to the more serious specification limit.
- Take an initial random sample of size n , measure the characteristic x in each item and then calculate the sample mean $\bar{x}_1$ and the estimate s_1 of the process standard deviation.

16.2 Form k acceptance procedure for the “s” method — Single specification limit

For a single specification limit, the simplest procedure is as follows.

Calculate the quality statistic

$$Q_{U,1} = \frac{U - \bar{x}_1}{s_1} \text{ or } Q_{L,1} = \frac{\bar{x}_1 - L}{s_1}$$

as appropriate.

- If the quality statistic ($Q_{U,1}$ or $Q_{L,1}$) is greater than or equal to k_a , then, without a second random sample being required, the lot is acceptable.
- If the quality statistic is less than or equal to k_r , then, again without a second random sample being required, the lot is non-acceptable.
- If the quality statistic lies between k_r and k_a , then draw a second random sample of the same size from the lot, and calculate its mean $\bar{x}_2$ and standard deviation s_2 .

Next, calculate the combined sample mean,

$$\bar{x}_c = (\bar{x}_1 + \bar{x}_2) / 2$$

the combined standard deviation,

$$s_c = \sqrt{(s_1^2 + s_2^2) / 2}$$

and the combined quality statistic

$$Q_{U,c} = \frac{U - \bar{x}_c}{s_c} \text{ or } Q_{L,c} = \frac{\bar{x}_c - L}{s_c}$$

as appropriate. If the combined quality statistic is greater than or equal to k_c , then the lot shall be accepted; otherwise, it shall be non-accepted.

In summary, if only the upper specification limit U is given, the lot is

- acceptable if $Q_{U,1} \geq k_a$, or if $k_r < Q_{U,1} < k_a$ and $Q_{U,c} \geq k_c$; or
- non-acceptable if $Q_{U,1} \leq k_r$, or if $k_r < Q_{U,1} < k_a$ and $Q_{U,c} < k_c$;

or, if only the lower specification limit L is given, the lot is:

- acceptable if $Q_{L,1} \geq k_a$, or if $k_r < Q_{L,1} < k_a$ and $Q_{L,c} \geq k_c$; or
- non-acceptable if $Q_{L,1} \leq k_r$, or if $k_r < Q_{L,1} < k_a$ and $Q_{L,c} < k_c$.

EXAMPLE 1

“s” method, upper specification limit

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. The process standard deviation is unknown. Inspection level II and normal inspection with AQL = 2,5 % are to be used.

From Table 9, it is found that the sample size code letter is F. From Table 10, it is seen that samples of size 8 are required under normal inspection, and that the acceptability constants k_a , k_r and k_c are 1,677, 1,160 and 1,476 respectively. Suppose that the temperature measurements of the eight devices in the first sample are as follows: 58 °C; 59 °C, 54 °C; 58 °C; 50 °C; 50 °C; 55 °C; 54 °C. Conformance to the acceptability criteria is to be determined. The analysis is shown in Table 1.

Table 1 — Example of “s” method analysis for an upper specification limit

Information needed	Values obtained
Size of first sample: n	8
Upper specification limit: U	60 °C
Form k acceptability constant at the first sample: k_a	1,677
Form k non-acceptability constant at the first sample: k_r	1,160
Mean of first sample: $\bar{x}_1$	54,75 °C
Standard deviation of first sample: s_1	3,495 °C
Quality statistic at the upper specification limit for the first sample: $Q_{U,1} = (U - \bar{x}_1)/s_1$	1,502
As $k_r < Q_{U,1} < k_a$, a second sample of 8 devices is required in order to determine lot acceptability. Suppose that the measurements for the second sample are 56 °C; 58 °C, 55 °C; 55 °C; 56 °C; 52 °C; 51 °C; 59 °C.	
Form k acceptability constant for the combined first and second samples: k_c	1,476
Mean of second sample: $\bar{x}_2$	55,25 °C
Standard deviation of second sample: s_2	2,712 °C
Combined sample mean: $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$	55,00 °C
Combined sample standard deviation: $s_c = \sqrt{(s_1^2 + s_2^2)/2}$	3,128 °C
Quality statistic at the upper specification limit for the combined sample: $Q_{U,c} = (U - \bar{x}_c)/s_c$	1,598
As $Q_{U,c} > k_c$, the lot meets the acceptability criteria and is therefore acceptable.	

EXAMPLE 2

“s” method, lower specification limit, requiring the following of an arrow in the master table

A certain pyrotechnic delay mechanism has a specified minimum delay time of four seconds. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. The process standard deviation is unknown.

From Table 9, it is seen that the sample size code letter is J. However, on entering Table 10 with sample size code letter J and AQL 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample sizes 18 and acceptability constants $k_a = 2,923$, $k_r = 2,389$ and $k_c = 2,562$. A random sample of size 18 is drawn. Suppose that the sample delay times, in seconds, are as follows: 5,05, 4,14, 4,78, 4,73, 4,75, 4,62, 4,69, 4,96, 4,67, 5,01, 4,50, 4,54, 4,44, 4,24, 4,25, 4,39, 4,73, 4,80.

Conformance to the acceptability criterion is to be determined. Details of the analysis are given in Table 2.

Table 2 — Example of “s” method analysis for a lower specification limit

Information needed	Values obtained
Size of first sample: n	18
Lower specification limit: L	4,0 s
Form k acceptability constant at the first sample: k_a	2,923
Form k non-acceptability constant at the first sample: k_r	2,389
Mean of first sample: $\bar{x}_1 = \sum x/n$	4,627 2 s
Standard deviation of first sample: $s_1 = \sqrt{\sum (x_j - \bar{x})^2 / (n-1)}$	0,263 0 s
Quality statistic at the lower specification limit for the first sample: $Q_{L,1} = (\bar{x}_1 - L) / s_1$	2,385
Initial acceptability criterion: Is $Q_{L,1} \geq k_a$?	No
Initial non-acceptability criterion: Is $Q_{L,1} \leq k_r$?	Yes
As the quality statistic is less than k_r , the lot is deemed to be non-acceptable without the need to select a second random sample.	

NOTE The lot is non-acceptable even though all the sampled delay times are within specification.

16.3 Form k acceptance procedure for the “s” method — Separate control of double specification limits

Under separate control of double specification limits, the Form k acceptability constants at L and U will generally be different. Denote them by $k_{L,a}$, $k_{L,r}$ and $k_{L,c}$, and by $k_{U,a}$, $k_{U,r}$ and $k_{U,c}$ respectively. In this case, the lot is acceptable if

$$Q_{U,1} \geq k_{U,a}, \text{ or } k_{U,r} < Q_{U,1} < k_{U,a} \text{ and } Q_{U,c} \geq k_{U,c}; \text{ and}$$

$$Q_{L,1} \geq k_{L,a}, \text{ or } k_{L,r} < Q_{L,1} < k_{L,a} \text{ and } Q_{L,c} \geq k_{L,c}.$$

Otherwise, the lot is non-acceptable, i.e. if

$$Q_{U,1} \leq k_{U,r}; \text{ or}$$

$$Q_{L,1} \leq k_{L,r}; \text{ or}$$

$$k_{U,r} < Q_{U,1} < k_{U,a} \text{ and } Q_{U,c} < k_{U,c}; \text{ or}$$

$$k_{L,r} < Q_{L,1} < k_{L,a} \text{ and } Q_{L,c} < k_{L,c}.$$

The required sample sizes at the two limits might well be unequal. In such cases, either draw separate samples or use the larger sample size and identify the sample items as to their order of selection so that the mean and standard deviation of the smaller sample can also be determined.

EXAMPLE 3

“s” method, separate control of double specification limits, unequal sample sizes at the limits

Car battery acid is supplied separately from the dry batteries in plastic cartons with nominal contents of 500 cl. If there is insufficient acid, the battery electrodes will be insufficiently covered, whereas, if there is too much, the consumer will have a problem of disposing of the surplus acid. Past evidence supports the view that the machine used to fill the cartons supplies a quantity of acid that is normally distributed from carton to carton within each lot. A lower specification limit of 495 cl has been set with an AQL of 0,40 %, and an upper specification limit of 505 cl with an AQL of 1,5 %. The limits are to be controlled separately, and the process standard deviation is unknown. Lots of 250 cartons are to be inspected at inspection level II.

From Table 9, it is found that the sample size code letter is G. Details of the determination of the acceptability of the first lot are given in Table 3.

Table 3 — Example of “s” method analysis for separate control of double specification limits

Information needed	Value obtained
Lower specification limit: L	495 cl
Sample size code letter (from Table 9)	G
AQL for lower specification limit	0,40 %
Required initial sample size (from Table 10): n_L	10
Acceptability constant at lower limit for first sample: $k_{L,a}$	2,463
Non-acceptability constant at lower limit for first sample: $k_{L,r}$	1,863
Upper specification limit: U	505 cl
AQL for upper specification limit	1,5 %
Required initial sample size: n_U	12
Acceptability constant at upper limit for first sample: $k_{U,a}$	1,907
Non-acceptability constant at upper limit for first sample: $k_{U,r}$	1,439
A sample of 12 cartons is selected at random from the first lot. In order of selection, the sampled cartons were found to contain 497,2 cl, 504,0 cl, 503,7 cl, 499,5 cl, 498,2 cl, 501,3 cl, 501,8 cl, 500,1 cl, 502,4 cl, 499,9 cl, 496,4 cl and 498,7 cl.	
Mean of initial sample for lower specification limit: $\bar{x}_{L,1}$	500,79 cl
Standard deviation of initial sample for lower specification limit: $s_{L,1}$	2,266 9 cl
Quality statistic at lower specification limit: $Q_{L,1} = (\bar{x}_{L,1} - L) / s_{L,1}$	2,554 1
Is $Q_{L,1} \geq k_{L,a}$?	Yes
The lot is acceptable as far as the lower specification limit is concerned. Now consider the upper specification limit.	
Mean of initial sample for upper specification limit: $\bar{x}_{U,1}$	500,25 cl
Standard deviation of initial sample for upper specification limit: $s_{U,1}$	2,456 7 cl
Quality statistic at upper specification limit: $Q_{U,1} = (U - \bar{x}_{U,1}) / s_{U,1}$	1,933 5
Is $Q_{U,1} \geq k_{U,a}$?	Yes
The acceptability criteria at both limits are satisfied at the first sample, so the lot is acceptable.	

If $Q_{U,1}$ had turned out to be 1,8, say, then it would have been found that $Q_{U,1}$ lay between $k_{U,r}$ and $k_{U,a}$. In such a case, the acceptability of the lot with regard to the upper specification limit would have been unresolved at the first sample, and a second sample of size 12 would have been necessary. As acceptability of the lot with regard to the lower specification limit has already been established, this second sample would only be used for the purposes of resolving the acceptability at the upper limit.

16.4 Form p^* acceptance procedure for the “s” method — Combined control of double specification limits

16.4.1 Introduction

This part of ISO 3951 provides both Form k and Form p^* methods for determining lot acceptability. Form k applies only to a single quality characteristic with either a single specification limit or with double specification limits that are to be controlled separately. Form p^* may be applied much more generally to single or multiple quality characteristics with any combination of single or double specification limits and with combined, separate or complex control.

16.4.2 Maximum sample standard deviation (MSSD)

If combined or complex control of both the upper and lower specification limits is required, there will be an overall AQL for the total percentage of the process outside the two specification limits, so the first step is to check that the standard deviation s_1 of the initial sample is not so large that lot acceptability is impossible. If the value of s_1 exceeds the value of the maximum sample standard deviation (MSSD) determined as $s_{1,\max} = (U - L)f_{s,1}$ with the aid of Table 16, 17 or 18, no further calculation is necessary, for the lot shall be immediately judged non-acceptable.

16.4.3 Further procedures for combined control

16.4.3.1 Exact procedures

If the value of s_1 does not exceed $s_{1,\max}$, calculate the estimate $\hat{p}_1$ of the process fraction nonconforming from the initial sample as described in E.3.1, E.4.1, E.5, E.6 or E.7 of Annex E, and compare it with the appropriate Form p^* acceptability constants p_a^* and p_r^* provided in Table 23, 24 or 25. The lot is:

acceptable if $\hat{p}_1 \leq p_a^*$;

non-acceptable if $\hat{p}_1 \geq p_r^*$.

If $p_a^* < \hat{p}_1 < p_r^*$, select a second random sample of the same size and calculate the statistics $\bar{x}_c$ and s_c (see 16.2). Find the appropriate value of $f_{s,c}$ from Table 16, 17 or 18. If the value of s_c exceeds the value of the MSSD determined as $s_{c,\max} = (U - L)f_{s,c}$, no further calculation is necessary and the lot is non-acceptable.

If the value of s_c does not exceed the value of the MSSD, calculate the estimate $\hat{p}_c$ of the process fraction nonconforming from the combined sample as described in Annex E, and compare it with p_c^* . The lot is:

acceptable if $\hat{p}_c \leq p_c^*$;

non-acceptable if $\hat{p}_c > p_c^*$.

16.4.3.2 Simplified exact formulae for $\hat{p}$ for sample sizes 3 and 4

Clauses E.6 and E.7 of Annex E provide simplified exact formulae for the estimate of the process fraction nonconforming for samples of size 3 and 4 respectively.

16.4.3.2.1 Tabular method for evaluating $\hat{p}$ when the sample size is 3

Tables 10, 11 and 12 contain combinations of sample size code letter and AQL for which the required sample size is 3. For these cases, the estimate from the first sample of the process fractions nonconforming beyond the upper and lower specification limits may be found by entering Table 22 with $\sqrt{3}Q_{U,1}/2$ and $\sqrt{3}Q_{L,1}/2$ to find $\hat{p}_{U,1}$ and $\hat{p}_{L,1}$ respectively.

NOTE Negative values of Q correspond to estimates of the process percent nonconforming in excess of 50 % at that specification limit and will consequently always, except for small lots under reduced inspection with an AQL of 10 %, result in lot non-acceptance under the provisions of this part of ISO 3951. However, for the purposes of record-keeping under these circumstances, the estimate of the process fraction nonconforming may be obtained by entering Table 22 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. For example: if $Q_{U,1} = -0,156$ then $\sqrt{3}Q_{U,1}/2 = -0,135$; entering Table 22 with 0,135 gives an estimate of 0,456 9; subtracting this from 1,0 gives $\hat{p}_{U,1} = 0,543 1$.

EXAMPLE

"s" method, combined control, $n = 3$, simplified exact formula used on results of first sample, second sample required, normal approximation used on results of combined sample

Projectiles supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally non-acceptable, so a combined AQL requirement for double specification limits is appropriate. The process standard deviation is unknown. The specification limits are set at 10 m on either side of the point of aim at a distance of one kilometre from the firing point, with an AQL of 10 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-3 is to be used.

From Table 9, the sample size code letter is found to be C. From Table 10, it is seen that samples of size 3 are required under normal inspection. Three projectiles are tested, yielding deviations from the point of aim of -5,0 m, 6,7 m and 8,8 m. Conformance to the acceptability criterion under normal inspection is to be determined.

Details of the acceptance sampling procedure are provided in Table 4.

Table 4 — Example of "s" method analysis for combined control of double specification limits ($n = 3$)

Information needed	Value obtained
Size of initial sample: n	3
Mean of initial sample: $\bar{x}_1$	3,5 m
Standard deviation of initial sample: s_1	7,435 7 m
Value of $f_{s,1}$ for MSSD (Table 16)	0,712 4
Lower specification limit: L	-10 m
Upper specification limit: U	10 m
$MSSD = s_{1,max} = (U - L)f_{s,1} = 0,7124 \times [10 - (-10)]$	14,248 m
As $s_1 = 7,435 7 < s_{1,max} = 14,248$, the lot might be acceptable, so continue with the calculations.	
$Q_{U,1} = (U - \bar{x}_1)/s_1 = (10 - 3,5) / 7,435 7$	0,874 16
$Q_{L,1} = (\bar{x}_1 - L)/s_1 = (3,5 + 10) / 7,435 7$	1,815 57
$\sqrt{3}Q_{U,1}/2$	0,757 04
$\sqrt{3}Q_{L,1}/2$	1,572 32
$\hat{p}_{U,1}$ (from Table 22)	0,226 6
$\hat{p}_{L,1}$ (from Table 22)	0,000 0
$\hat{p}_1 = \hat{p}_{U,1} + \hat{p}_{L,1}$	0,226 6
p_a^* (from Table 23, as it is normal inspection)	0,202 9
p_r^* (from Table 23, as it is normal inspection)	0,430 8

Table 4 (continued)

Information needed	Value obtained
As $p_a^* < \hat{p}_1 < p_r^*$, a second sample is required in order to determine lot acceptability.	
Size of second sample: n	3
Suppose that the second sample consists of deviations -3,1 m, 2,8 m and -6,6 m.	
Mean of second sample: $\bar{x}_2$	-2,3 m
Combined sample mean: $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$	0,6 m
Standard deviation of second sample: s_2	4,750 79 m
Combined standard deviation: $s_c = \sqrt{(s_1^2 + s_2^2)/2}$	6,239 39 m
Value of $f_{s,c}$ for MSSD (Table 16)	0,472 1
MSSD = $s_{c,max} = (U - L)f_{s,c}$	9,442 m
As $s_c = 6,239 < s_{c,max} = 9,442$, the lot might be acceptable, so continue with the calculations.	
$Q_{U,c} = (U - \bar{x}_c)/s_c = (10 - 0,6)/6,23939$	1,506 56
$Q_{L,c} = (\bar{x}_c - L)/s_c = [0,6 - (-10)]/6,23939$	1,698 88
The example is continued using the procedure given in E.5.	
Combined sample size: $N = 2n$	6
Step b of E.5 for upper limit: $v_{U,c} = \frac{1}{2}(1 - Q_{U,c}\sqrt{N/\{(N-1)(N-2)\}})$	0,087 41
Step b of E.5 for lower limit: $v_{L,c} = \frac{1}{2}(1 - Q_{L,c}\sqrt{N/\{(N-1)(N-2)\}})$	0,034 74
Constant from Table E.1: a_n	0,731 350
Step c of E.5 for upper limit: $y_{U,c} = a_n \ln \{v_{U,c}/(1 - v_{U,c})\}$	-1,715 49
Step c of E.5 for lower limit: $y_{L,c} = a_n \ln \{v_{L,c}/(1 - v_{L,c})\}$	-2,431 34
Step d of E.5 for upper limit: $w_{U,c} = y_{U,c}^2 - 3$	-0,057 1
Step d of E.5 for lower limit: $w_{L,c} = y_{L,c}^2 - 3$	2,911 4
Step e of E.5 for upper limit: $m = N - 2$	4
Step f of E.5 for upper limit: $w_{U,c} < 0$, so $t_{U,c} = 12(m-1)y_{U,c}/[12(m-1) + w_{U,c}]$	-1,718 2
Step f of E.5 for lower limit: $w_{L,c} > 0$, so $t_{L,c} = 12my_{L,c}/[12m + w_{L,c}]$	-2,292 3
Step g of E.5 for upper limit (from tables of the normal distribution function): $\hat{p}_{U,c} = \Phi(t_{U,c})$	0,042 88
Step g of E.5 for lower limit (from tables of the normal distribution function): $\hat{p}_{L,c} = \Phi(t_{L,c})$	0,010 94
Estimate of the total fraction nonconforming: $\hat{p}_c = \hat{p}_{U,c} + \hat{p}_{L,c} = 0,042 88 + 0,010 94$	0,053 82
p_c^* (from Table 23, as it is normal inspection)	0,305 2
As $\hat{p}_c < p_c^*$, the lot is acceptable.	

16.4.3.2.2 Tabular method for evaluating $\hat{p}$ when the sample size is 4

Tables 10, 11 and 12 contain combinations of sample size code letter and AQL for which the required sample size is 4. For these cases, the estimate from the first sample of the process fractions nonconforming beyond the upper and lower specification limits may be found as $\hat{p}_{U,1} = \max\left\{0, \min\left(1, \frac{1}{2} - \frac{1}{3}Q_{U,1}\right)\right\}$ and $\hat{p}_{L,1} = \max\left\{0, \min\left(1, \frac{1}{2} - \frac{1}{3}Q_{L,1}\right)\right\}$, where $\max(x,y)$, $\min(x,y)$ are respectively the maximum and minimum of the two arguments x and y .

EXAMPLE

"s" method, combined control, $n = 4$

Items are being manufactured in lots of size 50. The lower and upper specification limits on their diameters are 82 mm and 84 mm. The process standard deviation is unknown. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming beyond either limit using an AQL of 6,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations.

From Table 9, the sample size code letter is found to be D. From Table 10, it is seen that samples of size 4 are required under normal inspection. The diameters of an initial sample of four items from the first lot are measured, yielding diameters 82,4 mm, 82,2 mm, 83,1 mm and 82,3 mm. Conformance to the acceptability criterion under normal inspection is to be determined.

Details of the analysis are given in Table 5.

Table 5 — Example of "s" method analysis for combined control of double specification limits ($n = 4$)

Information needed	Value obtained
Size of initial sample: n	4
Upper specification limit: U	84,0 mm
Lower specification limit: L	82,0 mm
Mean of first sample: $\bar{x}_1$	82,50 mm
Standard deviation of first sample: s_1	0,408 2 mm
Factor for maximum of first sample standard deviation (from Table 16): $f_{s,1}$	0,478 5
Maximum of first sample standard deviation: $s_{1,\max} = (U - L)f_{s,1} = 0,4785 \times (84,0 - 82,0)$	0,957 0 mm
As $s_1 = 0,4082 < s_{1,\max} = 0,9570$, the lot <i>might</i> be acceptable, so continue with the calculations.	
Quality statistic for upper limit: $Q_{U,1} = (U - \bar{x}_1)/s_1 = (84,0 - 82,5) / 0,4082$	3,675
Quality statistic for lower limit: $Q_{L,1} = (\bar{x}_1 - L)/s_1 = (82,5 - 82,0) / 0,4082$	1,225
Estimate from first sample of fraction nonconforming above U (from E.7, and as $Q_{U,1} > 3/2$): $\hat{p}_{U,1}$	0,000 0
Estimate from first sample of fraction nonconforming below L (from E.7, as $\frac{1}{2} - \frac{1}{3}Q_{L,1}$): $\hat{p}_{L,1}$	0,091 7
Estimate from first sample of total fraction nonconforming: $\hat{p}_1 = \hat{p}_{U,1} + \hat{p}_{L,1}$	0,091 7
Form p^* acceptability constant for first sample (from Table 23): p_a^*	0,100 3
As $\hat{p}_1 < p_a^*$, the lot is acceptable.	

16.4.3.3 Approximative formulae for $\hat{p}$ for a sample of size 5 or more

When the sample is of size 5 or more, accurate normal approximations to the exact estimates of the fraction nonconforming may be obtained using the procedure given in E.5.

EXAMPLE

"s" method, combined control, $n \geq 5$, approximative method used to evaluate $\hat{p}$.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 96 items. The process standard deviation is unknown. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used.

From Table 9, it is found that the sample size code letter is F. From Table 23, it is found that samples of size 11 are required under normal inspection, and that the Form p^* acceptance constants are $p_a^* = 0,017\ 50$, $p_r^* = 0,069\ 94$ and $p_c = 0,038\ 08$. From Table 16, it is found that the value of $f_{s,1}$ for the MSSD of the initial sample under normal inspection is 0,293 4, and that $f_{s,c}$ for the combined sample is 0,251 3. The measurements obtained for the initial sample are as follows: 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C. Conformance to the acceptability criterion is to be determined.

Details of the analysis are given in Table 6.

**Table 6 — Example of “s” method analysis for combined control of double specification limits
($n \geq 5$): approximative method**

Information needed	Value obtained
Size of initial sample: n	11
Upper specification limit: U	70 °C
Lower specification limit: L	60 °C
Mean of first sample: $\bar{x}_1$	64,46 °C
Standard deviation of first sample: s_1	2,877 °C
Factor for maximum of first sample standard deviation (from Table 16): $f_{s,1}$	0,293 4
Maximum acceptable first sample standard deviation: $s_{1,max} = (U - L)f_{s,1} = (70 - 60) \times 0,2934$	2,934 °C
As $s_1 = 2,877 < s_{1,max} = 2,934$, the lot <i>might</i> be acceptable, so the calculations are continued.	
Quality statistic for upper limit: $Q_{U,1} = (U - \bar{x}_1) / s_1 = (70 - 64,46) / 2,877$	1,926
Quality statistic for lower limit: $Q_{L,1} = (\bar{x}_1 - L) / s_1 = (64,46 - 60,0) / 2,877$	1,550
Step b of E.5 for upper limit: $v_{U,1} = \frac{1}{2} \{1 - Q_{U,1} \sqrt{n} / (n - 1)\}$	0,180 6
Step b of E.5 for lower limit: $v_{L,1} = \frac{1}{2} \{1 - Q_{L,1} \sqrt{n} / (n - 1)\}$	0,243 0
Constant from Table E.1: a_n	1,417 833
Step c of E.5 for upper limit: $y_{U,1} = a_n \ln \{v_{U,1} / (1 - v_{U,1})\}$	-2,144
Step d of E.5 for upper limit: $w_{U,1} = y_{U,1}^2 - 3$	1,597
Step e of E.5 for upper limit: $m = n - 1$	10
Step f of E.5 for upper limit: $w_{U,1} > 0$, so $t_{U,1} = 12my_{U,1} / (12m + w_{U,1})$	-2,116
Step g of E.5 for upper limit (from tables of the normal distribution function): $\hat{p}_{U,1} = \Phi(t_{U,1})$	0,017 2
Step c of E.5 for lower limit: $y_{L,1} = a_n \ln \{v_{L,1} / (1 - v_{L,1})\}$	-1,611
Step d of E.5 for lower limit: $w_{L,1} = y_{L,1}^2 - 3$	-0,404 7
Step e of E.5 for lower limit: $m = n - 1$	10
Step f of E.5 for lower limit: $w_{L,1} < 0$, so $t_{L,1} = 12(m - 1)y_{L,1} / \{12(m - 1) + w_{L,1}\}$	-1,617
Step g of E.5 for lower limit (from tables of the normal distribution function): $\hat{p}_{L,1} = \Phi(t_{L,1})$	0,052 9
Estimate of the total fraction nonconforming at first sample: $\hat{p}_1 = \hat{p}_{U,1} + \hat{p}_{L,1} = 0,017\ 2 + 0,052\ 9$	0,070 1
Acceptability constant p_a^* (from Table 23)	0,017 50
Non-acceptability constant p_r^* (from Table 23)	0,069 94
As $\hat{p}_1 > p_r^*$ the lot is deemed to be non-acceptable without the need for a second sample.	

16.5 Form p^* acceptance procedure for the “s” method — Complex control of double specification limits

Complex control consists of combined control of both specification limits and simultaneous separate control of one of the limits using a separate and smaller AQL.

For example, suppose that the separate control applies to the lower specification limit, for which the required double samples are each of size n_L with acceptability constants $p_{L,a}^*$, $p_{L,r}^*$ and $p_{L,c}^*$. Suppose that the required double sample sizes for combined control are each of size n with acceptability constants p_a^* , p_r^* and p_c^* . Select a random sample of a size that is the larger of n_L and n , noting the order of selection of the items. For the first n items selected, calculate the estimate $\hat{p}_{U,1}$ of the process fraction nonconforming at the upper specification limit, the estimate $\hat{p}_{L,1}$ of the process fraction nonconforming at the lower specification limit and their sum $\hat{p}_1 = \hat{p}_{U,1} + \hat{p}_{L,1}$. For the first n_L items selected, calculate the (second) estimate $\hat{p}_{L,1}^{(2)}$ of the process fraction nonconforming at the lower specification limit.

- If $\hat{p}_1 \leq p_a^*$ and $\hat{p}_{L,1}^{(2)} \leq p_{L,a}^*$, the lot is acceptable without drawing a second sample.
- If $\hat{p}_1 \geq p_r^*$ or $\hat{p}_{L,1}^{(2)} \geq p_{L,r}^*$, the lot is non-acceptable without drawing a second sample.
- If $p_a^* < \hat{p}_1 < p_r^*$ and $\hat{p}_{L,1}^{(2)} < p_{L,r}^*$, or if $\hat{p}_1 < p_r^*$ and $p_{L,a}^* < \hat{p}_{L,1}^{(2)} < p_{L,r}^*$, draw a second sample of the same size as the first. Calculate the estimate $\hat{p}_c$ of the total process fraction nonconforming beyond both specification limits from the combined samples each of size n . Calculate the estimate $\hat{p}_{L,c}$ of the process fraction nonconforming beyond the lower specification limit from the combined samples each of size n_L . If both $\hat{p}_c \leq p_c^*$ and $\hat{p}_{L,c} \leq p_{L,c}^*$, the lot is acceptable. Otherwise, the lot is non-acceptable.
- If $p_a^* < \hat{p}_1 < p_r^*$ and $\hat{p}_{L,1}^{(2)} < p_{L,a}^*$, only the acceptability of the combined component of the complex control specification remains to be resolved. Draw a second sample of size n . Calculate the estimate $\hat{p}_c$ of the total process fraction nonconforming beyond both specification limits from the combined samples each of size n . If $\hat{p}_c \leq p_c^*$, the lot is acceptable. Otherwise, the lot is non-acceptable.
- If $\hat{p}_1 < p_a^*$ and $p_{L,a}^* < \hat{p}_{L,1}^{(2)} < p_{L,r}^*$, only the acceptability of the single specification limit component of the complex control specification remains to be resolved. Draw a second sample of size n_L . Calculate the estimate $\hat{p}_{L,c}$ of the process fraction nonconforming beyond the lower specification limit from the combined samples each of size n_L . If $\hat{p}_{L,c} \leq p_{L,c}^*$, the lot is acceptable. Otherwise, the lot is non-acceptable.

NOTE 1 If $n_L = n$, $\hat{p}_{L,1}^{(2)}$ will be equal to $\hat{p}_{L,1}$.

NOTE 2 If the separate control applies to the upper specification limit, replace “L” by “U” and “lower” by “upper” in the foregoing text of this subclause.

17 Standard univariate “σ” method procedures

17.1 Obtaining a plan, sampling and preliminary calculations

The “σ” method shall only be used when, in the view of the responsible authority, there is sufficient evidence that the standard deviation of the process can be considered constant and taken to be σ .

For double specification limits, before sampling begins, determine the maximum process standard deviation (MPSD) as

$$\sigma_{\max} = (U - L)f_{\sigma}$$

where the factor f_{σ} is obtained

- a) for combined control by entering Table 19 with the single AQL; or

- b) for separate control by entering Table 20 with both AQLs; or
- c) for complex control by entering Table 21 with both AQLs.

Compare the value of the process standard deviation σ with d_σ . If σ exceeds d_σ , the process is non-acceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

If, and only if, $\sigma \leq d_\sigma$, obtain the sample size code letter from Table 9. Then, depending on the severity of inspection and the type of control required (see following subclauses), and for each AQL, enter either

- i) Table 13, 14 or 15 with the sample size code letter and the specified AQL to obtain the sample sizes n and acceptability constants k_a , k_r and k_c or
- ii) Table 26, 27 or 28 with the sample size code letter and the specified AQL to obtain the sample sizes n and acceptability constants p_a^* , p_r^* and p_c^* .

Take an initial random sample of size n , measure the characteristic under inspection, x , for all items of the sample and calculate the mean $\bar{x}_1$. (The standard deviation, s_1 , of the initial sample should also be calculated, but only for the purpose of checking the continued stability of the process standard deviation. See Clause 20.)

It will be seen that the remaining steps are similar to those for the “s” method except that s_1 and s_c have been replaced by σ .

17.2 Form k acceptance procedure for the “ σ ” method — Single specification limit

17.2.1 General procedure

For a single specification limit, Form k is the simplest procedure to use. Calculate the quality statistic

$$Q_{U,1} = \frac{U - \bar{x}_1}{\sigma} \text{ or } Q_{L,1} = \frac{\bar{x}_1 - L}{\sigma}$$

as appropriate. If the quality statistic ($Q_{U,1}$ or $Q_{L,1}$ or) is greater than or equal to k_a then, without drawing a second random sample, the lot is acceptable. If the quality statistic is less than or equal to k_r , then, without drawing a second random sample, the lot is non-acceptable.

If the quality statistic lies between k_r and k_a , then draw a second random sample of the same size from the lot, and calculate its mean $\bar{x}_2$. Next, calculate the combined sample mean

$$\bar{x}_c = (\bar{x}_1 + \bar{x}_2) / 2$$

and the combined quality statistic

$$Q_{U,c} = \frac{U - \bar{x}_c}{\sigma} \text{ or } Q_{L,c} = \frac{\bar{x}_c - L}{\sigma}$$

as appropriate. If the combined quality statistic is greater than or equal to k_c , then the lot is acceptable; otherwise, it is non-acceptable.

Thus, if only the upper specification limit U is given, the lot is

acceptable if $Q_{U,1} \geq k_a$, or if $k_r < Q_{U,1} < k_a$ and $Q_{U,c} \geq k_c$; or

non-acceptable if $Q_{U,1} \leq k_r$, or if $k_r < Q_{U,1} < k_a$ and $Q_{U,c} < k_c$;

or, if only the lower specification limit L is given, the lot is

acceptable if $Q_{L,1} \geq k_a$, or if $k_r < Q_{L,1} < k_a$ and $Q_{L,c} \geq k_c$; or

non-acceptable if $Q_{L,1} \leq k_r$, or if $k_r < Q_{L,1} < k_a$ and $Q_{L,c} < k_c$.

17.2.2 Simplified general procedure

Note that, for an upper specification limit, the acceptability criteria may be written as inequalities on $\bar{x}$, i.e. the lot is

acceptable if $\bar{x}_1 \leq U - k_a\sigma$, or if $U - k_r\sigma < \bar{x}_1 < U - k_a\sigma$ and $\bar{x}_c \leq U - k_c\sigma$; or

non-acceptable if $\bar{x}_1 \geq U - k_r\sigma$, or $U - k_a\sigma < \bar{x}_1 < U - k_r\sigma$ and $\bar{x}_c > U - k_c\sigma$.

As U , k_a , k_r , k_c and σ are all known in advance, the values of $x_{U,a}[=U - k_a\sigma]$, $x_{U,r}[=U - k_r\sigma]$ and $x_{U,c}[=U - k_c\sigma]$ should therefore be determined before inspection begins, in order to avoid the need to calculate the values of quality statistics. For a single, upper specification limit, a lot will be

acceptable if $\bar{x}_1 \leq x_{U,a}$, or if $x_{U,a} < \bar{x}_1 < x_{U,r}$ and $\bar{x}_c \leq x_{U,c}$; or

non-acceptable if $\bar{x}_1 \geq x_{U,r}$, or if $x_{U,a} < \bar{x}_1 < x_{U,r}$ and $\bar{x}_c > x_{U,c}$.

Similarly, for a single, lower specification limit, the values of $x_{L,a}[=L + k_a\sigma]$, $x_{L,r}[=L + k_r\sigma]$ and $x_{L,c}[=L + k_c\sigma]$ should be determined in advance. A lot will be

acceptable if $\bar{x}_1 \geq x_{L,a}$, or if $x_{L,r} < \bar{x}_1 < x_{L,a}$ and $\bar{x}_c \geq x_{L,c}$; or

non-acceptable if $\bar{x}_1 \leq x_{L,r}$, or $x_{L,r} < \bar{x}_1 < x_{L,a}$ and $\bar{x}_c < x_{L,c}$.

EXAMPLE

“ σ ” method, single specification limit

The specified minimum yield point for certain steel castings is 400 N/mm². Lots of 500 items are submitted for inspection. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. The value of σ is considered to be 21 N/mm².

From Table 9, it is seen that the sample size code letter is H. Then, from Table 13, it is seen that for an AQL of 1,5 % the sample size n is 8 and the Form k acceptability constants are $k_a = 1,776$, $k_r = 1,357$ and $k_c = 1,638$. Suppose the yield points in N/mm² of the 8 sample specimens from the initial sample from the current lot are 431, 417, 469, 407, 442, 452, 427 and 411. Conformance to the acceptability criterion is to be determined. The analysis is given in Table 7.

Table 7 — Example of “ σ ” method analysis for a lower specification limit

Information needed	Value obtained
Sample size: n	8
Acceptability constant at the first sample: k_a	1,776
Non-acceptability constant at the first sample: k_r	1,357
Presumed value of the process standard deviation: σ	21 N/mm ²
Lower specification limit: L	400 N/mm ²
Acceptance value for the first sample: $x_{L,a} = L + k_a\sigma$	437,3 N/mm ²
Non-acceptance value for the first sample: $x_{L,r} = L + k_r\sigma$	428,5 N/mm ²
Sum of measurements for the first sample: Σx_1	3 464 N/mm ²
Mean of first sample: $\bar{x}_1$	433,0 N/mm ²
Initial acceptance test: Is $\bar{x}_1 \geq x_{L,a}$?	No
The lot does not meet the initial acceptability criterion, so we test to see if it satisfies the non-acceptability criterion.	
Initial non-acceptance test: $\bar{x}_1 \leq x_{L,r}$	No
The lot does not meet the initial non-acceptability criterion either, so a second sample of 8 items is drawn. Suppose the yield points in N/mm ² of the second sample are 439, 422, 415, 425, 432, 430, 410 and 428.	
Acceptability constant for the combined first and second samples: k_c	1,638
Acceptance value for the combined samples: $x_{L,c} = L + k_c\sigma$	434,4 N/mm ²
Sum of measurements for the second sample: Σx_2	3 456 N/mm ²
Mean of second sample: $\bar{x}_2$	432,0 N/mm ²
Combined mean: $\bar{x}_c = (\bar{x}_1 + \bar{x}_2)/2$	432,5 N/mm ²
Combined acceptance test: Is $\bar{x}_c \geq x_{L,c}$?	No
The lot fails the combined acceptance test and so the lot is non-accepted.	

NOTE This is another example in which the lot is non-accepted despite no nonconforming items being found in either sample.

17.3 Form k acceptance procedure for the “ σ ” method — Separate control of double specification limits

For double specification limits with separate control, the lot may at once be declared to be non-acceptable if σ is greater than the MPSD derived from Table 20. If $\sigma \leq \text{MPSD}$, enter Table 13, 14 or 15 as appropriate with the sample size code letter and the AQL for the upper specification limit to determine the sample size n_U and the relevant acceptability constants $k_{U,a}$, $k_{U,r}$ and $k_{U,c}$; repeat for the lower specification limit to determine the sample size n_L and the relevant acceptability constants $k_{L,a}$, $k_{L,r}$ and $k_{L,c}$. Denote the larger of n_U and n_L by n . Randomly select a sample of size n from the lot, with the order of selection recorded. Compute $\bar{x}_{U,1}$ from the measurements of the first n_U items selected and $\bar{x}_{L,1}$ from the first n_L items selected. The lot will be acceptable if

$$\bar{x}_{U,1} \leq x_{U,a} [= U - k_{U,a}\sigma] \text{ and } \bar{x}_{L,1} \geq x_{L,a} [= L + k_{L,a}\sigma],$$

and non-acceptable if

$$\bar{x}_{U,1} \geq x_{U,r} [= U - k_{U,r}\sigma] \text{ and/or } \bar{x}_{L,1} \leq x_{L,r} [= L + k_{L,r}\sigma].$$

If $x_{U,a} < \bar{x}_{U,1} < x_{U,r}$ and $x_{L,r} < \bar{x}_{L,1} < x_{L,a}$, select a second random sample of the same size and in the same way as the first sample from the lot, measure the quality characteristic on each item and calculate the sample means $\bar{x}_{U,2}$ and $\bar{x}_{L,2}$. Then calculate the combined sample means $\bar{x}_{U,c} = (\bar{x}_{U,1} + \bar{x}_{U,2})/2$ and $\bar{x}_{L,c} = (\bar{x}_{L,1} + \bar{x}_{L,2})/2$. If both $\bar{x}_{U,c} \leq x_{U,c} [= U - k_{U,c}\sigma]$ and $\bar{x}_{L,c} \geq x_{L,c} [= L + k_{L,c}\sigma]$, the lot is acceptable; otherwise, it is non-acceptable.

Alternatively, if $x_{U,a} < \bar{x}_{U,1} < x_{U,r}$ but $\bar{x}_{L,1} \geq x_{L,a}$, control at the lower specification limit may be deemed to be acceptable but further information is required relating to the upper limit. Select a second sample of size n_U and determine $\bar{x}_{U,2}$ and $\bar{x}_{U,c}$. The lot is acceptable if $\bar{x}_{U,c} \leq x_{U,c} [= U - k_{U,c}\sigma]$; otherwise, it is non-acceptable.

The final possibility is that $x_{L,r} < \bar{x}_{L,1} < x_{L,a}$ but $\bar{x}_{U,1} \leq x_{U,a}$. Here, further information is required relating to the lower specification limit before a decision as to acceptability can be made. Select a second sample of size n_L and determine $\bar{x}_{L,2}$ and $\bar{x}_{L,c}$. The lot is acceptable if $\bar{x}_{L,c} \geq x_{L,c} [= L + k_{L,c}\sigma]$; otherwise, it is non-acceptable.

17.4 Form p^* acceptance procedure for the “ σ ” method — Combined control of double specification limits

If there is a combined AQL requirement for the upper and the lower specification limits, i.e. one overall AQL for the percentage of the process outside both specification limits, the following procedure is recommended.

- Before sampling, determine the value of the factor f_σ by entering Table 19 with the AQL. Calculate the maximum allowable value $\sigma_{\max}$ of the process standard deviation (the MPSD) from the formula $\sigma_{\max} = (U - L)f_\sigma$.
- Compare the value of the process standard deviation σ with $\sigma_{\max}$. If σ exceeds $\sigma_{\max}$, the process is non-acceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.
- If $\sigma \leq \sigma_{\max}$, then use the lot size and given inspection level to determine the sample size code letter from Table 9.
- From the sample size code letter and inspection severity (i.e. whether inspection is normal, tightened or reduced), determine the sample sizes, n , and acceptability constants, p_a^* , p_r^* and p_c^* , from Table 26, 27 or 28.
- Select an initial random sample of size n from the lot and calculate the initial sample mean $\bar{x}_1$.
- Calculate the quality statistics $Q_{U,1} = (U - \bar{x}_1)/\sigma$ and $Q_{L,1} = (\bar{x}_1 - L)/\sigma$.
- Using the method described in E.3.2, calculate $\hat{p}_{U,1}$, $\hat{p}_{L,1}$ and $\hat{p}_1 = \hat{p}_{U,1} + \hat{p}_{L,1}$.
- If $\hat{p}_1 \leq p_a^*$, the lot is acceptable and no further samples, calculations or comparisons are required.
- If $\hat{p}_1 \geq p_r^*$, the lot is non-acceptable and no further samples, calculations or comparisons are required.
- If $p_a^* < \hat{p}_1 < p_r^*$, select a second random sample of size n and calculate the second sample mean $\bar{x}_2$ and the combined sample mean $\bar{x}_c$.
- Calculate the combined quality statistics $Q_{U,c} = (U - \bar{x}_c)/\sigma$ and $Q_{L,c} = (\bar{x}_c - L)/\sigma$.
- Using the method described in E.4.2, calculate $\hat{p}_{U,c}$, $\hat{p}_{L,c}$ and $\hat{p}_c = \hat{p}_{U,c} + \hat{p}_{L,c}$.
- If $\hat{p}_c \leq p_c^*$, the lot is acceptable; otherwise, it is non-acceptable.