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Shewhart control charts

Cartes de contrôle de Shewhart



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 8258 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*.

Annex A of this International Standard is for information only.

Introduction

The traditional approach to manufacturing is to depend on production to make the product and on quality control to inspect the final product and screen out items not meeting specifications. This strategy of detection is often wasteful and uneconomical because it involves after-the-event inspection when the wasteful production has already occurred. Instead, it is much more effective to institute a strategy of prevention to avoid waste by not producing unusable output in the first place. This can be accomplished by gathering process information and analysing it so that action can be taken on the process itself.

The control chart as a graphical means of applying the statistical principles of significance to the control of a production process was first proposed by Dr. Walter Shewhart in 1924^[1]. Control chart theory recognizes two kinds of variability. The first kind is random variability due to "chance causes" (also known as "common causes"). This is due to the wide variety of causes that are consistently present and not readily identifiable, each of which constitutes a very small component of the total variability but none of which contributes any significant amount. Nevertheless, the sum of the contributions of all of these unidentifiable random causes is measurable and is assumed to be inherent to the process. The elimination or correction of common causes requires a management decision to allocate resources to improve the process and system.

The second kind of variability represents a real change in the process. Such a change can be attributed to some identifiable causes that are not an inherent part of the process and which can, at least theoretically, be eliminated. These identifiable causes are referred to as "assignable causes" or "special causes" of variation. They may be attributable to the lack of uniformity in material, a broken tool, workmanship or procedures or to the irregular performance of manufacturing or testing equipment.

Control charts aid in the detection of unnatural patterns of variation in data resulting from repetitive processes and provide criteria for detecting a lack of statistical control. A process is in statistical control when the variability results only from random causes. Once this acceptable level of variation is determined, any deviation from that level is assumed to be the result of assignable causes which should be identified and eliminated or reduced.

The object of statistical process control is to serve to establish and maintain a process at an acceptable and stable level so as to ensure conformity of products and services to specified requirements. The major statistical tool used to do this is the control chart, which is a graphical method of presenting and comparing information based on a sequence of samples representing the current state of a process against limits established after consideration of inherent process variability. The control chart method helps first to evaluate whether or not a process has attained, or continues in, a state of statistical control at the proper

specified level and then to obtain and maintain control and a high degree of uniformity in important product or service characteristics by keeping a continuous record of quality of the product while production is in progress. The use of a control chart and its careful analysis leads to a better understanding and improvement of the process.

Shewhart control charts

1 Scope

This International Standard establishes a guide to the use and understanding of the Shewhart control chart approach to the methods for statistical control of a process.

This International Standard is limited to the treatment of statistical process control methods using only the Shewhart system of charts. Some supplementary material that is consistent with the Shewhart approach, such as the use of warning limits, analysis of trend patterns and process capability is briefly introduced. There are, however, several other types of control chart procedures, a general description of which can be found in ISO 7870.

2 Symbols

n Subgroup size; the number of sample observations per subgroup

k Number of subgroups

X Value of measured quality characteristic (individual values are expressed as $(X_1, X_2, X_3, \dots)$). Sometimes the symbol Y is used instead of X .

$\bar{X}$ (X bar) Subgroup average value:

$$\bar{X} = \frac{\sum X_i}{n}$$

$\bar{\bar{X}}$ (X double bar) Average value of the subgroup averages

μ True process mean value

Me Median value of a subgroup. For a set of n numbers $X_1, X_2, \dots, X_n$ arranged in ascending or descending order of magnitude, the median is the middle number of the set if n is odd and the mean of the two middle numbers if n is even

$\overline{Me}$ Average value of the subgroup medians

R Subgroup range: difference between the largest and smallest observations of a subgroup.

NOTE 1 In the case of charts for individuals, R represents the moving range, which is the absolute value of the difference between two successive values $|X_1 - X_2|$, $|X_2 - X_3|$, etc.

$\bar{R}$ Average value of the R values for all subgroups

s Sample standard deviation:

$$s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}}$$

$\bar{s}$ Average value of the subgroup sample standard deviations

σ True within-subgroup process standard deviation value

$\hat{\sigma}$ Estimated within-subgroup process standard deviation value

p Proportion or fraction of nonconforming units in a subgroup:

p = number of nonconforming units in a subgroup/subgroup size

$\bar{p}$ Average value of the proportion or fraction nonconforming:

$\bar{p}$ = number of nonconforming units in all subgroups/total number of inspected units

np Number of nonconforming units in a subgroup

c Number of nonconformities in a subgroup

$\bar{c}$ Average value of the c values for all subgroups

u Number of nonconformities per unit in a subgroup

$\bar{u}$ Average value of the u values:

$\bar{u}$ = number of nonconformities in all units/total number of inspected units

3 Nature of Shewhart control charts

A shewhart control chart requires data obtained by sampling the process at approximately regular intervals. The intervals may be defined in terms of time (for example hourly) or quantity (every lot). Usually, each subgroup consists of the same product or service with the same measurable units and the same subgroup size. From each subgroup, one or more subgroup characteristics are derived, such as the subgroup average, $\bar{X}$, and the subgroup range, R , or the standard deviation, s . A Shewhart control chart is a graph of the values of a given subgroup characteristic versus the subgroup number. It consists of a central line (CL) located at a reference value of the plotted characteristic. In evaluating whether or not a state of statistical control exists, the reference value is usually the average of the data being considered. For process control, the reference value is usually the long-term value of the characteristic as stated in the product specifications or a nominal value of the characteristic being plotted based on past experience with the process or from implied product or service target values. The control chart has two statistically determined control limits, one on either side of the central line, which are called the upper control limit (UCL) and the lower control limit (LCL) (see figure 1).

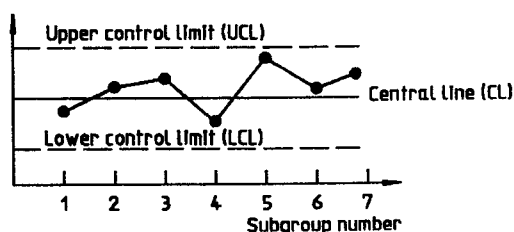


Figure 1 — Outline of a control chart

The control limits on the Shewhart charts are at a distance of 3σ on each side of the central line, where σ is the population within-subgroup standard deviation of the statistic being plotted. The within-subgroup variability is used as a measure of the random variation. Sample standard deviations or appropriate multiples of sample ranges are computed to give an estimate of σ . This measure of σ

does not include subgroup-to-subgroup variation but only the within-subgroup components. The 3σ limits indicate that approximately 99,7 % of the subgroup values will be included within the control limits, provided the process is in statistical control. Interpreted another way, there is approximately a 0,3 % risk, or an average of three times in a thousand, of a plotted point being outside of either the upper or lower control limit when the process is in control. The word “approximately” is used because deviations from underlying assumptions such as the distributional form of the data will affect the probability values.

It should be noted that some practitioners prefer to use the factor 3,09 instead of 3 to provide a nominal probability value of 0,2 % or an average of one spurious observation per thousand, but Shewhart selected 3 so as not to lead to attempts to consider exact probabilities. Similarly, some practitioners use actual probability values for the charts based on non-normal distributions such as for ranges and fraction nonconforming. Again, the Shewhart control chart used $\pm 3\sigma$ limits, instead of the probabilistic limits, in view of the emphasis on empirical interpretation.

The possibility that a violation of the limits is really a chance event rather than a real signal is considered so small that when a point appears outside of the limits, action should be taken. Since action is required at this point, the 3σ control limits are sometimes called the “action limits”.

Many times, it is advantageous to mark 2σ limits on the chart also. Then, any sample value falling beyond the 2σ limits can serve as a warning of an impending out-of-control situation. As such, the 2σ control limits are sometimes called “warning limits”.

Two types of error are possible when control charts are applied. The first is referred to as a type 1 error, which occurs when the process involved remains in control but a point falls outside the control limits due to chance. As a result, it is incorrectly concluded that the process is out of control, and a cost is then incurred in an attempt to find the cause of a non-existent problem.

The second error is referred to as a type 2 error. It occurs when the process involved is out of control but the point generated falls within the control limits due to chance. In this case, it is incorrectly concluded that the process is in statistical control, and there is a cost associated with failing to detect an increase in nonconforming output. The risk of a type 2 error, however, is a function of three things: the width of the control limits, the degree to which the process is out of control, and the sample size. The nature of these items is such that only a generalization can be made about the size of the risk of a type 2 error.

The Shewhart system takes into account only the type 1 error and the size of this error is 0,3 % with 3σ limits. Since it is generally impractical to make a meaningful estimate of the cost of a type 2 error in a given situation, and since it is convenient to arbitrarily select a small subgroup size, such as 4 or 5, it is appropriate and feasible to use 3σ limits and focus attention on controlling and improving the performance of the process itself.

When a process is in statistical control, the control chart provides a method for continuously testing a statistical null hypothesis that the process has not changed and remains in statistical control. Since the specific departures of the process characteristic from the target value that may be of concern are not usually defined in advance, along with the risk of a type 2 error, and the sample size is not calculated to satisfy appropriate risk levels, the Shewhart control chart should not be considered in the sense of a test of hypothesis. (See ISO 7966 and ISO 7870). Shewhart emphasized the empirical usefulness of the control chart for recognizing departures from an "in-control" process and de-emphasized probabilistic interpretation. Some users do examine operating characteristic curves as a means of making a hypothesis test interpretation.

When a plotted value falls outside of either control limit or a series of values reflect unusual patterns such as discussed in clause 7, the state of statistical control can no longer be accepted. When this occurs, an investigation is initiated to locate the assignable cause and the process may be stopped or adjusted. Once the assignable cause is determined and eliminated, the process is ready to continue. As discussed above, for a type 1 error, on rare occasions, no assignable cause can be found and it must be concluded that the point outside the limits represents the occurrence of a very rare event, a random cause which has resulted in a value outside of the control limits even though the process is in control.

When control charts are first set up for a process, it frequently occurs that the process will be found to be out of control. Control limits calculated from data from an out-of-control process would lead to erroneous conclusions because they would be too far apart. Consequently, it is always necessary to bring an out-of-control process into control before permanent control chart parameters are established. The procedure for establishing control charts for a process will be discussed in the following clauses.

4 Types of control charts

Shewhart control charts are basically of two types: variables control charts and attributes control charts. For each of the control charts, there are two distinct situations:

- a) when no standard values are given, and
- b) when standard values are given.

The standard values are some specified requirement or target values.

4.1 Control charts where no standard values are given

The purpose here is to discover whether observed values of the plotted characteristics, such as $\bar{X}$, R or any other statistic, vary among themselves by an amount greater than that which should be attributed to chance alone. Control charts based entirely on the data collected from samples are used for detecting those variations caused other than by chance.

4.2 Control charts with respect to given standard values

The purpose here is to identify whether the observed values of $\bar{X}$, etc., for several subgroups of n observations each, differ from the respective standard values X_0 (or μ_0), etc. by amounts greater than that expected to be due to chance causes only. The difference between charts with standards given and those where no standards are given is the additional requirement concerning the location of the centre and variation of the process. The specified values may be based on experience obtained by using control charts with no prior information or specified standard values. They may also be based on economic values established upon consideration of the need for service and cost of production or be nominal values designated by the product specifications.

Preferably, the specified values should be determined through an investigation of preliminary data that is supposed to be typical of all future data. The standard values should be compatible with the inherent process variability for effective functioning of the control charts. Control charts based on such standard values are used particularly during manufacture to control processes and to maintain product uniformity at the desired level.

4.3 Types of variables and attributes control charts

The following control charts are considered.

- a) Variables control charts:
 - 1) average ($\bar{X}$) chart and range (R) or standard deviation (s) chart;
 - 2) charts for individuals (X) and moving range (R);

3) median (Me) chart and range (R) chart.

b) Attributes control charts:

- 1) fraction nonconforming (p) chart or number of nonconforming units (np) chart;
- 2) number of nonconformities (c) chart or non-conformities per unit (u) chart.

5 Variables control charts

Variables data represent observations obtained by measuring and recording the numerical magnitude of a characteristic for each of the units in the subgroup under consideration. Examples of variables measurements are length in metres, resistance in ohms, noise in decibels, etc. Variables charts — and especially their most customary forms, the $\bar{X}$ and R charts — represent the classic application of control charting to process control.

Control charts for variables are particularly useful for several reasons.

- a) Most processes and their output have characteristics that are measurable, so the potential applicability is broad.
- b) A measurement value contains more information than a simple yes—no statement.
- c) The performance of a process can be analysed without regard to the specification. The charts start with the process itself and give an independent picture of what the process can do. Afterwards, the process may or may not be compared with the specification.
- d) Although obtaining one piece of measured data is generally more costly than obtaining one piece of go/no go data, the subgroup sizes for variables are almost always much smaller than

those for attributes, and so are more efficient. This helps to reduce the total inspection cost in some cases and to shorten the time gap between the production of parts and corrective action.

A normal (Gaussian) distribution is assumed for within-sample variability for all variables control chart applications considered in this International Standard and departures from this assumption will affect the performance of the charts. The factors for computing control limits were derived using the assumption of normality. Since most control limits are used as empirical guides in making decisions, reasonably small departures from normality should not cause concern. In any case, because of the central limit theorem, averages tend to be normally distributed even when individual observations are not; this makes it reasonable to assume normality for $\bar{X}$ charts, even for sample sizes as small as 4 or 5 for evaluating control. When dealing with individual observations for capability study purposes, the true form of the distribution is important. Periodic checks on the continuing validity of such assumptions are advisable, particularly for ensuring that only data from a single population are being used. It should be noted that the distribution of the ranges and standard deviations are not normal, although approximate normality was assumed in the estimation of the constants for the calculation of control limits, which is satisfactory for an empirical decision procedure.

5.1 Mean ($\bar{X}$) chart and range (R) or standard deviations (s) chart

Variables charts can describe process data in terms of both spread (piece-to-piece variability) and location (process average). Because of this, control charts for variables are almost always prepared and analysed in pairs — one chart for location and another for spread. The most commonly used pair is the $\bar{X}$ and R charts. Table 1 and table 2 give the control limit formulae and the factors for variables control charts respectively.

Table 1 — Control limit formulae for Shewhart variables control charts

Statistic	No standard values given		Standard values given	
	Central line	UCL and LCL	Central line	UCL and LCL
$\bar{X}$	$\bar{\bar{X}}$	$\bar{\bar{X}} \pm A_2 \bar{R}$ or $\bar{\bar{X}} \pm A_3 \bar{s}$	X_0 or μ	$X_0 \pm A \sigma_0$
R	$\bar{R}$	$D_3 \bar{R}, D_4 \bar{R}$	R_0 or $d_2 \sigma_0$	$D_1 \sigma_0, D_2 \sigma_0$
s	$\bar{s}$	$B_3 \bar{s}, B_4 \bar{s}$	s_0 or $c_4 \sigma_0$	$B_5 \sigma_0, B_6 \sigma_0$
NOTE — X_0, R_0, s_0, μ and σ_0 are given standard values.				

Table 2 — Factors for computing control chart lines

Observations in subgroup n	Factors for control limits											Factors for central line			
	A	A_2	A_3	B_3	B_4	B_5	B_6	D_1	D_2	D_3	D_4	C_4	$1/C_4$	d_2	$1/d_2$
2	2,121	1,880	2,659	0,000	3,267	0,000	2,606	0,000	3,686	0,000	3,267	0,797 9	1,253 3	1,128	0,886 5
3	1,732	1,023	1,954	0,000	2,568	0,000	2,276	0,000	4,358	0,000	2,574	0,886 2	1,128 4	1,693	0,590 7
4	1,500	0,729	1,628	0,000	2,266	0,000	2,088	0,000	4,698	0,000	2,282	0,921 3	1,085 4	2,059	0,485 7
5	1,342	0,577	1,427	0,000	2,089	0,000	1,964	0,000	4,918	0,000	2,114	0,940 0	1,063 8	2,326	0,429 9
6	1,225	0,483	1,287	0,030	1,970	0,029	1,874	0,000	5,078	0,000	2,004	0,951 5	1,051 0	2,534	0,394 6
7	1,134	0,419	1,182	0,118	1,882	0,113	1,806	0,204	5,204	0,076	1,924	0,959 4	1,042 3	2,704	0,369 8
8	1,061	0,373	1,099	0,185	1,815	0,179	1,751	0,388	5,306	0,136	1,864	0,965 0	1,036 3	2,847	0,351 2
9	1,000	0,337	1,032	0,239	1,761	0,232	1,707	0,547	5,393	0,184	1,816	0,969 3	1,031 7	2,970	0,336 7
10	0,949	0,308	0,975	0,284	1,716	0,276	1,669	0,687	5,469	0,223	1,777	0,972 7	1,028 1	3,078	0,324 9
11	0,905	0,285	0,927	0,321	1,679	0,313	1,637	0,811	5,535	0,256	1,744	0,975 4	1,025 2	3,173	0,315 2
12	0,866	0,266	0,886	0,354	1,646	0,346	1,610	0,922	5,594	0,283	1,717	0,977 6	1,022 9	3,258	0,306 9
13	0,832	0,249	0,850	0,382	1,618	0,374	1,585	1,025	5,647	0,307	1,693	0,979 4	1,021 0	3,336	0,299 8
14	0,802	0,235	0,817	0,406	1,594	0,399	1,563	1,118	5,696	0,328	1,672	0,981 0	1,019 4	3,407	0,293 5
15	0,775	0,223	0,789	0,428	1,572	0,421	1,544	1,203	5,741	0,347	1,653	0,982 3	1,018 0	3,472	0,288 0
16	0,750	0,212	0,763	0,448	1,552	0,440	1,526	1,282	5,782	0,363	1,637	0,983 5	1,016 8	3,532	0,283 1
17	0,728	0,203	0,739	0,466	1,534	0,458	1,511	1,356	5,820	0,378	1,622	0,984 5	1,015 7	3,588	0,278 7
18	0,707	0,194	0,718	0,482	1,518	0,475	1,496	1,424	5,856	0,391	1,608	0,985 4	1,014 8	3,640	0,274 7
19	0,688	0,187	0,698	0,497	1,503	0,490	1,483	1,487	5,891	0,403	1,597	0,986 2	1,014 0	3,689	0,271 1
20	0,671	0,180	0,680	0,510	1,490	0,504	1,470	1,549	5,921	0,415	1,585	0,986 9	1,013 3	3,735	0,267 7
21	0,655	0,173	0,663	0,523	1,477	0,516	1,459	1,605	5,951	0,425	1,575	0,987 6	1,012 6	3,778	0,264 7
22	0,640	0,167	0,647	0,534	1,466	0,528	1,448	1,659	5,979	0,434	1,566	0,988 2	1,011 9	3,819	0,261 8
23	0,626	0,162	0,633	0,545	1,455	0,539	1,438	1,710	6,006	0,443	1,557	0,988 7	1,011 4	3,858	0,259 2
24	0,612	0,157	0,619	0,555	1,445	0,549	1,429	1,759	6,031	0,451	1,548	0,989 2	1,010 9	3,895	0,256 7
25	0,600	0,153	0,606	0,565	1,435	0,559	1,420	1,806	6,056	0,459	1,541	0,989 6	1,010 5	3,931	0,254 4

Source: ASTM, Philadelphia, PA, USA.

5.2 Control chart for individuals ($\bar{X}$)

In some process control situations, it is either impossible or impractical to take rational subgroups. The time or cost required to measure a single observation is so great that repeat observations cannot be considered. This would typically occur when the measurements are expensive (e.g. in a destructive test) or when the output at any time is relatively homogeneous. In other situations there is only one possible value, e.g. an instrument reading or a property of a batch of input material. In such situations, it is necessary for process control to be based on individual readings.

In the case of charts for individuals, since there are no rational subgroups to provide an estimate of within-batch variability, control limits are based on a variation measure obtained from moving ranges of, often, two observations. A moving range is the absolute difference between successive pairs of measurements in a series; i.e. the difference between the first and second measurements, then be-

tween the second and third, and so on. From the moving ranges, the average moving range $\bar{R}$ is calculated and used for the construction of control charts. Also, from the entire data, the overall average $\bar{X}$ is calculated. Table 3 gives the control limit formulae for control charts for individuals.

Some caution should be exercised with respect to control charts for individuals.

- The charts for individuals are not as sensitive to process changes as are the $\bar{X}$ and R charts.
- Care shall be taken in the interpretation of charts for individuals if the process distribution is not normal.
- Charts for individuals do not isolate the piece-to-piece repeatability of the process and, therefore, it may be better in some applications to use a conventional $\bar{X}$ and R chart with small subgroup sample sizes (2 to 4) even if this requires a larger period between subgroups.