

INTERNATIONAL STANDARD

ISO/CIE 10527

First edition
1991-12-15

CIE standard colorimetric observers

Observateurs de référence colorimétrique CIE



Reference number
ISO/CIE 10527 : 1991 (E)

AVA Documents
Document Sharing Hub

Contents

	Page
Introduction	v
1 Scope	1
2 Normative references	1
3 Definitions	1
4 Specifications	2
4.1 Colour-matching functions	2
4.2 Spectral chromaticity coordinates	2
5 Derivation of the colour-matching functions for the CIE 1931 standard colorimetric observer	2
5.1 Experimental basis	2
5.2 Transformation procedures	2
5.3 Transformation properties	3
5.4 Comparison with earlier data	3
6 Derivation of the colour-matching functions for the CIE 1964 supplementary standard colorimetric observer	3
6.1 Experimental basis	3
6.2 Transformation procedures	3
6.3 Transformation properties	3
6.4 Comparison with earlier data	3
7 Practical applications of colour-matching functions for CIE standard colorimetric observers	3
7.1 Obtaining tristimulus values	3
7.2 The basis for integration	4
7.3 Rod activity	4
7.4 The use of restricted data	4
7.5 Standard of reflectance	4
Annex	
A Bibliography	25

© ISO 1991

All rights reserved. No part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from the publisher.

International Organization for Standardization

Case postale 56 • CH-1211 Genève 20 • Switzerland

Printed in Switzerland

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The International Commission on Illumination (abbreviated as CIE from its French title) is an organization devoted to international cooperation and exchange of information among its member countries on all matters relating to the science and art of lighting.

The objectives of the CIE are

- a) to provide an international forum for the discussion of all matters relating to science, technology and art in the fields of light and lighting and for the interchange of information between countries in these fields;
- b) to develop basic standards and procedures of metrology in the fields of light and lighting;
- c) to provide guidance on the application of principles and procedures in the development of International Standards and national standards in the fields of light and lighting;
- d) to prepare and publish standards, reports and other publications concerned with all matters relating to science, technology and art in the fields of light and lighting;
- e) to maintain liaison and technical interaction with other international organizations concerned with matters relating to science, technology, standardization and art in the fields of light and lighting.

Within these objectives, light and lighting embrace fundamental subjects such as vision, photometry and colorimetry, involving natural and man-made radiations in the ultraviolet, visible and infrared regions of the spectrum, and also applications covering all uses of light, indoors and out, including environmental and aesthetic effects, and also means for the production and control of light and radiation.

The technical activities of the CIE are covered by seven divisions, each being responsible for a major subject area of interest to the CIE. Technical Committees consisting of small groups of experts are established in each division to work on separate subjects. The text of this International Standard was prepared by Division 1: *Vision and Colour*. The ratification of a CIE Standard requires the approval of the division members, the Council and national member bodies of the CIE.

Standards produced by the CIE are a concise documentation of data defining aspects of light and lighting, for which international harmony requires a unique definition. CIE Standards are therefore a primary source of internationally accepted and agreed data, which can be taken, essentially unaltered, into universal standard systems.

ISO/CIE 10527 : 1991 (E)

International Standard ISO/CIE 10527 was prepared as Standard CIE S002 by the International Commission on Illumination, which has been recognized by the ISO Council as an international standardizing body. It was adopted by ISO under a special procedure which requires approval by at least 75 % of the member bodies casting a vote, and is published as a joint ISO/CIE edition.

International Standard ISO/CIE 10527 was prepared by Technical Committee CIE/TC 1.3, *Colorimetry*.

Annex A of this International Standard is for information only.

Introduction

Colours with different spectral compositions can look alike. An important function of colorimetry is to determine whether a pair of such metameric colours will look alike. The use of visual colorimeters for this purpose is handicapped by variations in the colour matches made amongst observers classified as having normal colour vision. Visual colorimetry also tends to be time-consuming. For these reasons, it has long been the practice in colorimetry to make use of sets of colour-matching functions to calculate tristimulus values for colours: equality of tristimulus values for a pair of colours indicates that the colour appearances of the two colours match, when they are viewed in the same conditions by an observer for whom the colour-matching functions apply. The use of standard sets of colour-matching functions makes the comparison of tristimulus values obtained at different times and locations possible.

CIE standard colorimetric observers

1 Scope

This International Standard specifies colour-matching functions for use in colorimetry. Two sets of colour-matching functions are specified.

- a) Colour-matching functions for the CIE 1931 standard colorimetric observer

This set of colour-matching functions is representative of the colour-matching properties of observers with normal colour vision for visual field sizes of angular subtense from about 1° to about 4° , for vision at photopic levels of adaptation.

- b) Colour-matching functions for the CIE 1964 supplementary standard colorimetric observer

This set of colour-matching functions is representative of the colour-matching properties of observers with normal colour vision for visual field sizes of angular subtense greater than about 4° , for vision at sufficiently high photopic levels and with spectral power distributions such that no participation of the rod receptors of the retina is to be expected.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

CIE Publication 15.2 : 1986, *Colorimetry*.

CIE Publication 17.4 : 1987, *International lighting vocabulary* (IEC/CIE joint publication).

3 Definitions

For the purposes of this International Standard, the following definitions apply. These terms are taken from CIE Publication 17.4, where other relevant terms will be found.

3.1 colour stimulus function, $\phi_\lambda(\lambda)$: Description of a colour stimulus by the spectral concentration of a radiometric quantity (such as radiance or radiant power) as a function of wavelength.

3.2 relative colour stimulus function, $\phi(\lambda)$: Relative spectral power distribution of the colour stimulus function.

3.3 metametric colour stimuli; metamers: Spectrally different colour stimuli that have the same tristimulus values.

3.4 monochromatic stimulus; spectral stimulus: A stimulus consisting of a monochromatic radiation.

3.5 equi-energy spectrum: Spectrum of a radiation whose spectral concentration of a radiometric quantity as a function of wavelength is constant throughout the visible region.

3.6 additive mixture of colour stimuli: Method of stimulation that combines on the retina the actions of various stimuli in such a manner that they cannot be perceived individually.

3.7 colour matching: Action of making a colour stimulus appear the same in colour as a given colour stimulus.

3.8 trichromatic system: System for specifying colour stimuli in terms of tristimulus values based on matching colours by additive mixture of three suitably chosen reference colour stimuli.

3.9 reference colour stimuli, [R], [G], [B]; [X], [Y], [Z]; [X]₁₀, [Y]₁₀, [Z]₁₀; etc.: The set of three colour stimuli on which a trichromatic system is based.

3.10 tristimulus values, R, G, B; X, Y, Z; X₁₀, Y₁₀, Z₁₀; etc.: Amounts of three reference colour stimuli, in a given trichromatic system, required to match the colour of the stimulus considered.

3.11 colour-matching functions, $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$; $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$; $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$; etc.: The tristimulus values of monochromatic stimuli of equal radiant power.

3.12 CIE 1931 standard colorimetric system (X, Y, Z): A system for determining the tristimulus values of any spectral power distribution using the set of reference colour stimuli [X], [Y], [Z], and the three CIE colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ adopted by the CIE in 1931.

3.13 CIE 1964 supplementary standard colorimetric system (X₁₀, Y₁₀, Z₁₀): A system for determining the tristimulus values of any spectral power distribution using the set of reference colour stimuli [X₁₀], [Y₁₀], [Z₁₀], and the three CIE colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ adopted by the CIE in 1964.

3.14 CIE colour-matching functions: The functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ in the CIE 1931 standard colorimetric system and $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ in the CIE 1964 supplementary standard colorimetric system.

3.15 CIE 1931 standard colorimetric observer: An ideal observer whose colour-matching properties correspond to the CIE colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$.

3.16 CIE 1964 supplementary standard colorimetric observer: An ideal observer whose colour-matching properties correspond to the CIE colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$.

3.17 chromaticity coordinates, *r, g, b*; *x, y, z*; *x₁₀, y₁₀, z₁₀*; etc.: Ratio of each of a set of three tristimulus values to their sum.

3.18 spectral chromaticity coordinates, *r(λ), g(λ), b(λ)*; *x(λ), y(λ), z(λ)*; *x₁₀(λ), y₁₀(λ), z₁₀(λ)*; etc.: Chromaticity coordinates of monochromatic stimuli.

3.19 spectral luminous efficiency, *V(λ)*: Ratio of the radiant flux at wavelength λ_m to that at wavelength λ , such that both radiations produce an equal visual response under specified photometric conditions and λ_m is chosen so that the maximum value of this ratio is equal to 1.

3.20 perfect reflecting diffuser: Ideal isotropic diffuser with a reflectance equal to unity.

4 Specifications

4.1 Colour-matching functions

The colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ of the CIE 1931 standard colorimetric observer are defined by the values given in table 1, and those $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$, of the CIE 1964 supplementary standard colorimetric observer are defined by the values given in table 2. The values are given at 1 nm wavelength intervals from 360 nm to 830 nm. If values are required at closer wavelength intervals than 1 nm, they should be derived by linear interpolation.

4.2 Spectral chromaticity coordinates

Tables 1 and 2 also give values for the spectral chromaticity coordinates, $x(\lambda)$, $y(\lambda)$, $z(\lambda)$; $x_{10}(\lambda)$, $y_{10}(\lambda)$, $z_{10}(\lambda)$; these have been derived from the appropriate colour-matching functions by forming the ratios:

$$x(\lambda) = \bar{x}(\lambda) / [\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)]$$

$$y(\lambda) = \bar{y}(\lambda) / [\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)]$$

$$z(\lambda) = \bar{z}(\lambda) / [\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)]$$

$$x_{10}(\lambda) = \bar{x}_{10}(\lambda) / [\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)]$$

$$y_{10}(\lambda) = \bar{y}_{10}(\lambda) / [\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)]$$

$$z_{10}(\lambda) = \bar{z}_{10}(\lambda) / [\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)]$$

NOTE — All wavelengths are for a vacuum.

5 Derivation of the colour-matching functions for the CIE 1931 standard colorimetric observer

5.1 Experimental basis

The CIE 1931 colour-matching functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, were derived from experimental work carried out by Wright^[6] and by Guild^[7] in which a total of 17 observers matched the monochromatic stimuli of the spectrum, over the range of about 400 nm to 700 nm, with additive mixtures of red, green and blue lights, using observing fields of 2° angular subtense.

5.2 Transformation procedures

The experimental results were converted into those that would have been obtained if the matching had been carried out using, as reference colour stimuli, monochromatic radiations of wavelengths 700 nm for the red [R], 546,1 nm for the green [G] and 435,8 nm for the blue [B], measured in units such that equal quantities of [R], [G] and [B] were required to match the equi-energy spectrum.

The results for the 17 observers were averaged and then slightly adjusted so that by adding together suitable proportions of the [R], [G], [B] colour-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, it was possible to obtain a function identical to that of the CIE spectral luminous efficiency, $V(\lambda)$; the proportions used were in the ratios of 1,000 0 to 4,590 7 to 0,060 1, and these were then the relative luminances of unit quantities of [R], [G] and [B]. The CIE 1931 colour-matching functions were then determined by the following equations:

$$\bar{x}(\lambda) = [0,49\bar{r}(\lambda) + 0,31\bar{g}(\lambda) + 0,20\bar{b}(\lambda)]n$$

$$\bar{y}(\lambda) = [0,176\,97\bar{r}(\lambda) + 0,812\,40\bar{g}(\lambda) + 0,010\,63\bar{b}(\lambda)]n$$

$$\bar{z}(\lambda) = [0,00\bar{r}(\lambda) + 0,01\bar{g}(\lambda) + 0,99\bar{b}(\lambda)]n$$

where n is a normalising constant given by

$$n = V(\lambda) / [0,176\,97\bar{r}(\lambda) + 0,812\,40\bar{g}(\lambda) + 0,010\,63\bar{b}(\lambda)].$$

n is a constant, not a function of wavelength, because the coefficients 0,176 97, 0,812 40, and 0,010 63 are in the same

ratios to one another as the ratio of 1,000 0 to 4,590 7 to 0,060 1; n is equal to:

$$(1,000\ 0 + 4,590\ 7 + 0,060\ 1) / (0,176\ 97 + 0,812\ 40 + 0,010\ 63) = 5,650\ 8$$

The values of $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ given in table 1 from 360 nm to 400 nm and from 700 nm to 830 nm are extrapolations.

5.3 Transformation properties

The transformation given in the above equations was chosen to achieve the following objectives. First, the $\bar{y}(\lambda)$ function is identical to the $V(\lambda)$ function. Second, the values of $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are all positive for all wavelengths of the spectrum (unlike $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, one of which is negative at most wavelengths because of the need to desaturate spectral stimuli when matching them with red, green, and blue reference stimuli). Third, the values of $\bar{z}(\lambda)$ are zero for wavelengths longer than 650 nm. Fourth, the values of $\bar{x}(\lambda)$ are nearly zero at wavelengths around 505 nm. Fifth, the values of $\bar{x}(\lambda)$ and $\bar{y}(\lambda)$ are small at the short-wavelength end of the spectrum. Sixth, the equi-energy spectrum is specified by equal amounts of X , Y and Z .

Because the $\bar{y}(\lambda)$ function is identical to the $V(\lambda)$ function, the Y tristimulus value is proportional to luminance.

5.4 Comparison with earlier data

The values of $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ given in table 1 for the spectral range of 380 nm to 780 nm at 5 nm intervals, when rounded to four decimal places, agree closely with those originally published in 1931. There are only three minor differences: at $\lambda = 775$ nm the new value of $\bar{x}(\lambda)$ is 0,000 1 instead of 0,000 0; at $\lambda = 555$ nm, $\bar{y}(\lambda) = 1,000 0$ instead of 1,000 2 and at $\lambda = 740$ nm, $\bar{y}(\lambda) = 0,000 2$ instead of 0,000 3. These changes are considered insignificant in most colorimetric computations.

When the relative luminances of unit quantities of $[R]$, $[G]$ and $[B]$ are deduced from the data of table 1, the values obtained are 1,000 0 to 4,588 8 to 0,060 3 instead of 1,000 0 to 4,590 7 to 0,060 1, the relative radiances being 71,893 8 to 1,374 7 to 1,000 0 instead of 72,096 2 to 1,379 1 to 1,000 0. These changes are also considered insignificant in practice.

6 Derivation of the colour-matching functions for the CIE 1964 supplementary standard colorimetric observer

6.1 Experimental basis

The CIE 1964 colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$, were derived from experimental work carried out by Stiles and Burch^[8] and by Speranskaya^[9] in which a total of 67 observers matched monochromatic stimuli of the spectrum from approximately 390 nm to 830 nm with additive mixtures of red, green, and blue lights, using observing fields of 10° angular subtense (but ignoring the central 4° or so).

6.2 Transformation procedures

The experimental results were converted into those that would have been obtained if the matching had been carried out using,

as reference colour stimuli, monochromatic radiations of wavenumbers $15\ 500\ \text{cm}^{-1}$ for the red $[R_{10}]$, $19\ 000\ \text{cm}^{-1}$ for the green $[G_{10}]$, and $22\ 500\ \text{cm}^{-1}$ for the blue $[B_{10}]$, corresponding approximately to wavelengths 645,2 nm, 526,3 nm and 444,4 nm respectively. The units used for the quantities of $[R_{10}]$, $[G_{10}]$ and $[B_{10}]$ were such that equal amounts were required to match the equi-energy spectrum. A weighted average of the results for the 67 observers was used to provide a set of colour-matching functions $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$. The CIE 1964 colour-matching functions were then derived by the following equations:

$$\bar{x}_{10}(\nu) = 0,341\ 080\ \bar{r}_{10}(\nu) + 0,189\ 145\ \bar{g}_{10}(\nu) + 0,387\ 529\ \bar{b}_{10}(\nu)$$

$$\bar{y}_{10}(\nu) = 0,139\ 058\ \bar{r}_{10}(\nu) + 0,837\ 460\ \bar{g}_{10}(\nu) + 0,073\ 316\ \bar{b}_{10}(\nu)$$

$$\bar{z}_{10}(\nu) = 0,000\ 000\ \bar{r}_{10}(\nu) + 0,039\ 553\ \bar{g}_{10}(\nu) + 2,026\ 200\ \bar{b}_{10}(\nu)$$

In table 2, the CIE 1964 colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ are given on a wavelength basis and were obtained by interpolation from the frequency-based functions given above. The values in the range of 360 nm to 390 nm are extrapolations.

6.3 Transformation properties

The transformation given in the equations in 6.2 was chosen to achieve a colorimetric system (X_{10} , Y_{10} , Z_{10}) having a coordinate system broadly similar to that of the CIE 1931 (X , Y , Z) system. However, in the 1964 system, the data were not constrained to fit the CIE $V(\lambda)$ spectral luminous efficiency function, and the Y_{10} tristimulus value is not proportional to luminance.

6.4 Comparison with earlier data

The values given in table 2 are the same as those given in CIE Publication No. 15 (1971).

7 Practical application of colour-matching functions for CIE standard colorimetric observers

7.1 Obtaining tristimulus values

The data given in tables 1 and 2 provide the tristimulus values and chromaticity coordinates of all monochromatic stimuli directly or by interpolation. For stimuli consisting of radiation of various wavelengths, the tristimulus values X , Y , Z and X_{10} , Y_{10} , Z_{10} are calculated by integration over the spectral range 360 nm to 830 nm using the following equations:

$$X = k \int \varphi_\lambda(\lambda) \bar{x}(\lambda) d\lambda \quad X_{10} = k_{10} \int \varphi_\lambda(\lambda) \bar{x}_{10}(\lambda) d\lambda$$

$$Y = k \int \varphi_\lambda(\lambda) \bar{y}(\lambda) d\lambda \quad Y_{10} = k_{10} \int \varphi_\lambda(\lambda) \bar{y}_{10}(\lambda) d\lambda$$

$$Z = k \int \varphi_\lambda(\lambda) \bar{z}(\lambda) d\lambda \quad Z_{10} = k_{10} \int \varphi_\lambda(\lambda) \bar{z}_{10}(\lambda) d\lambda$$

where

$\phi_\lambda(\lambda)$ is the colour stimulus function of the stimulus considered;

$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda), \bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ are the appropriate CIE colour-matching functions;

k and k_{10} are constants.

Tristimulus values are usually evaluated on a relative basis, and the constants, k and k_{10} are then chosen according to agreed conventions; however, it is essential that, for stimuli that will be considered together, the same value for k (or for k_{10}) be adopted, so that all the tristimulus values involved are assessed on the same basis. For reflecting object-colours, k and k_{10} shall be chosen so that Y and Y_{10} are equal to 100 for the perfect reflecting diffuser, and, for transmitting object-colours, so that Y and Y_{10} are equal to 100 for the perfect transmitter. In the case of primary light sources, if it is required that Y be equal to the absolute value of the photometric quantity, k shall be equal to K_m , the maximum spectral luminous efficacy (which is equal to 683 lm/W) and $\phi_\lambda(\lambda)$ shall then be the spectral concentration of the radiometric quantity corresponding to the photometric quantity required.

7.2 The basis for integration

The integration step in the equations in 7.1 implies additivity of colour matches: that is, if two colour stimuli $[C_1]$ and $[C_2]$ have

tristimulus values X_1, Y_1, Z_1 and X_2, Y_2, Z_2 , respectively, then the additive mixture of $[C_1]$ and $[C_2]$ will have tristimulus values $X_1 + X_2, Y_1 + Y_2, Z_1 + Z_2$. Experimental investigations have shown that, although additivity of this type sometimes fails to occur, the principle of additivity is sufficiently valid for predicting colour matches in most cases of importance in practical colorimetry.

7.3 Rod activity

The tristimulus values in the CIE 1964 supplementary standard colorimetric system are relevant only to those observing conditions where the luminances are sufficiently high and the spectral power distributions are such that no significant participation of the rod receptors of the retina is to be expected.

7.4 The use of restricted data

For most practical applications of colorimetry, it is sufficient to use values of colour-matching functions at less frequent intervals of wavelength than every 1 nm, covering a more restricted range of wavelengths than from 360 nm to 830 nm, and using fewer decimal places than are given in tables 1 and 2. Data and guidelines that facilitate such practice are given in CIE Publication No. 15.2, together with various other recommended procedures for practical colorimetry.

7.5 Standard of reflectance

The perfect reflecting diffuser is the CIE reference standard for the colorimetry of reflecting samples.

Table 1 — Colour-matching functions and chromaticity coordinates of CIE 1931 standard colorimetric observer

Wave-length λ nm	CIE colour-matching functions			Chromaticity coordinates		
	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$x(\lambda)$	$y(\lambda)$	$z(\lambda)$
360	0.000 129 900 0	0.000 003 917 000	0.000 606 100 0	0.175 56	0.005 29	0.819 15
61	0.000 145 847 0	0.000 004 393 581	0.000 680 879 2	0.175 48	0.005 29	0.819 23
62	0.000 163 802 1	0.000 004 929 604	0.000 765 145 6	0.175 40	0.005 28	0.819 32
63	0.000 184 003 7	0.000 005 532 136	0.000 860 012 4	0.175 32	0.005 27	0.819 41
64	0.000 206 690 2	0.000 006 208 245	0.000 966 592 8	0.175 24	0.005 26	0.819 50
365	0.000 232 100 0	0.000 006 965 000	0.001 086 000	0.175 16	0.005 26	0.819 58
66	0.000 260 728 0	0.000 007 813 219	0.001 220 586	0.175 09	0.005 25	0.819 66
67	0.000 293 075 0	0.000 008 767 336	0.001 372 729	0.175 01	0.005 24	0.819 75
68	0.000 329 388 0	0.000 009 839 844	0.001 543 579	0.174 94	0.005 23	0.819 83
69	0.000 369 914 0	0.000 011 043 23	0.001 734 286	0.174 88	0.005 22	0.819 90
370	0.000 414 900 0	0.000 012 390 00	0.001 946 000	0.174 82	0.005 22	0.819 96
71	0.000 464 158 7	0.000 013 886 41	0.002 177 777	0.174 77	0.005 23	0.820 00
72	0.000 518 986 0	0.000 015 557 28	0.002 435 809	0.174 72	0.005 24	0.820 04
73	0.000 581 854 0	0.000 017 442 96	0.002 731 953	0.174 66	0.005 24	0.820 10
74	0.000 655 234 7	0.000 019 583 75	0.003 078 064	0.174 59	0.005 22	0.820 19
375	0.000 741 600 0	0.000 022 020 00	0.003 486 000	0.174 51	0.005 18	0.820 31
76	0.000 845 029 6	0.000 024 839 65	0.003 975 227	0.174 41	0.005 13	0.820 46
77	0.000 964 526 8	0.000 028 041 26	0.004 540 880	0.174 31	0.005 07	0.820 62
78	0.001 094 949	0.000 031 531 04	0.005 158 320	0.174 22	0.005 02	0.820 76
79	0.001 231 154	0.000 035 215 21	0.005 802 907	0.174 16	0.004 98	0.820 86
380	0.001 368 000	0.000 039 000 00	0.006 450 001	0.174 11	0.004 96	0.820 93
81	0.001 502 050	0.000 042 826 40	0.007 083 216	0.174 09	0.004 96	0.820 95
82	0.001 642 328	0.000 046 914 60	0.007 745 488	0.174 07	0.004 97	0.820 96
83	0.001 802 382	0.000 051 589 60	0.008 501 152	0.174 06	0.004 98	0.820 96
84	0.001 995 757	0.000 057 176 40	0.009 414 544	0.174 04	0.004 98	0.820 98
385	0.002 236 000	0.000 064 000 00	0.010 549 99	0.174 01	0.004 98	0.821 01
86	0.002 535 385	0.000 072 344 21	0.011 965 80	0.173 97	0.004 97	0.821 06
87	0.002 892 603	0.000 082 212 24	0.013 655 87	0.173 93	0.004 94	0.821 13
88	0.003 300 829	0.000 093 508 16	0.015 588 05	0.173 89	0.004 93	0.821 18
89	0.003 753 236	0.000 106 136 1	0.017 730 15	0.173 84	0.004 92	0.821 24
390	0.004 243 000	0.000 120 000 0	0.020 050 01	0.173 80	0.004 92	0.821 28
91	0.004 762 389	0.000 134 984 0	0.022 511 36	0.173 76	0.004 92	0.821 32
92	0.005 330 048	0.000 151 492 0	0.025 202 88	0.173 70	0.004 94	0.821 36
93	0.005 978 712	0.000 170 208 0	0.028 279 72	0.173 66	0.004 94	0.821 40
94	0.006 741 117	0.000 191 816 0	0.031 897 04	0.173 61	0.004 94	0.821 45
395	0.007 650 000	0.000 217 000 0	0.036 210 00	0.173 56	0.004 92	0.821 52
96	0.008 751 373	0.000 246 906 7	0.041 437 71	0.173 51	0.004 90	0.821 59
97	0.010 028 88	0.000 281 240 0	0.047 503 72	0.173 47	0.004 86	0.821 67
98	0.011 421 70	0.000 318 520 0	0.054 119 88	0.173 42	0.004 84	0.821 74
99	0.012 869 01	0.000 357 266 7	0.060 998 03	0.173 38	0.004 81	0.821 81
400	0.014 310 00	0.000 396 000 0	0.067 850 01	0.173 34	0.004 80	0.821 86
01	0.015 704 43	0.000 433 714 7	0.074 486 32	0.173 29	0.004 79	0.821 92
02	0.017 147 44	0.000 473 024 0	0.081 361 56	0.173 24	0.004 78	0.821 98
03	0.018 781 22	0.000 517 876 0	0.089 153 64	0.173 17	0.004 78	0.822 05
04	0.020 748 01	0.000 572 218 7	0.098 540 48	0.173 10	0.004 77	0.822 13
405	0.023 190 00	0.000 640 000 0	0.110 200 0	0.173 02	0.004 78	0.822 20
06	0.026 207 36	0.000 724 560 0	0.124 613 3	0.172 93	0.004 78	0.822 29
07	0.029 782 48	0.000 825 500 0	0.141 701 7	0.172 84	0.004 79	0.822 37
08	0.033 880 92	0.000 941 160 0	0.161 303 5	0.172 75	0.004 80	0.822 45
09	0.038 468 24	0.001 069 880	0.183 256 8	0.172 66	0.004 80	0.822 54