
Identification cards — Test methods —

Part 6: Proximity cards

AMENDMENT 5: Bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ from PCD to PICC

Cartes d'identification — Méthodes d'essai —

Partie 6: Cartes de proximité

*AMENDEMENT 5: Débits binaires de $3fc/4$, fc , $3fc/2$ et $2fc$ de PCD à
PICC*



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Foreword

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The main task of the joint technical committee is to prepare International Standards. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

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Amendment 5 to ISO/IEC 10373-6:2011 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 17, *Cards and personal identification*.

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Part 6:

Proximity cards

AMENDMENT 5: Bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ from PCD to PICC

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Add new subclause

7.3 Test methods for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ from PCD to PICC

See [Annex J](#).

After Annex I

Add following new annex:

Annex J (normative)

Test methods for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ from PCD to PICC

J.1 Overview

This annex specifies the test methods for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ from PCD to PICC.

NOTE Future revisions of ISO/IEC 14443 and ISO/IEC 10373-6 may specify new NPV tolerance and phase noise values with corresponding test methods.

J.2 Test of ISO/IEC 14443-2 parameters

J.2.1 PCD Tests

All the tests described below will be done in the operating volume as defined by the PCD manufacturer.

J.2.1.1 PCD phase range and waveform characteristics

J.2.1.1.1 Purpose

This test is used to determine the PR as well as the normalized differential phase noise and inter-symbol interference parameters, ISI_m and ISI_d , as defined in ISO/IEC 14443-2:2010/Amd.5.

J.2.1.1.2 Test procedure

Apply the procedure defined in 7.1.4.2 with the following adaptations:

- After the activation of a bit rate of $3fc/4$, fc , $3fc/2$ or $2fc$, the PCD shall transmit an $I(0)_0$ (TEST_COMMAND1(1)).
- In steps a) and f) of 7.1.4.2, the waveform characteristics shall be determined using the analysis tool defined in [J.3](#).

J.2.1.1.3 Test report

The test report shall give the measured PR, ISI_m , ISI_d and the normalized differential phase noise values of the PCD field, within the defined operating volume in unloaded and loaded conditions.

NOTE [J.3.12](#) gives some example test reports.

J.2.2 PICC Tests

J.2.2.1 PICC reception

J.2.2.1.1 Purpose

The purpose of this test is to verify the ability of the PICC to receive PCD commands for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$.

J.2.2.1.2 Test conditions

Four test conditions are defined at the border of the PICC signal parameters as defined in ISO/IEC 14443-2:2010/Amd.5. A low pass filtered pseudo-random white noise as defined in [J.4.3](#) is added to the transmitted APVs such that the normalized differential phase noise (rms) is the maximum value as defined in ISO/IEC 14443-2:2010/Amd.5. The test conditions are created using the test PCD assembly in combination with digital pre-conditioning of the transmitted APVs as shown in [J.4](#):

- Condition 1: the test PCD signal is digitally pre-conditioned to have the maximum ISI_m value for the ISI_d value of 45° as defined in ISO/IEC 14443-2:2010/Amd.5;
- Condition 2: the test PCD signal is digitally pre-conditioned to have the maximum ISI_m value for the ISI_d value of -45° as defined in ISO/IEC 14443-2:2010/Amd.5;
- Condition 3: the test PCD signal is digitally pre-conditioned to have the maximum ISI_m value for the ISI_d value of 120° as defined in ISO/IEC 14443-2:2010/Amd.5;
- Condition 4: the test PCD signal is digitally pre-conditioned to have the maximum ISI_m value for the ISI_d value of 0° as defined in ISO/IEC 14443-2:2010/Amd.5.

NOTE 1 These conditions are applied after switching to the bit rate under test.

NOTE 2 [J.4](#) informatively describes how to create the above 4 conditions in the base-band domain (on the complex envelope of the signal).

These 4 test conditions shall be tested at least using H_{min} and H_{max} .

J.2.2.1.3 Test procedure

For each supported bit rate of $3fc/4$, fc , $3fc/2$ and $2fc$, the PICC shall operate under the defined conditions after the selection of a bit rate. This PICC shall respond correctly to an $I(0)_0(\text{TEST_COMMAND1}(1))$ transmitted at the specified bit rate.

The activation of the bit rates uses $S(\text{PARAMETERS})$ mechanism as defined in ISO/IEC 14443-4:2008/Amd.3.

NOTE For a frame size higher than 256 bytes a frame with error correction as defined in ISO/IEC 14443-4:2008/Amd.4 should be used.

J.2.2.1.4 Test report

The test report shall confirm the intended operation at the bit rates under test. Used test conditions shall be mentioned in the test report.

J.3 PCD waveform characteristics analysis tool for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$

J.3.1 Overview

The working principle of the analysis tool for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ is illustrated in [Figure J.1](#).

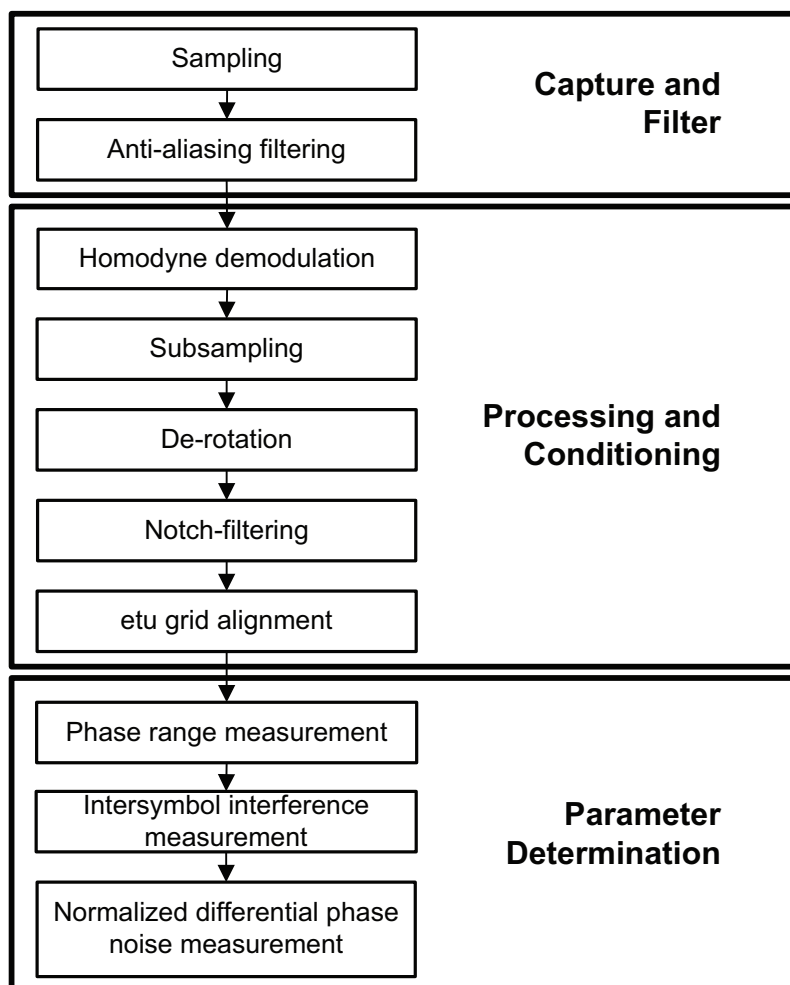


Figure J.1 — Block diagram of the analysis tool for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$

Each block is separately described in the subsequent clauses.

J.3.2 Sampling

The oscilloscope used for signal capturing shall fulfill the requirements defined in 5.1.1. The time and voltage data of at least 1000 non-modulated carrier periods followed by one data frame, followed by at least 10 non-modulated carrier periods (see illustration in [Figure J.2](#)) shall be transferred to a suitable computer.

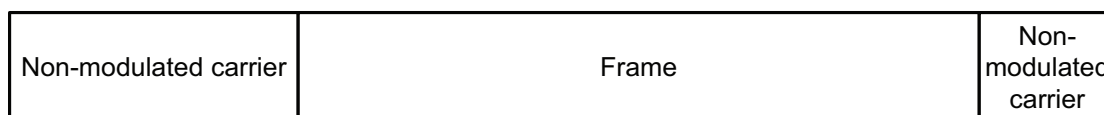


Figure J.2 — Non-modulated carrier followed by one frame, followed by non-modulated carrier

J.3.3 Anti-aliasing filtering

A 4th order, Butterworth type low pass filter with 3-dB cut off frequency at 120 MHz shall be used for filtering higher frequency components. The filter characteristic is illustrated in [Figure J.3](#).

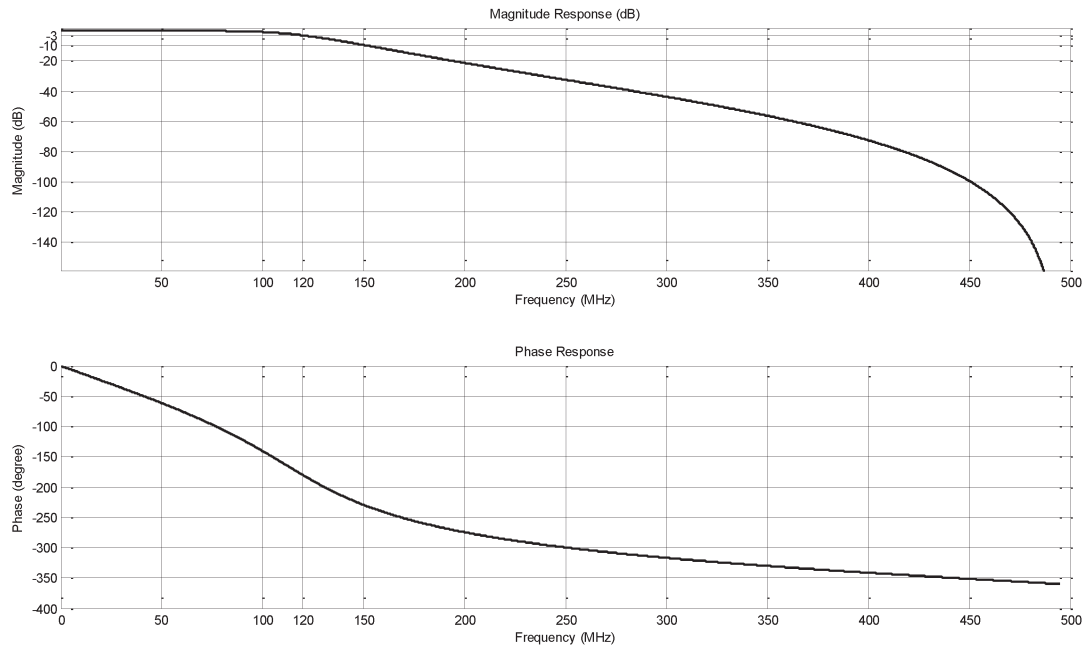


Figure J.3 — Anti-aliasing filter characteristics

J.3.4 Homodyne demodulation

The signal shall be demodulated using a homodyne demodulator (IQ demodulator) and the argument of this complex transform represents the phase signal over time (see [Figure J.4](#)).

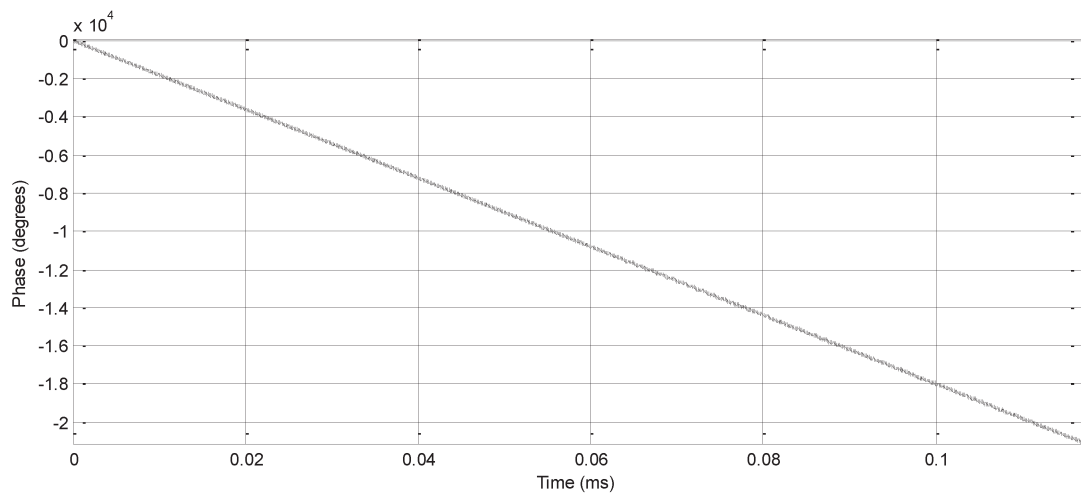


Figure J.4 — Example phase signal over time after homodyne demodulation

J.3.5 Subsampling

The phase signal shall be sub-sampled to an integer number multiple of f_c using linear interpolation. The integer number shall be at least 32.

J.3.6 De-rotation

This phase signal over time is continuously changing due to the difference between the modulated RF carrier frequency and the demodulator frequency. This frequency mismatch is computed from the constant phase slope of the phase signal during the time of the non-modulated carrier. The complete

phase signal is multiplied by a carrier signal whose frequency is the computed frequency difference (see [Figure J.5](#)).

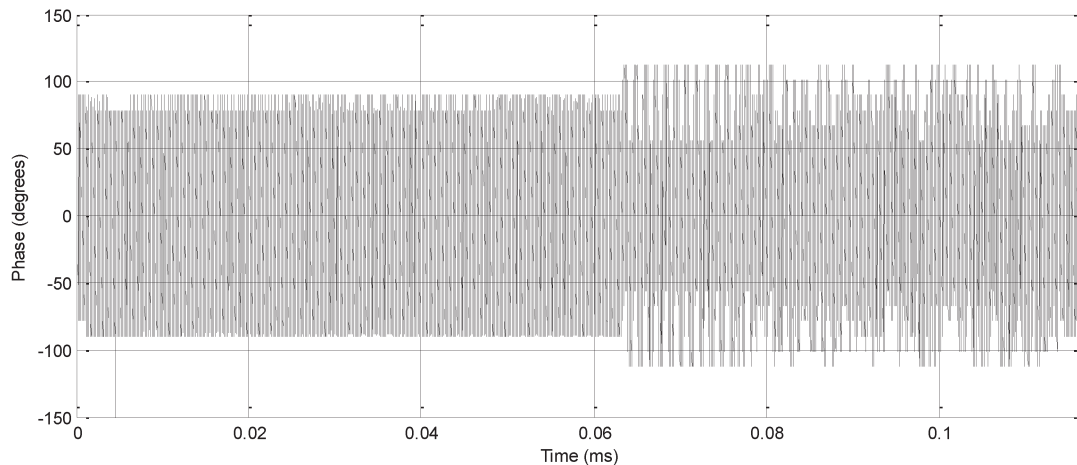


Figure J.5 — Example phase signal after de-rotation

J.3.7 Notch-filtering

The phase signal contains the second harmonic due to demodulation. The phase signal shall be smoothed with a moving average filter having a filter period of $2/f_c$ (see [Figure J.6](#)).

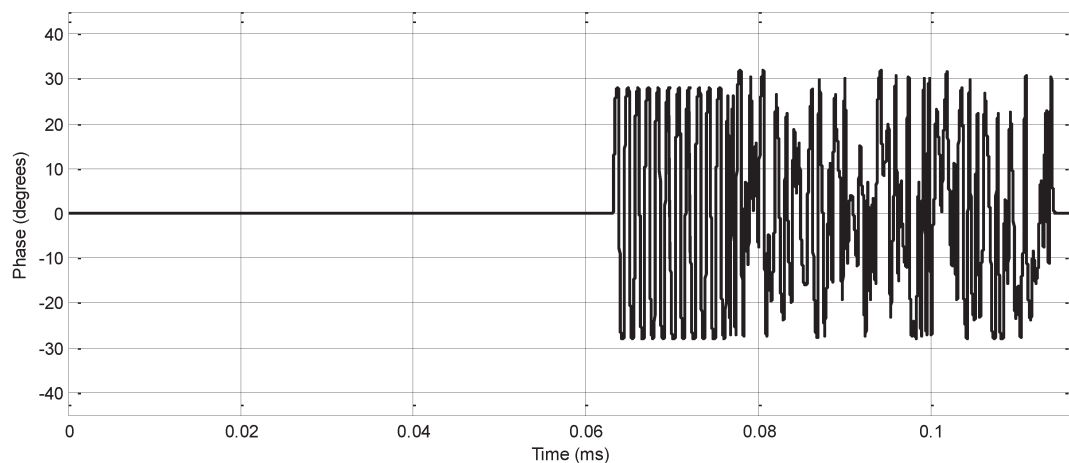


Figure J.6 — Example phase signal after filtering

J.3.8 etu grid alignment

The phase signal shall be aligned to the etu grid of the reference phase signal. The reference phase signal is computed from the SOC using the method defined in ISO/IEC 14443-2:2010/Amd.5. The etu grid alignment is carried out by maximizing the computed correlation, using the phase signal and the reference phase signal.

J.3.9 Phase range measurement

The PR parameter shall be determined as defined in ISO/IEC 14443-2:2010/Amd.5.

J.3.10 Intersymbol interference measurement

The ISI_m and ISI_d parameters shall be determined from the system identification coefficients. The system identification coefficients shall be determined by solving the system identification problem given by the phase signal and the reference phase signal using the Linear Least Squares method. ISI_m and ISI_d values shall be computed for every sampling time within the last carrier period of an etu. The maximum ISI_m value shall be selected with the related ISI_d .

J.3.11 Normalized differential phase noise measurement

The normalized differential phase noise shall be determined during a section of a non-modulated carrier of at least 500 carrier periods according to the definition in ISO/IEC 14443-2:2010/Amd.5.

J.3.12 Program of the PCD waveform characteristics analysis tool for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$ (informative)

The following program written in ANSIC language gives an example for the implementation of the analysis tool for bit rates of $3fc/4$, fc , $3fc/2$ and $2fc$.

This ANSIC implementation consists of 7 files which should be placed in the same folder.

[illegible]

```

7, 7, 1, 7, 1, 7, 0, 0, 7, 3, 6, 1, 5, 3,
6, 2, 2, 2, 7, 1, 0, 3, 5, 2, 3, 5, 2, 3,
6, 0, 7, 2, 3, 3, 7, 6, 4, 5, 6, 1, 6, 5,
2, 6, 1, 3, 4, 0, 2, 0, 6, 6, 7, 0, 5, 7,
3, 7, 3, 0, 3, 6, 6, 1, 1, 0, 6, 4, 0, 6,
3, 5, 6, 1, 1, 1, 2, 6, 7, 0, 7, 0, 7, 3,
1, 2, 4, 2, 1, 5, 7, 4, 0, 3, 3, 2, 3, 4};

```

```

static const psk_uint32 PSK16[] = {1, 1, 15, 15, 1, 1, 15, 15, 1, 1,
15, 15, 1, 1, 15, 15, 1, 1, 15, 15,
1, 1, 15, 15, 1, 1, 15, 15, 1, 1,
15, 15, 1, 1, 15, 15, 1, 1, 15, 15,
1, 1, 15, 15, 1, 15, 1, 15, 0, 0,
15, 6, 11, 0, 8, 4, 10, 1, 0, 15,
9, 13, 11, 1, 4, 13, 14, 2, 11, 13,
3, 7, 4, 10, 12, 12, 4, 1, 13, 15,
0, 6, 15, 12, 5, 12, 1, 4, 6, 13,
0, 11, 7, 7, 9, 11, 4, 7, 15, 6,
13, 7, 12, 1, 0, 6, 5, 3, 14, 9,
0, 12, 6, 10, 11, 0, 15, 14, 15, 6,
15, 0, 15, 0, 15, 7, 2, 4, 8, 3,
0, 7, 11, 5, 13, 2, 1, 14, 15, 0};

```

```

static const psk_int32 SOC_PSK8_deg[] = { 0, 24, 24, -24, -24, 24, 24,
-24, -24, 24, 24, -24, -24, 24,
24, -24, -24, 24, 24, -24, -24,
24, 24, -24, -24, 24, 24, -24,
-24, 24, 24, -24, -24, 24, 24,
-24, -24, 24, 24, -24, -24, 24,
24, -24, -24, 24, -24, 24, -24,
32, 32, -24, 8, -16, 24, -8,
8, -16, 16, 16, 16, -24, 24,
32, 8, -8, 16, 8, -8, 16,
8, -16, 32, -24, 16, 8, 8,
-24, -16, 0, -8, -16, 24, -16,
-8, 16, -16, 24, 8, 0, 32,
16, 32, -16, -16, -24, 32, -8,
-24, 8, -24, 8, 32, 8, -16,
-16, 24, 24, 32, -16, 0, 32,
-16, 8, -8, -16, 24, 24, 24,
16, -16, -24, 32, -24, 32, -24,
8, 24, 16, 0, 16, 24, -8,
-24, 0, 32, 8, 8, 16, 8, 0};

```

```

static const psk_int32 SOC_PSK16_deg[] = { 0, 28, 28, -28, -28, 28, 28,
-28, -28, 28, 28, -28, -28, 28,
28, -28, -28, 28, 28, -28, -28,
28, 28, -28, -28, 28, 28, -28,
-28, 28, 28, -28, -28, 28, 28,
-28, -28, 28, 28, -28, -28, 28,
28, -28, -28, 28, -28, 28, -28,
32, 32, -28, 8, -12, 32, 0,
16, -8, 28, 32, -28, -4, -20,
-12, 28, 16, -20, -24, 24, -12,
-20, 20, 4, 16, -8, -16, -16,
16, 28, -20, -28, 32, 8, -28,
-16, 12, -16, 28, 16, 8, -20,
32, -12, 4, 4, -4, -12, 16,
4, -28, 8, -20, 4, -16, 28,
32, 8, 12, 20, -24, -4, 32,
-16, 8, -8, -12, 32, -28, -24,
-28, 8, -28, 32, -28, 32, -28,
4, 24, 16, 0, 20, 32, 4,
-12, 12, -20, 24, 28, -24, -28, 32};

```

```
static const psk_uint32 PSK_len = 141;
```

```

// output data types
enum TYPE
{

```

```

    COMPLEX,
    DOUBLE,
    INTEGER,
    BUTTERCFS
};

/*****
**
*   The order of the input psk signal is either 16 or 8
*/
extern psk_uint32 ORDER;

/*****
**
*   The bit rate of the input signal is a user defined parameter and can be
*   3/4 * fc
*   fc
*   3/2 * fc
*   2 * fc
*   where fc is the carrier frequency as defined in #FC.
*/
extern psk_double BIT_RATE;

/*****
**
*   The phase range is the difference between the highest an lowest phase value
*   and can be either 56 deg or 60 deg
*/
extern psk_uint32 PR;

/*****
**
*   The elementary phase interval. 8 deg or 4 deg depending on #ORDER
*/
extern psk_uint32 EPI;

/*****
**
*   The elementary time unit is always a multiple of #FC.
*   2/FC for bit rates 1.5*FC and 2.0*FC and 4/FC else
*/
extern psk_double ETU;

#endif // PSK_DEFINES_H

/*****
***   psk_types.h                                     ***
***   DESCRIPTION:                                     ***
***   Definition of used types in psk analysis tool     ***
***                                                    ***
*****/

#ifndef PSK_TYPES_H
#define PSK_TYPES_H

#define RE(z) ( (z).re )
#define IM(z) ( (z).im )
#define ABS(a) ( (a > 0) ? (a) : (-a) )
#define MAX(a, b) ( (a > b) ? (a) : (b) )

#define BUTTER_SIZE_A 5
#define BUTTER_SIZE_B 5

typedef double psk_double;
typedef int psk_int32;
typedef unsigned int psk_uint32;

typedef struct

```

```

{
    psk_double re;
    psk_double im;
} psk_complex;

typedef struct
{
    psk_double a[BUTTER_SIZE_A];
    psk_double b[BUTTER_SIZE_B];
} psk_butter_coefs;

#endif // PSK_TYPES_H

/*****/
/** psk_math.h ****/
/** DESCRIPTION: header of psk_math.c ****/
/** It contains the function declaration of used mathematical functions ****/
/** for the PSK waveform characteristics analysis tool ****/
/*****/

#ifndef PSK_MATH_H
#define PSK_MATH_H

#include "psk_types.h"

/*****/
/**
 * psk_mean
 * calculate the arithmetic mean of a given vector
 * @param vec Calculate the mean of the values in this vector
 * @param len The vector's number of elements
 * @return The arithmetic mean or zero, if len < 2
 */
psk_double psk_mean( psk_double* vec /*[in]*/, psk_uint32 len /*[in]*/);

/*****/
/**
 * psk_cmpl_mean
 * calculate the arithmetic mean of a given vector for both, real and
 * imaginary parts. The result is also a complex number
 * @param vec Calculate the mean of the values in this vector
 * @param len The vector's number of elements
 * @return The arithmetic mean or zero, if len < 2
 */
psk_complex psk_cmpl_mean( psk_complex* vec/*[in]*/, psk_uint32 len /*[in]*/);

/*****/
/**
 * psk_diff
 * The resulting vector's elements are the differences of two consecutive
 * elements of a given vector. The resulting vector has a length of len-1
 * @param vec Calculate the consecutive differences of this vector's values
 * @param len The vector's number of elements
 * @return The arithmetic mean or zero, if len < 2
 */
psk_double* psk_diff( psk_double* vec /*[in]*/, psk_uint32 len /*[in]*/);

/*****/
/**
 * psk_max
 * Find the maximal value in a vector and it's index
 * @param vec An array of values
 * @param vec_len The vector's number of elements
 * @param max_val Pointer where to store the maximum's value
 * @param max_idx Pointer where to store the maximum's index
 */
void psk_max( psk_double* vec,          /*[in]*/

```

```

        psk_uint32 vec_len,      /*[in]*/
        psk_double* max_val,    /*[out]*/
        psk_uint32* max_idx ); /*[out]*/

/****
* psk_min
* Find the minimal value in a vector and it's index
* @param vec An array of values
* @param vec_len The vector's number of elements
* @param min_val Pointer where to store the minimum's value
* @param min_idx Pointer where to store the minimum's index
*/
void psk_min( psk_double* vec,      /*[in]*/
              psk_uint32 vec_len,  /*[in]*/
              psk_double* min_val, /*[out]*/
              psk_uint32* min_idx ); /*[out]*/

/****
* psk_add
* Calculate the sum of two complex numbers
* @param a First summand
* @param b Second summand
* @return The complex result
*/
psk_complex psk_add( psk_complex a /*[in]*/, psk_complex b /*[in]*/ );

/****
* psk_sub
* Calculate the difference of two complex numbers
* @param a Minuend
* @param b Subtrahend
* @return The complex result
*/
psk_complex psk_sub( psk_complex a /*[in]*/, psk_complex b /*[in]*/ );

/****
* psk_cmpl_mult
* Calculate the product of two complex numbers
* @param a First factor
* @param b Second factor
* @return The complex result
*/
psk_complex psk_cmpl_mult( psk_complex a /*[in]*/, psk_complex b /*[in]*/ );

/****
* psk_cmpl_div
* Calculate the quotient of two complex numbers
* @param a Dividend
* @param b Divisor
* @return The complex result
*/
psk_complex psk_cmpl_div( psk_complex a /*[in]*/, psk_complex b /*[in]*/ );

/****
* psk_cmpl_conj
* Get the complex conjugate of a number
* @param a The complex number
* @return The complex conjugate of a
*/
psk_complex psk_cmpl_conj( psk_complex a /*[in]*/ );

```

```

/*****/
/**
 * psk_abs
 * Get the absolute value of a complex number: sqrt(RE^2+IM^2)
 * @param num The complex number
 * @return The absolute value
 */
psk_double psk_abs( psk_complex num /*[in]*/ );

/*****/
/**
 * psk_cmpl_vec_mult
 * Calculate the product of vector elements with the same index and return
 * the result in a vector. This function makes use of #psk_cmpl_mult
 * The two vectors must have the same length
 * @param a Factors of first vector
 * @param b Factors of second vector
 * @param len The number of elements. Must be the same for a and b
 * @return The complex product of a and b
 */
psk_complex* psk_cmpl_vec_mult( psk_complex* a,      /*[in]*/
                               psk_complex* b,      /*[in]*/
                               psk_uint32 len ); /*[in]*/

/*****/
/**
 * psk_vec_abs
 * Apply #psk_abs on every complex element of a vector
 * @param a The complex input values
 * @param len The number of elements in vector a
 * @return The absolute values
 */
psk_double* psk_vec_abs( psk_complex* a /*[in]*/, psk_uint32 len /*[in]*/ );

/*****/
/**
 * psk_cmpl_vec_conj
 * Apply #psk_cmpl_conj on every complex element of a vector
 * @param vec The complex input values
 * @param len The number of elements in vector vec
 * @return The complex conjugate of vec
 */
psk_complex* psk_cmpl_vec_conj( psk_complex* vec, /*[in]*/
                                psk_uint32 len); /*[in]*/

/*****/
/**
 * psk_vec_angle
 * Apply the built in function atan2 on every complex element of a vector
 * @param vec The complex input values
 * @param len The number of elements in vector vec
 * @return The angle of every input element
 */
psk_double* psk_vec_angle( psk_complex* vec /*[in]*/, psk_uint32 len /*[in]*/ );

/*****/
/**
 * psk_variance
 * Calculate the variance of the elements in a vector
 * @param vec The real number input values
 * @param len The number of elements in vector vec
 * @return The variance of the vector's elements
 */
psk_double psk_variance( psk_double* vec /*[in]*/, psk_uint32 len /*[in]*/ );

```

```

/*****
**
* psk_std
* Calculate the standard deviation of the elements in a vector
* This function uses #psk_variance and returns it's square root
* @param vec The real number input values
* @param len The number of elements in vector vec
* @return The standard deviation of the vector's elements
*/
psk_double psk_std( psk_double* vec /*[in]*/, psk_uint32 len /*[in]*/ );

/*****
**
* psk_normalize
* Normalize a vector's elements. This function uses #psk_vec_abs and #psk_max
* @param vec The real number input values.
* @param len The number of elements in vector vec
* @return The normalized vector
*/
psk_double* psk_normalize( psk_double* vec, /*[in]*/
                          psk_uint32 len ); /*[in]*/

/*****
**
* psk_cmpl_normalize
* Normalize a vector's complex elements both real and imaginary part separatly
* @param vec The complex number input values.
* @param len The number of elements in vector vec
* @return The normalized vector
*/
psk_complex* psk_cmpl_normalize( psk_complex* vec, /*[in]*/
                                 psk_uint32 len ); /*[in]*/

/*****
**
* psk_linspace
* Build a vector with a given start and stop value and defined step size
* @param start The value of the first element
* @param step The step size. Difference between two consecutive elements
* @param stop The value of the last element
* @param len Pointer where to save the length of the vector
* @return The resulting vector or NULL, if start >= stop
*/
psk_double* psk_linspace( psk_double start, /*[in]*/
                          psk_double step, /*[in]*/
                          psk_double stop, /*[in]*/
                          psk_uint32* len ); /*[out]*/

/*****
**
* psk_idx_linspace
* Build a vector with a given start and stop value and defined step size.
* This function uses integer values, so the resulting vector can be used
* as a indexing vector for signals.
* @param start The value of the first element
* @param step The step size. Difference between two consecutive elements
* @param stop The value of the last element
* @param len Pointer where to save the length of the vector
* @return The resulting vector or NULL, if start >= stop
*/
psk_uint32* psk_idx_linspace( psk_uint32 start, /*[in]*/
                              psk_uint32 step, /*[in]*/
                              psk_uint32 stop, /*[in]*/
                              psk_uint32* len ); /*[out]*/

#endif //PSK_MATH_H

```

```

/*****
*** psk_math.c ***
*** DESCRIPTION: Implementation of psk_math.h ***
*** Contains the function implementation of used mathematical functions ***
*** for the VHBR PSK wave shape analysis tool ***
*****/

#include <stdlib.h>
#include <math.h>

#include "psk_defines.h"
#include "psk_types.h"
#include "psk_math.h"

//-----
psk_double psk_mean( psk_double* vec, psk_uint32 len )
{
    psk_double sum = 0.0;
    psk_uint32 idx;

    if( len < 2 )
        return sum;

    for( idx = 0; idx < len; idx++ )
        sum += vec[idx];

    sum /= (double)len;

    return( sum );
}

//-----
psk_complex psk_cmpl_mean( psk_complex* vec, psk_uint32 len )
{
    psk_complex mean;
    RE( mean ) = 0.0;
    IM( mean ) = 0.0;
    psk_uint32 idx;

    if( len < 2 )
        return mean;

    for( idx = 0; idx < len; idx++ )
    {
        RE( mean ) += RE( vec[idx] );
        IM( mean ) += IM( vec[idx] );
    }

    RE( mean ) /= (double)len;
    IM( mean ) /= (double)len;

    return( mean );
}

//-----
psk_double* psk_diff( psk_double* vec , psk_uint32 len )
{
    psk_double* diff = calloc( ( len - 1 ), sizeof(psk_double) );
    psk_uint32 idx;

    for( idx = 0; idx < ( len - 1 ); idx++ )
        diff[idx] = vec[idx + 1] - vec[idx];

    return( diff );
}

//-----
void psk_max( psk_double* vec,
              psk_uint32 vec_len,

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        psk_double* max_val,
        psk_uint32* max_idx )
{
    if( vec == NULL )
        return;
    psk_uint32 idx;
    *max_val = vec[0];
    *max_idx = 0;

    for( idx = 1; idx < vec_len; idx++ )
    {
        if( vec[idx] > *max_val )
        {
            *max_val = vec[idx];
            *max_idx = idx;
        }
    }
}

//-----
void psk_min( psk_double* vec,
              psk_uint32 vec_len,
              psk_double* min_val,
              psk_uint32* min_idx )
{
    psk_uint32 idx;
    *min_val = vec[0];
    *min_idx = 0;

    for( idx = 1; idx < vec_len; idx++ )
    {
        if( vec[idx] < *min_val )
        {
            *min_val = vec[idx];
            *min_idx = idx;
        }
    }
}

//-----
psk_complex psk_add( psk_complex a, psk_complex b )
{
    psk_complex sum;
    RE( sum ) = RE( a ) + RE( b );
    IM( sum ) = IM( a ) + IM( b );

    return( sum );
}

//-----
psk_complex psk_sub( psk_complex a, psk_complex b )
{
    psk_complex diff;
    RE( diff ) = RE( a ) - RE( b );
    IM( diff ) = IM( a ) - IM( b );

    return( diff );
}

//-----
psk_complex psk_cmpl_mult( psk_complex a, psk_complex b )
{
    psk_complex prod;
    RE( prod ) = RE( a ) * RE( b ) - IM( a ) * IM( b );
    IM( prod ) = IM( a ) * RE( b ) + RE( a ) * IM( b );

    return( prod );
}

//-----
psk_complex psk_cmpl_div( psk_complex a, psk_complex b )

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{
    psk_complex quot;
    RE( quot ) = RE( a ) * RE( b ) + IM( a ) * IM( b );
    RE( quot ) = RE( quot ) / ( RE( b ) * RE( b ) + IM( b ) * IM( b ) );

    IM( quot ) = IM( a ) * RE( b ) - RE( a ) * IM( b );
    IM( quot ) = IM( quot ) / ( RE( b ) * RE( b ) + IM( b ) * IM( b ) );
    return( quot );
}

//-----
psk_complex psk_cmpl_conj( psk_complex a )
{
    psk_complex conj;
    RE( conj ) = RE( a );
    IM( conj ) = -IM( a );

    return( conj );
}

//-----
psk_double psk_abs( psk_complex num )
{
    return( sqrt( pow( RE( num ), 2 ) + pow( IM( num ), 2 ) ) );
}

//-----
psk_complex* psk_cmpl_vec_mult( psk_complex* a,
                                psk_complex* b,
                                psk_uint32 len )
{
    psk_complex *prod_vec = calloc( len, sizeof(psk_complex) );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )
        prod_vec[idx] = psk_cmpl_mult( a[idx], b[idx] );

    return( prod_vec );
}

//-----
psk_double* psk_vec_abs( psk_complex* a, psk_uint32 len )
{
    psk_double *abs_vec = calloc( len, sizeof(psk_double) );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )
        abs_vec[idx] = psk_abs( a[idx] );

    return( abs_vec );
}

//-----
psk_complex* psk_cmpl_vec_conj( psk_complex* vec, psk_uint32 len )
{
    psk_complex *conj_vec = calloc( len, sizeof(psk_complex) );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )
        conj_vec[idx] = psk_cmpl_conj( vec[idx] );

    return( conj_vec );
}

//-----
psk_double* psk_vec_angle( psk_complex* vec, psk_uint32 len )
{
    psk_double *angle_vec = calloc( len, sizeof(psk_double) );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )

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    angle_vec[idx] = atan2( IM( vec[idx] ), RE( vec[idx] ) );

    return( angle_vec );
}

//-----
psk_double psk_variance( psk_double* vec, psk_uint32 len )
{
    psk_double variance = 0.0;
    psk_double mean = psk_mean( vec, len );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )
        variance += pow( vec[idx] - mean, 2.0 );
    variance /= (double)len;

    return( variance );
}

//-----
psk_double psk_std( psk_double* vec, psk_uint32 len )
{
    psk_double std_deviation = 0.0;
    std_deviation = psk_variance( vec, len );
    return( sqrt( std_deviation ) );
}

//-----
psk_double* psk_normalize( psk_double* vec, psk_uint32 len )
{
    psk_double max_val = ABS( vec[0] );
    psk_double* norm_vec = calloc( len, sizeof(psk_double) );
    psk_uint32 idx;

    for( idx = 0; idx < len; idx++ )
    {
        if ( ABS( vec[idx] ) > max_val )
            max_val = ABS( vec[idx] );
    }

    for( idx = 0; idx < len; idx++ )
        norm_vec[idx] = vec[idx] / max_val;

    return( norm_vec );
}

//-----
psk_complex* psk_cmpl_normalize( psk_complex* vec, psk_uint32 len )
{
    psk_double max_abs = psk_abs( vec[0] );
    psk_complex* norm_vec = calloc( len, sizeof(psk_complex) );
    psk_uint32 idx;
    psk_double tmp = 0.0;

    // find absolute maxima
    for( idx = 1; idx < len; idx++ )
    {
        tmp = psk_abs( vec[idx] );
        if( tmp > max_abs )
            max_abs = tmp;
    }

    // normalize
    for( idx = 0; idx < len; idx++ )
    {
        RE( norm_vec[idx] ) = RE( vec[idx] ) / max_abs;
        IM( norm_vec[idx] ) = IM( vec[idx] ) / max_abs;
    }

    return( norm_vec );
}

```