



Liquid flow measurement in open channels — Velocity-area methods — Investigation of total error

Mesure du débit des liquides dans les canaux découverts — Méthode d'exploration du champ des vitesses — Recherche de l'erreur globale

Technical Report ISO/TR 7178 has been drawn up by Technical Committee ISO/TC 113, *Measurement of liquid flow in open channels*. It summarizes the results of investigations of the total error in the measurement of flow by velocity-area methods. Although this information is not considered to be a suitable subject for publication as an International Standard, it has been decided, in view of the wide interest which this information occasions, to publish it in the form of a Technical Report.

Contents

	Page
0 Introduction	2
1 Scope and field of application	3
2 Symbols	3
Section one : Procedure and types of errors	
3 General principles of velocity-area methods and accuracy requirements	5
4 Composition of total error	5
5 Errors in mean flow and depth	6
5.1 General	6
5.2 Error type I	6
5.3 Error type II	7
5.4 Error type III	8

UDC 532.57 : 532.543 : 627.133

Ref. No. ISO/TR 7178-1983 (E)

Descripteurs : liquid flow, water flow, open channel flow, flow measurement, velocity measurement, error analysis.

© International Organization for Standardization, 1983 •

Printed in Switzerland



Documents

Document Sharing Hub
Price based on 27 pages

6	General conclusions and recommendations	9
6.1	General	9
6.2	Error type I	9
6.3	Error type II	10
6.4	Error type III	10
6.5	General conclusions	10
Section two : Recommendations for the collection of data for investigation of errors		
7	Measurements and data	12
7.1	Individual point velocity	12
7.2	Mean velocity in the vertical	12
7.3	Finite number of verticals in the cross-section	12
7.4	General data	12
8	Data processing	13
8.1	General	13
8.2	Error type I	13
8.3	Error type II	15
8.4	Error type III	17
8.5	Total error	19
Annexes		
A	Characteristics of rivers from which data were collected	20
B	Example of calculation of total stochastic error	22
C	Effect of increasing measuring time on error	25
D	Some criteria used in choosing verticals	26
Bibliography		27

0 Introduction

All measurements of physical quantities are subject to uncertainties, which may be due to bias errors in the equipment used for calibration and measurement, or to random scatter caused by a lack of sensitivity of the equipment used for the measurements, etc.

During the preparation of ISO 748^[1], much discussion was given over to the question of the magnitude of errors in measurements, and it was concluded that recommendations could only be formulated on the basis of an analysis of sufficient data. Moreover, it was recognized that to be able to analyse such data statistically, it was essential that the data be collected and recorded on a standardized basis and in a systematic manner, and this recognition lead to the preparation of ISO 1088^[2].

On the basis of the procedures given in these two International Standards, data were subsequently collected and processed from the following rivers (see annex A for the characteristics of the rivers) :

- a) rivers Ganga, Jalangi, Yamuna, and Visvesvaraya Canal, in India;
- b) river IJssel, in the Netherlands;
- c) rivers Derwent, Eden, Lambourn, Ouse, Tyne and Usk, in the United Kingdom;
- d) rivers Columbia and Mississippi, in the USA.

Further data obtained on the rivers Ganga and Krishna, in India, and the Spey, Tay, Tweed, Tyne, Gala Water, Yarrow Water, Ettrick Water and the Clyde, in the United Kingdom, were received later, but could not be included in the processing exercise.

1 Scope and field of application

This Technical Report summarizes the results of investigations of the total error in measurements of flow by velocity-area methods. It describes the procedure used and types of errors (section one), and gives recommendation for the collection of data for investigations of errors (section two) with a view to supplementing the information given in ISO 1088.

2 Symbols

- a = coefficient of linear regression
- b = coefficient of linear regression
- b_i = unobservable true width of section i
- d_i = unobservable true depth in vertical of section i
- h_{rel} = relative depth, measured from the surface
- i = number of series of measurements (error types II and III)
- j = number of measurements per series (error type II)
- k = time displacement autocorrelation function (of time interval, etc.)
- m = number of verticals or sections per cross-section
- n = number of time intervals of measured velocities (error type I)
- q = unobservable true discharge
- q_i = discharge of section i
- s_i = stochastic sampling error of mean velocity in vertical (error type II)
- t_i = time i
- t_0 = initial measuring time
- v_i = velocity at time i or in section i
- V_i = actual velocity at time i or in section i
- V_{corr_i} = actual velocity from which trend is removed
- μ_s = mean sampling error (error type II)
- μ_m = mean error when measurements are made in m verticals (error type III)
- $\rho(k)$ = autocorrelation function for time displacement k

σ = standard deviation (general)

σ_l = relative standard deviation of the total stochastic instrumental and sampling error

σ_{B_i} = relative standard deviation due to the random instrumental error determining width of section i

σ_{D_i} = relative standard deviation due to the random instrumental error determining depth of section i

σ_{F_i} = relative standard deviation due to the random fluctuation (error type I)

σ_{S_d} = relative standard deviation due to the random sampling error of the depth profile

σ_{S_h} = relative standard deviation due to the random sampling error of the horizontal velocity profile

$\sigma_{S_{hd}}$ = relative standard deviation combining σ_{S_h} and σ_{S_d} respectively (error type III) :

$$\sigma_{S_{hd}}^2 = \sigma_{S_h}^2 + \sigma_{S_d}^2$$

σ_{S_v} = relative standard deviation of the mean velocity due to random instrumental error

NOTE — Observations, or results of calculations using observations, are indicated by a capital letter. Statistical quantities obtained from observations are indicated by a small letter with a circumflex. A mean value is indicated by overlining the symbol.

Section one : Procedure and types of errors

3 General principles of velocity-area methods and accuracy requirements

Velocity-area methods for the determination of discharge in open channel flow consist of measurements of velocity and depth at a number of points in the cross-section and calculation of the discharge using this data.

The velocity is measured in a number of verticals and, in each vertical, the mean velocity is determined from measurements at a number of points selected for that purpose.

The discharge per unit width is the product of mean velocity and depth in the vertical considered. Each vertical is assumed to be representative for a section of the cross-sectional area.

The accuracy of the determination of the discharge is of special importance where there is either restricted or abundant discharge. Usually, the discharge has to serve several purposes. If the discharge is restricted, the relative importance of the different purposes will have to be considered and the distribution for each of these purposes will be a matter of water management policy. In addition, demands relating to navigation requirements, as well as sediment transport phenomena, have to be considered.

For the statistical prediction of high floods in the framework of a "flood control" policy, it is very important to have accurate stage-discharge relations available. For a certain location, this relation can be read from a rating curve drawn through a number of points representing the results of the determination of discharge from measurements at various stages. As the stage can easily be determined with relatively high accuracy, the accuracy of the rating curve will be the criterion for the accuracy of the determination of actual discharges.

There remains, however, the question of the degree of accuracy required in such determinations.

Usually, the degree of accuracy required will be based on a number of considerations, according to needs from the viewpoints of research, design, construction, economy, management, etc. An important consideration is the extent to which the accuracy can be improved with reasonably increased effort. Also, if improved accuracy obtained in this way is not at the time needed under the existing conditions, the costs of the increased efforts may prove to have been a good investment in the future.

It has been stated^[3] that, generally, an error with a standard deviation of 2 to 3 % can be regarded as satisfactory. These figures are in agreement with experience and can be accepted.¹⁾

4 Composition of total error

For the determination of a discharge, a number of quantities or components have to be measured. To optimize the measurements, it is necessary to know the accuracy that can be achieved when measuring each of the components. As the total error is composed of uncertainties in the measurement of the individual components, it is evident that, if one of the components is measured relatively inaccurately, this may affect the total uncertainty to such an extent that very accurate measurement of the remaining components becomes impossible.

In general, a distinction can be made between errors of a systematic, and those of a stochastic (random), nature.

According to their origin, errors can be distinguished as being due to the instrument used, the measuring procedures and the processing of data. They can be systematic as well as stochastic.

When using velocity-area methods, three quantities have to be measured, i.e. width, depth and flow velocity. Each of these measurements will be subject to the uncertainties mentioned.

In this investigation, an examination of the accuracy of the instruments was not included.

The systematic bias error of an instrument is related to the characteristic properties of the instrument.

The stochastic instrumental error, however, has to be included in the calculation of the total stochastic error. For this reason, standard deviations of stochastic instrumental errors, known from existing literature and research, are given below :

- a) In ISO 748, annex E, for the measurement of distance, a relative error of 0,3 % is indicated for a distance between 0 and 100 m, and 0,5 % for a distance of 250 m. When the distance is measured electronically, an error as a percentage of the distance (for instance, 0,5 to 1 %), in addition to a fixed error of 0,5 to 2 m, has to be considered.

1) All values of errors stated in this Technical Report are at one standard deviation.

b) The instrumental error in the measured depth depends, to a large extent, on the composition of the river bed, which is critical if the sounding rod, lead or acoustic pulse of the echo-sounder penetrates into the bed. An error of 1 % is considered to be a reasonable approximation.

c) For the determination of flow velocity, two types of instrument are used : the cup-type and the screw-type (propeller) current meter. They are calibrated by moving the current meter through still water in a calibrating tank at accurately known speeds. Although it remains questionable whether this method adequately simulates the reversed situation that the water moves and the meter is at rest, this method of calibration is generally accepted.^[4]

Although the cup-type meter seems to be more sensitive to turbulence, simultaneous measurements with cup-type and screw-type current meters made by Townsend and Blust,^[5] Carter and Anderson^[6] and Grindley^[4] have shown identical results. The standard deviation of the stochastic calibration error of cup-type current meters is less than 1 %.

Other investigations using a screw-type current meter showed a relative standard deviation of 4,9 % for a flow velocity of 0,2 m/s decreasing to 0,44 % for a flow velocity of 2,5 m/s. The absolute standard deviation showed a minimum value for a flow velocity of approximately 1 m/s ($\sigma = 0,83 \times 10^{-2}$ m/s).

5 Errors in mean flow velocity and depth

5.1 General

In the investigation, special attention was concentrated on stochastic errors in the measurements due to the methods used to determine the mean flow velocity in the cross-section and to the methods used to determine the depth in the section considered.

Apart from the instrumental error, the error in the mean flow velocity component can be considered as comprising three independent types of errors :

- error type I (measuring time), due to the restricted measuring time of the local point velocity in the vertical;
- error type II (number of points in the vertical), arising from the use of a restricted number of sampling points in the vertical. The calculated mean velocity in a vertical is, therefore, an approximation of the true mean velocity in that vertical;
- error type III (number of verticals), of the same nature as error type II, due to the restricted number of verticals in the cross-section. The horizontal velocity profile and the bed profile between two verticals have to be determined by interpolation and, therefore, errors will be introduced.

NOTE — The types of errors referred to in this Technical Report are not related to statistical type I and type II errors.

5.2 Error type I

5.2.1 Although steady flow conditions are assumed, the instantaneous local point velocity, owing to turbulence, will be a random phenomenon, and can, therefore, be regarded as a stochastic process.

The mean flow velocity at a certain point, determined from measurements during a finite measuring time, will be an approximation of the true mean flow velocity at that point. Repetition of the same measurement, using the same measuring time, will show a deviation in the result. These deviations will become smaller as the measuring time becomes longer or when the fluctuations are smaller. In general, it can be stated that the fluctuations have less influence on the measured mean flow velocity when the measuring time is increased, and this manifests itself in a decrease of the standard deviation.

In order to investigate the influence of the measuring time on fluctuations of flow velocity, and, therefore, its influence on the accuracy of the mean local point velocity, the following measuring procedure is described in ISO 1088.

Three verticals of the cross-section are selected at the deepest point and at places where the depths are 60 % and 30 % of the greatest depth, respectively, both located on the wider side of the vertical containing the deepest point.

In each of these verticals, the velocity is measured at 20, 60, 80 and 90 % of the depth, measured from the surface, each measurement consisting of uninterrupted observation using a current meter for a period of 50 min, and taking a reading every 30 s.

The fluctuations are not independent of each other. This means that the velocity at time t_2 is influenced by the velocity at time t_1 . This influence will decrease as the time interval $t_2 - t_1$ increases.

This interdependence is important for the selection of the measuring time, because an improved approximation of the true flow velocity, which might be expected when the measuring time is increased, will, due to the interdependence, not be obtained entirely (see annex C).

If decreasing the measuring time is considered, it is necessary to know the physical background of the damping process, or, in the absence of this knowledge, to have a mathematical model of the (non-deterministic) process. The following model has been tested :

$$\varrho(\tau) = e^{-\lambda\tau}$$

where

$\varrho(\tau)$ is the autocorrelation function;

λ is a constant;

τ is a time displacement.

For this purpose, special measurements were made with a measuring time of 10 s. The initial measuring time was 30 s.

For the fluctuations of the flow velocity, a normal (gaussian) distribution was assumed and tested.

5.2.2 The following conclusions concerning error type I have been drawn :

- a) In general, fluctuations of velocity behave according to normal (gaussian) distribution.
- b) The extent of fluctuations of velocity is related to depth. The absolute value of the standard deviation (σ_{abs}) of the point velocities increases with depth.
- c) Due to the increase of σ_{abs} and the decrease with depth of the point velocities, the relative standard deviation (σ_{rel}) which is the absolute standard deviation divided by the point velocity considered, (σ_{abs}/v_{point}), increases rapidly with depth.
- d) No relation is found between σ_{rel} on the one hand and the location of the vertical in the cross-section, the discharge and the width-depth relation, respectively, on the other hand.
- e) The hypothesis $\varrho(\tau) = e^{-\lambda\tau}$ as a model for the autocorrelation function has no general validity; the hypothesis was not in contradiction with the results derived from measurements in nine cases, but was rejected in six other cases. However, the results appeared to agree with the hypothesis in one case, when a measuring time of 10 s was used.

5.3 Error type II

5.3.1 Usually, the mean flow velocity in a vertical is calculated by the use of one of the existing computation rules. These rules result in an approximation of the true mean velocity at a certain moment.

A complication is that the error due to velocity fluctuations (error type I) is included in addition to the sampling error (error type II). Assuming steady flow conditions, and considering the dispersion of a number of measured mean velocities by means of their standard deviation, it is possible to determine the influence of error type I.

ISO 1088 specifies that the determination of velocity in the vertical be carried out at ten points, with a measuring time of 60 s, repeated five times. Using the ten observations, the velocity profile in the vertical can be drawn and the mean velocity can be determined using a planimeter. In this investigation, the mean velocity, determined in this way, was assumed to be the true mean velocity.

The mean velocities calculated by using computation rules were compared with this true mean velocity. The following computation rules were examined :

$$\bar{v} = v_{0,6} \quad \dots (1)$$

$$\bar{v} = 0,96 v_{0,5} \quad \dots (2)$$

$$\bar{v} = 0,5 (v_{0,2} + v_{0,8}) \quad \dots (3)$$

$$\bar{v} = 0,25 v_{0,2} + 0,5 v_{0,6} + 0,25 v_{0,8} \quad \dots (4)$$

$$\bar{v} = 0,4 v_{0,2} + 0,3 v_{0,6} + 0,25 v_{0,8} \quad \dots (5)$$

$$\bar{v} = 1/3 (v_{0,2} + v_{0,6} + v_{0,8}) \quad \dots (6)$$

$$\bar{v} = 1/4 (v_{0,2} + v_{0,4} + v_{0,7} + v_{0,9}) \quad \dots (7)$$

$$\bar{v} = 0,1 v_{surf.} + 0,3 v_{0,2} + 0,3 v_{0,6} + 0,2 v_{0,8} + 0,1 v_{bed} \quad \dots (8)$$

$$\bar{v} = 1/6 (v_{\text{surf.}} + v_{0,2} + v_{0,4} + v_{0,6} + v_{0,8} + v_{\text{bed}}) \quad \dots (9)$$

$$\bar{v} = 0,1 v_{\text{surf.}} + 0,2 v_{0,2} + 0,2 v_{0,4} + 0,2 v_{0,6} + 0,2 v_{0,8} + 0,1 v_{\text{bed}} \quad \dots (10)$$

The results of this examination are shown in table 1.

Table 1

Rule	Number of points	Mean error $\hat{\mu}_s$ %	Standard deviation of mean error $\hat{\sigma}_{SV}$ %	$\sqrt{\text{M.S.E.}}$ * %	Standard deviation of the mean error including error type I $\hat{\sigma}_s + f$ %
(1)	1	1,6	7,5	7,7	8,2
(2)	1	3,3	4,8	5,9	6,5
(3)	2	2,2	3,4	4,0	4,9
(4)	3	1,9	4,4	4,8	4,8
(5)	3	-0,8	3,3	3,4	3,9
(6)	3	2,0	3,7	4,2	4,2
(7)	4	-0,9	2,2	2,4	3,0
(8)	5	0,2	2,2	2,2	2,7
(9)	6	-1,6	2,5	3,0	2,8
(10)	6	0,9	2,1	2,3	2,4

* $\sqrt{\text{M.S.E.}} = \sqrt{\hat{\mu}_s^2 + \hat{\sigma}_{SV}^2}$ (root sum squared error)

The non-systematic character of the mean error with respect to zero is taken into account using $\sqrt{\text{M.S.E.}}$ in a way which enables mutual comparison with the standard deviation and of the various rules.

5.3.2 The following conclusions concerning error type II have been drawn :

- a) The results for a rule differ from river to river. The rules have a more general validity for larger rivers ($Q > 120 \text{ m}^3/\text{s}$) than for smaller rivers ($Q < 120 \text{ m}^3/\text{s}$). (The criterion of $120 \text{ m}^3/\text{s}$ was chosen in such a way that both groups were represented by a sufficient number of rivers.)
- b) The nature of the velocity profile in the vertical is sufficiently fixed by measurements at four points (rule No. 7). The result can be improved by increasing the total measuring time, either by measurements at more than four points or by increasing the measuring time at each of the four points.

5.4 Error type III

5.4.1 Error type III is due to the approximation by interpolation of the bed profile and the horizontal velocity distribution between the verticals.

In practice, both factors usually occur simultaneously. The measurement of flow velocity and depth takes place in a restricted number of verticals located in the cross-section. The selection of the number and location of the verticals is mainly based on personal judgment, taking into account the shape of the bed profile in the cross-section.

In general, it is known that the selection of too few verticals may lead to a considerable error, but the extent of the approximations and the relation with errors of different origin are unknown.

In this investigation, an attempt was made to enable a comparison between the error involved in the normal (subjective) practice of measurement and the error which remains after optimum selection of the verticals. For this purpose, a number of (objective) criteria were adopted.

For the comparison, it is obvious that the horizontal velocity distribution as well as the bed profile must be known as accurately as possible. According to ISO 1088, either the continuous profile of the cross-section, or at least the depth at intervals of not more than 2 % of the total width, must be measured. The horizontal velocity distribution must be observed by taking velocity readings (measuring time of 120 s) at 60 % of the depth at the intervals mentioned. Some of the criteria used in choosing verticals are described in annex D.

In order to determine the influence of the number of verticals on the accuracy achieved, the number of verticals used for the determination of the discharge was decreased successively in a way depending on the criterion under consideration.

The results are given in table 2 for the criteria described in annex D. They show the standard deviation of error type III deduced from regression curves drawn through the points observed.

Table 2

Number of verticals	Relative standard deviation of error, %		
	Criterion 1 : Bed profile in the cross-section	Criterion 2 : Verticals equidistant	Criterion 3 : Sections of equal flow
5	7,70		
6	7,00		4,52
10	4,40	2,60	3,35
15	3,02	1,98	2,60
20	2,20	1,65	2,08
25	1,70	1,45	1,76
30	1,28	1,30	1,60
35	1,02		1,55
40	0,80		
45	0,68		

5.4.2 The following conclusions concerning error type III have been drawn :

- a) Calculating the discharge from a restricted number of verticals gives results which are systematically too low.
 - b) The selection of the verticals based on the profile in the cross-section (criterion 1) leads to good results compared with the use of the various other criteria.
 - c) Selection of the verticals according to the equidistant criterion leads to results which seem to be slightly better than those obtained when using the criterion "sections of equal flow". The difference, however, should be considered to be insignificant.
 - d) For large rivers ($Q > 120 \text{ m}^3/\text{s}$), the interpolation of the horizontal velocity profile affects the extent of the error more than the interpolation of the bed profile. The difference, however, is small.
- However, for small rivers ($Q < 120 \text{ m}^3/\text{s}$), the interpolation of the bed profile influences the error much more than the interpolation of the horizontal velocity profile.
- e) Errors in discharge, caused by the interpolation of the velocity profile and depth, respectively, are related. This relation is based on the interdependence between flow velocity and depth in the vertical.
 - f) The error in discharge can be decreased considerably by using knowledge of the continuous profile (an echogram) when determining the discharge, instead of using only the depth in the verticals where the flow velocity is observed.

6 General conclusions and recommendations

6.1 General

A basic rule for the setting-up of discharge measurements is that the accuracy of the various components must harmonize with each other.

For instance, selection of the number of verticals will not only influence the extent of error type III, but will also determine the influence of errors of types I and II, respectively, on the total error.

6.2 Error type I

6.2.1 Conclusions

6.2.1.1 For the measurement of individual point velocities, a measuring time of at least 60 s is desirable. From measurements using a measuring time of 30 s, it appears that dependence, as well as independence, can occur between velocity fluctuations in consecutive