



Explanatory notes on ISO 281/1-1977

Notes explicatives sur l'ISO 281/1-1977

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- type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

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The reasons which led to the decision to publish this document in the form of a technical report type 3 are explained in the Introduction.

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1. Introduction

This Technical Report gives supplementary background information regarding the derivation of formulae and factors given in ISO 281/I, Rolling bearings - Dynamic load ratings and rating life - Part I : Calculation methods.

2. Brief History

2.1 ISO/R281-1962

A first discussion on an international level of the question of standardizing calculation methods for load ratings of rolling bearings took place at the 1934 conference of the International Federation of the National Standardizing Associations (ISA). When ISA held its last conference in 1939 no progress had been made. However, in its 1945 report on the state of rolling bearing standardization, the ISA 4 Secretariat included proposals for definition of concepts being fundamental for load rating and life calculation standards. This report was distributed in 1949 as document ISO/TC 4 (Secretariat-1)1, and the definitions it contained are in essence those given in ISO 281/I for the concepts "life" and "basic dynamic load rating".

In 1946, on the initiative of the Anti-Friction Bearing Manufacturers Association (AFBMA), New York, discussions of load rating and life calculation standards were started between bearing industries in U.S.A. and Sweden. Chiefly on the basis of results of scientific investigations by G. Lundberg and A. Palmgren, published in 1947 [1]*, an AFBMA Standard "Method of Evaluating Load Ratings of Annular Ball Bearings" was worked out and published in 1949. On the same basis, the member body of Sweden presented in Feb., 1950 a first proposal to ISO, "Load Rating of Ball Bearings", doc. ISO/TC 4/SC 1 (Sweden-1)1.

* figures in brackets indicate literature references in References.

In view of results of further research, of a modification of the AFBMA Standard in 1950, and of the interest also in roller bearing rating standards, the member body of Sweden submitted in 1951 a modified proposal for rating of ball bearings, doc. ISO/TC 4/SC 1 (Sweden-6) 20, as well as a proposal for rating of roller bearings, doc. ISO/TC 4/SC 1 (Sweden-7) 21.

Load rating and life calculation methods were then studied by ISO/TC 4, ISO/TC 4/SC 1 and ISO/TC 4/WG 3 at eleven different meetings during 1951 - 1959. An additional paper by Lundberg-Plamgren published in 1952 [2] was of considerable use, serving as a major basis for the sections regarding roller bearing rating.

The framework for the Recommendation was settled at TC 4/WG 3 meeting in 1956. At the time, deliberation of the draft for revision of AFBMA Standards was concluded in U.S.A. and ASA B3 approved the revised standard. It was proposed to the meeting by U.S.A. and discussed in detail, together with the Secretariat's proposal. At the meeting, WG3 proposal was prepared which adopted many parts of the U.S.A. proposal.

In 1957, Draft Proposal (doc. TC 4 N145) based on the WG proposal was issued. At the next year's WG3 meeting, this Draft Proposal was investigated in detail, and at the following TC4 meeting, the adoption of TC 4 N145, with some minor amendments, was concluded. Then, Draft ISO Recommendation No. 278 as TC 4 N188 was issued in 1959, and ISO/R281 was accepted by ISO Council in 1962.

2.2 ISO 281/I-1977

In 1964 the member body of Sweden suggested that, in view of the development of improved bearing steels, the time had come

to review R281 and submitted a proposal, ISO/TC 4/WG3 (Sweden-1) 9. However, at this time WG3 was not in favour of a revision.

In 1969, on the other hand, TC 4 followed a suggestion by the member body of Japan (doc. TC 4 N627) and reconstituted its WG3, giving it the task of revising R281. The AFBMA load rating working group had at this time started to work on a revised standard, and the member body of U.S.A. submitted the Draft AFBMA Standard "Load ratings and fatigue life for ball bearings" for consideration, ISO/TC 4/WG3 (USA-1) 11, in 1970 and "Load ratings and fatigue life for roller bearings", ISO/TC4/WG3 (USA-3) 19, in 1971.

In 1972, TC 4/WG3 was reorganized and became TC 4/SC8. This proposal was investigated in detail at the five meetings during 1971 - 1974. The final proposal, Third Draft Proposal (doc. TC 4/SC 8 N23), with some amendments, was circulated as Draft International Standard in 1976 and ISO 281/I was accepted by ISO Council in 1977.

The major part of this International Standard constitutes a re-edition of R281, the substance of which was only very slightly modified. However, based mainly on American investigations during the 1960's, a new clause was added, dealing with adjustment of rating life for reliability other than 90% and for material and operating conditions.

Furthermore, supplementary background information regarding the derivation of formulae and factors given in ISO 281/I was to be published, preliminarily as ISO 281/II Explanatory Notes. In 1979, however, TC 4/SC 8 and TC 4 decided to publish it as Technical Report.

3. Basic Dynamic Load Rating

The background of basic dynamic load ratings according to the standard ISO 281 of rolling bearings is in the Lundberg and Palmgren papers listed as references [1] and [2].

The formulae for calculation of basic dynamic load ratings of rolling bearings develop from a power equation that can be written as follows:

$$\ln \frac{1}{S} \sim \frac{\tau_0^{cN^eV}}{z_0^h} \dots\dots\dots (3-1)$$

where S = probability of survival
 τ_0 = maximum orthogonal subsurface shear stress
 N = number of stress applications to a point on the raceway
 V = volume representative of the stress concentration
 z_0 = depth of the maximum orthogonal subsurface shear stress
 c, h = exponents determined experimentally
 e = measure of life scatter, i.e. Weibull slope determined experimentally.

For "point" contact conditions (ball bearings) it is assumed that the volume (V) representative of the stress concentration in equation (3-1) is proportional to major axis of the projected contact ellipse ($2a$), the circumference of the raceway (l) and the depth (z_0) of the maximum orthogonal subsurface shear stress (τ_0):

$$V \sim az_0l. \dots\dots\dots (3-2)$$

Substituting (3-2) into relationship (3-1):

$$\ln \frac{1}{S} \sim \frac{\tau_0^{cN^eal}}{z_0^{h-1}}, \dots\dots\dots (3-3)$$

"Line" contact was considered by Lundberg and Palmgren to be approached under conditions where the major axis of the calculated Hertz contact ellipse is 1,5 times the effective roller contact length:

$$2a = 1,5 L_{we} \dots\dots\dots (3-4)$$

In addition, b/a should be small enough to permit the introduction of the limit value of ab^2 for b/a approaching 0:

$$ab^2 = \frac{2}{\pi} \cdot \frac{3Q}{E_o \Sigma \rho} \dots\dots\dots (3-5)$$

(for notation see 3.1).

3.1 Basic dynamic radial load rating C_r for radial ball bearings

From the Hertz's theory, the maximum orthogonal subsurface shear stress τ_o and the depth z_o can be expressed in terms of a radial load Fr , i.e. a maximum rolling element load Q_{max} or a maximum contact stress σ_{max} and dimensions for the contact area between a rolling element and the raceways. The relationships are given as follows:

$$\begin{aligned} \tau_o &= T \sigma_{max}, \\ z_o &= \zeta b, \\ T &= \frac{(2t - 1)^{1/2}}{2t(t + 1)}, \\ \zeta &= \frac{1}{(t + 1)(2t - 1)^{1/2}}, \\ a &= \mu \left[\frac{3Q}{E_o \Sigma \rho} \right]^{1/3}, \\ b &= \nu \left[\frac{3Q}{E_o \Sigma \rho} \right]^{1/3} \end{aligned}$$

where $\sigma_{\max}$ = maximum contact stress
 t = auxiliary parameter
 a = semimajor axis of the projected contact ellipse
 b = semiminor axis of the projected contact ellipse
 Q = normal force between a rolling element and the raceways
 E_0 = modulus of elasticity
 $\Sigma\rho$ = curvature sum
 μ, ν = auxiliary quantities introduced by Hertz.

Consequently, for a given rolling bearing τ_0 , a , l and z_0 can be expressed in terms of bearing geometry, load and revolutions. The relationship (3-3) is changed to an equation by inserting a constant of proportionality. Inserting a specific number of revolutions (e.g. 10^6) and a specific reliability (e.g. 0,9), the equation is solved for a rolling element load for basic dynamic load rating which is designated to point contact rolling bearings introducing a constant of proportionality A_1 :

$$Q_c = \frac{1,3}{4} \frac{2c+h-2}{c-h+2} \frac{3e}{0,5} \frac{3e}{c-h+2} A_1 \left[\frac{2r}{2r-D_w} \right]^{0,41} \frac{(1+\gamma) \frac{1,59c+1,41h-5,82}{c-h+2}}{(1\pm\gamma) \frac{3e}{c-h+2}} \times \left(\frac{\gamma}{\cos\alpha} \right)^{\frac{3}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}} z^{-\frac{3e}{c-h+2}} \dots \dots \dots (3-6)$$

where Q_c = rolling element load for the basic dynamic load rating of the bearing
 D_w = ball diameter
 $\gamma = D_w \cos\alpha / D_{pw}$
 D_{pw} = pitch diameter of ball set
 α = nominal contact angle
 z = number of balls per row.

(x is used as multiplication symbol.)

The basic dynamic radial load rating C_1 of a rotating ring is given as follows:

$$C_1 = Q_{c1} Z \cos \alpha \frac{J_r}{J_1} = 0,407 Q_{c1} Z \cos \alpha \quad \dots\dots\dots (3-7)$$

The basic dynamic radial load rating C_2 of a stationary ring is given as follows:

$$C_2 = Q_{c2} Z \cos \alpha \frac{J_r}{J_2} = 0,389 Q_{c2} Z \cos \alpha \quad \dots\dots\dots (3-8)$$

where Q_{c1} = rolling element load for the basic dynamic load rating of a ring rotating relative to the applied load

Q_{c2} = rolling element load for the basic dynamic load rating of a ring stationary relative to the applied load

$J_r = J_r(0,5)$ = radial load integral (see table 4-1)

$J_1 = J_1(0,5)$ = factor relating mean equivalent load on a rotating ring to Q_{max} (see table 4-1)

$J_2 = J_2(0,5)$ = factor relating mean equivalent load on a stationary ring to Q_{max} (see table 4-1).

The relationship among C_r for an entire radial ball bearing, C_1 and C_2 is expressed in terms of the product law of probability as follows:

$$C_r = C_1 \left[1 + \left(\frac{C_1}{C_2} \right) \frac{c-h+2}{3} \right]^{-\frac{3}{c-h+2}} \quad \dots\dots\dots (3-9)$$

Substituting equations (3-7), (3-8) and (3-6) into equation (3-9), the basic dynamic radial load rating C_r for an entire

ball bearing is expressed as follows:

$$\begin{aligned}
 C_r = & 0,41 \frac{1,3}{\frac{2c+h-2}{4c-h+2} \frac{3e}{0,5c-h+2}} A_1 \left[\frac{2r_i}{2r_i-D_w} \right]^{0,41} \frac{(1-\gamma)}{(1+\gamma)} \frac{\frac{1,59c+1,41h-5,82}{c-h+2}}{\frac{3e}{c-h+2}} \\
 & \times \gamma^{\frac{3}{c-h+2}} \\
 & \times \left[1 + \left\{ 1,04 \left(\frac{r_i}{r_e} \times \frac{2r_e-D_w}{2r_i-D_w} \right)^{0,41} \left(\frac{1-\gamma}{1+\gamma} \right)^{\frac{1,59c+1,41h+3e-5,82}{c-h+2}} \right\}^{\frac{c-h+2}{3}} - \frac{3}{c-h+2} \right] \\
 & \times (i \cos \alpha)^{\frac{c-h-1}{c-h+2}} z^{\frac{c-h-3e+2}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}} \dots \dots \dots (3-10)
 \end{aligned}$$

where A_1 = proportionality constant determined experimentally
 r_i = cross-sectional raceway groove radius of inner ring
 r_e = cross-sectional raceway groove radius of outer ring
 i = number of rows of balls.

Here, a contact angle α , number of rolling elements (balls) Z and the diameter D_w depend on bearing design. On the other hand, the ratios of raceway groove radii r_i and r_e to a half diameter of a rolling element (ball) $D_w/2$ and $\gamma = D_w \cos \alpha / D_{pw}$ are not dimensional, therefore it is convenient in practice that the value for the first three lines in the right side of equation (3-10) is designated as a factor f_c .

Consequently,

$$C_r = f_c (i \cos \alpha)^{\frac{c-h-1}{c-h+2}} z^{\frac{c-h-3e+2}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}} \dots \dots \dots (3-11)$$

With radial ball bearings we must consider the faults in bearings resulting from the manufacturing, and a reduction factor λ is introduced to reduce the value for a basic dynamic radial load rating for radial ball bearings from its theoretical value, and it is convenient to contain the factor λ in the factor f_c . The value for the factor λ is determined experimentally.

Consequently the factor f_c is given as follows:

$$f_c = 0,41\lambda \frac{1,3}{4^{c-h+2}} \frac{3e}{0,5^{c-h+2}} A_1 \left[\frac{2r_i}{2r_i - D_w} \right]^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h-5,82}{c-h+2}}}{(1+\gamma)^{\frac{3e}{c-h+2}}} \\ \times \gamma^{\frac{3}{c-h+2}} \\ \times \left[1 + \left\{ 1,04 \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h+3e-5,82}{c-h+2}}}{(1+\gamma)} \right\}^{\frac{c-h+2}{3}} - \frac{3}{c-h+2} \right] \\ \dots\dots\dots (3-12)$$

Based on the original experimental work by Lundberg and Palmgren with ball bearings the following values were assigned to the experimental constants in the load rating equations:

$$\begin{aligned} e &= 10/9 \\ c &= 31/3 \\ h &= 7/3. \end{aligned}$$

Substituting the numerical values into equation (3-11) gives the following, however, a sufficient number of test results

are only available for small balls, i.e. up to a diameter of about 25 mm, and these show that the load rating may be taken as being proportional to $D_w^{1,8}$. In the case of larger balls the load rating appears to increase even more slowly in relation to the ball diameter, and $D_w^{1,4}$ can be assumed where $D_w > 25,4$ mm:

$$C_r = f_c (\cos \alpha)^{0,7} Z^{2/3} D_w^{1,8} \quad (D_w \leq 25,4 \text{ mm}), \dots (3-13)$$

$$C_r = 3,647 f_c (\cos \alpha)^{0,7} Z^{2/3} D_w^{1,4} \quad (D_w > 25,4 \text{ mm}), \dots (3-14)$$

$$f_c = 0,089 A_1 \times 0,41 \lambda \left[\frac{2r_i}{2r_i - D_w} \right]^{0,41} \frac{\gamma^{0,3} (1-\gamma)^{1,39}}{(1+\gamma)^{1/3}} \\ \times \left[1 + \left\{ 1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{1,72} \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \right\}^{10/3} \right]^{-3/10} \\ \dots \dots \dots (3-15)$$

Values for f_c on table 1 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-15).

The value for $0,089 A_1$ is 98,0665 to calculate C_r in Newtons.

3.2 Basic dynamic axial load rating C_a for single row thrust ball bearings

3.2.1 Thrust ball bearing with contact angle $\alpha \neq 90^\circ$

Similarly according to 3.1, for thrust ball bearings with contact angle $\alpha \neq 90^\circ$:

$$C_a = f_c (\cos \alpha)^{\frac{c-h-1}{c-h+2}} \tan \alpha Z^{\frac{c-h-3e+2}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}} \dots \dots (3-16)$$

For most thrust ball bearings the theoretical value of a

basic dynamic axial load rating must be reduced on the basis of unequal distribution of load among the rolling elements in addition to the reduction factor λ which is introduced in radial ball bearing load ratings. This reduction factor is designated as η .

Consequently, the factor f_c is given as follows:

$$f_c = \lambda \eta \frac{1,3}{4^{c-h+2}} \frac{3e}{0,5^{c-h+2}} A_1 \left[\frac{2r_i}{2r_i - D_w} \right]^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h-5,82}{c-h+2}}}{(1+\gamma)^{\frac{3e}{c-h+2}}} \gamma^{\frac{3}{c-h+2}} \\ \times \left[1 + \left\{ \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h+3e-5,82}{c-h+2}}}{(1+\gamma)} \right\}^{\frac{c-h+2}{3}} \right] - \frac{3}{c-h+2} \\ \dots\dots\dots (3-17)$$

Substituting experimental constants $e = 10/9$, $c = 31/3$ and $h = 7/3$ into equations (3-16) and (3-17), however, considering the effect of ball size similarly,

$$C_a = f_c (\cos \alpha)^{0,7} \tan \alpha \, z^{2/3} D_w^{1,8} \quad (D_w \leq 25,4 \text{ mm}), \dots (3-18)$$

$$C_a = 3,647 f_c (\cos \alpha)^{0,7} \tan \alpha \, z^{2/3} D_w^{1,4} \quad (D_w > 25,4 \text{ mm}), \dots (3-19)$$

$$f_c = 0,089 A_1 \lambda \eta \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \frac{\gamma^{0,3(1-\gamma)} 1,39}{(1+\gamma)^{1/3}} \\ \times \left[1 + \left\{ \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \frac{(1-\gamma)^{1,72}}{(1+\gamma)} \right\}^{10/3} \right]^{-3/10} \\ \dots\dots\dots (3-20)$$

The value for $0,089A_1$ is 98,0665 to calculate C_a in Newtons. Values for f_c on the right column of table 3 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-20).

3.2.2 Thrust ball bearings with contact angle $\alpha = 90^\circ$

Similarly according to 3.1, for thrust ball bearings with contact angle $\alpha = 90^\circ$:

$$C_a = f_c Z^{\frac{c-h-3e+2}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}}, \quad \dots\dots\dots (3-21)$$

$$f_c = \lambda \eta \frac{1,3}{\frac{2c+h-2}{4^{c-h+2}} \frac{3e}{0,5^{c-h+2}}} A_1 \left[\frac{2r_i}{2r_i - D_w} \right]^{0,41} \gamma^{\frac{3}{c-h+2}} \\ \times \left[1 + \left\{ \left(\frac{r_i}{r_e} \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \right\}^{\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}} \quad \dots\dots (3-22)$$

in which $\gamma = D_w/D_{pw}$.

Substituting experimental constants $e = 10/9$, $c = 31/3$ and $h = 7/3$ into equations (3-21) and (3-22), however, considering the effect of ball size similarly:

$$C_a = f_c Z^{2/3} D_w^{1,8} \quad (D \leq 25,4 \text{ mm}), \quad \dots\dots\dots (3-23)$$

$$C_a = 3,647 f_c Z^{2/3} D_w^{1,4} \quad (D > 25,4 \text{ mm}), \quad \dots\dots\dots (3-24)$$

$$f_c = 0,089 A_1 \lambda \eta \left(\frac{2r_i}{2r_i - D_w} \right)^{0,41} \gamma^{0,3} \\ \times \left[1 + \left\{ \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w} \right)^{0,41} \right\}^{10/3} \right]^{-3/10} \quad \dots\dots (3-25)$$

The value for $0,089A_1$ is 98,0665 to calculate C_a in Newtons. Values for f_c on the left column of table 3 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-25).

3.3 Basic dynamic axial load rating C_a for thrust ball bearings with two or more rows of balls

According to the product law of probability, relationships between the basic axial load rating of an entire thrust ball bearing and of both the rotating and stationary washers are given as follows:

$$C_{ak} = \left[C_{alk}^{-\frac{c-h+2}{3}} + C_{a2k}^{-\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}}, \dots\dots (3-26)$$

$$\left. \begin{aligned} C_{alk} &= Q_{c1} \sin \alpha Z_k, \\ C_{a2k} &= Q_{c2} \sin \alpha Z_k, \end{aligned} \right\} \dots\dots\dots (2-27)$$

$$C_a = \left[C_{a1}^{-\frac{c-h+2}{3}} + C_{a2}^{-\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}}, \dots\dots\dots (3-28)$$

$$\left. \begin{aligned} C_{a1} &= Q_{c1} \sin \alpha \sum_{k=1}^n Z_k, \\ C_{a2} &= Q_{c2} \sin \alpha \sum_{k=1}^n Z_k \end{aligned} \right\} \dots\dots\dots (3-29)$$

where C_{ak} = basic dynamic axial load rating as a row k of an entire thrust ball bearing

C_{alk} = basic dynamic axial load rating as a row k of the rotating washer of an entire thrust ball bearing

C_{a2k} = basic dynamic axial load rating as a row k of stationary washer of an entire thrust ball bearing

C_a = basic dynamic axial load rating of an entire thrust ball bearing

C_{a1} = basic dynamic axial load rating of the rotating washer of an entire thrust ball bearing

C_{a2} = basic dynamic axial load rating of the stationary washer of an entire thrust ball bearing

Z_k = number of balls as a row k .

Substituting equations (3-29), (3-27) and (3-26) into equation (3-28), and rearrangement of equation (3-28) gives:

$$\begin{aligned}
 C_a &= \sum_{k=1}^n Z_k \left[\frac{(Q_{c1} \sin \alpha \sum_{k=1}^n Z_k)^{-\frac{c-h+2}{3}} + (Q_{c2} \sin \alpha \sum_{k=1}^n Z_k)^{-\frac{c-h+2}{3}}}{\left(\sum_{k=1}^n Z_k \right)^{-\frac{c-h+2}{3}}} \right]^{-\frac{3}{c-h+2}} \\
 &= \sum_{k=1}^n Z_k \left[\frac{\sum_{k=1}^n \left[\left\{ (Q_{c1} \sin \alpha Z_k)^{-\frac{c-h+2}{3}} + (Q_{c2} \sin \alpha Z_k)^{-\frac{c-h+2}{3}} \right\}^{-\frac{3}{c-h+2}} \right]}{Z_k^{-\frac{c-h+2}{3}}} \right]^{-\frac{3}{c-h+2}} \\
 &= \sum_{k=1}^n Z_k \left[\sum_{k=1}^n \left(\frac{Z_k}{C_{ak}} \right)^{\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}}
 \end{aligned}$$

Substituting experimental constants $c = 31/3$ and $h = 7/3$,

$$\begin{aligned}
 C_a &= (Z_1 + Z_2 + Z_3 + \dots + Z_n) \\
 &\times \left[\left(\frac{Z_1}{C_{a1}} \right)^{10/3} + \left(\frac{Z_2}{C_{a2}} \right)^{10/3} + \left(\frac{Z_3}{C_{a3}} \right)^{10/3} + \dots + \left(\frac{Z_n}{C_{an}} \right)^{10/3} \right]^{-3/10}
 \end{aligned}$$

..... (3-30)

The load ratings C_{a1} , C_{a2} , C_{a3} , ... C_{an} for the rows with z_1 , z_2 , z_3 , ... z_n balls are calculated from the appropriate single row thrust ball bearing formula in 3.2.

3.4 Basic dynamic radial load rating C_r for radial roller bearings

By a procedure similar to that used to obtain equation (3-10) for point contact in 3.1, but applying (3-4) and (3-5), the basic dynamic radial load rating of radial roller bearings (line contact) is obtained :

$$C_r = 0,377 \frac{1}{2^{\frac{c+h-1}{c-h+1}} 0,5^{\frac{2e}{c-h+1}}} B_1 \frac{(1-\gamma)^{\frac{c+h-3}{c-h+1}}}{(1+\gamma)^{\frac{2e}{c-h+1}}} \gamma^{\frac{2}{c-h+1}} \\ \times \left[1 + \left\{ 1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{\frac{c+h+2e-3}{c-h+1}} \right\}^{\frac{c-h+1}{2}} \right]^{\frac{2}{c-h+1}} \\ \times (i L_{we} \cos \alpha)^{\frac{c-h-1}{c-h+1}} z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}} \\ \dots\dots\dots (3-31)$$

where B_1 = proportional constant determined experimentally
 $\gamma = D_{we} \cos \alpha / D_{pw}$
 D_{we} = mean roller diameter
 α = nominal contact angle
 D_{pw} = pitch diameter of roller set
 L_{we} = effective contact length of roller
 i = number of rows of rollers
 z = number of rollers per row.

Here, a contact angle α , number of rollers z , the mean diameter D_{we} , and the effective contact length L_{we} depend

on bearing design. On the other hand, $\gamma = D_{we} \cos \alpha / D_{pw}$ is not dimensional, therefore it is convenient in practice that the value for the first two lines in the right side of equation (3-31) is designated as a factor f_c .

Consequently,

$$C_r = f_c (i L_{we} \cos \alpha)^{\frac{c-h-1}{c-h+1}} z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}} \dots (3-32)$$

For the basic dynamic radial load rating for radial roller bearings adjustments are made to take account of stress concentration (e.g. edge loading) and of the use of a constant instead of a varying life formula exponent (see clause 5). Adjustment for stress concentration is a reduction factor λ and for exponent variation a factor v . It is convenient to contain both factors - which are determined experimentally - in the factor f_c , which consequently is given as follows:

$$f_c = 0,377 \lambda v \frac{1}{2^{\frac{c+h-1}{c-h+1}} 0,5^{\frac{2e}{c-h+1}}} B_1 \frac{(1-\gamma)^{\frac{c+h-3}{c-h+1}}}{(1+\gamma)^{\frac{2e}{c-h+1}}} \gamma^{\frac{2}{c-h+1}} \times \left[1 + \left\{ 1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{\frac{c+h+2e-3}{c-h+1}} \right\}^{\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}} \dots (3-33)$$

The Weibull slope e , the constants c and h are determined experimentally. Based on the original experimental work by Lundberg and Palmgren with ball bearings and subsequent verification tests with spherical, cylindrical and tapered roller bearings the following values were assigned to the experimental constants in the rating equations:

$$\begin{aligned} e &= 9/8 \\ c &= 31/3 \\ h &= 7/3 . \end{aligned}$$

Substituting experimental constants $e = 9/8$, $c = 31/3$ and $h = 7/3$ into equations (3-32) and (3-33),

$$C_r = f_c (i L_{we} \cos \alpha)^{7/9} z^{3/4} D_{we}^{29/27}, \dots\dots\dots (3-34)$$

$$\begin{aligned} f_c &= 0,483 B_1 \times 0,377 \lambda v \frac{\gamma^{2/9} (1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \\ &\times \left[1 + \left\{ 1,04 \left(\frac{1-\gamma}{1+\gamma} \right)^{143/108} \right\}^{9/2} \right]^{-2/9} . \dots\dots (3-35) \end{aligned}$$

The value for $0,483 B_1$ is 551,13373 to calculate C_r in Newtons. Values for f_c on table 5 in ISO 281/I are calculated from substituting reduction factor which is given in table 3-2 into equation (3-35).

3.5 Basic dynamic axial load rating C_a for single row thrust roller bearings

3.5.1 Thrust roller bearings with contact angle $\alpha \neq 90^\circ$

Extension of 3.1 gives:

$$\begin{aligned} C_a &= f_c (L_{we} \cos \alpha)^{\frac{c-h-1}{c-h+1}} \tan \alpha z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}} . \\ &\dots\dots\dots (3-36) \end{aligned}$$

For thrust roller bearings the theoretical value of a basic dynamic axial load rating must be reduced on the basis of