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Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences

*Spécifications géométriques des produits (GPS) — Erreurs
systématiques et contributions à l'incertitude de mesure de la longueur,
dues aux influences thermiques*



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 16015 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

Introduction

This Technical Report is a geometrical product specification (GPS) document and is to be regarded as a global GPS document (see ISO/TR 14638). It influences chain links 4, 5 and 6 of the chains of standards.

For more detailed information on the relationship of this Technical Report to other standards and to the GPS matrix model, see Annex C.

Uncertainty in temperature measurement and measurement at other than the standard reference temperature lead to uncertainty in the length measurement results. In addition, measurements at other than the standard reference temperature leads to systematic error in the measurement result.

The principle addressed by this Technical Report is that most materials expand or contract when their temperatures are changed. If the temperature at which the measurement is made is the standard reference temperature, the nominal thermal expansion is zero but uncertainty in the measurement of temperature leads to uncertainty in the measurement result. If length measurements are made at temperatures other than the standard reference temperature, there will be a resulting differential thermal expansion. This can arise both when the measuring instrument is adjusted, as by comparison with a working standard, and when it is used to measure the workpiece.

If the temperatures and the response to thermal changes of the workpiece, the working standard, and the measuring instrument are known, a correction can be made for differential thermal expansion. It is impossible to know exactly either the temperatures or the response; thus, there will be an uncertainty in the correction and in the measurement result. This Technical Report shows how to calculate the relevant systematic error and evaluate the thermal contribution to the measurement uncertainty.

The resulting standard uncertainty component due to thermal effects shall be combined in the usual manner (see GUM) in order to evaluate the combined standard uncertainty for a measurement.

When necessary, an appropriate decision rule (e.g. an acceptable fraction of workpiece tolerance or that embodied in ISO 14253-1) can be invoked so that the consequence of the thermally-induced dimensional uncertainty on workpiece conformance decisions can be determined.

ISO/TR 16015 is developed in support of ISO 1.

It is recognized that this Technical Report, developed in support of ISO 1, will have to be brought in line with ISO/TS 17450-2, but at the time of publication this presentation was the only practical one possible.

Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences

1 Scope

This Technical Report defines procedures for the determination of errors, appropriate corrections and the evaluation of measurement uncertainty contributors due to thermal influences in the performance of length measurements for geometrical product specification (GPS), when

- the mean temperature is the standard reference temperature,
- the mean temperature is not the standard reference temperature and where the mean is taken over time and space,
- temperature varies with time.

Three cases are considered:

- a) the mean temperature is the standard reference temperature;
- b) the mean temperature is not the standard reference temperature and the user makes corrections;
- c) the mean temperature is not the standard reference temperature and the user makes no corrections.

This Technical Report provides the specialized details for the analysis of thermal influence when broader analyses, such as those in accordance with ISO/TS 14253-2, show that thermal effects are significant contributors to the measurement uncertainty, systematic error or both. While temperature gradients are not covered in detail, some information is given (see Annex A).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1, *Geometrical Product Specifications (GPS) — Standard reference temperature for geometrical product specification and verification*

Guide to the Expression of Uncertainty in Measurement (GUM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1st edition, 1993, corrected and reprinted in 1995

International Vocabulary of Basic and General Terms in Metrology (VIM). BIPM, IFCC, IEC, ISO, IUPAC, IUPAP, OIML, 2nd edition, 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in GUM and VIM and the following apply.

3.1 Terms concerning coefficient of thermal expansion

3.1.1

coefficient of thermal expansion

α

ratio of the fractional change of length to the change in temperature

NOTE 1 In general, this coefficient is a function of temperature. For the purposes of this Technical Report, however, a temperature range averaged value of the coefficient is used in accordance with

$$\alpha(20, T) = \frac{L_T - L_{20}}{L_{20}(T - 20)} \quad (1)$$

where the temperature range is relative to 20°C in accordance with ISO 1.

NOTE 2 To the symbol α are added subscripts to denote the coefficient of thermal expansion for the workpiece (i.e. α_w) and the working standard (i.e. α_s).

NOTE 3 *Coefficient of thermal expansion* can describe the behaviour of a non-homogeneous body such as a composite working standard or a workpiece that is an assembly of different materials. No distinction is made in this document between coefficients of expansion for homogeneous and composite working standards or workpieces.

3.1.2

measured coefficient of thermal expansion

α_m

experimentally-determined coefficient of thermal expansion of a specific individual object

NOTE 1 This coefficient can be obtained from the calibration service of an accredited measurement laboratory or from properly performed experiments.

NOTE 2 To the symbol α_m are added subscripts to denote the measured coefficient of thermal expansion for the workpiece (i.e. α_{mw}) and the working standard (i.e. α_{ms}).

3.1.3

nominal coefficient of thermal expansion

α_n

approximate value for the coefficient of thermal expansion over a range of temperature from 20 °C to T

NOTE To the symbol α_n are added subscripts to denote the nominal coefficient of thermal expansion for the workpiece (i.e. α_{nw}) and the working standard (i.e. α_{ns}). Estimated values for α_{nw} and α_{ns} can be obtained from experiments on like objects or from published data.

3.1.4

uncertainty of coefficient of thermal expansion

$u(\alpha)$

parameter that characterizes the dispersion of the values that could reasonably be attributed to the coefficient of thermal expansion

NOTE 1 To the symbol $u(\alpha)$ are added subscripts to denote the uncertainty of coefficient of thermal expansion for the workpiece [i.e. $u(\alpha_w)$] and the working standard [i.e. $u(\alpha_s)$]. Such values, like that of α itself, shall either be estimates, based on an assumed probability distribution, or the results of experiments performed to measure α .

NOTE 2 The uncertainty for α_n is usually much larger than the uncertainty for α_m .

3.2 Terms concerning thermal expansion

3.2.1

thermal expansion

Δ_E

change in length of a body, such as a workpiece or working standard, in response to a temperature change

3.2.2

thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nE} or Δ_{mE}

estimate of the thermal expansion of an object from 20 °C to its average temperature at the time of measurement

NOTE 1 The estimate is based on either nominal or measured coefficients of thermal expansion:

$$\Delta_{nE} = \alpha_n L (T - 20\text{ °C}) = \alpha_n L \theta$$

$$\Delta_{mE} = \alpha_m L (T - 20\text{ °C}) = \alpha_m L \theta$$

$$\Delta_E = \Delta_{nE} \text{ or } \Delta_{mE} \quad (2)$$

NOTE 2 To the symbol Δ_{nE} or Δ_{mE} are added subscripts to denote the thermal expansion based on nominal/measured coefficients of thermal expansion for the workpiece (i.e. Δ_{nEw} or Δ_{mEw}) and the working standard (i.e. Δ_{nEs} or Δ_{mEs}).

3.2.3

corrected length

L_c

measured length adjusted for the computed thermal expansion based on either the measured or nominal thermal expansion

NOTE The corrected length can be calculated to

$$L_c = L_m - \Delta_{mE}$$

or

$$L_c = L_m - \Delta_{nE} \quad (3)$$

3.2.4

differential thermal expansion

difference between the changes in the lengths of the workpiece and the working standard in response to temperature changes from 20 °C to their temperatures at the times of the measurements

NOTE If not corrected, differential thermal expansion can cause a systematic error in a measurement result.

3.2.5

differential thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nDE} or Δ_{mDE}

difference between the thermal expansion of the workpiece and the working standard(s)

NOTE The differential thermal expansion based on nominal or measured coefficients of thermal expansion can be calculated by

$$\Delta_{nDE} = \Delta_{nEw} - \Delta_{nEs}$$

or

$$\Delta_{mDE} = \Delta_{mEw} - \Delta_{mEs} \quad (4)$$

3.2.6

uncertainty of thermal expansion due to uncertainty of α

$u_E(L)$

uncertainty in the thermal expansion arising from uncertainty in the coefficient of thermal expansion

NOTE 1 The uncertainty of thermal expansion due to uncertainty of α can be calculated by

$$u_E(L) = L \theta u(\alpha) \quad (5)$$

NOTE 2 An additional subscript is added to each symbol to specify values for a workpiece ("..._w") or working standard ("..._s").

3.2.7

uncertainty of differential thermal expansion due to uncertainties of α_w and α_s

$u_{DE}(L)$

combined standard uncertainty of the uncertainties of thermal expansion of the workpiece and working standard due to uncertainties in their coefficients of expansion

NOTE 1 Assuming that the coefficients of thermal expansion of the workpiece and the working standard are uncorrelated, the value of $u_{DE}(L)$ is given by

$$\begin{aligned} u_{DE}(L) &= \sqrt{u_E^2(L_w) + u_E^2(L_s)} \\ &= \sqrt{\theta_w^2 L_w^2 u^2(\alpha_w) + \theta_s^2 L_s^2 u^2(\alpha_s)} \end{aligned} \quad (6)$$

NOTE 2 The estimates of the coefficients of thermal expansion of the workpiece and working standard can be assumed to be uncorrelated when they are obtained from different sources.

NOTE 3 When nominal values are used to estimate the coefficients of thermal expansion, this quantity has historically been called the *uncertainty of nominal differential expansion (UNDE)*.

3.3 Dimensional consequences of temperature variation

3.3.1

dimensional thermal response

amplitude of length variation of an object as a thermal response to the magnitude and time-dependency of a temperature fluctuation

3.3.2

thermal response time

soak-out time

time interval between the instant when the environmental temperature is subjected to a specified abrupt change and the instant when the temperature of an object reaches and remains within specified limits around its final steady value

NOTE When a change in environmental temperature is experienced, such as occurs when an object is transported from one room to another, there will be some period of time during which the object equilibrates, within specified limits, to its new environmental temperature.

3.3.3

differential thermal response

length difference between any two objects measured simultaneously in an environment with varying temperature and caused solely by the variation in temperature of the objects with time

3.3.4**dimensional variation due to environmental temperature variation** E_{ETV}

estimate of the possible length measurement variation induced solely by deviation of the environment from average conditions over a time interval equivalent to the adjustment cycle time

NOTE The dimensional variation due to environmental temperature variation, E_{ETV} , is usually determined from the results of two **drift test(s)** (3.4.5), one of the working standard and comparator and the other of the workpiece and comparator. However, drift tests performed at a single position cannot reveal all thermally-induced errors, particularly where spatial temperature gradients exist. It is the responsibility of the instrument user to ensure that such effects are detected and the appropriate uncertainties added.

3.3.5**uncertainties of workpiece and working standard temperatures** $u(\theta_w)$ and $u(\theta_s)$

uncertainty of time-averaged temperature of the workpiece and working standard

NOTE 1 These uncertainties arise from the calibration of the thermometer, thermometer mounting procedures, and instrumental variations.

NOTE 2 The time over which the temperature is averaged is usually the period of a measurement cycle.

3.3.6**uncertainty of length due to temperature measurement** $u_{\text{TM}}(L)$

uncertainty in a measured length due to uncertainty in the measurement of the temperature at which the length measurement was made

NOTE 1 In the case where there is a workpiece and a working standard, it is given by

$$u_{\text{TM}}(L) = \sqrt{\alpha_w^2 L_w^2 u^2(\theta_w) + \alpha_s^2 L_s^2 u^2(\theta_s)} \quad (7)$$

where $u(\theta_w)$ and $u(\theta_s)$ are the **uncertainties of the workpiece and the working standard temperatures** (3.3.5) in measurement. The temperature measurements of the workpiece and the working standard are assumed to be uncorrelated, an assumption which is valid if the two temperatures are measured using two different thermometers, each traceable to the international temperature scale (ITS-90) through different calibration processes.

NOTE 2 In cases where the standard is a laser interferometer, the coefficient relating wavelength to the temperature of the propagation medium is used for the coefficient of expansion of the working standard. In the case where the medium is "standard" air, the value for α_{ns} is $0,93 \times 10^{-6}/^\circ\text{C}$.

3.3.7**uncertainty of length due to environmental temperature variation** $u_{\text{ETV}}(L)$

uncertainty in the estimate of the possible length measurement variation induced solely by deviation of the environmental temperature from average conditions over a time interval equivalent to the adjustment cycle time

NOTE 1 The dimensional uncertainty of length due to environmental temperature variation, $u_{\text{ETV}}(L)$, arises from the range of the drift of the instrument/working standard/workpiece system over times equivalent to the adjustment cycle time. It is established from measurements or estimates of dimensional variation due to environmental temperature variation, E_{ETV} .

NOTE 2 In computing $u_{\text{ETV}}(L)$, note that the maximum error is the range observed in the drift test, depending on whether the setting was done at the lowest temperature and the measuring at the highest temperature, or vice versa. The probability density function may be "U-shaped" as, for example, for a sinusoidally varying temperature. However, the probability that the setting and measuring are exactly displaced in time by the period of the sinusoidally varying temperature is low. The setting-measuring process involves taking pairs of points from the U-shaped probability distribution function and then differencing them. The consequence of this procedure is that the measurement process has

a different probability distribution function from “U-shaped”. The default is to use a uniform distribution [see 5.4 and Equation (13)].

3.4 Measuring instruments, measuring procedures and metrology

3.4.1

comparator

measuring device used to perform a comparison of a workpiece and a working standard

NOTE A comparator can be a simple short-range indicating device such as a gauge block comparator or a complex comparator such as a coordinate measuring machine.

3.4.2

working standard

standard that is used routinely to calibrate or check material measures, measuring instruments or reference materials

[VIM:1993, definition 6.7]

NOTE 1 A working standard is usually calibrated against a *reference standard* (see VIM:1993, definition 6.6).

NOTE 2 A working standard used routinely to ensure that measurements are being carried out correctly is called a check standard. While it is recognized that the use of *working standard* versus *check standard* has not yet been perfectly resolved, at the time of publication this terminology remained the only practical solution possible. The subject is to be re-examined at the next revision of this Technical Report.

NOTE 3 For procedures in accordance with this Technical Report, such working standards are dimensional and can be in the form of the wavelength of light, a gauge block, a line standard, a lead screw, etc.

3.4.3

adjustment

operation of bringing a measuring instrument into a state of performance suitable for its use

[VIM:1993, definition 4.30]

EXAMPLE The action of nulling or setting a comparator with a working standard.

NOTE Adjustment may be automatic, semi-automatic or manual.

3.4.4

adjustment cycle time

period between successive adjustments of a comparator

3.4.5

drift test

experiment conducted to determine the slow change in the metrological characteristic of a measuring instrument

3.4.6

tolerance

TOL

difference between the upper and lower tolerance limits

[ISO 3534-2:1993, definition 1.4.4]

NOTE For the purpose of this Technical Report, the abbreviation TOL is used instead of the standard “T” (for target value), in order to avoid confusion with other symbols or abbreviations using the same letter.

3.4.7**target uncertainty** U_T

uncertainty determined as the optimum for the measuring task

[ISO/TS 14253-2:1999, definition 3.10]

3.4.8**systematic error**

mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand

NOTE 1 Systematic error is equal to error minus random error.

NOTE 2 Like true value, systematic error and its causes cannot be completely known.

NOTE 3 For a measuring instrument, see *bias* (VIM:1993, definition 5.25).

[VIM:1993, definition 3.14]

3.5 Dimensional quantities related to thermal effects**3.5.1****standard uncertainty component due to thermal effects** $u_{cT}(L)$

standard uncertainty component for a length measurement made at a temperature other than 20 °C

NOTE The standard uncertainty component due to thermal effects can be calculated by

$$u_{cT}(L) = \sqrt{u_{ETV}^2(L) + u_{DE}^2(L) + u_{TM}^2(L)} \quad (8)$$

A simplified form arises in the case where: $\alpha_s = \alpha_{ns} = \alpha_w = \alpha$ with a standard uncertainty $u(\alpha)$; $\theta_w = \theta_s = \theta$ with a standard uncertainty $u(\theta)$; and where the nominal lengths $L_w = L_s = L$. In this case, $A_{nDE} = 0$ and

$$u_{cT}(L) = \sqrt{u_{ETV}^2(L) + 2\theta^2 L^2 u^2(\alpha) + 2\alpha^2 L^2 u^2(\theta)}$$

3.5.2**thermal error**

TE

estimate of the maximum error that might reasonably be expected to occur if a length measurement is not corrected for a nominal differential expansion

NOTE 1 The thermal error can be calculated by

$$TE = [A_{nDE} + 2u_{cT}(L)] \quad (9)$$

NOTE 2 If length measurements are made at other than 20 °C, and in the special case where corrections are not made for the nominal differential thermal expansion between working standard and workpiece, then, in order to be in accordance with this Technical Report, the thermal error has to be evaluated and reported.

3.5.3**thermal error index**

TEI

fraction of the tolerance that is consumed by the thermal error

NOTE 1 The thermal error index can be calculated by

$$TEI = 2 \times \frac{TE}{TOL} \times 100 \% \quad (10)$$

NOTE 2 In the case where a target uncertainty, U_T , exists, the thermal error index is computed using Equation (10), with TOL replaced by $2 U_T$.

NOTE 3 The TEI is a component of uncertainty management; large values of the TEI may require that a measurement result has to be corrected for the differential thermal expansion in order to prove workpiece conformance.

4 Symbols and abbreviated terms

The standard reference temperature is 20 °C (see ISO 1). All temperatures are assumed to be in degrees Celsius. The majority of the symbols used in this Technical Report are based on the conventions given in Table 1, while Table 2 provides an overview of the symbols used.

Table 1 — Symbol conventions

Symbol	Explanation
α_{ij}	Coefficient of thermal expansion, where: i is the subscript m or n, where m is measured and n is nominal, and $j = w$ or s , where w represents the workpiece and s the working standard
$\Delta(\dots)_{DE}$	Difference between thermally-induced expansions of the workpiece and working standard, where (...) may contain the subscript m or n, where m is for measured and n for nominal
$\Delta(\dots)_E$	Thermally-induced expansions, where (...) may contain the subscript m or n, where m is for measured and n for nominal, and the subscript w or s, where w represents the workpiece and s the working standard
L_j	Length where j is the subscript: m for measured, uncorrected for temperature; n for nominal; c for corrected for nominal expansion; T is at temperature T ; or w or s, where w represents the workpiece and s the working standard
T_j	Temperature where j is the subscript w or s, where w represents the workpiece and s the working standard
θ_j	Temperature difference ($T_j - 20$ °C) where j is the subscript w or s, where w represents the workpiece and s the working standard
u (symbol)	Uncertainty of the quantity associated with the symbol given in parentheses

Table 2 — List of symbols and abbreviations

Symbol	Explanation
d	Reading of instrument
E_{ETV}	Dimensional variation due to environmental temperature variation
k	Coverage factor
L	Length
L_c	Corrected length
L_i	Length of a measuring instrument or comparator measured at temperature, T , and corrected for nominal thermal expansion
L_m	Length of an object measured at temperature, T , and not corrected for nominal thermal expansion
L_n	Nominal length of a workpiece
L_w	Length of a workpiece at 20 °C
L_s	Length of a working standard at 20 °C