



Recommendation ITU-R M.1851-1
(01/2018)

**Mathematical models for
radiodetermination radar systems antenna
patterns for use in interference analyses**

M Series
**Mobile, radiodetermination, amateur
and related satellite services**



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Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

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RECOMMENDATION ITU-R M.1851-1

**Mathematical models for radiodetermination radar systems antenna patterns
for use in interference analyses**

(2009-2018)

Scope

This Recommendation describes radiodetermination radar systems antenna patterns to be used for single-entry and aggregate interference analysis. Dependent on the antenna 3 dB beamwidth and first peak side-lobe level, the proper set of equations for both azimuth and elevation patterns may be selected. Both peak, for single interferer, and average patterns, for multiple interferers, are defined.

Keywords

Antenna patterns, current distribution, illumination field, peak and average mask pattern equations

Abbreviations/Glossary

ADP Antenna directivity pattern

Related ITU-R Recommendations

Recommendations ITU-R F.699, ITU-R F.1245, ITU-R M.1638, ITU-R M.1652, ITU-R M.1849

The ITU Radiocommunication Assembly,

considering

that a mathematical model is required for generalized patterns of antennas for interference analyses when no specific pattern is available for the radiodetermination radar systems,

recommends

1 that, if antenna patterns and/or pattern equations applicable to a radar(s) are available in other ITU-R Recommendations dealing with radiodetermination radar system characteristics, then those should be used;

2 that, in the absence of information concerning the antenna patterns of the radiodetermination radar system antenna involved, one of the mathematical reference antenna models described in Annex 1 should be used for interference analysis.

Annex 1

Mathematical models for radiodetermination radar systems antenna patterns for use in interference analyses

1 Introduction

A generalized mathematical model for radiodetermination radar systems antenna patterns is required when these patterns are not defined in ITU-R Recommendations applicable to the radiodetermination radar system under analysis. Generalized antenna pattern models could be used in analyses involving single and multiple interferer entries, such as that from other radar and communication systems.

This text describes proposed antenna patterns to be used. Given knowledge about beamwidth and the first peak side-lobe level, the proper set of equations for both azimuth and elevation patterns may be selected.

The result of surveyed antenna parameter ranges from ITU-R Recommendations are recorded in Table 1.

TABLE 1
Surveyed antenna parameter limits

Antenna parameter	Units	Description	Minimum value	Maximum value
Transmit and receive frequencies	MHz		420	33 400
Antenna polarization type		Horizontal, vertical, circular		
Antenna type		Omni, yagi element array, parabolic reflector, phased array		
Beam type – most common		Fan, pencil, cosecant squared		
Transmit and receive gain	dBi		25.6	54
Elevation beamwidth (–3dB)	degrees	Pencil beam	0.25	5.75
		Cosecant squared (CSC^2) (equation (12) and Table 4)	3.6 (θ_3) 20 (θ_{Max})	3.6 (θ_3) 44 (θ_{Max})
Azimuth beamwidth (–3dB)	degrees	Pencil beam	0.4	5.75
Elevation scan angle limit	degrees		–60	+90
Azimuth scan angle limit	degrees		30 sector	360
First side-lobe level below main lobe peak	dB		–35	–15.6

Table 1 was used to guide the development of the antenna types and patterns proposed.

2 Mathematical formulae

2.1 Radar antenna with a narrow beamwidth

2.1.1 Background

In the absence of neither particular information concerning the 3 dB beamwidth or the shape of the current distribution or illumination field on the antenna aperture, but in the presence of information given about the size of the antenna, the θ_3 half power beam-width (degrees) is approximated by $70 \lambda/D$, where λ the wavelength and D the antenna length are expressed in the same units; see Recommendation ITU-R F.699, *recommends* 4.1.

If the information about the shape of the current distribution or illumination field on the antenna aperture is accessible, then a more accurate model of the antenna pattern can be used.

To simplify the analysis, the antenna current distribution is considered as a function of either the elevation or azimuth coordinates. The directivity pattern, $F(\mu)$, of a given distribution is found from the finite Fourier transform as:

$$F(\mu) = \frac{1}{2} \int_{-1}^{+1} f(x) \cdot e^{j\mu x} dx \quad (1)$$

where:

$f(x)$: relative shape of field distribution, see Table 2 and Fig. 1

μ : provided in the table below = $\pi \left(\frac{l}{\lambda} \right) \sin(\alpha)$

l : overall length of aperture

λ : wavelength

ω : beam elevation or azimuth pointing (scan) angle relative to aperture normal

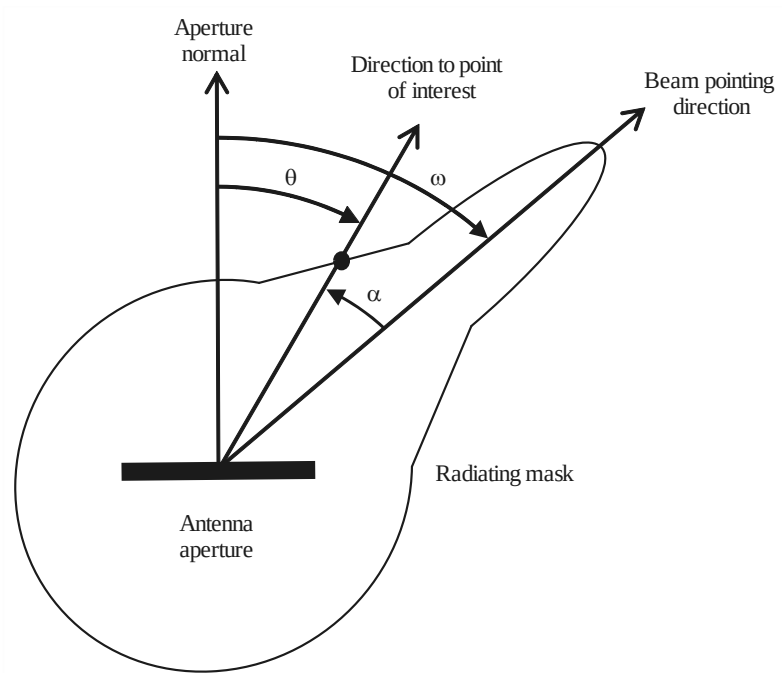
θ : point of interest direction angle relative to aperture normal

α : point of interest direction angle relative to pointing angle direction ($\alpha = \theta - \omega$)

x : normalized distance along aperture $-1 \leq x \leq 1$

j : complex number notation.

FIGURE 1
Antenna polar diagram mask



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The proposed theoretical antenna patterns for antennas having uniform phase field distribution are provided in Table 2.

The proposed theoretical antenna patterns for phased array antennas are given in § 7, taking into account specific sidelobes effects arising at large scan ω angles.

The parameters and formulae for determining antenna directivity patterns (ADP) that are presented in Table 2 (and thereafter in the related Table 3 and Figures) are correct only in the case where the field amplitude at the edge of the antenna aperture is equal to zero and within the bounds of the main lobe and first two side lobes of the ADP.

With other values of field amplitude at the edge of the antenna aperture, the form of the ADP and its parameters may differ significantly from the theoretical ones presented in this Recommendation.

In absence of any other information, a simplified antenna pattern fitting with the theoretical main lobe and a mask in other directions may be considered for sharing and compatibility studies with radar systems. Peak or average masks are recommended for doing such studies with respectively single or multiple interferers. The mask departs at break point issued from theoretical pattern and decreases over sidelobes down to a floor mask to represent antenna far side lobes and back lobes, as described in § 2.1.3.

If real radar antenna patterns are available, then those should be digitized and used.

2.1.2 Theoretical antenna equations

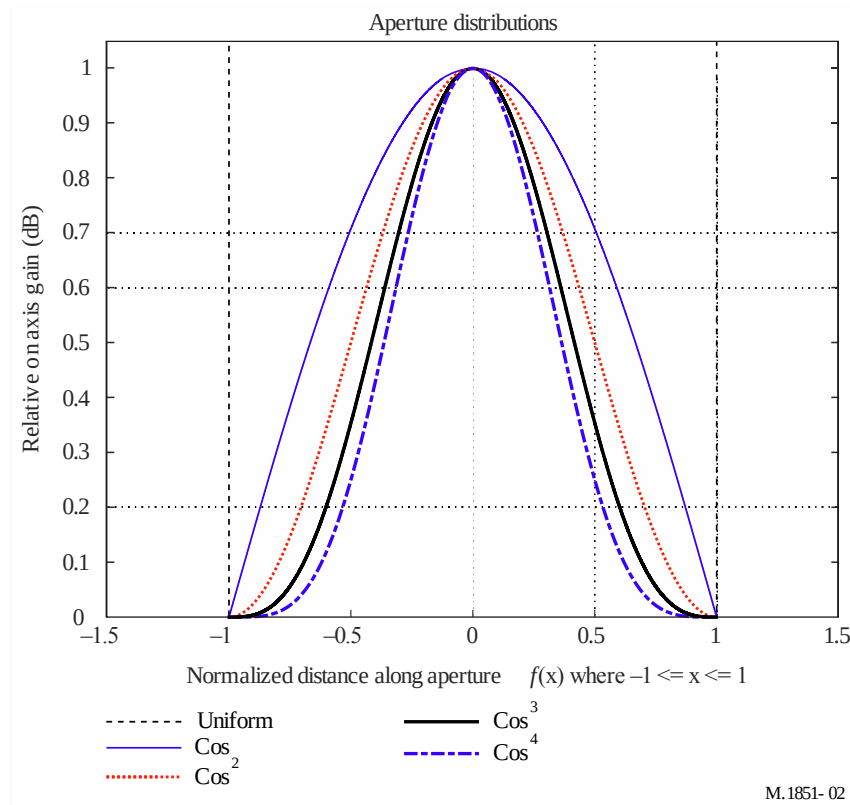
Equations of directivity patterns and associated parameters are given in Table 2 for different shape of field distribution on the antenna aperture.

TABLE 2
Theoretical antenna directivity parameters

Relative shape of field distribution $f(x)$ where $-1 \leq x \leq 1$	Directivity pattern $F(\mu)$	θ_3 half power beam-width (degrees)	μ as a function of θ_3	First side-lobe level below main lobe peak (dB)	Equation No.
Uniform value of 1	$\frac{\sin(\mu)}{\mu}$	$50.8 \left(\frac{\lambda}{l} \right)$	$\frac{\pi \cdot 50.8 \cdot \sin(\theta)}{\theta_3}$	-13.2	(2)
$\cos(\pi \cdot x/2)$	$\frac{\pi}{2} \left[\frac{\cos(\mu)}{\left(\frac{\pi}{2} \right)^2 - \mu^2} \right]$	$68.8 \left(\frac{\lambda}{l} \right)$	$\frac{\pi \cdot 68.8 \cdot \sin(\theta)}{\theta_3}$	-23	(3)
$\cos^2(\pi \cdot x/2)$	$\frac{\pi^2}{2 \cdot \mu} \left[\frac{\sin(\mu)}{\pi^2 - \mu^2} \right]$	$83.2 \left(\frac{\lambda}{l} \right)$	$\frac{\pi \cdot 83.2 \cdot \sin(\theta)}{\theta_3}$	-32	(4)
$\cos^3(\pi \cdot x/2)$	$\frac{3 \cdot \pi \cdot \cos(\mu)}{8} \left[\frac{1}{\left(\frac{\pi}{2} \right)^2 - \mu^2} - \frac{1}{\left(\frac{3 \cdot \pi}{2} \right)^2 - \mu^2} \right]$	$95 \left(\frac{\lambda}{l} \right)$	$\frac{\pi \cdot 95 \cdot \sin(\theta)}{\theta_3}$	-40	(5)
$\cos^4(\pi \cdot x/2)$	$\frac{3\pi^4 \sin(\mu)}{2\mu(\mu^2 - \pi^2)(\mu^2 - 4\pi^2)}$	$106 \left(\frac{\lambda}{l} \right)$	$\frac{\pi \cdot 106 \cdot \sin(\theta)}{\theta_3}$	-47	(6)

where θ_3 is the 3 dB antenna half-power beamwidth (degrees). The relative shapes of the field distribution functions $f(x)$, as defined in Table 2, are plotted in Fig. 2.

FIGURE 2
Aperture distributions



Given that the half power beamwidth, θ_3 , is provided, the value of μ can be redefined as a function of the half-power antenna beamwidth. This is done by replacing the quantity $\left(\frac{l}{\lambda}\right)$ in $\mu = \pi \left(\frac{l}{\lambda}\right) \sin(\theta)$ by a constant that depends on the relative shape of the field distribution; divided by the half-power beamwidth, θ_3 , as shown in Table 2. These constant values of 50.8, 68.8, 83.2, 95 and 106, shown in Table 2, can be derived by setting the equation for $F(\mu)$ equal to -3 dB, and solving for the angle θ .

Figure 3 shows various linear aperture antenna patterns for uniform, cosine (COS), cosine-squared (COS^2) and cosine-cubed (COS^3) and cosine-to-the-fourth power (COS^4) field distribution functions. As the patterns are mathematically symmetric, they have been partially traced on the diagram. For comparison, all patterns are set to a same 3 dB beamwidth of 6.0 degrees, meaning different ratios for λ/l .