



Recommendation ITU-R P.834-9
(12/2017)

**Effects of tropospheric refraction
on radiowave propagation**

P Series
Radiowave propagation



Foreword

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Series	Title
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BT	Broadcasting service (television)
F	Fixed service
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P	Radiowave propagation
RA	Radio astronomy
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SA	Space applications and meteorology
SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
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V	Vocabulary and related subjects

Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

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RECOMMENDATION ITU-R P.834-9

Effects of tropospheric refraction on radiowave propagation

(Question ITU-R 201/3)

(1992-1994-1997-1999-2003-2005-2007-2015-2016-2017)

Scope

Recommendation ITU-R P.834 provides methods for the calculation of large-scale refractive effects in the atmosphere, including ray bending, ducting layers, the effective Earth radius, the apparent elevation and boresight angles in Earth-space paths and the effective radio path length.

Keywords

Tropospheric excess path length, Earth-space link, GNSS, numerical weather product, digital maps

The ITU Radiocommunication Assembly,

considering

- a) that for the proper planning of terrestrial and Earth-space links it is necessary to have appropriate calculation procedures for assessing the refractivity effects on radio signals;
- b) that procedures have been developed that allow the calculation of some refractive propagation effects on radio signals on terrestrial and Earth-space links,

recommends

that the information in Annex 1 should be used for the calculation of large-scale refractive effects.

Annex 1**1 Ray bending**

A radio ray passing through the lower (non-ionized) layer of the atmosphere undergoes bending caused by the gradient of the refractive index. Since the refractive index varies mainly with altitude, only the vertical gradient of the refractive index is generally considered. The curvature at a point is therefore contained in the vertical plane and is expressed by:

$$\frac{1}{\rho} = - \frac{\cos \varphi}{n} \frac{dn}{dh} \quad (1)$$

where:

- ρ : radius of curvature of the ray path
- n : refractive index of the atmosphere
- dn/dh : vertical gradient of refractive index
- h : height of the point above the Earth's surface

φ : angle of the ray path with the horizontal at the point considered.

This ray curvature is defined as positive for ray bending towards the Earth's surface. This phenomenon is virtually independent of frequency, if the index gradient does not vary significantly over a distance equal to the wavelength.

2 Effective Earth radius

If the path is approximately horizontal, φ is close to zero. However, since n is very close to 1 equation (1) is simplified as follows:

$$\frac{1}{\rho} = - \frac{dn}{dh} \quad (2)$$

It is therefore clear that if the vertical gradient is constant, the trajectories are arcs of a circle.

If the height profile of refractivity is linear, i.e. the refractivity gradient is constant along the ray path, a transformation is possible that allows propagation to be considered as rectilinear. The transformation is to consider a hypothetical Earth of effective radius $R_e = k a$, with:

$$\frac{1}{ka} = \frac{1}{a} + \frac{dn}{dh} = \frac{1}{R_e} \quad (3)$$

where a is the actual Earth radius, and k is the effective earth radius factor (k -factor). With this geometrical transformation, ray trajectories are linear, irrespective of the elevation angle.

Strictly speaking, the refractivity gradient is only constant if the path is horizontal. In practice, for heights below 1000 m the exponential model for the average refractive index profile (see Recommendation ITU-R P.453) can be approximated by a linear one. The corresponding k -factor is $k = 4/3$.

3 Modified refractive index

For some applications, for example for ray tracing, a modified refractive index or refractive modulus is used, defined in Recommendation ITU-R P.310. The refractive modulus M is given by:

$$M = N + \frac{h}{a} \quad (4)$$

h being the height of the point considered expressed in metres and a the Earth's radius expressed in thousands of kilometres. This transformation makes it possible to refer to propagation over a flat Earth surmounted by an atmosphere whose refractivity would be equal to the refractive modulus M .

4 Apparent boresight angle on slant paths

4.1 Introduction

In sharing studies it is necessary to estimate the apparent elevation angle of a space station taking account of atmospheric refraction. An appropriate calculation method is given below.

4.2 Visibility of space station

As described in § 1 above, a radio beam emitted from a station on the Earth's surface (h (km) altitude and θ (degrees) elevation angle) would be bent towards the Earth due to the effect of atmospheric refraction. The refraction correction, τ (degrees), can be evaluated by the following integral:

$$\tau = - \int_h^\infty \frac{n'(x)}{n(x) \cdot \tan \varphi} dx \quad (5)$$

where φ is determined as follows on the basis of Snell's law in polar coordinates:

$$\cos \varphi = \frac{c}{(r + x) \cdot n(x)} \quad (6)$$

$$c = (r + h) \cdot n(h) \cdot \cos \theta \quad (7)$$

r : Earth's radius (6 370 km)

x : altitude (km).

Since the ray bending is very largely determined by the lower part of the atmosphere, for a typical atmosphere the refractive index at altitude x may be obtained from:

$$n(x) = 1 + a \cdot \exp(-bx) \quad (8)$$

where:

$$a = 0.000315$$

$$b = 0.1361.$$

This model is based on the exponential atmosphere for terrestrial propagation given in Recommendation ITU-R P.453. In addition, $n'(x)$ is the derivative of $n(x)$, i.e. $n'(x) = -ab \exp(-bx)$.

The values of $\tau(h, \theta)$ (degrees) have been evaluated under the condition of the reference atmosphere and it was found that the following numerical formula gives a good approximation:

$$\tau(h, \theta) = 1/[1.314 + 0.6437 \theta + 0.02869 \theta^2 + h(0.2305 + 0.09428 \theta + 0.01096 \theta^2) + 0.008583 h^2] \quad (9)$$

The above formula has been derived as an approximation for $0 \leq h \leq 3$ km and $\theta_m \leq \theta \leq 10^\circ$, where θ_m is the angle at which the radio beam is just intercepted by the surface of the Earth and is given by:

$$\theta_m = -\arccos \left(\frac{r}{r + h} \cdot \frac{n(0)}{n(h)} \right) \quad (10)$$

or, approximately, $\theta_m = -0.875\sqrt{h}$ (degrees).

Equation (9) also gives a reasonable approximation for $10^\circ < \theta \leq 90^\circ$.

Let the elevation angle of a space station be θ_0 (degrees) under free-space propagation conditions, and let the minimum elevation angle from a station on the Earth's surface for which the radio beam is not intercepted by the surface of the Earth be θ_m . The refraction correction corresponding to θ_m is $\tau(h, \theta_m)$. Therefore, the space station is visible only when the following inequality holds:

$$\theta_m - \tau(h, \theta_m) \leq \theta_0 \quad (11)$$