

REPORT 944

THEORETICAL NETWORK PLANNING

(Question 46/10, Study Programme 46L/10)

(1982)

1. Introduction

Broadcasting-transmitter networks should be planned in such a way that the required coverage of the area is provided using the minimum number of frequencies. From the purely technical standpoint, the coverage area of each transmitter depends upon a number of factors, for example: transmitter power, minimum usable field strength, radio-frequency protection ratio, the distance between transmitters sharing the same or adjacent channels, channel spacing, bandwidth of emission and other factors influencing wave propagation. It also depends on the channel distribution scheme.

When a large number of channels are to be planned or replanned for a particular AM or FM sound or television service, it has been found that effecting an efficient use of the spectrum can prove difficult when employing empirical methods only. For this reason, a theory of uniform transmitter networks was developed during the late 1950s and early 1960s [EBU, 1960]. This method can be applied with success when some uniformity of standards exists for the service to be planned. Furthermore, the frequency band to be planned should be constrained as little as possible, i.e. there should ideally be complete freedom in assigning any frequency to any transmitter.

The theory is not only useful in designing new or remodelling actual transmitter networks, but also provides a powerful tool for determining optimal technical parameters such as channel spacing, transmitter characteristics, etc., and identifying the best attainable coverage possible.

The method described below has already been used during the VHF/UHF European Broadcasting conference, Stockholm 1961, during the African VHF/UHF Broadcasting Conference, 1963, and helped in preliminary studies for the Geneva 1975 LF/MF Broadcasting Conference.

2. The theory of regular networks

It should be noted that the networks which will be studied in the present section are purely theoretical in the following sense:

- all the transmitters are identical: their power and antenna height are the same;
- they are equipped with non-directional antennas;
- propagation is isotropic and independent of frequency, at least within the band to be planned.
- for the purpose of calculating distances, the planning area is assumed to be flat and the population evenly spread over its surface; there are neither political nor natural boundaries.

In those conditions, and provided interference is negligible, the coverage area, i.e., the region where a good reception is achievable with a normal domestic receiver, is limited by a contour within which the electromagnetic field strength is greater than or equal to the value necessary to obtain a given signal-to-noise ratio. In the ideal situation described above, this contour is a circle whose radius depends on the type of service and the propagation laws valid for the frequency range considered. If it is desired to cover the whole of the planning area using such circular coverage areas, it is quite obvious that the number of transmitters per unit surface will be minimized if they are situated at the vertices of a lattice of equilateral triangles (Fig. 1). It should be noted that there is not one transmitter per triangle, but rather one transmitter for each pair of triangles forming a rhombus (shaded area of Fig. 1). This remark is of importance when computing the network efficiency.

With this configuration, every location on the planning area is served by at least one transmitter and overlapping of service areas is minimized. In what follows, we will take as a unit of length the side of the elementary equilateral triangle. The transmitter density is then one per elementary rhombus, the area of which is $\sqrt{3}/2$. The network efficiency can be expressed as the ratio of the total area where the service is acceptable (the area of the elementary rhombus) to the sum of the (overlapping) coverage areas. A regular lattice of equilateral triangles gives the optimum value, i.e., 0.83 (each transmitter has a coverage area of $\pi/3$ with a radius of $\sqrt{3}/3$). The ideal value would obviously be 1, but it is not attainable as it is impossible to cover a surface with circles without overlapping areas. A regular lattice of squares (or of isosceles right triangles) only achieves a ratio of 0.64 (each transmitter has a coverage area of $\pi/2$ with a radius of $\sqrt{2}/2$).

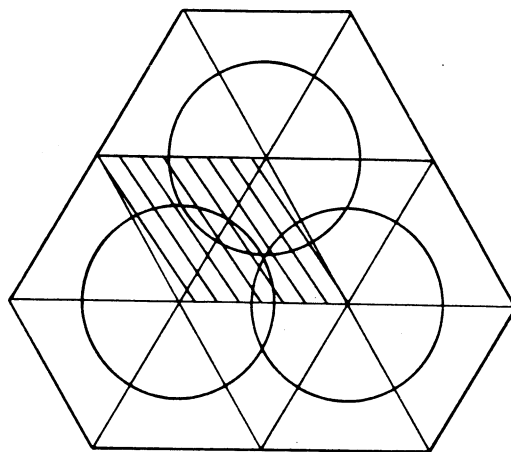


FIGURE 1 – Location and coverage areas of identical broadcasting transmitters in a regular network

It should be noted that actual transmitter networks are neither geometrically regular, nor do their technical characteristics correspond to those of the theoretical network. Deviations from geometrical regularity and from the power and antenna height of the theoretical network towards lower values will inevitably result in lower network efficiency. Nevertheless it is possible to derive, from the results of studies concerning theoretical networks, a fairly clear picture of the relationship between these factors and network efficiency.

3. An example of theoretical network

It is assumed first of all that the available frequency band has been divided, in an appropriate manner, into channels sufficiently wide for a sound or television programme to be located within its limits and adequately spaced (9 kHz for AM sound broadcasting; 100 or 200 kHz for FM sound broadcasting; 6 or 8 MHz for television (200 kHz and 6 MHz correspond to the American channels)) to make co-channel interference predominant vis-à-vis adjacent-channel interference.

It is then easy to derive, from this channel spacing, the number of channels that can be used when planning. As this number is obviously finite, it is inevitable that, at some more or less large distance from the starting point, the first channel assigned has to be re-used. The smaller the distance D , between two transmitters operating on the same channel, the smaller the number of channels needed to cover the whole planning area. But it is necessary that D be large enough for the two co-channel transmitters not to interfere unduly with each other.

It is not difficult to evaluate D . With the help of suitable propagation curves, field strengths of the wanted transmitter and of the interferer are computed, for given percentages of time (see Recommendations 368 and 370), on the circle defined in § 2, limiting the coverage area of the wanted transmitter. D is chosen sufficiently large so that the difference, expressed in dB, between the two field strengths is larger than the protection ratio which is to be found in the relevant Recommendations 560, 412, 418 and Report 306. The case of multiple interference will be dealt with later.

With N channels, it is possible to cover N elementary rhombi, that is an area equal to $N\sqrt{3}/2$, taking as unit area the surface of a square whose side is equal to the side of the elementary triangle. So another kind of network arises using rhombi consisting of N elementary rhombi, and it is logical to stick to the conclusion found in § 2: a given channel will be repeated at the nodes of a lattice of rhombi whose shorter diagonal is equal to the side, the area of which is $N\sqrt{3}/2$. This rhombus will be called the "co-channel rhombus" and the length D of its side is $\sqrt{N}$ ($\sqrt{N}$ times the side of the elementary triangle). The characteristics of rhombic lattices will be shown in a simple example, where $N = 13$ (see Fig. 2).

- **Period:** all the co-channel rhombi are identical. A two-dimensional periodicity appears. It may remind the reader of the theory of elliptical functions.
- **Channel distribution:** within a co-channel rhombus, channels might be distributed at random, but in every rhombus their locations should be identical. However, a random distribution is not suitable if it is necessary to take into account interference other than co-channel interference, which is usually the case. Assuming that a homogenous network leads to the most efficient utilization of the spectrum, it is necessary for the interfering signals on any channel to have the same strength, wherever the transmitter is.

Hence if, taking account of the protection ratio between adjacent channel transmitters (referred to here as 1-channel transmitters; likewise 2-channel transmitters are spaced by 2 channels, etc.), channel 1 is placed at a point which is sufficiently far from channel 0, this defines for channel 1 a co-channel rhombic which is offset from the co-channel rhombic for channel 0, by the vector $\overline{01}$. To meet the requirement referred to above, for homogeneity, channel 2 will be placed such that $\overline{12} = \overline{01}$, and so on for the other channels. Channel N falls on channel 0. If we consider the 13 channel network in Fig. 2, it is seen that someone moving along the straight lines of the elementary triangle lattice will find the channel numbers are in an arithmetic progression, modulo 13. Starting from channel 0 and heading west, he will encounter channels: 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 0, etc. It is therefore very easy to define the entire network on the basis of the progression step in two distinct directions: + 2 towards the left and + 5 towards the top righthand corner.

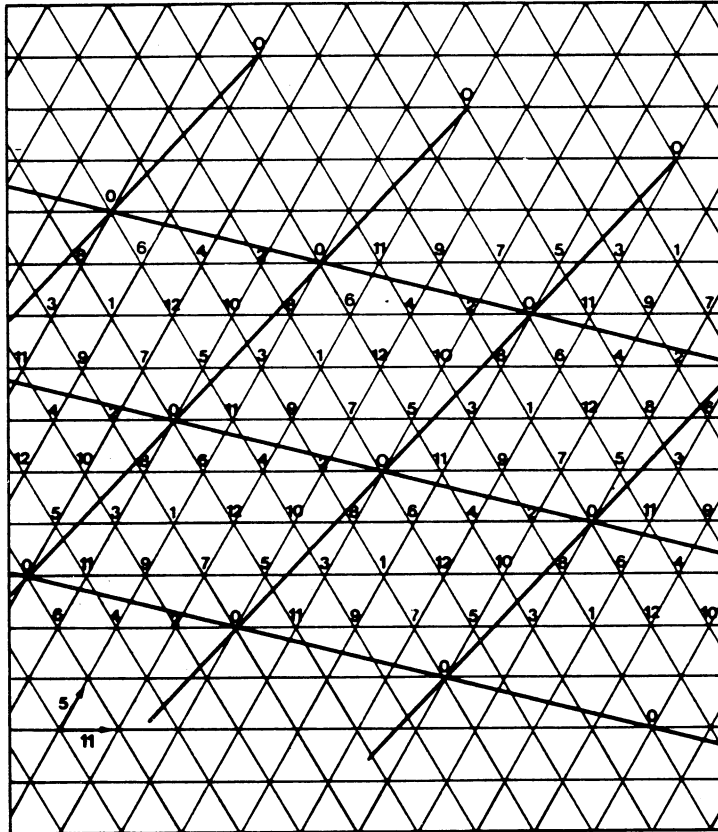


FIGURE 2 – Example of regular lattice for 13 channels

4. Ideal and non-ideal networks

To make the calculations easier, it is convenient to refer the plan to a system of two coordinate axes, bearing equal scales but making an angle of 60° rather than the conventional 90° (see Fig. 3).

We already know that the side of the co-channel rhombus has a length equal to $\sqrt{N}$, but it is necessary for the rhombus to coincide with the intersections of the elementary lattice. In the tilted coordinate system, the distance of any point (x, y) from the origin is:

$$(x^2 + xy + y^2)^{1/2} *$$

If two integers a and b can be found such that $a^2 + ab + b^2 = N$ (number of channels), then there is a co-channel rhombus whose vertices coincide with the intersections of the elementary lattice. $N = 13$ corresponds to the case where $a = 3$ and $b = 1$. In the following discussion, such numbers will be called "rhombic numbers". Some of them are given on Fig. 4.

* All the distances in this section are calculated on the base of this formula and expressed in the multiples of units equal to the length of a side of an elementary triangle.

There are other conditions for the choice of a and b : they shall not have any common divisor, either between themselves or with N , and neither of them can be equal to zero. For instance, if they are both even numbers, only the even-numbered channels will appear. The other restriction will be given later. The theory still works if N is not exactly a rhombic number. The co-channel rhombus becomes a co-channel parallelogram, with an area equal to N . In this case, the sides of the parallelogram are not equal; one of them is shorter than $\sqrt{N}$ and as a consequence this co-channel distance is slightly smaller than for the rhombus.

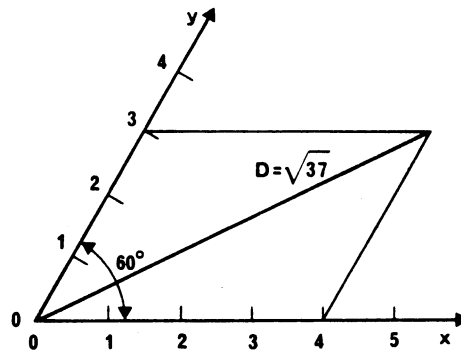


FIGURE 3 – Reference coordinates of regular lattices

$\begin{smallmatrix} a \\ b \end{smallmatrix}$	1	2	3	4	5	6	7	8	9	10	11	12
1	3	7	13	21	31	43	57	73	91	111	133	157
2			19		39		67		103		147	
3				37	49		79	97		139		
4					61		93		133			
5						91	109	129	151			
6							127					

FIGURE 4 – Table of rhombic numbers

Figure 5 shows a network with 26 channels, 26 not being a rhombic number. However, in this particular case, it is seen that the shortest co-channel distance is the shorter side of the parallelogram whose length equals 4.358. This is not much less than the distance $\sqrt{N} = 5.099$ which would have been obtained if 26 were a rhombic number. The lengths of other co-channel distances, e.g. the longer side of the parallelogram or the shorter diagonal of it, are 5.291 and 6.082 respectively. The next rhombic number is 31 an example of which is given in Fig. 6.

Figure 7 shows a network with 120 channels, 120 not being a rhombic number. However, in this particular case, it is seen that the shortest co-channel distance is the diagonal whose length equals 10.58. The lengths of the sides are 11.53 and 10.82, respectively. None of these numbers is much less than the distance $\sqrt{N} = 10.95$, which would have been obtained if 120 were a rhombic number. This lattice may not be optimum for VHF planning.

In the case of N not being a rhombic number, there are three possibilities:

- not to take note of this value N and consider the nearest rhombic number which could be used instead (e.g. 111; see Fig. 4);
- to consider the resulting regular lattice despite its consisting of co-channel parallelograms;
- to distort the lattice so as to transform co-channel parallelograms to rhombi. In these circumstances the elementary rhombi would become parallelograms, and network efficiency would decrease. This decrease will only be slight when N is fairly large.

The best choice among the three options depends on the circumstances to be taken into account.

It is not always easy to find the progression steps such that the channel assigned to the origin will reappear at the vertices of the co-channel rhombus or parallelogram. Referring Fig. 2 to the slant coordinate system of Fig. 3, the steps p along the x -axis and q along the y -axis must satisfy the following equations:

$$1 \cdot p + 3 \cdot q = kN \quad \text{and} \quad 4 \cdot p - 1 \cdot q = k'N$$

k and k' are integers.

In Fig. 2, $p = 11$, $q = 5$, $k = 2$, and $k' = 3$, for $N = 13$. These steps are indicated in Fig. 2. In the case of a non-rhombic number, there is more than one parallelogram and many attempts are necessary to find the best one [Arnaud, 1962]. Even in the case of rhombic numbers it may happen that two different co-channel rhombi exist, as it appears on Fig. 4 for 91 (9 and 1 or 6 and 5).

The case where a or $b = 0$ is a trivial one. If, for example, $b = 0$, the co-channel rhombus sides coincide with the x and y axes, and $a = \sqrt{N}$. The sum of steps along the axes from one vertex to the next must be N ; consequently their value is $\sqrt{N}$ and it is then only possible to write on the lattice channels that are multiples of $\sqrt{N}$.

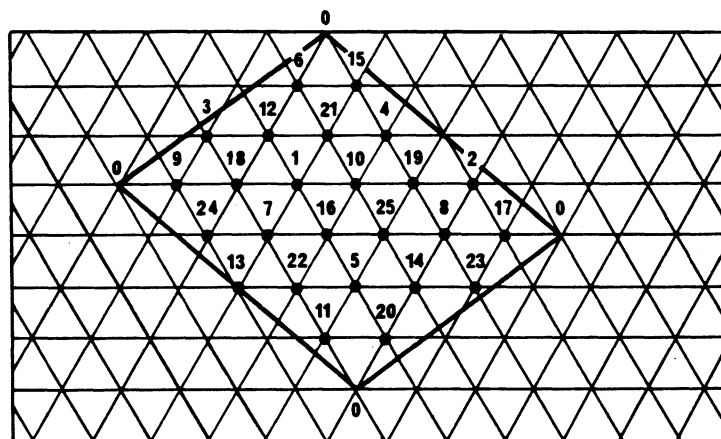


FIGURE 5 – Network with 26 channels

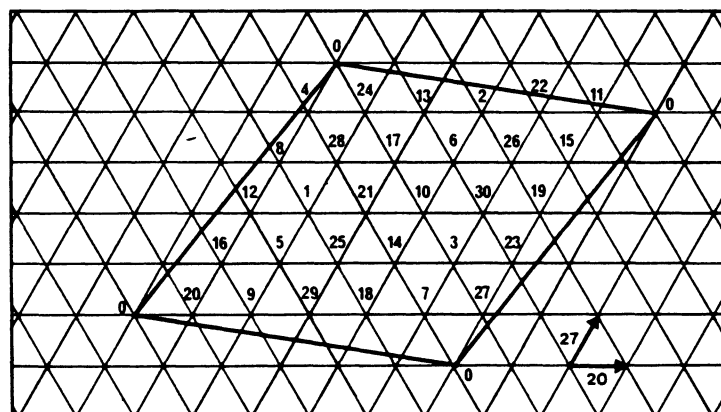


FIGURE 6 – Example of an optimum regular lattice for 31 channels

5. Evaluation of interference

5.1 Co-channel interference

It has been explained in § 3 how the minimum necessary number of channels derives from the consideration of propagation curves and protection ratios, but only one interfering transmitter has been taken into account. On Fig. 8 are drawn a few co-channel rhombi for a co-channel distance D , the wanted transmitter being at the centre and the eighteen others surrounding it are the interferers. In the case of VHF or UHF broadcasting, the coverage area is evaluated on the basis of the 50% or 1% of the time propagation curves of Recommendation 370 (for the wanted or interfering transmitter, respectively), taking due account of the characteristics of the