

REPORT ITU-R SA.2065

Protection of the space VLBI telemetry link

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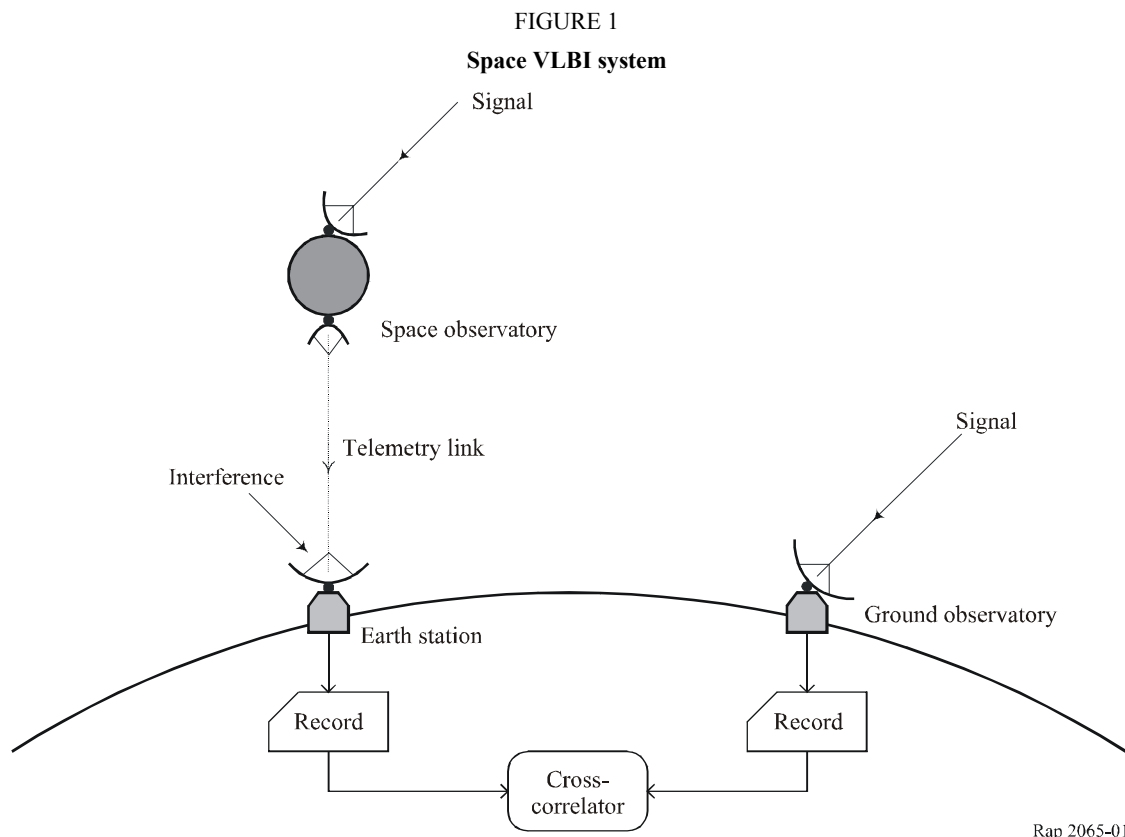
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1 Introduction

Very long baseline interferometry (VLBI) is used to achieve very high resolution of observed radio sources. VLBI has many scientific and engineering uses, from observing extragalactic radio sources to navigating and tracking of spacecraft [Thompson *et al.*, 2001].

2 Space VLBI system

Space VLBI pairs a space-borne observatory with a ground-based observatory to form an interferometer as shown in Fig. 1.



A telemetry link returns the signal record from the spaceborne observatory-to-Earth. This signal record is degraded by thermal and other internal noise sources in the space-to-Earth telemetry link and by external interference from other radio stations into the receiving earth station. An appropriate figure-of-merit for the overall space VLBI telemetry link is the degradation in the cross-correlation SNR ([Thompson *et al.*, 2001] and Recommendation ITU-R SA.1344 – Preferred frequency bands and bandwidths for the transmission of space VLBI data). This Report characterizes the degradation to the cross-correlation SNR by the interference present on the telemetry link.

2.1 Space VLBI telemetry signal, noise and interference

In the telemetry link analysis, we assume that signal $s(t)$ and noise $n(t)$ plus an interfering signal $I(t)$ are present, the received signal being $s(t)+n(t)+I(t)$.

2.1.1 Signal

The signal is a carrier with power P and frequency f_c . It is data modulated using differentially encoded quadriphase-shift keying (DQPSK) and rectangular data pulses. This data modulation suppresses the carrier. The signal is of the form:

$$s(t) = \sqrt{P} \left[\sum_k d_1[k] q(t - kT) \right] \cos(2\pi f_c t) + \sqrt{P} \left[\sum_k d_2[k] q(t - kT) \right] \sin(2\pi f_c t) \quad (1)$$

where T is the quaternary symbol period, and

$$q(t) = \begin{cases} 1, & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

is the rectangular data pulse. The quaternary symbol period, T , is the reciprocal of the quaternary symbol rate ($R = 1/T$), which equals one-half the binary symbol rate at the input to the DQPSK modulator. The sequences $d_1[k]$ and $d_2[k]$ are binary valued: $d_1[k] = \pm 1$ and $d_2[k] = \pm 1$, which are related to the data bits by a four-phase differential encoding.

2.1.2 Noise and interference

The noise $n(t)$ is, as always, assumed zero-mean, Gaussian, and white within the passband of the receiver. Its one-sided power spectral density in the receiver passband is N_0 .

The interfering signal is characterized by a one-sided power spectral density $S_i(f)$. This power spectral density is bandpass and its peak value is presumably located at a frequency near f_c , otherwise it would not be an interference threat. Since this power spectral density is one-sided, the integral $\int_0^\infty S_i(f) df$ accounts for all of the interfering signal power.

The noise spectral density, N_0 , and the interference power spectral density, $S_i(f)$, are referenced to the same point in the receiver chain as the signal power, P . Normally, the receiver reference point is the input terminal of the low-noise amplifier (LNA). The noise spectral density is based on the equivalent noise temperature at this point. The equivalent noise temperature accounts for antenna noise temperature and all noise that enters the receiver chain after the antenna.

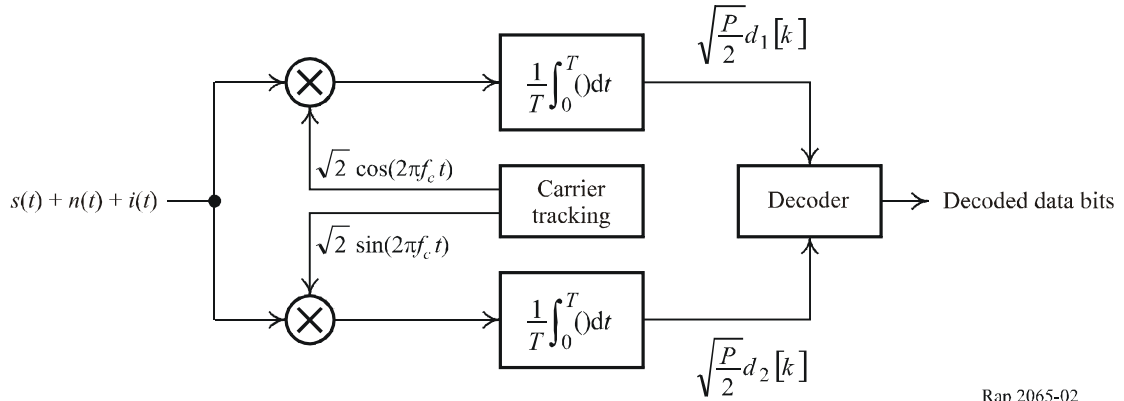
3 Space VLBI telemetry detection

3.1 Telemetry receiver

The receiver multiplies $s(t) + n(t) + I(t)$ by a local oscillator signal $\sqrt{2} \cdot \cos(2\pi f_c t + \phi)$ and, in parallel, by $\sqrt{2} \cdot \sin(2\pi f_c t + \phi)$ to create in-phase and quadrature channels as shown in Fig. 2.

To simplify the analysis, we assume here that the carrier tracking phase $\phi = 0$. In actual processing, the QPSK carrier synchronization loop can only reduce ϕ to one of four values. The differential encoding, however, ensures that this four-fold phase ambiguity does not affect data recovery. In the present analysis, the $\phi = 0$ assumption is fair because the effect of interference and noise on telemetry detection is same for all four possible values of ϕ .

FIGURE 2
Space VLBI telemetry receiver



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The desired signals that appear at baseband within the in-phase and quadrature receiver channels are (with $\varphi = 0$):

$$\sqrt{\frac{P}{2}} \left[\sum_k d_1[k] q(t - kT) \right], \text{ and } \sqrt{\frac{P}{2}} \left[\sum_k d_2[k] q(t - kT) \right]$$

3.1.1 Matched filter

The in-phase and quadrature receiver channels each have a matched filter, modelled simply as time-average over T seconds (see Fig. 2). The discrete-time signals available from these matched filters are therefore:

$$\sqrt{\frac{P}{2}} d_1[k], \text{ and } \sqrt{\frac{P}{2}} d_2[k]$$

The thermal noise, which contributes a term to each receiver channel output, is calculated as follows. The baseband noise in each channel, prior to matched filtering, is Gaussian with a one-sided noise spectral density N_0 within the baseband bandwidth. The unity power of the local oscillator signal ensures that the N_0 of the baseband noise is numerically equal to the N_0 of the pre-detection bandpass noise. The additive noise is modelled as a sequence of zero-mean, Gaussian random variables, each statistically independent of every other. The noise at the output of the second receiver channel has the same statistics. The noises of the two receiver channels are statistically independent because the local oscillator signals are orthogonal. The variance of the Gaussian random variables equals the noise power within the bandwidth of the matched filter as referenced to the input terminal of the low noise amplifier. This thermal noise variance σ_n^2 equals to the total thermal noise power N and is given by:

$$N = \sigma_n^2 = \frac{1}{2} \int_{-\infty}^{+\infty} N_0 \cdot \text{sinc}^2(fT) \cdot df = \frac{N_0}{2T} \quad (3)$$

Within the bandwidth of the matched filter, the interference variance σ_i^2 equals to the interference power, I , as referenced to the input terminal of the low-noise amplifier. It can be calculated from:

$$I = \sigma_i^2 = \frac{1}{2} \int_{-\infty}^{+\infty} S_i(f + f_c) \cdot \text{sinc}^2(fT) \cdot df \quad (4)$$

The $\text{sinc}()$ function in equations (3) and (4) is defined by $\text{sinc}(x) = \text{sinc}(\pi x)/(\pi x)$.

3.2 Telemetry bit error rate (BER)

When only thermal noise is present, the probability P_e that a differentially encoded bit (symbol) is incorrectly detected is:

$$P_e[n] = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{PT}{2N_0}} \right) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right) \quad (5)$$

where $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} dy$ is the complementary error function. Here, we used that:

$$\frac{PT}{2N_0} = \frac{E_b}{N_0} \quad (6)$$

since T is the quaternary symbol period.

The presence of interference changes the total variance at the output of each matched filter from σ_n^2 to $(\sigma_n^2 + \sigma_i^2)$. This raises the effective noise spectral density N_0 to $N_0(\sigma_n^2 + \sigma_i^2)/\sigma_n^2$; thus, the effective energy-per-bit-to-noise-spectral-density ratio decreases by $\sigma_n^2/(\sigma_n^2 + \sigma_i^2)$. In the presence of interference, the probability of symbol error now becomes:

$$P_e[n, i] = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0} \frac{\sigma_n^2}{\sigma_n^2 + \sigma_i^2}} \right) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0} \cdot \frac{N}{N+I}} \right) \quad (7)$$

since σ_i^2 is the total interference power, I , and σ_n^2 is the total noise power, N .

In the differential decoding process, the error probability of detecting a differentially encoded bit (symbol), P_e , and the error probability of a message bit, P_e , are related as:

$$BER = P_e = 2P_e(1 - P_e) \quad (8)$$

Note that telemetry BER equals P_e . From equation (8) we see that for DQPSK, when P_e is small, $P_e \approx 2P_e$; that is, an error in one differentially encoded bit typically leads to two errors in the message bits.

4 Space VLBI cross-correlation

Figure 3 below shows the space VLBI signal processing. The space observatory digitizes the observed signal record and transmits it to the earth station using the space-to-Earth telemetry link. The ground observatory also digitizes its observed signal record and cross-correlates it with the record received from the space observatory.