

1 Introduction

The increasing use of space stations in circular low-orbit in the space research service (and other services) necessitates the development of dynamic sharing models in which the potential interference from the space station can be treated as a time-varying function. This Report defines analytical tools for calculating visibility statistics for low-orbiting spacecraft in circular orbits (see Note 1) as seen from a specific point on the Earth's surface.

NOTE 1 – This Report only deals with circular satellite orbits in which the orbital period is not an even multiple of the Earth's rotational period.

Section 2 of this Report describes the factors affecting the visibility statistics, presents a bounding equation for determining the percentage of time that a low-orbiting satellite will occupy specified regions of the orbit shell visible to an earth station, and contains summary charts giving the maximum duration a low-orbiting satellite spends in certain regions of the orbit shell as a function of several parameters. Section 3 develops the probability density function (pdf) of a satellite occupying specific locations on the orbit shell, illustrates how the pdf may be used to calculate the statistical characteristics of interference to low-orbiting satellites resulting from emissions from stations in the FS, and demonstrates the computation of the pdf of the interference to FS systems assuming the power flux-density (pfd) of the emissions of the low-orbiting satellites conform to a specific profile. Section 4 proposes a simplified method to calculate the visibility statistics of earth stations or terrestrial stations using an antenna with a beam of circular cross section and also presents a manual visibility computation method based on the use of a spreadsheet to calculate the visibility statistics of earth stations or terrestrial stations employing an antenna with a beam of a more complex cross section. Finally, section 5 provides a means to calculate the coordinates in inertial space of the intersection of two orbital planes. This section is particularly useful for predicting the conjunction of satellites in sun-synchronous orbits whose orbital planes are offset.

2 Percentage of time and maximum duration for a low-orbiting spacecraft occupying a defined region

Even for the simplest of dynamic sharing models, at least six specific system parameters must be evaluated to define precisely the primary time dependent statistics of a low-orbit space station as seen from a location on the Earth's surface.

The time dependent statistics are:

- the longest time of passage of a space station through the main beam of a ground antenna (discussed in § 3);
- the long-term percentage of time that the space station spends in various areas of the orbit sphere as seen from the ground station.

The first statistic is important in that it defines the longest continuous duration of noise power into the ground receiving system from the space station. The second set of statistics, after convolution with transmit and receive antenna patterns, and range loss, can be used to develop interference-to-noise (I/N) relations as a function of time for the dynamic sharing model. In one sense then, I/N versus time relations can be treated in a method similar to the signal strength versus time relations derived from atmospheric propagation statistics. However, instead of a receiver experiencing change in the S/N ratio as a statistical function of time, it experiences a change in signal-to-noise-plus-interference ratio, as a statistical function of time, based upon the low-orbit space station model parameters.

The specific parameters which define the long-term visibility statistics of a space station in a low circular inclined orbit as seen from a receiving system on the Earth's surface are:

- altitude of the space station, H (km);

- inclination of the space station orbit, i (degrees);
- latitude of the ground station, La (degrees);
- pointing azimuth of the ground station antenna measured from North, Az (degrees);
- pointing elevation of the ground station antenna measured from the local horizontal plane, El (degrees);
- angular area of the region of interest, δA .

The last parameter may take on several different physical interpretations depending upon the purpose of the analysis. For instance, it may be the angular area of the main beam of the ground station antenna or it may be taken as an angular area expressed by an azimuth “width” of δAz (degrees) and an elevation height expressed as δEl (degrees).

2.1 Bounding equation for percentage of time spacecraft is in defined region

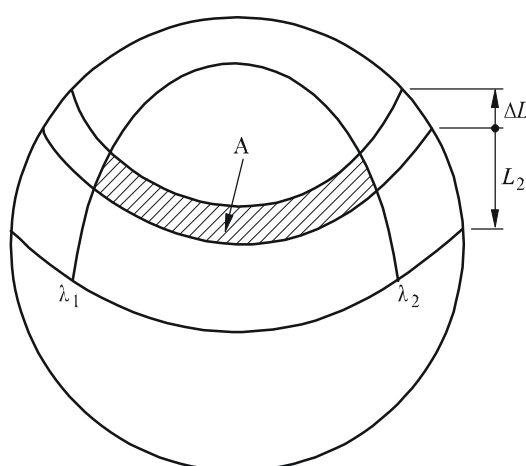
The bounding equation is given below and may be used to determine the percentage of time that a low-orbit spacecraft will reside in certain regions visible to a ground station over long periods of time:

$$T(\%) = \frac{\delta\lambda}{2\pi^2} \left(\sin^{-1} \left[\frac{\sin(L + \Delta L)}{\sin i} \right] - \sin^{-1} \left[\frac{\sin L}{\sin i} \right] \right) \times 100 \quad (1)$$

where:

- $L, \Delta L$: latitude limits of the region on the orbital shell (see Fig. 1)
- $\delta\lambda$: longitudinal extent of the region on the orbital shell, between the longitude limits of λ_1 and λ_2 (as seen in Fig. 1)
- i : inclination of the satellite orbit (all angles in rad).

FIGURE 1



Rap 2066-01

2.2 The maximum time a satellite spends in the beam of a ground station

This section provides worst case numerical data on one aspect of frequency sharing with low-orbit, inclined orbit satellites. Such sharing is influenced by the amount of time that an “unwanted” and potentially interfering satellite appears within the 3 dB beamwidth of a ground station. This

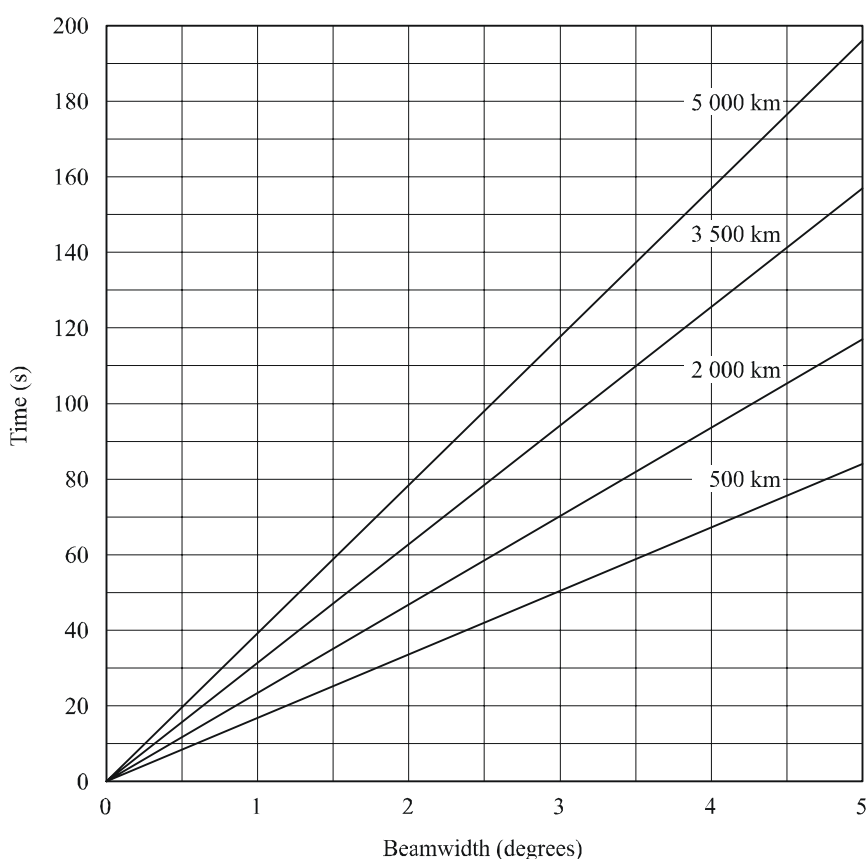
parameter is evaluated for several orbit altitudes and for two “bounding” elevations of the receiving antenna. The numerical results developed in this paper represent an upper bound on the length of time a spacecraft at a given altitude will appear within the beam of a ground station.

The time a satellite spends in a ground station’s beam is a function of the beam’s width, the elevation of the beam and the altitude of the satellite. The worst case, i.e. when the satellite spends the maximum possible time in the beam, occurs when the ground station is located at the equator with a beam of elevation = 0° and the satellite is traveling east along an orbit with 0° inclination. The time the satellite spends in the beam depends upon the satellite’s velocity relative to the velocity of the beam as it rotates with the Earth, and upon the length of the intersection of the orbit with the beam.

The maximum time that a spacecraft can spend in the main beam of an antenna is shown in Figs. 2 and 3 for antenna elevations of 0° and 90° respectively, and refers to a variety of orbital altitude and beamwidths.

FIGURE 2

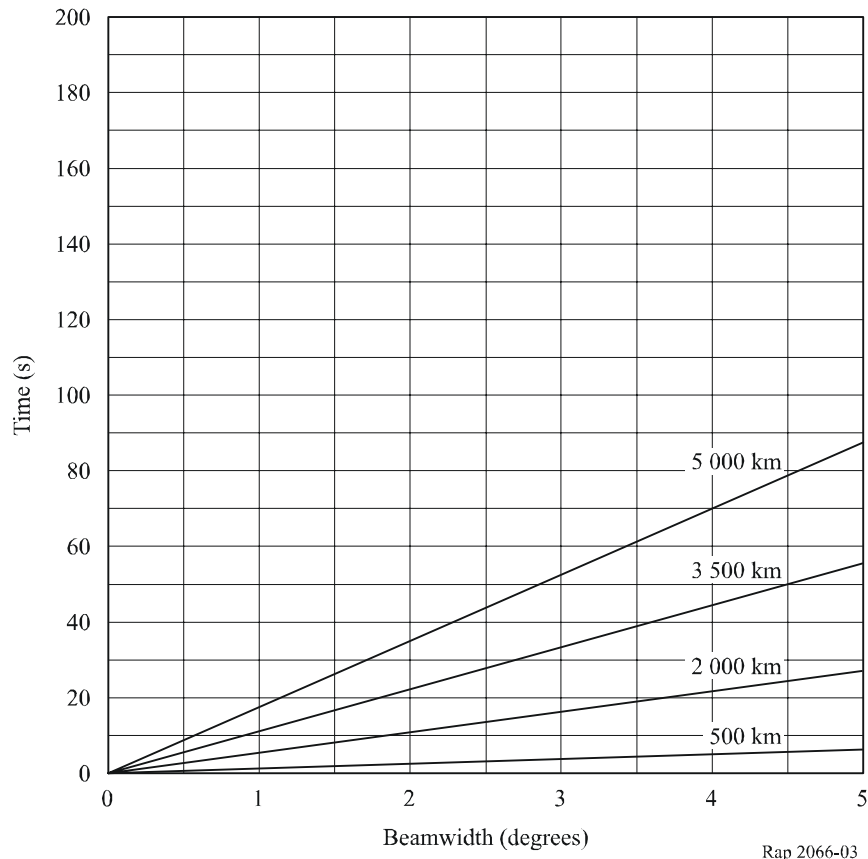
Maximum time in beam plotted against beamwidth at elevation of 0°



Rap 2066-02

FIGURE 3

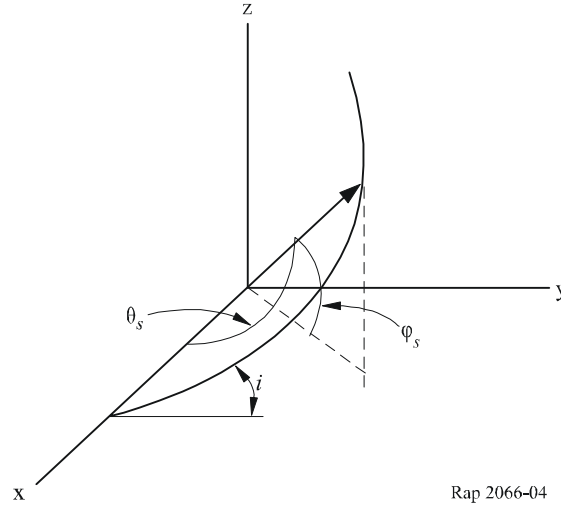
Maximum time in beam plotted against beamwidth at elevation of 90°



3 Probability density function pdf of the position of a low-orbiting satellite on the orbit shell

The position, i.e. the latitude and longitude of an orbiting satellite on the orbit shell relative to a fixed point on Earth, is a function of two independent parameters: the position of the satellite in its orbit plane; and the longitude of the observation point on Earth relative to the orbit plane. The geometry used for this analysis is shown in Fig. 4. It is assumed that the satellite is in a circular orbit at an altitude, h , the inclination of the orbit plane is i , and the period of rotation of the satellite and of the Earth are not directly related.

FIGURE 4
Simplified geometric model of an Earth-orbiting satellite



The coordinate system shown in Fig. 4 is a right-handed, geocentric system with the x-y plane corresponding to the equatorial plane, and the x-axis pointing in an arbitrary direction in space (usually the First Point of Aries).

For simplicity, assume that the intersection of the orbit plane and the equatorial plane is the x-axis. The latitude φ_s of the position of the satellite in space is given by:

$$\sin \varphi_s = \sin \theta_s \sin i \quad (2)$$

where θ_s is the central angle between the x-axis and the position vector of the satellite. For satellites in circular orbits, θ_s is a linear function of time t , i.e. $\theta_s = 2\pi t / \tau$, where τ is the period of the orbit. Equation (2) relates the latitude of the satellite as a function of the central angle θ_s and the orbit inclination angle i .

If the central angle of the position vector of a satellite in a circular orbit is sampled at random times, the angle θ_s will be found to be uniformly distributed between 0 and 2π radians. Stated as a probability density function $p(\theta_s)$:

$$p(\theta_s) = \frac{1}{2\pi} \quad (3)$$

The pdf of the latitude of the position vector of the satellite may be found using a straightforward transformation technique from probability theory. It may be shown for a random variable x with pdf $p(x)$ that undergoes the transformation $y = g(x)$, that the pdf $p(y)$ of the random variable y is given by:

$$p(y) = \frac{p(x_1)}{|g'(x_1)|} + \dots + \frac{p(x_n)}{|g'(x_n)|} \quad (4)$$

where:

$$g'(x) = \frac{dg(x)}{dx}$$

and $x_1, \dots, x_n$ are the real roots of $y = g(x)$.

Applying the procedure described above to equations (2) and (3) yields the pdf of the latitude of the position vector of the satellite in its orbital plane:

$$p(\varphi_s) = \frac{1}{\pi} \frac{\cos \varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (5)$$

Equation (5) represents the function that would be obtained if the latitude of the satellite were randomly sampled a large number of times. Inspection of equation (5) shows that the expression is defined only for real values of $|\varphi_s| \leq i$ as expected. It may also be shown that:

$$\int_{-i}^i p(\varphi_s) d\varphi_s = 1 \quad (6)$$

also as expected.

For the satellite to appear at a specific longitude λ_s on the orbit shell relative to the reference point on the surface of the Earth, the orbit plane must intersect the orbit shell at that longitude. The probability of this occurring is uniformly distributed over 2π radians, i.e.:

$$p(\lambda_s) = \frac{1}{2\pi} \quad (7)$$

Finally, since it has been assumed that the period of the satellite and the rotation of the Earth are not directly related, the pdf of the satellite position is the joint probability of two independent events which is given by the product of the individual pdfs:

$$p(\varphi_s, \lambda_s) = \frac{1}{2\pi^2} \frac{\cos \varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (8)$$

The probability $P(\Delta\varphi, \Delta\lambda)$ of the satellite occupying the region on the orbit shell bounded by latitude φ_s , $\varphi_s + \Delta\varphi_s$ and longitude $\Delta\lambda_s$ is given by:

$$P(\Delta\varphi, \Delta\lambda) = \frac{1}{2\pi^2} \int_0^{\Delta\lambda_s} \int_{\varphi_s}^{\varphi_s + \Delta\varphi_s} \frac{\cos \varphi_s d\lambda_s d\varphi_s}{\sqrt{\sin^2 i - \sin^2 \varphi_s}} \quad (9)$$

Carrying out the integration yields:

$$P(\Delta\varphi, \Delta\lambda) = \frac{\Delta\lambda_s}{2\pi^2} \left(\sin^{-1} \left[\frac{\sin(\varphi_s + \Delta\varphi_s)}{\sin i} \right] - \sin^{-1} \left[\frac{\sin \varphi_s}{\sin i} \right] \right) \quad (10)$$

3.1 Probability density function of interference to low-orbiting satellites caused by emissions from FS systems

The pdf of interference to low-orbiting satellites caused by emissions from FS systems is a function of the geometry and the pdf of the satellite position. If the interference can be expressed as a function of the coordinates (latitude and relative longitude) of the visible orbit shell, i.e. $I(\varphi_s, \lambda_s)$, then the pdf of the interference to the low-orbiting satellite $p(I)$ is given by:

$$P(I)dI = \iint_S p(\varphi_s, \lambda_s) d\varphi_s d\lambda_s \quad (11)$$

where S indicates that the integration is to be performed over the segment of the surface of the orbit sphere that contributes a level of interference between the values of I and $I + dI$.