

RECOMMENDATION ITU-R S.1257-3

Analytical method to calculate short-term visibility and interference statistics for non-geostationary satellite orbit satellites as seen from a point on the Earth's surface

(Questions ITU-R 206/4 and ITU-R 231/4)

(1997-2000-2001-2002)

The ITU Radiocommunication Assembly,

considering

- a) that there may be a need to calculate the probabilities of exceeding a given interference level in interference or sharing studies between non-GSO satellite orbit stations and other stations;
- b) that there are computer simulation programmes to derive the required statistics but such programmes may not be commonly available;
- c) that computer simulations can be used to provide a large amount of statistics with good accuracy, but they may need expertise in their use and may require considerable time to run;
- d) that in some cases analytical methods may be time saving and need less sophisticated means for calculation of limited amount of statistics,

recommends

- 1 that the analytical methodology given in Annex 1 can be used to obtain short-term visibility statistics for interference and sharing studies between non-GSO networks and other GSO FSS networks (The term “probability” is not used in the strict mathematical sense. It signifies “percentage of time”.);
- 2 that the methodology given in Annex 2 can then be used to obtain short-term interference statistics for interference and sharing studies between non-GSO networks and other GSO FSS networks;
- 3 that the information given in Annex 3 may be used to extend the methodology in Annex 1 to enable information on the frequencies and durations of short-term interference events to be derived;
- 4 that the following Notes should be regarded as part of this Recommendation:

NOTE 1 – The methodology can be used for interference calculations between non-GSO feeder links and GSO stations. They can also be applied to any case where one antenna is pointing to a fixed direction and the other is either on board a non-GSO satellite, or is tracking the non-GSO satellite.

NOTE 2 – The methodology gives an average value for a non-synchronous satellite constellation. It is applicable also for a synchronous orbit constellation (satellite tracks are repeating after a certain period of time) if there are several satellite tracks through the area of interest.

NOTE 3 – The limitations of the methodology described in Annex 1 have to be taken into account.

ANNEX 1

Appendix 1 gives a method to define an area for which the probability that a satellite falls within the area to be calculated.

Appendix 2 gives the method to calculate the probability.

Appendix 3 gives the derivation of the formulae used in Appendix 2.

APPENDIX 1

TO ANNEX 1

Calculation of the needed discrimination angle for a given interference level

The case considered here is non-GSO/GSO sharing.

The area is defined for interference:

- a) from non-GSO earth station to GSO satellite;
- b) from non-GSO satellite to GSO earth station;
- c) from GSO earth station to non-GSO satellite;
- d) from GSO satellite to non-GSO earth station.

In the following equations:

- ε : elevation angle of a satellite from a station
- r : earth radius = 6 378 km
- H : altitude of the GEO satellite (km)
- h : altitude of the non-GSO satellite (km)
- d_I : distance from the interference source to the interfered receiver (km)
- d_N : distance to the non-GSO satellite from a station (km)
- d_G : distance to the GSO satellite from a station (km)
- f : frequency (GHz)
- E : e.i.r.p. spectral density (dB(W/Hz))
- N_0 : noise spectral density (dB(W/Hz))
- $G(\varphi)$: antenna gain to the direction φ (degrees) from the main beam axis (dB)
- G : antenna on-axis gain (dB)
- Suffix G refers to the GSO network
- Suffix N refers to the non-GSO network
- Suffix E refers to an earth station
- Suffix S refers to a satellite
- Suffix T refers to transmitting parameter
- Suffix R refers to receiving parameter
- PR : protection ratio

Distances to a non-GSO and GSO satellite from a station for the in-line case are:

$$d_N = \sqrt{r^2 \sin^2 \varepsilon + 2r h + h^2} - r \sin \varepsilon \quad (1)$$

$$d_G = \sqrt{r^2 \sin^2 \varepsilon + 2r H + H^2} - r \sin \varepsilon \quad (2)$$

C_0/I_0 method

$$(C_0/I_0)_{in-line} = E_{GE} - E_{NE} \quad \text{for case a)} \quad (3)$$

$$(C_0/I_0)_{in-line} = E_{GS} - E_{NS} - 20 \log d_G + 20 \log d_N \quad \text{for case b)} \quad (4)$$

$$(C_0/I_0)_{in-line} = E_{NE} - E_{GE} \quad \text{for case c)} \quad (5)$$

$$(C_0/I_0)_{in-line} = E_{NS} - E_{GS} - 20 \log d_N + 20 \log d_G \quad \text{for case d)} \quad (6)$$

The GSO and non-GSO earth stations should be either co-located or, in cases a) and d), the GSO satellite antenna discrimination has to be taken into account, and in cases b) and c) the non-GSO satellite antenna discrimination has to be taken into account.

The required antenna discrimination for a non-GSO earth station, in cases a) and d), and for a GSO earth station in cases b) and c), is:

$$G - G(\varphi) = (C_0/I_0)_{in-line} - RP \quad (7)$$

To calculate the required avoidance angle φ , the appropriate antenna gain patterns has to be used. For GSO earth stations the side lobe pattern used in equation (8) is given in Recommendation ITU-R S.580. Avoidance angle at side lobe area is:

$$\varphi = 10^{(29 - G(\varphi))/25} \quad (8)$$

For the main beam area, the discrimination angle may be calculated by:

$$\varphi = \varphi_0 \sqrt{\frac{G - G(\varphi)}{12}} \quad (9)$$

In case b) the distance from the non-GSO satellite to the GSO earth station changes as the satellite moves on its orbit. For small values of φ and for high elevation angles ε it need not be taken into account but in other cases the distance d_N has to be calculated separately for the highest elevation ($\varepsilon + \varphi$) and for the lowest elevation ($\varepsilon - \varphi$) and the new values have to be used in equation (2).

For calculations in Appendices 2 and 3 the angle φ is the radius of the area for which the probability is to be calculated. For the calculation of statistics $\varepsilon_1 = \varepsilon - \varphi/2$ and $\varepsilon_2 = \varepsilon + \varphi/2$.

I_0/N_0 method

To calculate the required avoidance angle φ for the case based on I_0/N_0 requirement, the following equations are used:

$$G - G(\varphi) = (I_0/N_0)_{in-line} - (I_0/N_0)_{required} \quad (10)$$

I_0/N_0 for the in-line case can be calculated by:

$$(I_0/N_0)_{in-line} = E - N_0 - 20 \log d_I - 20 \log f - 92.5 \quad (11)$$

where:

- E : e.i.r.p. of the interfering transmitter
 N_0 : noise spectral density of the wanted receiving system.

APPENDIX 2

TO ANNEX 1

Calculation of in-area statistics for non-GSO satellites

1 Introduction

The method given in this Appendix can be used in calculating the probability to find a satellite of a constellation in a circular or rectangular area (azimuth/elevation or latitude/longitude). A circular area may be for example a radio-relay link or satellite earth station antenna main beam or side lobe area. If the area is based on an offset angle where certain interference level is achieved, the result is the probability that a given interference level is exceeded. The method can be used to calculate the interference between non-GSO and GSO networks. The method can also be used in calculating the probability of interference between a non-GSO earth station and a receiving fixed satellite station or FSS earth station. The calculation uses the discrimination angle to define an area where the permissible interference level is exceeded.

The method can be used for any observation point (e.g. earth station) latitude and for any satellite altitude, inclination, azimuth and elevation, but only in the case that the satellite may be visible in the defined area. (See § 5.)

2 Symbols used (in Appendix 2 and Appendix 3)

- T_s : Satellite orbital period (min)
 T_e : Earth rotation time (min)
 L_0 : Latitude of the observation point (rad)
 L : Latitude of the area (rad)
 l_m : Medium length of the tracks through a circle
 b : Length of the area in longitudinal direction (rad)
 i : Inclination of satellite orbit (rad)
 r_c : Radius of the circular area (rad)
 α : Angle between ground track and latitude line (rad)
 A : Area on a spherical surface (sterad)
 A_c : Area of a circle on a spherical surface (sterad)
 P_h : Probability to hit the area (one satellite calculation)
 P_i : Probability to be inside the area if hit during the revolution (one satellite calculation)

P :	Probability for a satellite to be inside the defined area (one satellite calculation)
P_c :	Probability that any one of the satellites in a constellation is inside the area
N :	Number of satellites in the constellation
$k =$	$r / (r + h)$
Λ :	Azimuth of the centre of the area (rad)
ε :	Elevation of the centre of the area (rad)
r :	Radius of the Earth
h :	Altitude of the satellite
γ :	Nadir angle from subsatellite point (see Fig. 3) (rad)
θ_ε :	Geocentric angle in elevation direction corresponding to ε (rad)
θ_{ε_2} :	Geocentric angle for the highest point of the area in elevation direction (rad)
θ_{ε_1} :	Geocentric angle for the lowest point of the area in elevation direction (rad)
$\Delta\theta_\beta$:	Geocentric angle difference in direction perpendicular to θ_ε (rad)
β :	Width of the area in azimuthal direction (e.g. antenna beamwidth in azimuthal direction)
$\varepsilon_1, -\varepsilon_2$:	The highest and lowest elevation of the area (rad) ($\varepsilon_1, -\varepsilon_2$ is e.g. the antenna beamwidth in the elevation direction)

3 Calculation of probability

The following formulae are a collection of those in Appendix 3 and given here are only those which are needed for the probability calculation. The numbering of formulae is the same as in Appendix 3, explanation is given for the calculated parameters.

$$k = \frac{r}{r + h} \quad (21)$$

$$\theta_{\varepsilon_1} = \pi - \gamma - (\pi/2 + \varepsilon_1) = \arccos(k \cos \varepsilon_1) - \varepsilon_1 \quad (22)$$

$$\theta_{\varepsilon_2} = \pi - \gamma - (\pi/2 + \varepsilon_2) = \arccos(k \cos \varepsilon_2) - \varepsilon_2 \quad (22)$$

$$\Delta\theta_\varepsilon = \theta_{\varepsilon_2} - \theta_{\varepsilon_1} \quad (23)$$

$$\theta_\varepsilon = \frac{\theta_{\varepsilon_1} + \theta_{\varepsilon_2}}{2} \quad (24)$$

$$\Delta\theta_\beta = 2 \arctan \frac{\tan(\beta/2) \sin \theta_\varepsilon}{\cos \varepsilon} \quad (25)$$

$$L = \arcsin(\cos \theta_\varepsilon \cdot \sin L_0 + \sin \theta_\varepsilon \cdot \cos L_0 \cdot \cos \Lambda) \quad (27)$$

$$\alpha = \arccos \frac{\cos i}{\cos L} \quad (15)$$

$$P = \frac{A}{2\pi^2} \frac{1}{\sin \alpha} \frac{1}{\cos L} \quad (19a)$$

$$P_c = N P$$

For a circular area:

$$A = A_c$$

$$A_c = \frac{\pi}{4} \Delta\theta_\epsilon \Delta\theta_\beta$$

For a rectangular area:

$$A = \beta (\epsilon_2 - \epsilon_1) \quad (26)$$

APPENDIX 3*

TO ANNEX 1

Derivation of the formulae given in Appendix 2

1 Probability that a satellite is in a given area

Figure 1 shows a non-GSO satellite orbit around the Earth. A simple case is the orbit over the poles. If the satellite is projected on the Earth surface, ground tracks are created. If the Earth would be stationary, there would be only one track over the poles. However, as the Earth rotates during the time it takes the satellite to make one revolution, the next ground track will be shifted by a longitude difference equal to:

$$T_s / T_e \ 2\pi \quad (12)$$

Figure 1 shows several tracks over a longer period of time. The rectangular area is an area for which the probability is to be calculated. The probability that the satellite hits the area is the portion of length $2b$ from the whole length of the shadowed band around the Earth (see Fig. 1). The multiple 2 takes into account the fact that the satellite crosses the band twice during one revolution around the Earth. The length of the band is $2\pi \cos L$ and the probability to hit a rectangular area as in Fig. 2 is:

$$P_h = \frac{b}{\pi} \frac{1}{\cos L} \quad (13)$$

In this equation, the value of b is in radians and it is the actual length expressed as a geocentric angle. If the longitude difference is used, then:

$$b = (\text{Longitude difference}) \times \cos L \quad (13a)$$

It should be noted that the area used in calculations is the area at the satellite orbit shell. For a circular antenna beam that area is an ellipsoid whose major axis is in elevation direction. The latitude L used in calculations is not the station latitude but the latitude of a point from the orbit shell projected to the Earth surface. In this case, the projected point is the centre of the area.

* The symbols used in this Appendix are given in § 2 of Appendix 2.

For a common case, when the satellite orbit inclination is different than 90° , the probability of a satellite to be inside the defined area is the probability to hit the area multiplied by the medium length of the tracks inside the area and divided by the length of one revolution. The probability to hit the area is dependent on the length of its projection on a latitude line crossing the middle of the area. The projection is made parallel to the ground tracks. Calculations are here presented for a circular area case, because it is more illustrative.

FIGURE 1
Tracks of polar orbit satellite
(Rectangular area for calculation)

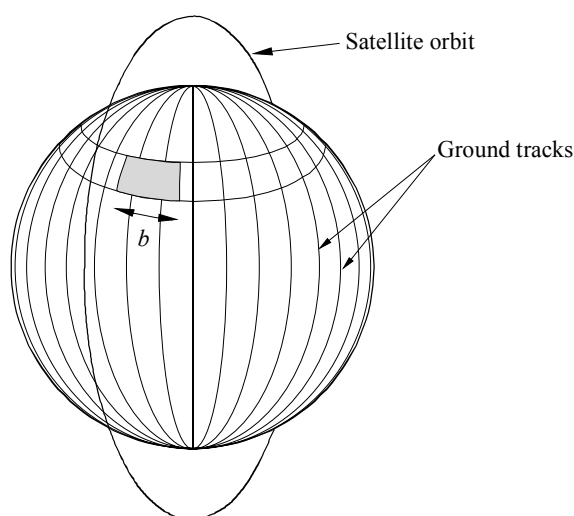
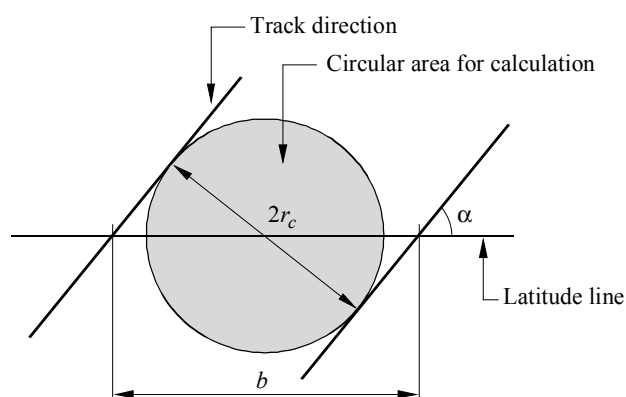


FIGURE 2
Length b of a circular
area projection on latitude line



1257-01

According to Fig. 2, the length of b is:

$$b = \frac{2r_c}{\sin \alpha} \quad (14)$$

$$\alpha = \arccos \frac{\cos i}{\cos L} \quad (15)$$

Angle α is the angle between the satellite track and latitude line.

The rotation speed of the Earth does not need to be taken into account in the probability calculation, but it should be taken into account if the real angle, in relation to the rotating Earth, is needed.

The average length of a large number of equally spaced paths traversing a circle is:

$$l_m = \frac{\pi r_c}{2} \quad (16)$$

If the satellite hits the area during one revolution, the probability that it is inside the area is the length of the track inside the area divided by the length of one revolution.

$$P_i = \frac{l_m}{2\pi} = \frac{r_c}{4} \quad (17)$$

The total probability is then:

$$P = P_h P_i = \frac{r_c}{\pi} \frac{1}{\sin \alpha} \frac{1}{\cos L} \frac{r_c}{2} \quad (18)$$

FIGURE 3a

Projection of circular beam on orbit shell

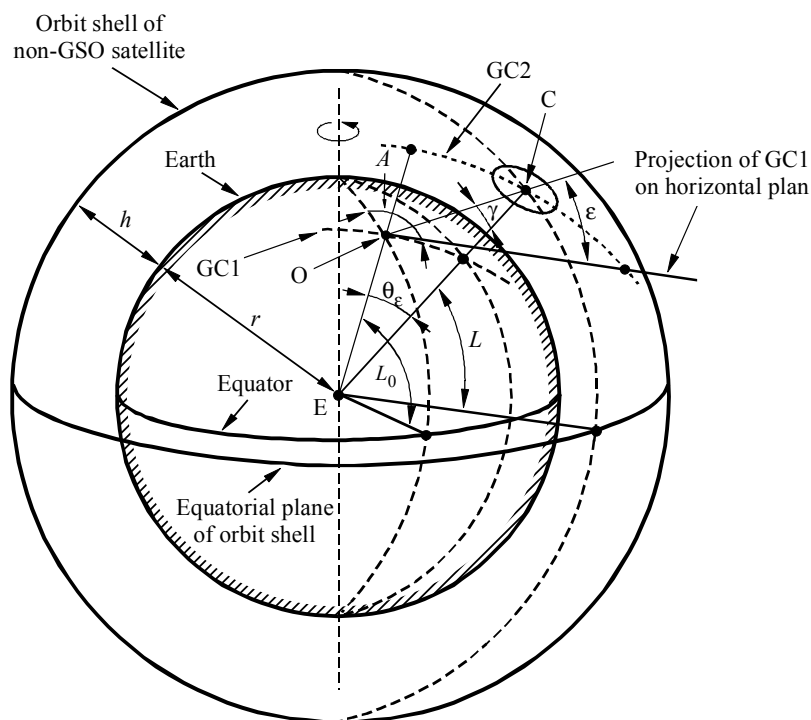


FIGURE 3b

Area size in vertical direction

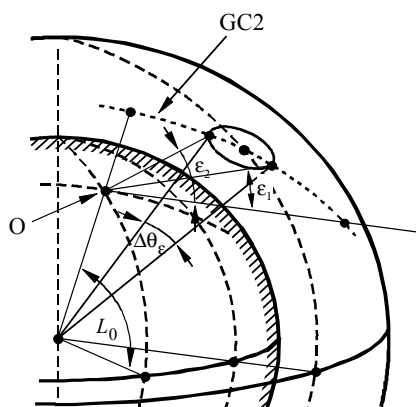
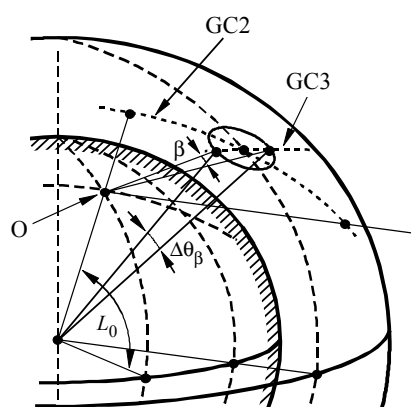


FIGURE 3c

Area size in horizontal direction



- E: Earth's centre
- O: observation point
- C: centre of the area
- GC1: great circle line between observation point and sub-satellite point of the centre of the area
- GC2: projection of vertical line on orbit shell
- GC3: projection of horizontal line on orbit shell

1257-03



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