

## RECOMMENDATION ITU-R S.1323-2

**Maximum permissible levels of interference in a satellite network  
(GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links)\*  
in the fixed-satellite service caused by other  
codirectional FSS networks below 30 GHz**

(Questions ITU-R 205/4, ITU-R 206/4 and ITU-R 231/4)

(1997-2000-2002)

The ITU Radiocommunication Assembly,

*considering*

- a) that emissions from the earth stations as well as from the space station of a satellite network (geostationary-satellite orbit (GSO)/fixed-satellite service (FSS); non-GSO/FSS; non-GSO/mobile-satellite service (MSS) feeder links) in the FSS may result in interference to another such network when both networks operate in the same bands;
- b) that the system designer and its operator should have control over the overall performance of a network and have the capability to provide the required quality of service;
- c) that it is necessary to protect a network of the FSS (GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links) from interference by other such networks and that the inclusion of additional link margin above that necessary to compensate for rain fading, e.g. to compensate for equipment aging, is not to be considered as part of that protection;
- d) that Article 5 of the Radio Regulations (RR), Nos. 5.441, 5.484A, 5.487A, 5.516 indicate that in certain frequency bands “Non-geostationary-satellite systems in the fixed-satellite service shall not claim protection from geostationary-satellite networks in the fixed-satellite service operating in accordance with the Radio Regulations, irrespective of the dates of receipt by the Bureau of the complete coordination or notification information, as appropriate, for the non-geostationary-satellite systems in the fixed-satellite service and of the complete coordination or notification information, as appropriate, for the geostationary-satellite networks, and No. 5.43A does not apply.”;
- e) that in the frequency bands specified in RR No. 22.5I, the limits contained therein apply to non-GSO FSS systems;
- f) that to allow an operator to exercise control over the quality of service there needs to be a limit on the aggregate interference a network must be able to tolerate from emissions of all other networks;

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\* The methodologies for determination of short-term interference criteria contained in this Recommendation are intended to address interference to GSO/FSS, non-GSO/FSS and non-GSO/MSS feeder links. However, the applicability of these methodologies for all such networks requires further verification.

- g) that to limit the aggregate interference from all other networks, there needs to be a limit on the interference a network should be expected to tolerate from any one other network and this single entry interference should allow accommodation of an appropriate number of interfering systems;
- h) that in frequency bands above 10 GHz where very high signal attenuation may occur for short periods of time, it may be desirable for systems to make use of some form of fade compensation to counteract signal fading;
- j) that in interference situations involving non-GSO systems, FSS networks (GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links) are potentially exposed to high levels of interference for short periods of time which could affect the short-term performance or availability of these networks;
- k) that the long-term interference allowance from non-GSO systems to GSO FSS networks should be a small percentage of the existing long-term allowance into a GSO FSS network; and in addition to that allowance;
- l) that if not limited short-term interference events may cause loss of synchronization or other unstable conditions even under clear-sky conditions which may cause a degradation or loss of service for periods longer than the interference event;
- m) that the permissible interference resulting from short-term interference events has to be specified differently for FSS operation in different frequency bands due to the different propagation characteristics of signals in these different bands;
- n) that the effect of non-GSO interference into GSO systems that employ adaptive downlink coding is not the same as the effects due to rain, and that studies performed so far indicate the need to consider these non-GSO interference effects on at least a per-beam basis (in the GSO system) rather than on a per-link basis;
- o) that propagation effects should account for no more than 90% of the unavailability of an FSS link,

*recommends*

1 that a GSO FSS network operating in the frequency bands below 30 GHz should be designed and operated in such a manner that in any satellite link performance objectives can be met when the aggregate interfering power from the earth and space station emissions of all other GSO FSS networks operating in the same frequency band or bands, assuming clear-sky conditions on the interference paths, does not exceed at the input to the demodulator:

1.1 25% of the total system noise power under clear-sky conditions when the network does not practice frequency reuse;

1.2 20% of the total system noise power under clear-sky conditions when the network does practice frequency reuse;

2 that for a GSO FSS network as mentioned in *recommends* 1, the internetwork interference caused by the earth and space station emissions of any one other GSO FSS network operating in the same frequency band or bands should be limited to 6% of the total system noise power under clear-sky conditions;

**3** that for a GSO/FSS network the internetwork interference caused by the earth and space station emissions of all other satellite networks operating in the same frequency band and that can potentially cause interference of time-varying nature, should:

**3.1** be responsible for at most 10% of the time allowance for the BER (or  $C/N$  value) specified in the short-term performance objectives of the desired network and corresponding to the shortest percentage of time (lowest  $C/N$  value);

**3.2** in the case of networks using adaptive coding, be responsible for at most a 10% decrease in the amount of reserve capacity available to links that require heavier coding to compensate for rain fading, on the assumption that the network maintains, with the use of this reserve capacity the same level of performance as it did with no time-varying interference present. Further studies are needed for bands other than 30/20 GHz;

**4** that non-GSO FSS systems operating in frequency bands subject to RR Nos. 5.441, 5.484A, 5.487A and 5.516 should include in their interference budget, as a guide only, an allocation of 10% increase of the time allowance for the BER (or  $C/N$  value) specified in the short-term performance objectives of the desired network and corresponding to the shortest percentage of time (lowest  $C/N$  value) caused by the aggregate emissions from the earth and space stations of all GSO FSS networks;

**5** that for a non-GSO (non-GSO/FSS; non-GSO/MSS feeder links) network in frequency bands subject to RR No. 9.11A (which is not subject to the limits in RR Nos. 22.5C, 22.5D and 22.5F), the internetwork interference caused by the aggregate emissions from the earth and space stations of all GSO FSS networks operating in the same frequency band should:

**5.1** be responsible for at most 10% of the time allowance for the BER (or  $C/N$  value) specified in the short-term performance objectives of the desired network and corresponding to the shortest percentage of time (lowest  $C/N$  value);

**5.2** in the case of networks using adaptive coding, provisionally be responsible for at most a 10% (until review by further studies) decrease in the amount of reserve capacity available to links that require heavier coding to compensate for rain fading, on the assumption that the network maintains, with the use of this reserve capacity, the same level of performance as it did with no time-varying interference present. Further studies are needed to validate this approach for the case of interference from other non-GSO systems;

**6** that for a non-GSO (non-GSO/FSS; non-GSO/MSS feeder links) network the internetwork interference caused by the aggregate emissions from earth and space stations of all other non-GSO satellite networks operating in the same frequency band should:

**6.1** be responsible for at most 10% of the time allowance for the BER (or  $C/N$  value) specified in the short-term performance objectives of the desired network and corresponding to the shortest percentage of time (lowest  $C/N$  value);

**6.2** in the case of networks using adaptive coding, provisionally be responsible for at most a 10% (until review by further studies) decrease in the amount of reserve capacity available to links that require heavier coding to compensate for rain fading, on the assumption that the network

maintains, with the use of this reserve capacity, the same level of performance as it did with no time-varying interference present. Further studies are needed to validate this approach;

7 that, when applying Methodologies A and A' described in Annex 1 or Procedure D described in Annex 2, there is no need for a long-term allowance to be defined because, since simultaneous effects of fading and interference are taken into consideration, then a full characterization of the interference mask results from the conditions in *recommends* 3, 4, 5 and 6;

8 that, when applying Methodology B described in Annex 1, a long-term allowance should be additionally defined because simultaneous effects of fading and interference are not taken into account;

9 that this allowance corresponding to long-term interference, when used in addition to *recommends* 3, 4, 5 and 6, should be expressed by requiring that the aggregate interference should not exceed 6% of the total system noise power for more than 10% of the time;

10 that the verification of whether the internetwork interference caused by the earth and space station emissions of any given satellite network meets the requirements of *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable) or the derivation of an interference mask (interference levels and maximum percentages of time for which such levels could be exceeded) that would lead to *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable) being met may be conducted using the methodologies described in Annexes 1 and 2 in connection with an appropriate, assumed number of interfering networks;

11 that the maximum level of interference noise power caused to a GSO/FSS network should be calculated on the basis of the following values for the receiving earth station antenna gain, in a direction at an angle  $\varphi$  (degrees) referred to the main beam direction:

for GSO to GSO interference:

$$\begin{array}{lll} G = 32 - 25 \log \varphi & \text{dBi} & \text{for } 1^\circ \leq \varphi < 48^\circ \\ G = -10 & \text{dBi} & \text{for } 48^\circ \leq \varphi \leq 180^\circ \end{array}$$

for non-GSO to GSO interference, the antenna patterns contained in Recommendation ITU-R S.1428;

12 that the following Notes should be regarded as part of this Recommendation.

NOTE 1 – For the interference between GSO FSS networks, *recommends* 1 and 2 apply but *recommends* 3 does not apply.

NOTE 2 – The term “interference of time-varying nature” in *recommends* 3 includes the constant component that may be present throughout time.

NOTE 3 – For the calculation of the limits quoted in *recommends* 1.1, 1.2, 2, 3, 4, 5, 6 and 9 it should be assumed that the total system noise power at the input to the demodulator is of thermal nature and includes all intra-system noise contributions as well as interference noise from other systems.

In the event that the interference cannot be assumed to be thermal in nature the permissible level of interference into a digital carrier should be based upon the degradation of the BER (or  $C/N$ ) performance.

NOTE 4 – For the calculation of interference, in respect of *recommends* 1, 2, 3, 4, 5, 6 and 9 as applied to satellite networks operating in a fading environment, it should be assumed that the carrier power level of the interfered system is reduced, until the system performance coincides with the above long-term BER (or  $C/N$ ) and percentage of month (see Annex 1 of Recommendation ITU-R S.735 for clarification).

NOTE 5 – It is assumed in connection with *recommends* 1 and 2 that the interference from other satellite networks is of a continuous nature at frequencies below 10 GHz: further study is required with respect to cases where interference is not of a continuous nature above 10 GHz.

NOTE 6 – When interfering signals are characterized by a non-uniform spectral distribution there may be cases where, for design purposes, a greater interference allocation of total system noise may be made to narrow-bandwidth carriers by the system designer. One model developed to address this is presented in detail in Annex 2 of Recommendation ITU-R S.735.

NOTE 7 – For networks using 8-bit PCM encoded telephony see Recommendation ITU-R S.523.

NOTE 8 – In some cases it may be necessary to limit the single entry interference value to less than the value quoted in *recommends* 2 in order that the total value recommended in *recommends* 1 may not be exceeded. In other cases, particularly in congested arcs of the GSO, administrations may agree bilaterally to use higher single entry interference values than those quoted in *recommends* 2, but any interference noise power in excess of the value recommended in *recommends* 2 should be disregarded in calculating whether the total value recommended in *recommends* 1 is exceeded.

NOTE 9 – There is a need for study of the acceptability of an increase in the maximum total interference noise values recommended in *recommends* 1.

NOTE 10 – For frequencies above 10 GHz short-term propagation data are not available uniformly throughout the world and there is a continuing need to examine such data to confirm an appropriate interference allowance to meet the applicable performance objectives.

NOTE 11 – There is a need to continue the study of the interference noise allowances appropriate to systems operating at frequencies above 15 GHz. There is an urgent need to study the effect on the interference noise allowances when power control or adaptive coding is used for fade compensation.

NOTE 12 – In order to promote orbit efficiency, satellite networks operating in climates having heavy rain are encouraged to use some form of fade compensation.

NOTE 13 – In certain bands the off-axis equivalent isotropically radiated power (e.i.r.p.) density limits of Section VI of RR Article 22 may be used to assess the likely level of interference from GSO FSS networks. However, those limits do not apply to earth station antennas in service or ready to be in service prior to 2 June 2000, nor to earth stations associated with a satellite network in the FSS for which complete coordination or notification information has been received before 2 June 2000.

## ANNEX 1

**Methodologies for determining whether interference to a network in the FSS (GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links) meets *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable) or for deriving interference allowances that would meet *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable)**

This Annex includes three methodologies for verifying whether interference meets *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable) or for deriving interference allowances that would meet *recommends* 3, 4, 5 and 6 (and *recommends* 9, where applicable). They are referred to here as Methodologies A, A' and B. Application of these methodologies in the context of interference from an individual network (i.e., single-entry interference) requires allocation of the aggregate interference allowance of *recommends* 3, 4, 5 and 6 among the interfering networks. Determination of the appropriate number of interfering systems is beyond the scope of this Recommendation.

Methodologies A and A' consider simultaneous effects due to fading and interference. Verification of compliance with *recommends* 3, 4, 5 and 6 or derivation of interference allowances take into account that during certain percentages of time performance objectives are violated because of the combination of the two sources of degradation, while none of them would isolatedly cause such violation. However, modelling fading may be difficult, specially for links to or from non-GSO satellites where elevation and azimuth vary with time. Methodology A' is a special case of Methodology A in the sense that particular parametric models for the probability density functions of the degradations due to fading and interference are assumed. In Methodology A, the parametric representation of these probability density functions remains undefined and can be chosen to best fit the particular situation under consideration.

For systems operating in clear-sky with relatively small margins and relying heavily on power control or adaptive coding to combat fading, simultaneous effects due to fading and interference become less significant and may be neglected if the affected system so wishes. Methodology B explores this possibility (separate consideration of interference effects).

Methodology B is indeed a simplification of Methodology A where, in addition to considering interference separately, performance objectives are summarized by a threshold BER (or  $C/N$ ) and the percentage of time it can be exceeded.

A procedure implementing verification of compliance with *recommends* 3.1 and refinement of the interference mask is described in Annex 2. This procedure can be applied to verify compliance with *recommends* 3.1 for interference masks developed using any of the methodologies described in Annex 1.

Further study is needed to determine the nature of both short-term and long-term interference into a non-GSO network from multiple GSO networks.

## PART 1

**Methodology A****1 Basic assumptions**

The following basic assumptions are made in connection with the procedure proposed here for verifying whether interference meets the requirements in *recommends* 3, 4, 5 and 6 or for determining the interference allowances associated with any given desired carrier that would meet *recommends* 3, 4, 5 and 6.

*Assumption 1:* The two time-varying sources of degradation considered in the analysis are link fading plus any other time variations in the characteristics of the link and interference from other FSS networks.

The total  $C/N$  for a given carrier is:

$$C/N = C/(N_T + I)$$

where:

$C$ : wanted power (W), which varies as a function of the uplink and downlink fades and also as a function of the transmission configuration (multiple access, use of uplink power control, etc.) Thus  $C$  can be described as a function of  $A_{\uparrow}$ , the uplink rain attenuation, and  $A_{\downarrow}$ , the downlink rain attenuation as:

$$C = C_{CS} / F(A_{\uparrow}, A_{\downarrow})$$

$C_{CS}$ : wanted power in clear-sky conditions (long-term condition)

$N_T$ : total system noise (W) (i.e. the thermal power including uplink and downlink contributions at the demodulator input, the noise power resulting from the multi-carrier operation of the involved power amplifier – in the earth stations and in the space stations –, the cross polarization isolations of the different transmit and receive antennas, the thermal power increase due to the rain fades, Sun – and Moon if applicable – temperature), which also varies as a function of the transmission configuration and with the uplink and downlink fades.  $N_T$  also includes the long-term contributions from other networks. Thus  $N_T$  can be described as a function of  $A_{\uparrow}$  and  $A_{\downarrow}$  as:

$$N_T = N_{T,CS} \cdot G(A_{\uparrow}, A_{\downarrow})$$

$N_{T,CS}$ : noise power in clear-sky conditions (long-term condition) (W)

$I$ : time-varying interference power (W) generated by other networks.

*Assumption 2:* Due to fading plus other time variations in the characteristics of the link, carrier power reduction due to the uplink fade  $A_{\uparrow}$  and the downlink fade  $A_{\downarrow}$  i.e.  $F(A_{\uparrow}, A_{\downarrow})$ , and the noise increase,  $G(A_{\uparrow}, A_{\downarrow})$ , can be accounted for by substituting  $C/X$  for  $C$ , with  $X = H(A_{\uparrow}, A_{\downarrow}) = F(A_{\uparrow}, A_{\downarrow}) \cdot G(A_{\uparrow}, A_{\downarrow})$ , and the corresponding degradation  $x$  (dB), is:

$$x = 10 \log X = 10 \log (H(A_{\uparrow}, A_{\downarrow})) \quad (1)$$

The effect of interference can be represented by increasing the noise power from  $N_T$  to  $Y N_T$  and the corresponding degradation  $y$  (dB) is:

$$y = 10 \log Y \quad (2)$$

The total  $C/N$  degradation  $z$  (dB) is therefore:

$$z = x + y \quad (3)$$

The random variables  $x$  and  $y$  are assumed to be statistically independent and therefore the probability density function (pdf) of  $z$  is the convolution of the pdf of  $x$  and  $y$ . Independence between these two random variables is an approximation because the presence of fading may increase the noise level and also lead to a reduction of  $I$  (fading in the interference path). In both respects, the assumption of independence is conservative in the sense of over-estimating the effect of interference.

Further, it follows from the definition of  $y$  that:

$$Y = 1 + (I/N_T) \quad (4)$$

where  $I$  is the interfering power.

In order to permit the computation of the probability density function of the degradation  $x$ , it is necessary to identify, prior to the application of this methodology, the exact carrier parameters of the considered network, as well as the necessary parameters required to develop the computation of the uplink and downlink fades as well as the power reduction and noise increase functions ( $F$  and  $G$ ).

*Assumption 3:* The time allowances for each interference entry are obtained by dividing by  $N$  the time allowances associated with the total interference. This number  $N$  is related to the number of networks that can potentially cause time-varying interference and will be referred to as the equivalent number of networks.  $N$  may also vary with the time percentage considered.

*Assumption 4:* This analysis assumes that, during a fading event, the wanted carrier is attenuated but the interfering carrier is not. This assumption results in some over-estimation of the total downlink degradation under circumstances where interference peaks and downlink fading occur simultaneously.

## 2 Input data

The following data is required to verify compliance with *recommends* 3, 4, 5 and 6 or to determine the interference allowances that would meet *recommends* 3, 4, 5 and 6 corresponding to any specific desired carrier.

- a) The performance requirements of the desired carrier, as expressed by the values of BER associated with different percentages of time have to be known. In general, this will be a set of values  $\text{BER}_j$  ( $j = 1, \dots, J$ ) and the corresponding percentages of the year  $p_j$  ( $j = 1, \dots, J$ ) for which the BER can be worse than  $\text{BER}_j$ .
- b) The clear-sky carrier-to-noise ratio  $(C/N)_{CS}$ , as well as the carrier-to-noise ratio values  $(C/N)_j$  ( $j = 1, \dots, J$ ) corresponding to the BER values  $\text{BER}_j$  defined in a) above. In addition, if power control is used, information on the corresponding procedures is required.  $C/N$  values can be given directly without association with BER values, in which case only the values  $p_j$  ( $j = 1, \dots, J$ ) in a) are needed.

- c) The pdf,  $p_x(X)$  of the random variable  $x$  which expresses in dB the degradation in performance due to fading plus any other time variations in the characteristics of the link. This pdf is highly dependent on the presence of power control and its characteristics. This pdf has to be compatible with *recommends* 3.1 and therefore the degradation  $x$  cannot use more than 90% of the time allowances associated with each BER (or  $C/N$ ) level (see equation (6) for an expression of this condition).
- d) The equivalent number  $N$  of interfering networks that can potentially cause time-varying interference and that will be sharing the same frequency band with the desired network. For a GSO/FSS desired network,  $N$  is related to the number of non-GSO systems sharing the same frequency band. For a non-GSO desired network,  $N$  is related to the number of other non-GSO networks plus the number of 2° potentially interfering geostationary orbital positions visible, above the minimum elevation angle, by the earth station of the non-GSO network.

In addition, to verify compliance, the pdf,  $p_y(Y)$ , of the degradation due to interference must be provided. This pdf can be derived by the application of any of the methodologies described in this Recommendation or indeed by any other means that may be seen fit.

### 3 Proposed procedure

#### 3.1 Verification of compliance with *recommends* 3, 4, 5 and 6

For verification of compliance with *recommends* 3, 4, 5 and 6 it is necessary to obtain the pdf  $p_z(Z)$ , of the total degradation, given by:

$$p_z(Z) = p_x p_y(Z) \quad (4a)$$

where  $p_x(X)$  and  $p_y(Y)$  were given as input data. Conditions to be verified are:

$$P(z \geq z_j) \leq (0.9 + 0.1 / N) p_j / 100 \quad \text{for } j = 1, \dots, J \quad (4b)$$

Detailed procedures to implement this verification of compliance with *recommends* 3, 4, 5 and 6 can be found in Annex 2, where examples of the application of these procedures are also given.

#### 3.2 Derivation of interference allowances

Based on the assumptions and required input data given above, the following steps define the procedure to determine the interference allowances corresponding to any given desired carrier.

*Step 1:* From a) and b) of the input data, the values  $z_i$  of the total degradation  $z$  which can be exceeded at most during  $p_i\%$  of the year can be determined from:

$$z_j = (C/N)_{cs} - (C/N)_j \quad \text{for } j = 1, \dots, J \quad (5)$$

As a consequence, the conditions on  $p_x(X)$  given in c) of the input data can be expressed as:

$$P(x \geq z_j) \leq (0.9 p_j) / 100 \quad (6)$$

*Step 2:* A parametric representation is chosen for the pdf,  $p_y(Y)$ , corresponding to the degradation due to interference. In the case of a transparent transponder, this includes uplink and downlink interference from all earth stations and space stations in the interfering network. When there is on-board processing, separate probability densities for uplink and downlink degradations are required. The trade-off here is, on one hand, to have a sufficiently detailed representation of  $p_y(Y)$  and, on the other hand, to keep computations simple enough. This representation will depend on a certain number  $K$  of parameters  $\alpha_k$  ( $k = 1, \dots, K$ ) and can be expressed as:

$$p_y(Y) = f(Y, \alpha_1, \dots, \alpha_K) \quad (7)$$

*Step 3:* A parametric representation for the total degradation  $z$  is obtained from:

$$p_z(Z) = p_x * p_y(Z) \quad (8)$$

where  $p_x(X)$  is given in c) of the input data and  $p_y(Y)$  was defined in Step 2. As  $p_y(Y)$  depends on the parameters  $\alpha_1, \dots, \alpha_K$ , according to equation (7), so does  $p_z(Z)$ . This function can therefore be written as:

$$p_z(Z) = h(Z, \alpha_1, \dots, \alpha_K) \quad (9)$$

*Step 4:* From equation (9), the probability that the total degradation  $z$  exceeds each of the values  $z_j$  obtained in Step 1 can be computed. Each of these probabilities is a function of the parameters  $\alpha_1, \dots, \alpha_K$  and can be written:

$$P(z \geq z_j) = r_j(\alpha_1, \dots, \alpha_K) \quad \text{for } j = 1, \dots, J \quad (10)$$

Finally, the parameters  $\alpha_1, \dots, \alpha_K$  can be obtained from the conditions:

$$r_j(\alpha_1, \dots, \alpha_K) \leq (0.9 + 0.1 / N) p_j / 100 \quad \text{for } j = 1, \dots, J \quad (11)$$

where the values of  $p_i$  are those in a) of the input data which are associated with the degradations  $z_i$  computed in Step 1 and  $n$  is the number of interference entries.

*Step 5:* From the parameters  $\alpha_1, \dots, \alpha_K$  computed in Step 4, the pdf of  $y$ , as defined in (7) is obtained. This pdf allows that a mask for the interference,  $I$ , produced by one interfering network, and expressed as a fraction of the total link noise  $N_T$ , be defined. For instance, if:

$$P(y \geq Y_m) \leq q_m \quad (12)$$

it follows that:

$$P(I \geq (10^{Y_m/10} - 1) N_T) \leq q_m \quad (13)$$

From  $p_y(Y)$ , a certain number  $M$  of pairs  $((10^{Y_m/10} - 1) N_T, q_m)$  can be computed, defining therefore a mask for the interference allowances from one interfering network.

#### 4 Interference into systems using transparent transponders: joint effects of uplink and downlink fading and interference

Let  $X_{\uparrow}$  denote the degradation due to fading in the uplink  $C/N$  and let  $X_{\downarrow}$  denote the degradation due to fading in the downlink  $C/N$ . In general,  $X_{\uparrow}$  will be made equal to the attenuation due to rain while  $X_{\downarrow}$  will further incorporate the effects of the increase in the receive noise temperature.

If  $N_{\uparrow}$  and  $N_{\downarrow}$  denote the total uplink and downlink noises in clear-sky and  $I_{\uparrow}$  and  $I_{\downarrow}$  denote the uplink and downlink time-varying interferences, the  $C/N$  in clear-sky and in the absence of any time-varying interference can be written as:

$$\left(\frac{C}{N}\right)_{cs} = \frac{C}{N_{\uparrow} + N_{\downarrow}} \quad (14)$$

while the  $C/N$  in the presence of uplink and downlink fading and interference can be written as:

$$\left(\frac{C}{N}\right) = \frac{1}{\frac{X_{\uparrow}(N_{\uparrow} + I_{\uparrow})}{C} + \frac{X_{\uparrow}X_{\downarrow}(N_{\downarrow} + I_{\downarrow})}{C}} \quad (15)$$

Therefore, the degradation due to uplink and downlink fading and interference can be expressed as:

$$Z = aX_{\uparrow}\left(1 + \frac{I_{\uparrow}}{N_{\uparrow}}\right) + (1 - a)X_{\uparrow}X_{\downarrow}\left(1 + \frac{I_{\downarrow}}{N_{\downarrow}}\right) = X_{\uparrow}[aY_{\uparrow} + (1 - a)X_{\downarrow}Y_{\downarrow}] \quad (16)$$

where:

$$Y_{\uparrow} = 1 + \frac{I_{\uparrow}}{N_{\uparrow}} \quad (17)$$

is the degradation in the clear-sky uplink  $C/N$  due to uplink interference,

$$Y_{\downarrow} = 1 + \frac{I_{\downarrow}}{N_{\downarrow}} \quad (18)$$

is the degradation in the clear-sky downlink  $C/N$  due to downlink interference and

$$a = \frac{N_{\uparrow}}{N_{\uparrow} + N_{\downarrow}} \quad (19)$$

If we write:

$$V = aY_{\uparrow} + (1 - a)Z_{\downarrow} \quad (20)$$

where:

$$Z_{\downarrow} = X_{\downarrow} Y_{\downarrow} \quad (21)$$

is the total degradation, due to interference and fading, in the downlink clear-sky  $C/N$ , then:

$$Z = X_{\uparrow} V \quad (22)$$

or (dB):

$$z = 10 \log Z = 10 \log X_{\uparrow} + 10 \log V = x_{\uparrow} + v \quad (23)$$

Equation (22) or (23), combined with equations (20) and (21), gives the total degradation due to interference and fading as a function of the degradations in the clear-sky uplink  $C/N$  due to fading ( $X_{\uparrow}$ ) and interference ( $I_{\uparrow}$ ) and of the degradations in the clear-sky downlink  $C/N$  due to fading ( $X_{\downarrow}$ ) and interference ( $I_{\downarrow}$ ).

In order to obtain the pdf of the total degradation  $z$  (dB) as given by equation (23), the pdf of the degradation due to uplink fading  $x_{\uparrow}$  (dB) has to be convolved with the pdf of the random variable  $v$ , defined by:

$$v = 10 \log[aY_{\uparrow} + (1 - a)Z_{\downarrow}] \quad (24)$$

In order to obtain the pdf of the random variable  $v$ , it is first necessary to convolve the pdfs of the random variables  $aY_{\uparrow}$  and  $(1 - a)Z_{\downarrow}$ .

### Example

As an example of the consideration of the joint effects of uplink and downlink interference and fading, interference from a non-GSO FSS system into a GSO FSS network is considered here.

The relevant parameters of the GSO link are:

- earth station location: 26° N, 128° E
- rain model: Recommendation ITU-R P.618, Region N
- elevation angle to GSO satellite (at 132° E): 59.28°
- $a = N_{\uparrow}/(N_{\uparrow} + N_{\downarrow}) = 0.0988$
- system margin: 11.5 dB.

The non-GSO earth station is co-located with the GSO earth station.

The pdfs of the degradations due to uplink fading,  $x_{\uparrow}$ , uplink interference,  $y_{\uparrow}$ , downlink fading,  $x_{\downarrow}$  and downlink interference,  $y_{\downarrow}$ , are shown in Fig. 1. Using the procedure described above allows us to obtain the pdf of the total degradation  $z$ , shown in Fig. 2, where the pdf of the downlink degradation,  $z_{\downarrow}$ , is also presented. From the probabilities of total degradation exceeding the system margin and downlink degradation exceeding the system margin, we note that in this case the effects of downlink degradation (fading plus interference) are dominant.

FIGURE 1  
pdf of rain and interference degradation

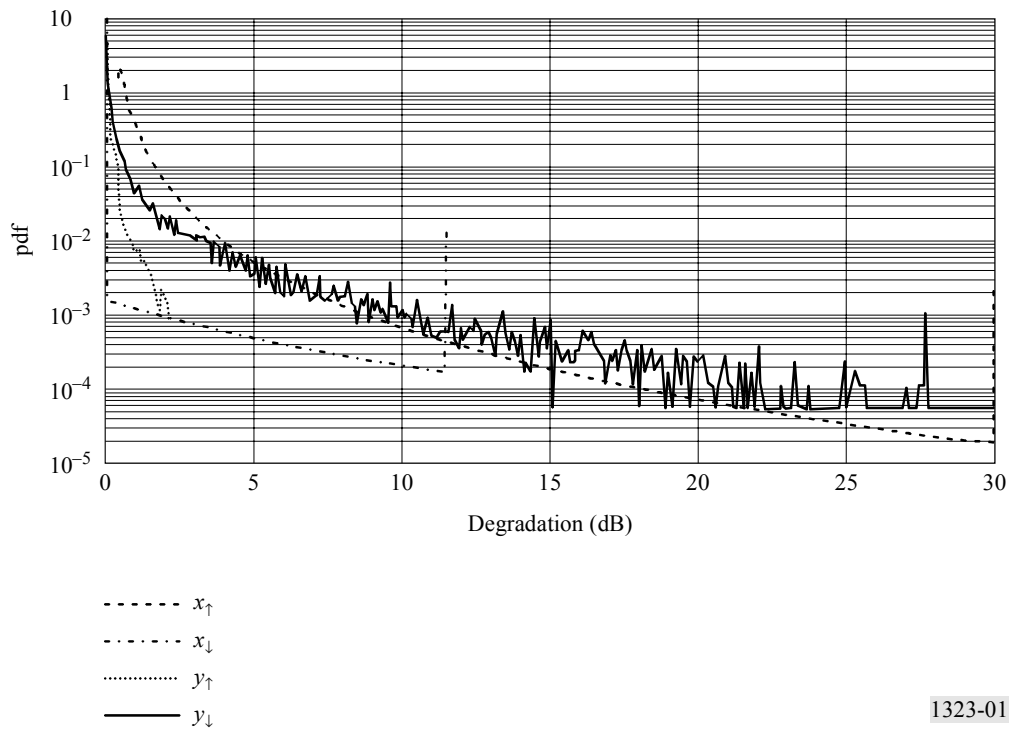
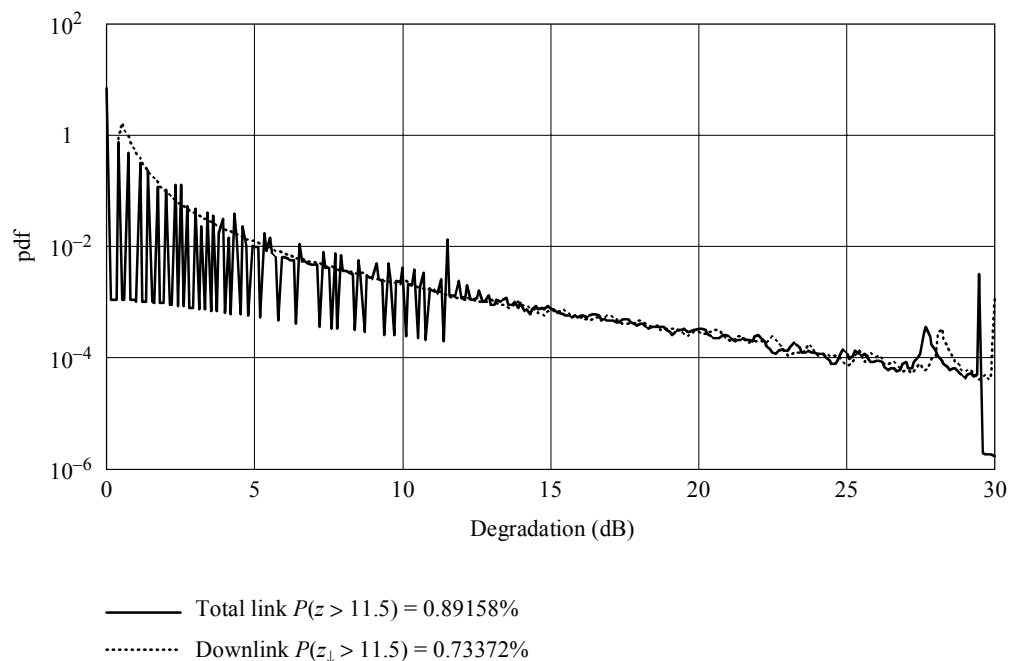


FIGURE 2  
pdf of total degradation



## 5 Consideration of degradation due to fading in links with variable elevation angle

In links to and from non-GSO satellites the degradation due to fading is also a function of the elevation angle  $\gamma$ . One approximate way of taking this into account consists in determining the pdf

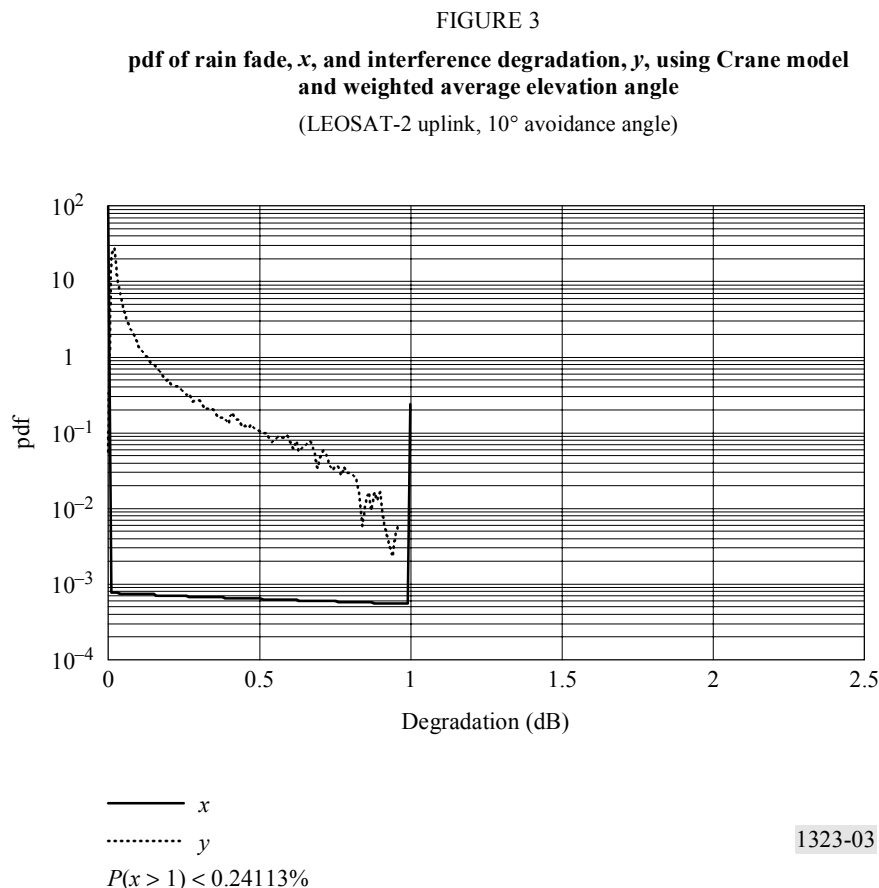
of the degradation due to fading for the average elevation angle  $\gamma_{av}$ . However, a more precise approach is to obtain the pdf  $p_\gamma(\Gamma)$  of the elevation angle and then express the pdf  $p_x(X)$  of the degradation as:

$$p_x(X) = \int_0^{\pi/2} p_{x|\gamma}(X|\gamma = Z) p_\gamma(Z) dZ \quad (25)$$

### Example

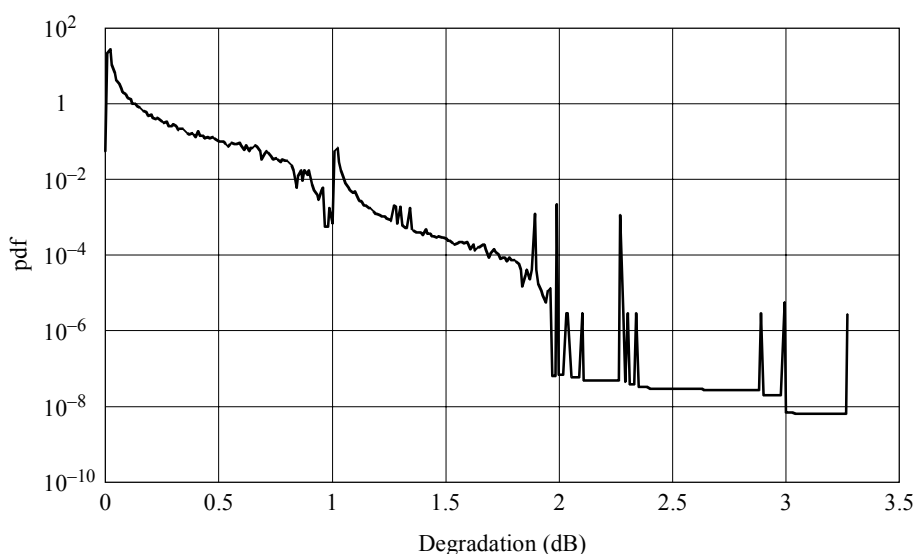
As an example, consider the interference between the uplinks of two non-GSO satellite systems. The interfered with non-GSO system avoids in-line events employing a  $10^\circ$  avoidance angle. This avoidance angle is just sufficient for the total degradation  $z$  to meet the allowable time percentage. The victim uplink uses power control with a dynamic range of 6.8 dB with a clear sky link margin of 1 dB and a heavy rain link margin of 0 dB. The corresponding pdf for the rain fading was therefore represented with an impulse at 0 dB corresponding to the probability of  $x$  (degradation due to fading) being between 0 and 5.8 dB and a second impulse at 1 dB corresponding to the probability of  $x$  exceeding 5.8 dB.

Figure 3 shows the rain fade  $x$  and interference degradation  $y$  pdfs for the uplink interference, where the  $x$  distribution is based on the average elevation angle. The Crane rain model is used. Figure 4 shows the corresponding total degradation  $z$ , derived from the convolution of the  $x$  and  $y$  pdfs.



1323-03

FIGURE 4  
pdf of total degradation,  $z$ , using Crane model and weighted average elevation angle  
(LEOSAT-2 uplink,  $10^\circ$  avoidance angle)



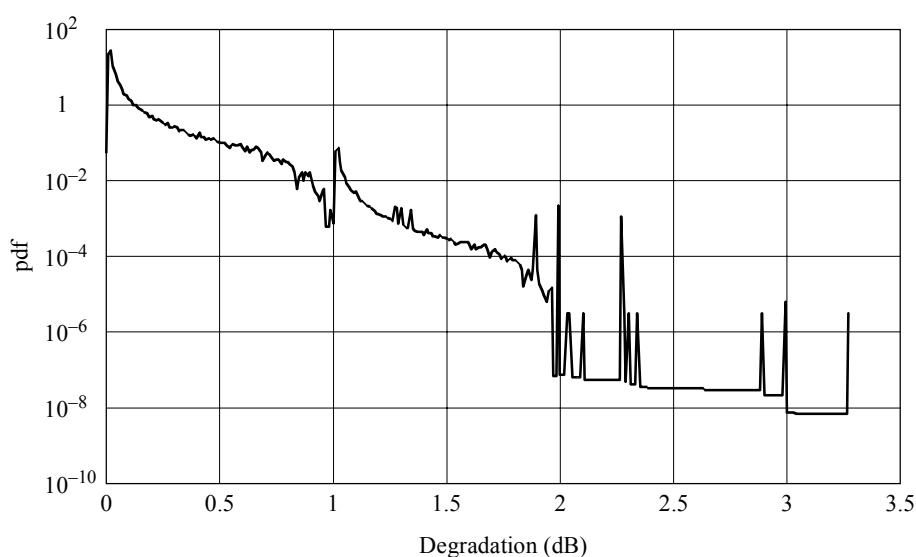
Threshold = 0.26792%

$P(z > 1) = 0.25783\%$

1323-04

Figure 5 shows the total degradation  $z$  distribution when the elevation angle distribution is used in generating the rain fade rather than using the average elevation angle. The total degradation just meets the time allowance, similarly to the results shown in Fig. 2.

FIGURE 5  
pdf of total degradation,  $z$ , using Crane model and elevation angle distribution  
(LEOSAT-2 uplink,  $10^\circ$  avoidance angle)



Threshold = 0.29394%

$P(z > 1) = 0.28159\%$

1323-05



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Other examples have confirmed that computing the degradation due to fading as proposed in equation (25), or basing this calculation on the average elevation angle  $\gamma_{av}$ , leads to essentially identical results. This justifies the use of the simpler approach i.e. to compute the degradation due to fading based on the average angle  $\gamma_{av}$ .

A procedure to consider the time variation of the parameters of a non-GSO link and also take into account any possible statistical dependence between fading and interference is described in Annex 3.

## 6 Examples of application of Methodology A

We consider here a GSO downlink that is supposed to operate in such a way that the received  $C/N$  is above a threshold value  $(C/N)_{thr}$  during at least 99.9% of the time.

It is assumed that the degradation due to fading includes the rain attenuation directly obtained from the Crane two-component model plus the effect of the increase in noise temperature due to rain. It is further assumed that the total downlink noise also includes interference (both intra-system and inter-system) and that the interference is faded by the same amount as the desired signal.

The degradation  $X$ , expressed as a factor, is given by:

$$X = \frac{(1 - \alpha) \left\{ L_R + \left[ (T_0 - T_B) / T_{sys} \right] [(L_R - 1 / L_A)] \right\} + (\alpha / L_A)}{(1 - \alpha) + (\alpha / L_A)} \quad (26)$$

where:

- $\alpha$ : fraction of the total downlink noise in clear-sky which is due to interference
- $L_R$ : attenuation due to rain
- $T_0$ : mean absorption temperature (274.8 K)
- $T_B$ : background temperature (2.76 K for the sky)
- $T_{sys}$ : downlink thermal noise temperature
- $L_A$ : loss due to atmospheric absorption (1.07, which corresponds to 0.3 dB).

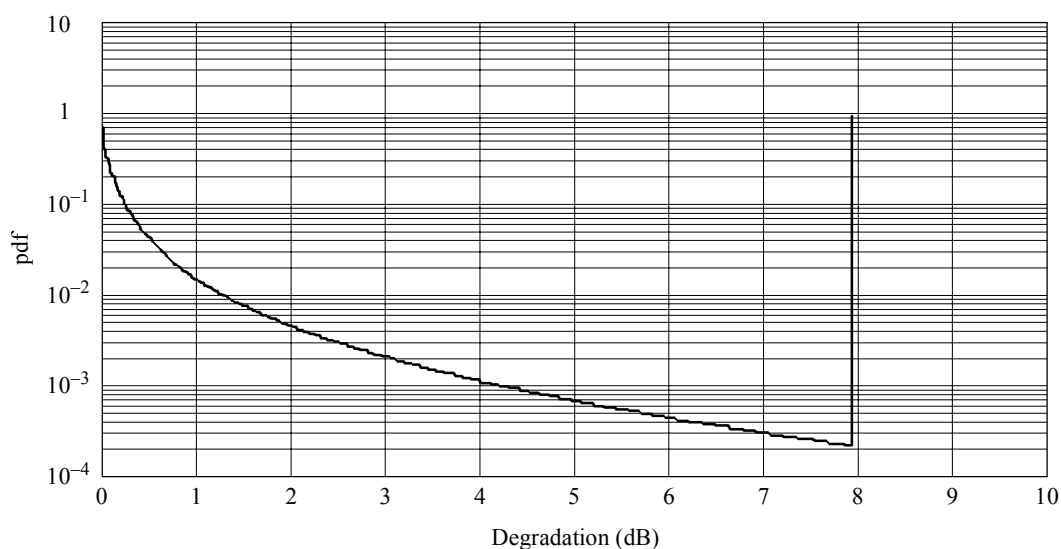
In order to be above a certain threshold  $(C/N)_{thr}$  during 99.9% of the time, the link is designed with a margin  $X_{max}$  – difference between  $(C/N)_{cs}$ , and  $(C/N)_{thr}$  – such that  $p(x > X_{max}) = 0.09\%$  (the remaining 0.01% will account for the effects of interference).

Assuming an earth-station located in New York City (latitude 41° N; longitude 74° W), receiving at 19 GHz with an elevation angle of 42.43°,  $\alpha = 0.2$  and  $T_{sys} = 323.6$  K; it turns out that  $X_{max} = 7.923$  dB and therefore:

$$(C/N)_{cs} - (C/N)_{thr} = 7.923 \quad (27)$$

The corresponding pdf  $p_x(X)$  of the degradation  $x$  due to fading is given in Fig. 6. This pdf has been clipped at  $X = X_{max} = 7.923$ .

FIGURE 6  
pdf of degradation,  $x$ , due to rain fading  
(for GSO receiver)

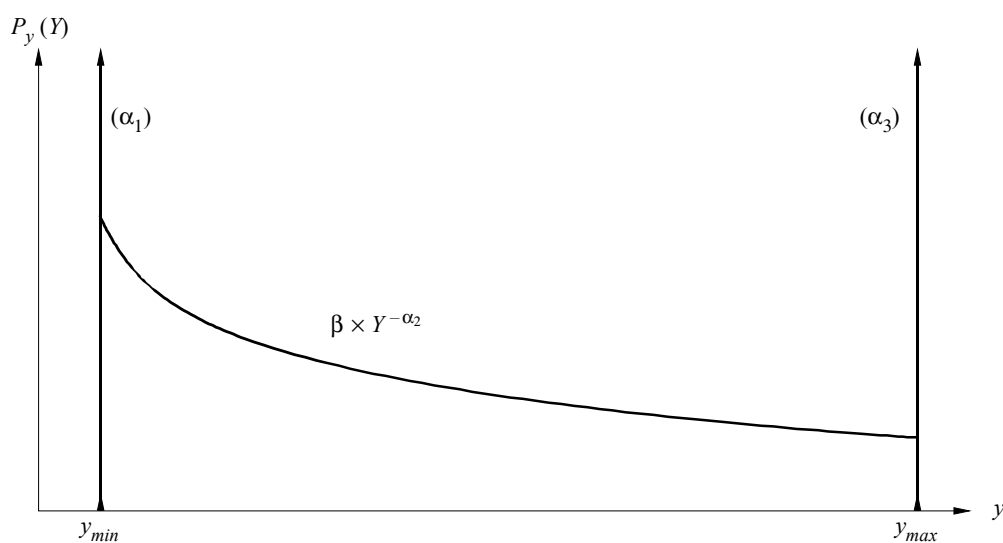


$X = 7.923$  for  $P(x > X) = 0.09\%$

1323-06

It has been verified that in some representative situations,  $p_y(Y)$  can be appropriately modelled by the function shown in Fig. 7.

FIGURE 7  
Parametric model for the degradation,  $y$ , due to interference expressed in dB



1323-07

At this point, we consider separately the use of Methodology A for the derivation of an interference mask and for the verification of whether the requirements in *recommends* 3, 4, 5 and 6 are met in a specific case.

## 6.1 Derivation of interference mask

The derivation of a probability mask would require convolving the pdfs in Figs. 6 and 7 and ensuring that the resulting pdf  $p_z(Z)$  is such that the condition in equation (11) of the description of Methodology A is met. In this example this condition becomes:

$$P(z > 7.923) \leq 0.1\% \quad (28)$$

Of course, given the clear difficulties involved in analytically convolving  $p_x(X)$  and  $p_y(Y)$ ,  $P(z > 7.923)$  cannot be analytically expressed as a function of the parameters  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  (note that the parameter  $\beta$  in Fig. 7 is a function of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ). Instead, the convolution has to be performed for several choices of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ , so that sets of possible values of these parameters can be determined (when performing these convolutions  $y_{min}$  was made very small, 0.04, and  $y_{max}$  was made equal to 7.923). These possible values are those for which the inequality in equation (28) is satisfied. A sample of possible choices for  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ , is given in Table 1.

TABLE 1

Sets of values of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  that would meet inequality  $P(z > 7.923) \leq 0.1\%$

| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $P(z > 7.923)$ |
|------------|------------|------------|----------------|
| 0.25       | 2.5        | 0.00007    | 0.000998       |
|            | 2.0        | 0.00002    | 0.000999       |
| 0.50       | 2.5        | 0.00007    | 0.000992       |
|            | 2.0        | 0.00004    | 0.000962       |
| 0.75       | 2.5        | 0.00008    | 0.000996       |
|            | 2.0        | 0.00006    | 0.000993       |
| 0.90       | 2.5        | 0.00008    | 0.000992       |
|            | 2.0        | 0.00008    | 0.000999       |
|            | 1.5        | 0.00004    | 0.000994       |
| 0.95       | 2.5        | 0.00008    | 0.000991       |
|            | 2.0        | 0.00008    | 0.000994       |
|            | 1.5        | 0.00006    | 0.000992       |
|            | 1.0        | 0          | 0.000995       |

As expected, it follows from Table 1 that  $\alpha_1$  and  $\alpha_3$  by themselves do not ensure that inequality in equation (28) is met. The higher the value of  $\alpha_2$ , the more flexible is the choice of  $\alpha_1$  and  $\alpha_3$ . Therefore, conditions to be impose on  $p_y(Y)$  should include some intermediate point of the

distribution, according to the required value for  $\alpha_2$ . Of course, the larger the value of  $\alpha_2$ , the higher the probability of the occurrence of lower degradation due to interference. For example, from the set of values  $\alpha_1 = 0.90$ ;  $\alpha_2 = 1.5$  and  $\alpha_3 = 0.00004$ , conditions to be imposed on the interference  $I$  could be expressed as:

$$P(I \geq 0.01N_T) \leq 10\%$$

$$P(I \geq 0.1N_T) \leq 2.69\%$$

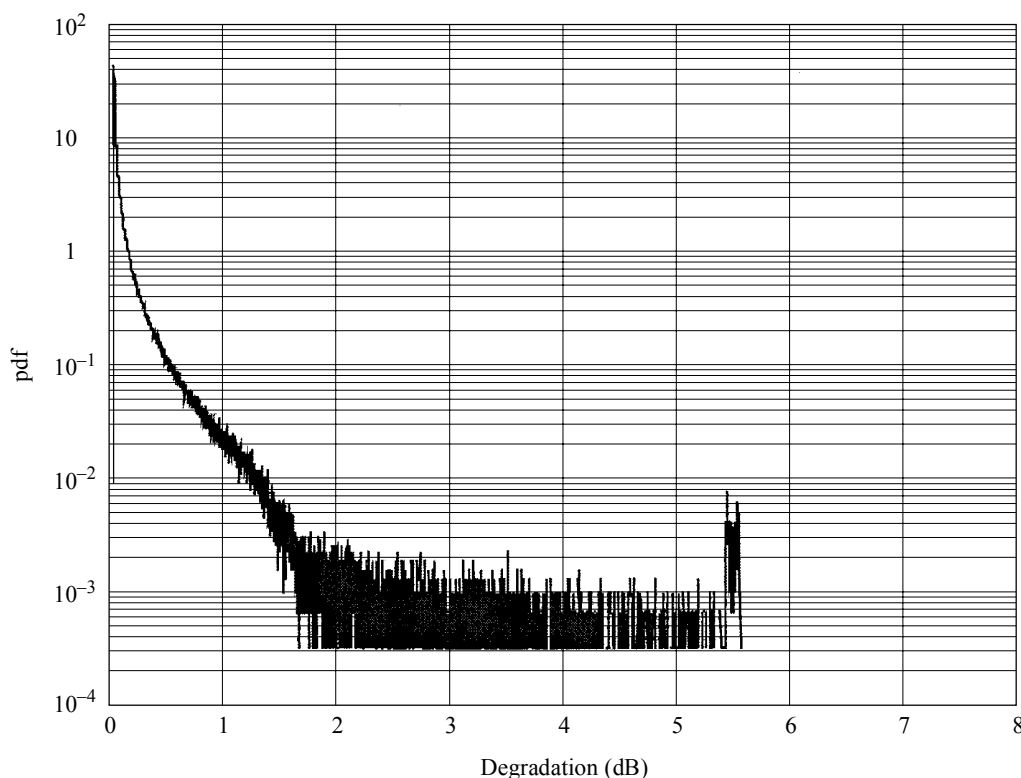
$$P(I \geq 5.2N_T) \leq 0.004\%$$

## 6.2 Verification of whether the requirements in *recommends* 3, 4, 5 and 6 are met

If we are dealing with a specific situation, and a pdf  $p_y(Y)$  of the degradation  $y$  due to interference is made available, the verification of whether the requirements in *recommends* 3, 4, 5 and 6 are met is straightforward. The pdfs  $p_x(X)$  and  $p_y(Y)$  have to be convolved, generating a pdf  $p_z(Z)$  associated with the total degradation  $z$ . Knowledge of  $p_z(Z)$  allows us to compute the left-hand side of equation (28) and check therefore whether the inequality is met.

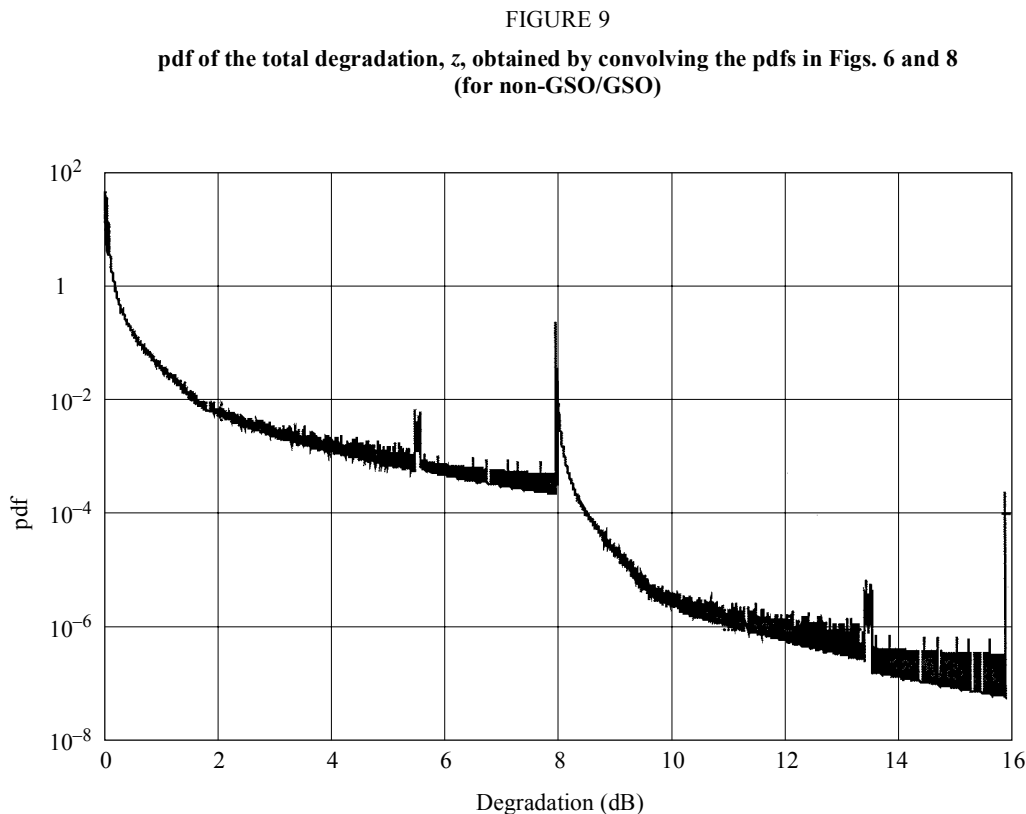
As an illustration, a pdf  $p_y(Y)$ , obtained by simulation and corresponding to the degradation due to interference from a non-GSO constellation into the GSO downlink considered in this example, is shown in Fig. 8.

FIGURE 8  
pdf of degradation due to interference,  $y$ , obtained by simulation  
(for non-GSO/GSO)



1323-08

By convolving the functions in Figs. 6 and 8, the pdf  $p_z(Z)$ , shown in Fig. 9, is obtained.



1323-09

$$P(z > 7.923) = 0.12035\%$$

From the pdf in Fig. 9, it can be computed that:

$$P(z > 7.293) = 0.12035\%$$

which means that the requirements of *recommends* 3, 4, 5 and 6 as expressed in equation (28) above, are not met.

## PART 2

### Methodology A'

#### 1 Introduction

Methodology A' is a simplification of Methodology A, in which specific parametric representations are chosen for the pdfs of rain fading and interference, in order to establish the joint probability of fading and interference and to ensure that the joint cumulative probability meets the specified link performance criteria, which is characterized by a set of degradations in  $C/N$  and the corresponding fractions of time  $p_i^{C/N}$  for which the degradations may be exceeded. The degradation in system

performance due to rain fading is characterized by the pdf  $p_x(x)$  that the degradation due to fading will be between  $x$  and  $x + \delta x$  dB, and the degradation due to interference is similarly characterized by the pdf  $p_y(y)$  that the degradation due to interference will be between  $y$  and  $y + \delta y$  dB. The overall performance objectives of the link will be achieved provided the joint cumulative probability distribution of both fading and interference does not exceed the specified  $C/N$  degradation objectives. The probability that a degradation  $z$  exceeds a value  $z_i$ ,  $P(z \geq z_i)$ , must be less than the specified performance criteria,  $p_i^{C/N}$ , i.e.:

$$P(z \geq z_i) = \int_{z_i}^{\infty} p_z(z) dz \leq p_i^{C/N} \quad (29)$$

with:

$$p_z(z) = p_x(x) * p_y(y) \quad (30)$$

where  $*$  represents the convolution of the two probabilities. The convolution is expressed mathematically by:

$$p_z(z) = \int_{-\infty}^{\infty} p_x(w) \cdot p_y(z - w) dw \quad (31)$$

Since the pdf of fading,  $p_x(x)$ , and several points on the curve of the probability distribution that the  $C/N$  performance must be achieved, which depends on  $p_z(z)$ , are predetermined, the levels of interference which can be allowed for various percentages of time, while still maintaining the link performance objectives, can be determined by finding a curve for the probability density  $p_y(y)$  which satisfies the above equation. In practice, the maximum allowable interference levels are specified when this equation is expressed as an equality. In essence, the problem reduces to finding a set of interference levels,  $y_j$ , which are not exceeded for  $P_y (= 100 p_y)$  per cent of the year, which satisfy this equation.

Methodology A' provides a first-order solution to equation (29) in which the pdfs are parameterized by two points, corresponding to:

- the percentage of time when a given level of rain fading or interference is exceeded
- the percentage of time beyond which there is little or no likelihood of rain fading or interference,

together with the linear slope of the distribution between these two points. This single-rectangle model yields an analytical solution to the integral in equation (29), which can readily be implemented in a spreadsheet.

## 2 Proposed procedure

The application of the following procedure will differ according to the nature of the satellite transponder. For a transparent transponder it is usual to refer the interference limits to the output terminals of the receiving earth-station antenna. In this case, since fading and interference on both uplinks and downlinks will degrade the wanted carrier, and for multi-carrier transponders the reductions in carrier level due to uplink fades will result in equal reductions in the downlink carrier level, strictly speaking it would be necessary to convolve both the uplink and downlink propagation statistics to determine the distribution  $p_x(x)$ . However, if no uplink power control is applied, the uplink fades may tend to dominate the short-term propagation statistics because rain attenuation for a given percentage of time is much greater at the higher uplink frequency than at the lower downlink frequency. Hence, for the present purposes, the simplification of using the predicted rain-attenuation statistics for the uplink frequency may be made. For carriers which are subject to up-path power control, further study is needed, but in the interim, estimates of the right order of magnitude may be obtained in this case by assuming that the net uplink fades are no more severe than the downlink fades, and thus using the predicted rain attenuation for the downlink frequency. There may be cases, however, even with power control, where the uplink fades dominate.

In the case of a re-modulating transponder, since the uplink  $C/N$  and  $C/I$  ratios are decoupled from the downlink  $C/N$  and  $C/I$  ratios, the interference criteria can be derived separately for the two segments of the connection, allocating the full short-term degradation to fading plus interference on each path.

Based on the assumptions and the required input data given above, the following steps define the procedure to determine the interference allowances corresponding to any given desired carrier.

### 2.1 Step 1: characterization of permitted degradation in $C/N$

First, determine the values  $z_j$  of the total degradation  $z$ , as defined in Methodology A, equation (32), which can be exceeded by no more than  $p_j\%$  of the year:

$$z_j = \left( \frac{C}{N} \right)_{cs} - \left( \frac{C}{N} \right)_j \quad j = 1, \dots, J \quad (32)$$

According to *recommends* 3.1, the interference should be responsible for at most 10% of the time allowance associated with the degradation values. Therefore, the degradation due to fading is allowed no more than 90% of the total degradation time. The probability of excessive degradation due to fading can be expressed as:

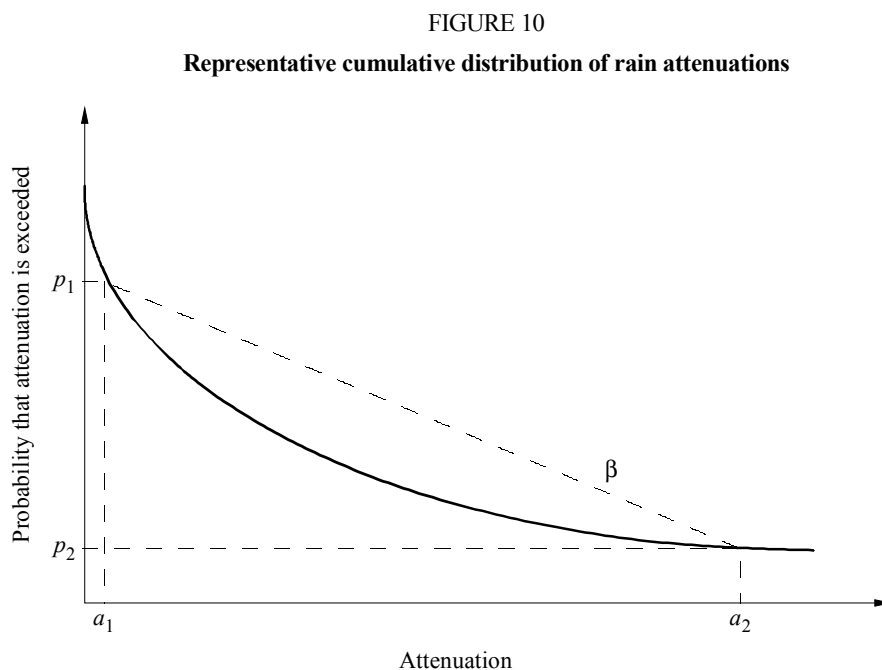
$$P_x(x \geq z_j) = \int_{z_j}^{\infty} p_x(x) dx \leq 0.9 p_j \quad (33)$$

## 2.2 Step 2: characterization of degradation due to fading and other short-term variations in link characteristics

This section currently addresses only degradations due to rain fading. The inclusion of the effects of other time variations in the characteristics of the link requires further study.

For each interfered network, a parametric representation is chosen for the pdf corresponding to the degradation in link performance due to rain fading,  $p_x(x)$ . In order to facilitate the calculations, it is necessary to simplify the form of these pdfs, reducing them to a set of rain attenuations  $A_p$  which can be exceeded for no more than  $p_x(x)$  percentage of time.

The parametric representation chosen for present purposes is to simplify the pdf to a set of gradients and an end-point, with the condition that the integral of this function be unity. The fading due to rain attenuation can be represented by the following cumulative distribution function in Fig. 10.



1323-10

This distribution curve can be divided into a single segment which is characterized by the gradient (or slope) of the line between two points, i.e., the gradient  $\beta = (p_2 - p_1)/(a_2 - a_1)$  while the upper bound can be established by the probability that the attenuation exceeds  $a_1$  dB,  $p = p_1$ . The lower bound is then constrained by the requirement that the total probability, i.e., the integral of the pdf, is unity. The purpose of this parametric representation is to transform the probability distributions of the factors affecting the link performance, i.e., the fading, the interference and the link performance requirements themselves, into pdfs in the form of a single rectangle plus two point probabilities, which can be readily convolved in order to derive the allowable distribution of interference levels.

The cumulative distribution of fading due to rainfall attenuation, can be found from the procedure in Recommendation ITU-R P.618, using the rain attenuation coefficients in Recommendation ITU-R P.838. The cumulative distribution is obtained from the basic input parameter,  $R_{0.01}$ , which is the rainfall rate for the earth station location for 0.01% of an average year. This parameter may be obtained from locally-measured meteorological data, or, in the absence of local data, of Recommendation ITU-R P.837 for the appropriate rain zone. Recommendation ITU-R P.618 then yields the rain attenuations  $A_{0.01}$  which will be exceeded for 0.01% of the year, and the cumulative distribution of rain attenuations between 0.001 and 1% of the year,  $A_p$ , are obtained at different time percentages  $p$  from:

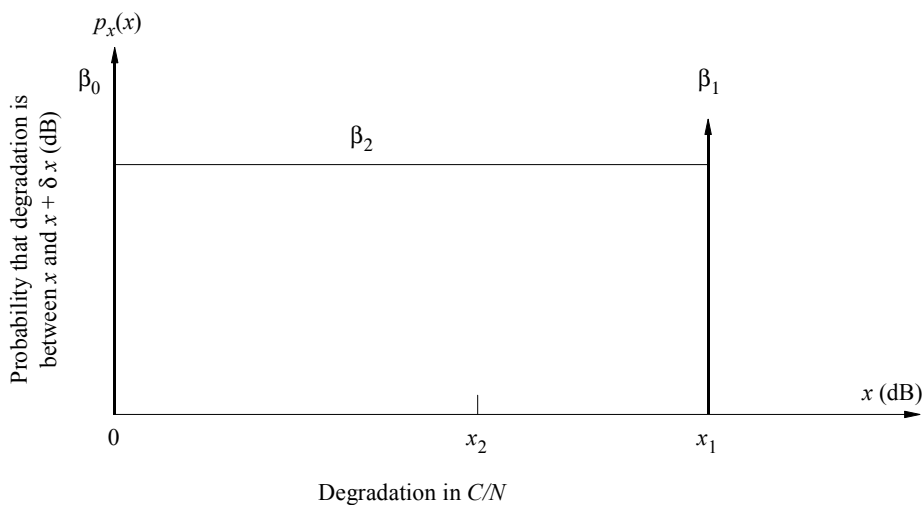
$$\frac{A_p}{A_{0.01}} = 0.12 p^{-(0.546 + 0.043 \log p)} \quad (34)$$

while the percentage of time for which an attenuation will be exceeded can be determined from the inverted form of equation (34).

$$p_A = 10^{11.628 \left( -0.546 + \sqrt{0.298 + 0.172 \log \left( 0.12 \frac{A_{0.01}}{A_p} \right)} \right)} \quad (35)$$

In the single-rectangle model, rain fading is parameterized by two points on the curve, corresponding to the percentage of time when a given level of attenuation is exceeded, the percentage of time beyond which there is little or no likelihood of rain, and the linear slope of the distribution between these two points. The cumulative distribution is thus approximated to a trapezoid, as shown in Fig. 10, and the probability density function to a single rectangle plus two point values, as shown in Fig. 11.

FIGURE 11  
Single-rectangle parameterization of fading pdf



1323-11

### 2.2.1 Determination of $\beta_i$

$\beta_1$  represents the probability that the degradation in  $C/N$  ratio,  $x_1$  dB, will be exceeded. This degradation can be determined from the  $(C/N)_{cs}$  and the  $(C/N)_i$  ratios required to achieve a given level of performance at  $p_i\%$  of time. The permitted degradations are thus given by:

$$x_i = \left( \frac{C}{N} \right)_{cs} - \left( \frac{C}{N} \right)_i \quad x_1 > x_2 \quad (36)$$

For a re-modulating satellite system, the maximum permitted degradation  $x_1$  can be associated with the fade level on the uplink and separately on the downlink, determined by the rain attenuations.

The fraction of time  $\beta_1$  for which the maximum degradation must not be exceeded can then be determined from equation (35), i.e.,  $\beta_1 = p_A$  at a fade of  $A_p \equiv x_1$ . Note that this fraction of time must comply with the requirement in *recommends* 3.1, that the degradations due to fading account for no more than 90% of the time allowances associated with each BER or  $C/N$  objective.

For a transparent satellite transponder, a fraction of the total system noise in the earth station receiver will arise from the uplink, and this noise will be reduced by the downlink fade. To take account of this, additional parameters for the system are required, and a procedure for calculating the resultant fade to use in equation (35) is given in Annex 3.

The parameter  $\beta_2$  represents the slope of the cumulative distribution of attenuation between the point at which  $\beta_1$  is determined (i.e. at a degradation of  $x_1$  dB), and another point on the distribution where the degradation  $x \approx 0$  dB. This point is the lower bound of the distribution and is essentially the fraction of time for which there is no attenuation due to rain. It can be identified with the fraction of time for which rain occurs (i.e. the raining time). Typically, this fraction  $p_0$  will be between 1% and 3% of the time. This number must satisfy certain conditions for use in this methodology (see below). The parameter  $\beta_2$  can then be determined from:

$$\beta_2 = \frac{p_0 - \beta_1}{x_1} \quad (37)$$

From the requirements of *recommends* 3.1, the time percentage for fading larger than  $x_2$  dB should be no more than 90% of  $p_2$ . Therefore:

$$\beta_1 + (x_1 - x_2) \beta_2 \leq 0.9 p_2 \quad (38)$$

From equations (37) and (38),  $p_0$  must satisfy:

$$p_0 \leq \frac{0.9 p_2 x_1 - \beta_1 x_2}{x_1 - x_2} \quad (39)$$

Since the integral of the pdf must equal unity,  $\beta_0$  can readily be determined from:

$$\beta_0 = 1 - x_1 \beta_2 - \beta_1 \quad (40)$$