

RECOMMENDATION ITU-R S.1529

Analytical method for determining the statistics of interference between non-geostationary-satellite orbit fixed-satellite service systems and other non-geostationary-satellite orbit fixed-satellite service systems or geostationary-satellite orbit fixed-satellite service networks

(Question ITU-R 231/4)

(2001)

The ITU Radiocommunication Assembly,

considering

- a) that emissions from the earth stations as well as from the space stations of satellite systems (geostationary-satellite orbit fixed-satellite service (GSO FSS), non-GSO FSS, non-GSO mobile-satellite service (MSS) feeder links) in the FSS may result in interference to another such system when both systems operate in the same frequency bands;
- b) that when non-GSO satellite systems are involved, the statistical behaviour of interference, especially that related to short-term events, constitutes an important factor in interference evaluation studies;
- c) that it is desirable to have reliable and precise tools for determining the statistical behaviour of interference between systems that have co-frequency links when the interference environment involves non-GSO satellite systems;
- d) that computer simulation methods (see Recommendation ITU-R S.1325) may require an excessively long computer time to ensure that all interference events are taken into account and thus statistically significant results are obtained,

recommends

- 1 that the analytical method given in Annex 1 should be considered as a possible method for use in obtaining aggregate interference cumulative probability distributions for assessing the interference between systems that have co-frequency links when the interference environment involves non-GSO satellite systems.

ANNEX 1

An analytical method for assessing interference in interference environments involving non-GSO satellite systems**1 Introduction**

Most of the existing methods to assess interference involving non-GSO satellite systems are based on direct computer simulation. These methods are usually time consuming and require a new lengthy simulation run each time a change is made in some of the system and system parameters. Also, in complex situations, involving a large number of earth stations and non-GSO satellites,

these methods may require a very long computer time to produce statistically significant results. This Annex presents an analytical method, that can be implemented through numerical techniques, intended to perform the evaluation of interference sensitivity to system and system parameters without requiring lengthy computer simulation runs. Also, as opposed to results generated through simulation, the results obtained with the analytical approach correspond to an infinite number of simulated days, and therefore, in this sense, they do not present the need for long running times, as may be required in computer simulation methods to assure statistically significant results.

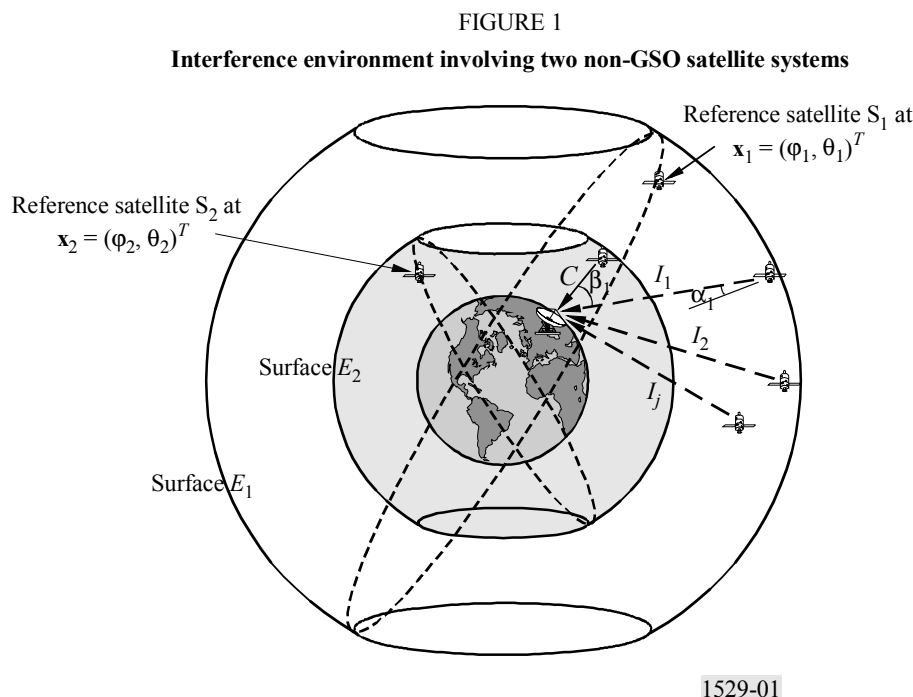
The method is based on the knowledge of the probability density function (pdf) of the position of a single satellite placed in an orbit with an arbitrary inclination. To illustrate the applicability of the proposed method to complex interference environments, results for some specific situations are presented. Comparisons of the results obtained using the proposed method and those generated by a widely used commercially available simulation software have indicated that the proposed method can generate reliable and precise results with less required computer time.

2 Methodology

Let us consider an interference environment involving several, say n , non-GSO systems. The approach being considered in this Recommendation to assess interference in such an environment takes into account the fact that, once the position of one particular satellite (here referred to as reference satellite) in each constellation is known, the aggregate interference levels affecting the receivers of any interfered-with system in the environment (considering that all systems parameters are given) can be uniquely determined. It further assumes that the positions of these reference satellites are characterized by statistically independent random vectors. Based on these assumptions, desired and interfering signal power levels can be seen as random variables that are deterministic functions of the positions of the reference satellites, and therefore their pdf's can be determined once the pdf's modelling the positions of each of the n reference satellites are known.

As an example, consider the situation illustrated in Fig. 1. This Figure shows two non-GSO satellite systems, both having circular orbits and arbitrary satellite constellations. Satellites of system 1 move on surface E_1 and satellites of system 2 move on surface E_2 . Reference satellites for both systems are also indicated. In this example the downlink aggregate interference from system 1 satellites into a given earth station in system 2 is considered. Given that reference satellite S_i of system i ($i = 1, 2$) is located at longitude φ_i and latitude θ_i the positions of all other satellites in both constellations can be uniquely determined as a function of the two location vectors $\mathbf{x}_1 = (\varphi_1, \theta_1)^T$ and $\mathbf{x}_2 = (\varphi_2, \theta_2)^T$. Therefore, considering for instance that the earth station antenna always points to the nearest satellite in the constellation of its system, and that all other systems parameters, such as satellite and earth station antenna radiation patterns, e.i.r.p., etc. are known, then both the downlink aggregate interference I and the desired signal level C at the considered earth station can be computed for each pair of values of the vectors $\mathbf{x}_i = (\varphi_i, \theta_i)^T$, $-\pi < \varphi \leq \pi$, $-\delta_i \leq \theta_i \leq \delta_i$, where $i = 1, 2$ (δ_i is the angle between the equatorial plane and the plane containing the orbit of S_i). If these two vectors are modelled as statistically independent random vectors with known pdf's

$p_{\mathbf{x}_i}(\Phi, \Theta), i=1, 2$ then the desired signal level $C(\mathbf{x}_1, \mathbf{x}_2)$ and the aggregate interference $I(\mathbf{x}_1, \mathbf{x}_2)$ correspond to random variables whose statistical characterization, e.g. pdf or cumulative distribution function (CDF), can be obtained from $p_{\mathbf{x}_i}(\Phi, \Theta), i=1, 2$ through analytical and/or numerical methods.



The pdf $p_{\mathbf{x}}(\Phi, \Theta)$ of the position (longitude and latitude) of a satellite placed in elliptical orbit around the Earth can be shown to be (see Appendix 1 to this Annex):

$$p_{\mathbf{x}}(\Phi, \Theta) = p_{\mathbf{x}|AM}(\Phi, \Theta)P(AM) + p_{\mathbf{x}|DM}(\Phi, \Theta)P(DM) \quad (1)$$

where $p_{\mathbf{x}|AM}(\Phi, \Theta)$ and $p_{\mathbf{x}|DM}(\Phi, \Theta)$ are the pdf's of the satellite position (longitude and latitude) given that the satellite is in ascending and descending mode, respectively. These conditional pdf's are given by:

$$p_{\mathbf{x}|AM}(\Phi, \Theta) = \begin{cases} \frac{1}{P(AM)} \frac{k(1+e)}{4\pi^2} \frac{\cos \Theta}{\sqrt{\sin^2 \delta - \sin^2 \Theta}} \left[\frac{2 \sin \delta}{(1+k^2) \sin \delta - (1-k^2)g(\Theta)} \right]^2 & \text{for } -\delta < \Theta \leq \delta \\ & -\pi < \Phi \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

and

$$p_{\mathbf{x}|DM}(\Phi, \Theta) = \begin{cases} \frac{1}{P(DM)} \frac{k(1+e)}{4\pi^2} \frac{\cos \Theta}{\sqrt{\sin^2 \delta - \sin^2 \Theta}} \left[\frac{2 \sin \delta}{(1+k^2) \sin \delta + (1-k^2)g(-\Theta)} \right]^2 & \text{for } -\delta < \Theta \leq \delta \\ & -\pi < \Phi \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

with $P(AM)$ and $P(DM)$ given by equations (60) and (61).

Note that equation (1) can be also written as:

$$p_x(\Phi, \Theta) = \begin{cases} \frac{k(1+e)}{4\pi^2} \frac{\cos \Theta}{\sqrt{\sin^2 \delta - \sin^2 \Theta}} \left[\left(\frac{2 \sin \delta}{(1+k) \sin \delta - (1-k^2)g(\Theta)} \right)^2 + \left(\frac{2 \sin \delta}{(1+k) \sin \delta + (1-k^2)g(-\Theta)} \right)^2 \right] & \text{for } -\delta < \Theta \leq \delta \\ & -\pi < \Phi \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

In equations (2), (3) and (4), δ is the angle between the orbital plane and the equatorial plane,

$$k = \sqrt{\frac{1+e}{1-e}} \quad (5)$$

with e denoting the orbit eccentricity, and

$$g(\Theta) = \cos \omega \sqrt{\sin^2 \delta - \sin^2 \Theta} + \sin \omega \sin \Theta \quad (6)$$

where ω is the argument of the perigee.

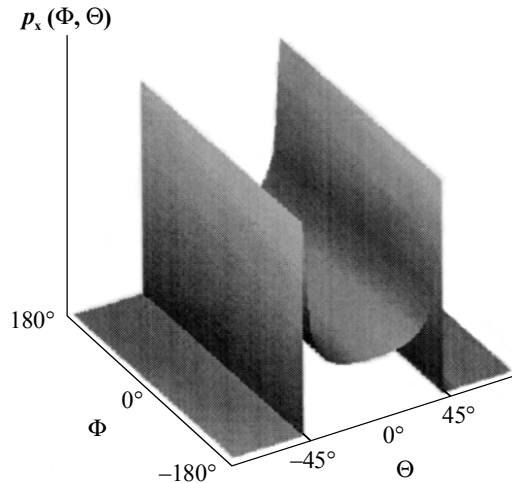
In the particular case of circular orbits ($e = 0 \rightarrow k = 1$), equation (1) is reduced to:

$$p_x(\Phi, \Theta) = \begin{cases} \frac{1}{2\pi^2} \frac{\cos \Theta}{\sqrt{\sin^2 \delta - \sin^2 \Theta}} & \text{for } -\delta < \Theta < \delta \\ & -\pi < \Phi \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

This pdf is illustrated in Fig. 2 for a satellite in a circular orbit on a plane inclined 45° with respect to the equatorial plane.

FIGURE 2

pdf $p_x(\Phi, \Theta)$ of the position of a satellite placed in a circular orbit on a plane inclined 45° with respect to the equatorial plane



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3 Procedure to obtain the interference CDF

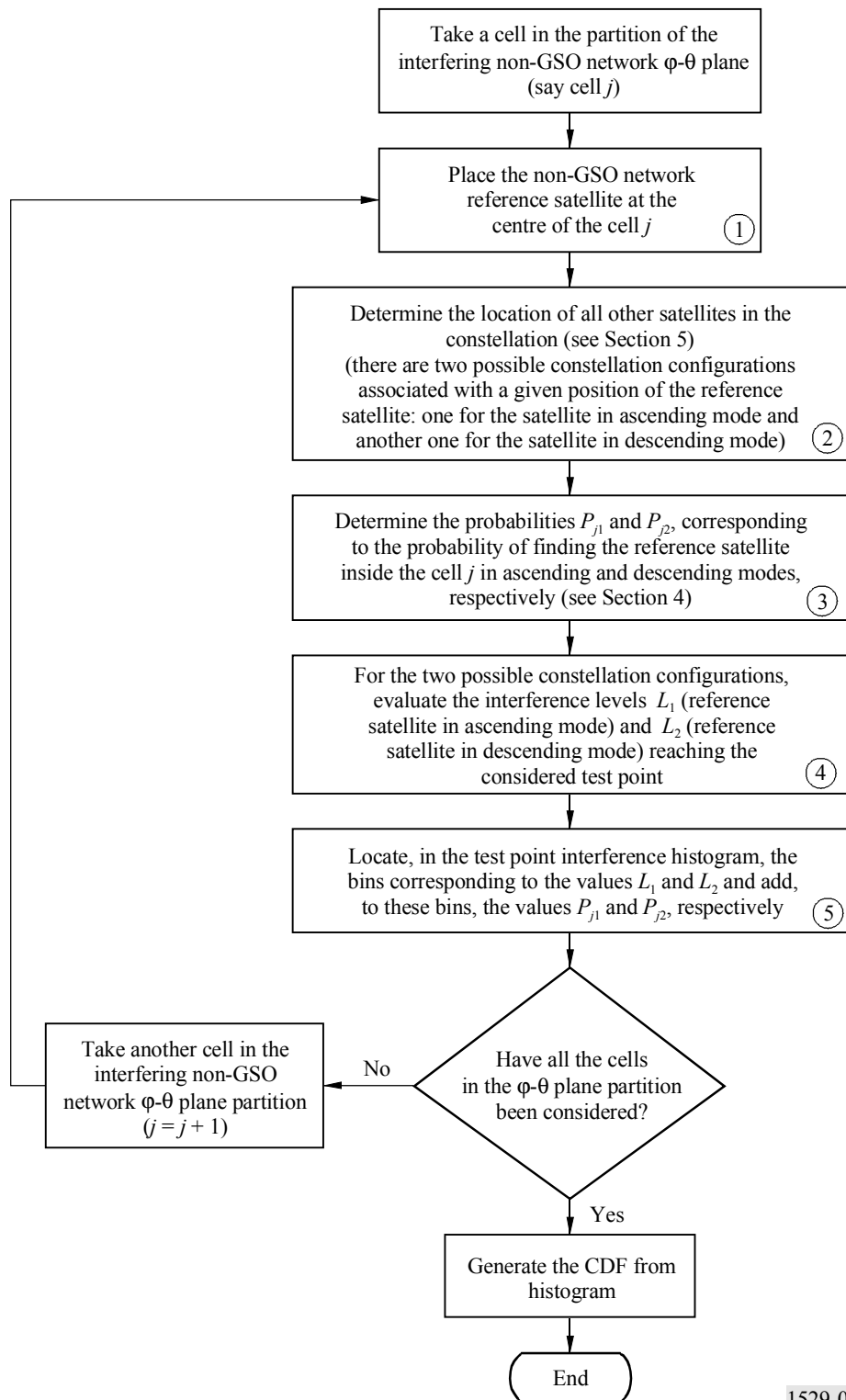
For simplicity, let us assume that only a single non-GSO system is involved in the interference environment. The longitude and latitude of the reference satellite of this non-GSO satellite system takes values on a φ - θ plane ($-\pi < \varphi \leq \pi$, $-\delta \leq \theta \leq \delta$). In a first step, this plane is finely partitioned into small rectangular cells. For each of these partition cells, it is assumed that the reference satellite is located at its centre and, for this condition, the position of all other satellites in the constellation are determined. We note that there are two possible constellation configurations associated with a given position of the reference satellite: one corresponding to the satellite in ascending mode and

another for the satellite in descending mode – both situations have to be taken into account. For each of the two constellation configurations, once the position of all satellites are known, the aggregate interference level into the desired test point is evaluated. To generate the probability distribution of a quantized version of the aggregate interference, the two obtained values (ascending mode and descending mode) are quantized to the nearest quantization levels and the corresponding probabilities of finding the reference satellite inside the considered cell, obtained using equations (2) and (3), are added to the current values of probability associated to the two corresponding quantization levels. This procedure is then repeated for all partition cells and the so obtained histogram is integrated to generate the desired CDF. The flowchart in Fig. 3 illustrates the procedure described above.

Concerning this procedure, the following additional comments are pertinent:

- Although the partition of the φ - θ plane into rectangular cells need not be a grid-type of partition, the use of grid-type partitions is convenient for implementation purposes. However, to avoid a prohibitive amount of required computer time when applying the proposed analytical method to complex situations involving a large number of earth stations and satellites, the following points should be taken into account:
 - The φ - θ plane quantization grid should be sufficiently fine to detect fast variations of the interference levels that occur near to “in-line” interference situations. However, a fine quantization of the whole φ - θ plane could result in an excessively large computer time. So, the numerical implementation of the analytical method could, as an option, be split in two parts. The first part performs the calculations in the regions of the φ - θ plane for which the interference level may have large variations (close to in-line interference), and where a fine quantization of the region is required. The second part of the numerical procedure performs the calculations in the regions of the φ - θ plane for which the interference level has smooth variations, allowing for a coarser quantization. Finding the φ - θ plane regions associated with potential quasi “in-line” interference corresponds to defining regions such that when the reference satellite is inside one of these regions, in-line interference events involving one or more of the satellites in the constellation may occur. The important point here is to guarantee that when the reference satellite is not inside any of these regions, “in-line” interference does not occur and a coarser quantization grid can be used. The regions of potential in-line interference (**RPII**) are usually defined as rectangular regions around points of potential in-line interference (**PPII**). These **PPII** can be found using the methodology described in Section 6.
 - Once the potential occurrence of quasi in-line interference is detected (the reference satellite is inside one of the potential in-line interference regions), it is important to identify which satellites and earth stations are involved in it. This way interference computations could be made considering that only a few interference entries (those associated with the in-line interference event) have to be re-computed when the reference satellite changes its location inside the considered potential in-line interference region. This measure could save a substantial amount of computer time when a large number of interference entries are present.

FIGURE 3
Flow chart of the analytical method



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- Given the position of the reference satellite, finding the position of all other satellites in the constellation (block ③ in the diagram in Fig. 3) is a problem that has two possible solutions. This is due to the fact that two different orbital planes, having the same inclination, can contain the reference satellite. In fact, one of the solutions assumes that the reference satellite is in ascending mode and the other assumes that the reference satellite is in descending mode. Both solutions have to be taken into account in the

proposed procedure. In the case of circular orbits, finding these solutions does not constitute a complex task, since the altitude of the satellites is previously known. For satellites in elliptical orbits, this is a more complex procedure, since the satellite altitudes change with time (see Section 5).

4 Probabilities of having the reference satellite inside a rectangular cell and in ascending and descending modes

Let a rectangular cell in the φ - θ plane, say cell j , be defined by $\varphi \in (\varphi_m, \varphi_M)$, $\theta \in (\theta_m, \theta_M)$. Also, let P_{j1} and P_{j2} represent respectively the probability of finding the reference satellite inside the rectangular cell j and in ascending mode, and the probability of finding the reference satellite inside the rectangular cell j and in descending mode. These two probabilities can be obtained by respectively integrating the two terms in the right hand side of equation (1), considering the values of $P(AM)$ and $P(DM)$ calculated through equations (60) and (61). It then results:

$$P_{jn} = \frac{\varphi_M - \varphi_m}{2\pi} \left[f(c_M, (n-1)\pi - (-1)^n \omega) - f(c_m, (n-1)\pi - (-1)^n \omega) \right] \quad \text{for } n = 1, 2 \quad (8)$$

with:

$$f(x, \omega) = \begin{cases} 0 & \text{for } -\infty < x \leq -\pi \\ Q_{ek}\left(\frac{x - \omega}{2}\right) - Q_{ek}\left(\frac{-\pi - \omega}{2}\right) & \text{for } -\pi < x \leq \pi \\ 1 & \text{for } \pi < x < \infty \end{cases} \quad (9)$$

where:

$$Q_{ek}(x) = \frac{1}{\pi} \arctan\left(\frac{\tan(x)}{k}\right) + \frac{1}{2} \left[\operatorname{sgn}\left(x + \frac{\pi}{2}\right) + \operatorname{sgn}\left(x - \frac{\pi}{2}\right) \right] - \frac{k e}{\pi} \frac{\tan(x)}{\tan^2(x) + k^2} \quad (10)$$

with $\operatorname{sgn}()$ denoting the signum function. In equation (8),

$$c_m = \arcsin\left(\frac{\sin \theta_m}{\sin \delta}\right) \quad (11)$$

and

$$c_M = \arcsin\left(\frac{\sin \theta_M}{\sin \delta}\right) \quad (12)$$

In the case of circular orbits, since $\omega = 0$ and $e = 0 \rightarrow k = 1$, equation (9) is reduced to:

$$f(x, 0) = \begin{cases} 0 & \text{for } -\infty < x \leq -\pi \\ \frac{1}{2} \left(1 + \frac{x}{\pi} \right) & \text{for } -\pi < x \leq \pi \\ 1 & \text{for } \pi < x < \infty \end{cases} \quad (13)$$

5 Finding the position of all satellites in the constellation

In this section, the following notations and definitions are used.

$\mathbf{u}$	Unitary vector in the direction of the reference satellite
δ	Orbit inclination angle (rad)
β	Satellite separation angle within the orbital plane (in terms of mean anomaly) (rad)
ψ	Angle between the intersections of adjacent orbit planes and the equatorial plane (rad)
λ	Satellite phasing between planes (rad)
r	Orbit radius (circular orbit) (km)
a	Major semi-axis of the elliptical orbit (km)
e	Orbit eccentricity
ω	Argument of the perigee (rad)
$\mathbf{k}_z$	Unitary vector in the z-axis direction
γ_ℓ	True anomaly of the reference satellite in constellation configuration ℓ measured from the line of nodes (rad)
v_ℓ	True anomaly of the reference satellite in constellation configuration ℓ (rad)
E_ℓ	Eccentric anomaly of the reference satellite in constellation configuration ℓ (rad)
M_ℓ	Mean anomaly of the reference satellite in constellation configuration ℓ (rad)
$(M_i^j)_\ell$	Mean anomaly of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ (rad)
$(E_i^j)_\ell$	Eccentric anomaly of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ (rad)
$J_n(\cdot)$	Bessel Function of first class and order n
$(v_i^j)_\ell$	True anomaly of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ (rad)
$(\mathbf{u}_i^j)_\ell$	Unitary vector in the direction of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ
$(\mathbf{p}_i^j)_\ell$	Vector of the position of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ
$(r_i^j)_\ell$	Distance, from the centre of the Earth, of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ (km)
$N_{Satperplane}$	Number of satellites per orbital plane
N_{Planes}	Number of orbital planes in the constellation

Consider a geocentric, GSO system of rectangular coordinates in which the x and y axis belong to the equatorial plane and the z-axis points to the north. Let $\mathbf{u}$ denote the unitary vector pointing to the reference satellite location and $(\mathbf{p}_i^j)_\ell$ be the vector characterizing the position of the i -th satellite in the j -th orbital plane, corresponding to constellation configuration ℓ , $\ell = 1, 2$. The

following procedure can be used to determine the locations $(\mathbf{p}_i^j)_\ell$ ($i=0, \dots, N_{Satplane}-1$, $j=0, \dots, N_{Planes}-1$, $\ell=1, 2$) of all satellites in the two constellation configurations. The described procedure includes a test to identify which of the two configurations corresponds to the reference satellite being in ascending mode and which corresponds to the reference satellite being in descending mode, as required by the proposed analytical method.

Step 1: Let $\mathbf{u} = (u_x, u_y, u_z)^T$ and calculate, for $\ell = 1, 2$, the unitary vectors $\mathbf{n}_\ell$ defined by:

$$\mathbf{n}_\ell = \begin{pmatrix} (-u_z \cos \delta - a_\ell u_y) / u_x \\ a_\ell \\ \cos \delta \end{pmatrix} \quad (14)$$

with

$$a_\ell = \frac{-u_y u_z \cos \delta + (-1)^\ell u_x \sqrt{(u_x^2 + u_y^2) \sin^2 \delta - u_z^2 \cos^2 \delta}}{u_x^2 + u_y^2}$$

Step 2: Let $\mathbf{k}_z$ be the unitary vector in the z-axis direction and calculate the following quantities for $\ell = 1, 2$:

$$\mathbf{w}_\ell = \mathbf{k}_z \times \mathbf{n}_\ell \quad (\times \text{ denotes the cross product})$$

$$\gamma_\ell = \arccos \left(\frac{\mathbf{u}^T \mathbf{w}_\ell}{|\mathbf{w}_\ell|} \right) \text{sgn}(u_z) \quad (15)$$

(T denotes transpose and $\text{sgn}(\cdot)$ denotes the signum function)

$$v_\ell = (\gamma_\ell - \omega)_{MOD \ 2\pi}$$

(throughout this Annex, $(x)_{MOD \ 2\pi}$ is defined in the interval $(-\pi, \pi)$)

$$E_\ell = 2 \arctan \left(\frac{\tan(v_\ell / 2)}{k} \right)$$

$$\text{where } k = \sqrt{\frac{1+e}{1-e}}$$

$$M_\ell = E_\ell - e \sin E_\ell$$

$$(M_i^j)_\ell = M_\ell + i\beta + j\lambda \quad (16)$$

$$(E_i^j)_\ell = (M_i^j)_\ell + 2 \sum_{n=1}^{\infty} \frac{1}{n} J_n(ne) \sin(n(M_i^j)_\ell)$$

$$(v_i^j)_\ell = 2 \arctan \left(k \tan \frac{(E_i^j)_\ell}{2} \right)$$

$$(\alpha_i^j)_\ell = (v_i^j)_\ell - v_\ell$$

Note that the quantity β in equation (16) is usually given by:

$$\beta = \frac{2\pi}{N_{Satperplane}}$$

Important Note: The values of γ_ℓ determined through equation (15) are used to identify which of the two configurations corresponds to the reference satellite being in ascending mode and which corresponds to the reference satellite being in descending mode. This is accomplished by the following test:

$$\begin{cases} \text{if } 0 \leq |\gamma_\ell| < \pi/2 \rightarrow \text{configuration } \ell \text{ corresponds to reference satellite in ascending mode} \\ \text{if } \pi/2 \leq |\gamma_\ell| \leq \pi \rightarrow \text{configuration } \ell \text{ corresponds to reference satellite in descending mode} \end{cases} \quad (17)$$

Step 3: Determine the location vectors of the satellites in the two constellation configurations by:

$$(\mathbf{p}_i^j)_\ell = (r_i^j)_\ell (\mathbf{u}_i^j)_\ell$$

where:

$$(r_i^j)_\ell = \frac{a(1-e^2)}{1-e\cos((v_i^j)_\ell)}$$

and

$$(\mathbf{u}_i^j)_\ell = \cos((\alpha_i^j)_\ell) \mathbf{M}_j \mathbf{u} - \sin((\alpha_i^j)_\ell) \mathbf{M}_j \mathbf{b}_\ell$$

with:

$$\mathbf{b}_\ell = \mathbf{u} \times \mathbf{n}_\ell$$

and

$$\mathbf{M}_j = \begin{pmatrix} \cos j\psi & -\sin j\psi & 0 \\ \sin j\psi & \cos j\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The angle ψ is usually given by:

$$\psi = \frac{2\pi}{N_{Planes}}$$

In the particular case of circular orbits, since the true anomaly, the eccentric anomaly and the mean anomaly are all the same, and since $a = r$ and $e = 0 \rightarrow k = 1$, Steps 2 and 3 simplify to:

Step 2:

$$(\alpha_i^j)_\ell = i\beta + j\lambda$$

Step 3:

$$(\mathbf{p}_i^j)_\ell = r (\mathbf{u}_i^j)_\ell$$

where:

$$(\mathbf{u}_i^j)_\ell = \cos((\alpha_i^j)_\ell) \mathbf{M}_j \mathbf{u} - \sin((\alpha_i^j)_\ell) \mathbf{M}_j \mathbf{b}_\ell$$