

RECOMMENDATION ITU-R TF.1010-1

RELATIVISTIC EFFECTS IN A COORDINATE TIME SYSTEM
IN THE VICINITY OF THE EARTH

(Question ITU-R 152/7)

(1994-1997)

The ITU Radiocommunication Assembly,

considering

- a) that it is desirable to maintain coordination of standard frequency and time-signal emissions in the vicinity of the Earth;
- b) that universal coordinated time (UTC) is the official coordinate time scale for the Earth defined on the rotating geoid;
- c) that atomic clocks are subject to path-dependent second-order motional frequency shifts and position-dependent gravitational frequency shifts;
- d) that the Consultative Committee for the Definition of the Second (CCDS) has recognized the need for well-defined procedures to account for relativistic effects in timing systems and time comparisons;
- e) that, since time comparisons in non-inertial frames require special consideration and the CCDS has recommended an appropriate set of equations which provide a consistent set of measurements of UTC in the vicinity of the Earth;
- f) that there is a growing trend to place accurate, stable clocks in Earth-bound orbits for time-keeping purposes;
- g) that there is a need for comparing frequency standards in the vicinity of the Earth with an accuracy of 10^{-14} ,

recommends

- 1 that for calculating coordinate time intervals in the vicinity of the Earth (out to at least geosynchronous radius) to an accuracy of 1 ns (or 10^{-14} of the integration time), the following procedures, based on the first order terms in the full general relativistic expressions, should be followed (some practical examples are given in Annex 1):

1.1 Clock transport in a rotating reference frame

When transferring time from point P to point Q by means of a portable clock, the coordinate time accumulated during transport is:

$$\Delta t = \int_P^Q ds \left[1 + \frac{\Delta U(\vec{r})}{c^2} + \frac{V^2}{2c^2} \right] + \frac{2\omega}{c^2} A_E \quad (1)$$

where:

- c : speed of light
- ω : angular velocity of rotation of the Earth
- V : velocity of the clock with respect to the ground
- $\vec{r}$: vector whose origin is at the centre of the Earth and whose terminus moves with the clock from P to Q
- A_E : equatorial projection of the area swept out during the time transfer by the vector $\vec{r}$ as its terminus moves from P to Q
- $\Delta U(\vec{r})$: gravitational potential difference (including the centrifugal potential) between the location of the clock at $\vec{r}$ and the geoid as viewed from an earth-fixed coordinate system, in agreement with the convention (Resolution A4, IAU, 1992) that $\Delta U(\vec{r})$ is negative when the clock is above the geoid

ds : increment of proper time accumulated on the portable clock. The increment of proper time is the time accumulated on the portable standard clock as measured in the "rest frame" of the clock; that is, in the reference frame travelling with the clock.

A_E is measured in an earth-fixed coordinate system. As the area A_E is swept, it is taken as positive when the projection of the path of the clock on the equatorial plane moves eastward. When the height h of the clock is less than 24 km above the geoid, $\Delta U(\vec{r})$ may be approximated by gh , where g is the total acceleration due to gravity (including the rotational acceleration of the Earth) evaluated at the geoid. This approximation applies to all aerodynamic and earthbound transfers.

When h is greater than 24 km, the potential difference $\Delta U(\vec{r})$ must be calculated to greater accuracy as follows:

$$\Delta U(\vec{r}) = GM_e/r + J_2 GM_e a_1^2 (1 - 3 \cos^2 \theta)/2r^3 + \omega^2 r^2 \sin^2 \theta/2 - U_g \quad (2)$$

where:

a_1 : equatorial radius of the Earth

$$a_1 = 6\,378.136 \text{ km}$$

r : magnitude of the vector $\vec{r}$

θ : colatitude

GM_e : product of the Earth's mass and the gravitational constant

$$GM_e = 398\,600 \text{ km}^3/\text{s}^2$$

J_2 : quadrupole moment coefficient of the Earth

$$J_2 = +1.083 \times 10^{-3}$$

ω : angular velocity of the Earth

$$\omega = 7.292115 \times 10^{-5} \text{ rad/s}$$

U_g : potential (gravitational and centrifugal) at the geoid

$$U_g = 62.63686 \text{ km}^2/\text{s}^2.$$

For time transfer at the 1 ns level of accuracy, this expression should not be used beyond a distance of about 50 000 km from the centre of the Earth.

1.2 Clock transport in a non-rotating reference frame

When transferring time from point P to point Q by means of a clock the coordinate time elapsed during the motion of the clock is:

$$\Delta t = \int_P^Q ds \left[1 + \frac{U(\vec{r}) - U_g}{c^2} + \frac{v^2}{2c^2} \right] \quad (3)$$

where:

$U(\vec{r})$: gravitational potential at the location of the clock excluding the centrifugal potential

v : velocity of the clock, both as viewed (in contrast to equation (1)) from the geocentric non-rotating reference frame

U_g : potential at the geoid

$$(U_g/c^2 = -6.9694 \times 10^{-10}), \text{ including the effect on the potential of the Earth's rotational motion.}$$

Note that $\Delta U(\vec{r}) \neq U(\vec{r}) - U_g$, since $U(\vec{r})$ does not include the effect of the Earth's rotation. This equation also applies to clocks in geostationary orbits but should not be used beyond a distance of about 50 000 km from the centre of the Earth.