



Recommendation ITU-R TF.2018
(08/2012)

**Relativistic time transfer in the vicinity of
the Earth and in the solar system**

TF Series
Time signals and frequency standards emissions



International
Telecommunication
Union
Documents
Document Sharing Hub

Foreword

The role of the Radiocommunication Sector is to ensure the rational, equitable, efficient and economical use of the radio-frequency spectrum by all radiocommunication services, including satellite services, and carry out studies without limit of frequency range on the basis of which Recommendations are adopted.

The regulatory and policy functions of the Radiocommunication Sector are performed by World and Regional Radiocommunication Conferences and Radiocommunication Assemblies supported by Study Groups.

Policy on Intellectual Property Right (IPR)

ITU-R policy on IPR is described in the Common Patent Policy for ITU-T/ITU-R/ISO/IEC referenced in Annex 1 of Resolution ITU-R 1. Forms to be used for the submission of patent statements and licensing declarations by patent holders are available from <http://www.itu.int/ITU-R/go/patents/en> where the Guidelines for Implementation of the Common Patent Policy for ITU-T/ITU-R/ISO/IEC and the ITU-R patent information database can also be found.

Series of ITU-R Recommendations

(Also available online at <http://www.itu.int/publ/R-REC/en>)

Series	Title
BO	Satellite delivery
BR	Recording for production, archival and play-out; film for television
BS	Broadcasting service (sound)
BT	Broadcasting service (television)
F	Fixed service
M	Mobile, radiodetermination, amateur and related satellite services
P	Radiowave propagation
RA	Radio astronomy
RS	Remote sensing systems
S	Fixed-satellite service
SA	Space applications and meteorology
SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
TF	Time signals and frequency standards emissions
V	Vocabulary and related subjects

Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

Electronic Publication
Geneva, 2012

© ITU 2012

All rights reserved. No part of this publication may be reproduced, by any means whatsoever, without written permission of ITU.

RECOMMENDATION ITU-R TF.2018

Relativistic time transfer in the vicinity of the Earth and in the solar system

(2012)

Scope

The purpose of this Recommendation is to establish common conventional algorithms and procedures to be used in comparing clocks on the surface of the Earth and on platforms far from the Earth but within the solar system. These expressions are explicitly determined in the general relativity theory that is presently accepted to form the basis of space-time reference systems. It is envisioned that these algorithms and procedures would be used for comparisons of clocks on Earth satellites, interplanetary spacecraft, and on the surfaces of solar system bodies.

The ITU Radiocommunication Assembly,

considering

- a) that it is desirable to maintain coordination of standard time and frequency on platforms operating in the vicinity of the Earth and in the solar system;
- b) that accurate means of transferring time and frequency are required to meet the future needs of timekeeping, navigation, science, and communication systems in the vicinity of the Earth and in the solar system;
- c) that clocks are subject to path-dependent time and frequency variations due to their motion and to the gravitational potential in which they operate;
- d) that the conceptual foundation for the transfer of time and frequency should be clearly outlined;
- e) that procedures for the transfer of time and frequency in the vicinity of the Earth and across celestial bodies and spacecraft in the solar system require the use of mathematical algorithms that account for relativistic effects;
- f) that requirements for precision and accuracy for the transfer of time and frequency in the vicinity of the Earth and in the solar system depend on the specific application,

recommends

that the mathematical algorithms that account for relativistic effects in the transfer of time and frequency as provided in Annex 1 should be used, as appropriate.

Annex 1

Objective

The purpose of this Recommendation is to raise the level of awareness for the need to address the effects of relativity in timekeeping, navigation, science, and communication systems. The Recommendation recalls the basic concepts and procedures to be applied in the analysis of such systems. No attempt is made to describe the details of any particular system. Rather, it is intended that the information presented here may serve as a convenient reference and a point of departure for specific applications.

One important application of this Recommendation is the comparison of the times registered by clocks on spacecraft in orbit around the Earth, in interplanetary space, and on planetary surfaces with the times recorded by clocks on Earth. An appropriate timescale for terrestrial measurements is Coordinated Universal Time (UTC). Therefore, the objective might be to relate the times registered by clocks, wherever they may be in the vicinity of the Earth and in the solar system, with the times of clocks on Earth that measure UTC.

The following discussion is based on the *IERS Conventions (2010)*, the ITU-R Handbook on *Satellite Time and Frequency Transfer and Dissemination (2010)*, Nelson, *Metrologia* (2011), and Petit and Wolf, *Metrologia* (2005). Users may consult those publications and references cited therein for further details.

Relativistic framework

The relativistic framework for space-time reference systems has been defined by Resolutions of international scientific organizations. The most important are:

- 1) IAU Resolution A4 (1991) defines the Geocentric Celestial Reference System (GCRS) and the Barycentric Celestial Reference System (BCRS) and their time coordinates. IAU Resolution B1 (2000) further refines the BCRS definition.
- 2) IUGG Resolution 2 (2007) defines the Geocentric Terrestrial Reference System (GTRS), along with the International Terrestrial Reference System (ITRS).

The nomenclature used in this document follows the one used in the past ITU-R Recommendations and may be related to the IAU/IUGG framework as follows: in this Recommendation the GCRS is termed the Earth-Centered Inertial (ECI) coordinate system, the GTRS (in practice, the ITRS) is termed the Earth-Centered Earth-Fixed (ECEF) coordinate system, and the BCRS is termed the barycentric coordinate system.

Definitions

Proper time

Proper time τ is the actual reading of a clock or the local time in the clock's own frame of reference.

Coordinate time

Coordinate time t is the independent variable in the equations of motion of material bodies and in the equations of propagation of electromagnetic waves. It is a mathematical coordinate in the four-dimensional space-time of the coordinate system. For a given event, the coordinate time has the same value everywhere. Coordinate times are not measured; rather, they are computed from the proper times of clocks.

Space-time interval

The relation between coordinate time and proper time depends on the clock's position and state of motion in its gravitational environment and is derived by integration of the space-time interval. In the comparison of the proper times of two clocks, the coordinate time is ultimately eliminated. Thus the relativistic transfer of time between clocks is independent of the coordinate system. The coordinate system may be chosen arbitrarily on the basis of convenience.

In general, the space-time interval is described by:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{00} c^2 dt^2 + 2 g_{0j} c dt dx^j + g_{ij} dx^i dx^j \quad (1)$$

where:

$g_{\mu\nu}$: components of the metric.

A Greek index assumes the range 0, 1, 2, 3 and a Latin index assumes the range 1, 2, 3. A repeated index implies summation on that index. The metric depends upon the gravitational potentials and upon the angular velocity and linear acceleration of the reference frame. Upon a transformation of the coordinates, the space-time interval remains invariant. Thus the metric $g_{\mu\nu}$ transforms as a second order covariant tensor.

The general expression for the relationship between proper time τ and the coordinates of the chosen coordinate system, comprising the coordinate time $x^0 \equiv ct$ and the spatial coordinates x^i , is given by:

$$ds^2 = g_{00} c^2 dt^2 + 2 g_{0j} c dt dx^j + g_{ij} dx^i dx^j = -c^2 d\tau^2 \quad (2)$$

where:

τ : the proper time.

Thus $dt = d\tau$ for a clock at rest in an inertial frame of reference, for which $dx^i = 0$ and $-g_{00} = 1$, $g_{0j} = 0$, and $g_{ij} = \delta_{ij}$. The elapsed coordinate time corresponding to the measured proper time as registered by a clock along a path between points A and B is:

$$\Delta t = \pm \int_A^B \frac{1}{\sqrt{-g_{00}}} \sqrt{1 + \frac{1}{c^2} \left(g_{ij} + \frac{g_{0i} g_{0j}}{-g_{00}} \right) \frac{dx^i}{d\tau} \frac{dx^j}{d\tau}} d\tau + \frac{1}{c} \int_A^B \frac{g_{0j}}{-g_{00}} \frac{dx^j}{d\tau} d\tau \quad (3)$$

For an electromagnetic signal, the space-time interval is:

$$ds^2 = g_{00} c^2 dt^2 + 2 g_{0j} c dt dx^j + g_{ij} dx^i dx^j = 0 \quad (4)$$

The speed of light is c in every inertial frame of reference. The elapsed coordinate time of propagation along a path between points A and B is:

$$\Delta t = \pm \frac{1}{c} \int_A^B \frac{1}{\sqrt{-g_{00}}} \sqrt{\left(g_{ij} + \frac{g_{0i} g_{0j}}{-g_{00}} \right) dx^i dx^j} + \frac{1}{c} \int_A^B \frac{g_{0j}}{-g_{00}} dx^j \quad (5)$$

The expression $\gamma_{ij} \equiv g_{ij} + g_{0i} g_{0j} / (-g_{00})$ represents the metric of three-dimensional space and $d\rho = \sqrt{\gamma_{ij} dx^i dx^j}$ represents the increment of three-dimensional distance.