



Recommendation ITU-R TF.538-4
(07/2017)

**Measures for random instabilities in
frequency and time (phase)**

TF Series
Time signals and frequency standards emissions



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SF	Frequency sharing and coordination between fixed-satellite and fixed service systems
SM	Spectrum management
SNG	Satellite news gathering
TF	Time signals and frequency standards emissions
V	Vocabulary and related subjects

Note: This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.

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RECOMMENDATION ITU-R TF.538-4

Measures for random instabilities in frequency and time (phase)

(1978-1990-1992-1994-2017)

Scope

Frequency and phase instabilities may be characterized by random processes that can be represented statistically in either the Fourier frequency domain or in the time domain. This Recommendation presents various methods and techniques for characterization of these frequency and phase instabilities.

Keywords

Random instabilities, Allan variance, time metrology, statistical measures, phase, frequency

The ITU Radiocommunication Assembly,

considering

- a) that there is a need for an adequate language with which to communicate the instability characteristics of standard frequency and time sources and measurement systems;
- b) that the classical variance does not converge for some of the kinds of random time and frequency instabilities;
- c) that major laboratories, observatories, industries and general users have already adopted some of the Recommendations of the Technical Committee on Frequency and Time of the IEEE Society on Instrumentation and Measurement and the existence of the IEEE Standard No. 1139-2008 on “IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology – Random instabilities”;
- d) that frequency and time instability measures should be based on sound theoretical principles, conveniently usable, and directly interpretable;
- e) that it is desirable to have frequency and time instability measures obtainable with simple instrumentation,

recommends

1 that the random instabilities of standard frequency and time signals should be characterized by the statistical measures $S_y(f)$, $S_\phi(f)$ or $S_x(f)$ in the frequency-domain, and $\sigma_y(\tau)$, Mod. $\sigma_y(\tau)$, $\sigma_x(\tau)$, $\sigma_y(t, \tau)$, and TheoBR in the time-domain as defined below:

1.1 the measure of the normalized frequency instabilities $y(t)$ in the frequency domain is $S_y(f)$; i.e. the one-sided spectral density ($0 < f < \infty$) of the normalized frequency instabilities $y(t) = (\nu(t) - \nu_0)/\nu_0$, where $\nu(t)$ is the instantaneous carrier frequency, and ν_0 is the nominal frequency;

1.2 the measure of the phase instabilities $\phi(t)$ in the frequency domain is $S_\phi(f)$; i.e. the one-sided spectral density ($0 < f < \infty$) of the phase instabilities $\phi(t)$ at a Fourier frequency f ;

1.3 the measure of the phase instabilities expressed in time units (phase-time) $x(t)$ in the frequency domain is $S_x(f)$; i.e. the one-sided spectral density ($0 < f < \infty$) of phase-time instabilities $x(t)$, where $x(t) = \phi(t)/2\pi\nu_0$; $x(t)$ being related to $y(t)$ by $y(t) = dx(t)/dt$;

1.4 the relationships of the above spectral densities are given below:

$$S_y(f) = \frac{f^2}{v_0^2} S_\phi(f) = 4\pi^2 f^2 S_x(f) \quad (1)$$

The dimensions of $S_y(f)$, $S_\phi(f)$ and $S_x(f)$ are respectively Hz^{-1} , $\text{Rad}^2 \text{Hz}^{-1}$ and $\text{s}^2 \text{Hz}^{-1}$;

1.5 the measure of the normalized frequency instabilities $y(t)$ in the time domain is the two-sample standard deviation, $\sigma_y(\tau)$, and the modified two-sample standard deviation, Mod. $\sigma_y(\tau)$ and TheoBR variance as defined in Annex 1;

1.6 the measure of time instabilities in the time-domain is $\sigma_x(\tau)$ as defined in Annex 1;

1.7 the measure of the variations of the normalized frequency instabilities $y(t)$ in the time domain is the two-sample standard deviation, $\sigma_y(t, \tau)$ as defined in Annex 1;

2 that, when stating statistical measures of frequency and time instability, non-random phenomena should be recognized, e.g.:

2.1 any observed time dependency of the statistical measures should be stated;

2.2 the method of measuring systematic behaviour should be specified (e.g. an estimate of the linear frequency drift was obtained from the coefficients of a linear least squares regression to M frequency measurements, each with a specified averaging or sample time τ and bandwidth f_h);

2.3 the environmental sensitivities should be stated (e.g. the dependence of frequency and/or phase on temperature, magnetic field, barometric pressure, etc.);

3 that, when stating a measure of frequency and time instability, all relevant measurement parameters should also be specified:

3.1 the method of measurements;

3.2 the characteristics of the reference signal;

3.3 the nominal signal frequency v_0 ;

3.4 the measurement system bandwidth f_h and the corresponding low pass filter response;

3.5 the total measurement time or number of measurements M ;

3.6 the calculation techniques (e.g. details of lag-windows when estimating power spectral densities from time domain data, or the assumption of the effect of dead-time in estimating the two-sample standard deviation $\sigma_y(\tau)$);

3.7 the confidence of the estimate;

4 that a graphic illustration or an analytic expression of the measures of the frequency and time instabilities should be provided and should include confidence intervals when appropriate (i.e. $S_y(f)$, $S_\phi(f)$ and $S_x(f)$ as a function of f ; $\sigma_y(\tau)$, Mod. $\sigma_y(\tau)$ and $\sigma_x(\tau)$ as a function of τ ; and/or $\sigma_y(t, \tau)$ as a function of t and τ).

Annex 1

Characterization of frequency and phase noise

1 Definition of terms

Frequency and phase instabilities may be characterized by random processes that can be represented statistically in either the Fourier frequency domain or in the time domain. The instantaneous, normalized frequency departure $y(t)$ from the nominal frequency ν_0 is related to the instantaneous-phase fluctuation $\varphi(t)$ about the nominal phase $2\pi \nu_0 t$ by:

$$y(t) = \frac{1}{2\pi \nu_0} \frac{d\varphi(t)}{dt} = \frac{\dot{\varphi}(t)}{2\pi \nu_0} \quad (2)$$

$$x(t) = \frac{\varphi(t)}{2\pi \nu_0}$$

where $x(t)$ is the phase variation expressed in units of time.

2 Fourier frequency domain

In the Fourier frequency domain, frequency instability may be defined by several one-sided (the Fourier frequency ranges from 0 to ∞) spectral densities such as:

$S_y(f)$ of $y(t)$, $S_\varphi(f)$ of $\varphi(t)$, $S_{\dot{\varphi}}(f)$ of $\dot{\varphi}(t)$, $S_x(f)$ of $x(t)$, etc.

These spectral densities are related by the equations:

$$S_y(f) = \frac{f^2}{\nu_0^2} S_\varphi(f) \quad (3)$$

$$S_{\dot{\varphi}}(f) = (2\pi f)^2 S_\varphi(f) \quad (4)$$

$$S_x(f) = \frac{1}{(2\pi \nu_0)^2} S_\varphi(f) \quad (5)$$

Power-law spectral densities are often employed as reasonable models of the random fluctuations in precision oscillators. In practice, it has been recognized that for many oscillators these random fluctuations are the sum of five independent noise processes and, with few limitations, the following equation is representative:

$$S_y(f) = \begin{cases} \sum_{\alpha=-2}^{+2} h_\alpha f^\alpha & \text{for } 0 < f < f_h \\ 0 & \text{for } f > f_h \end{cases} \quad (6)$$

where h_α 's are constants, α 's are integers, and f_h is the high frequency cut-off of a low pass filter. Equations (3), (4) and (5) are correct and consistent for stationary noises including phase noise. High frequency divergence is eliminated by the restrictions on f in equation (6). The identification and characterization of the five noise processes are given in Table 1, and shown in Fig. 1. In practice, only two or three noise processes are usually sufficient to describe the random frequency fluctuations in a specific oscillator; the others may be neglected.