



INTERNATIONAL TELECOMMUNICATION UNION

ITU-T

TELECOMMUNICATION
STANDARDIZATION SECTOR
OF ITU

E.507

TELEPHONE NETWORK AND ISDN

QUALITY OF SERVICE, NETWORK MANAGEMENT AND TRAFFIC ENGINEERING

MODELS FOR FORECASTING INTERNATIONAL TRAFFIC

ITU-T Recommendation E.507

(Extract from the *Blue Book*)

NOTES

1 ITU-T Recommendation E.507 was published in Fascicle II.3 of the *Blue Book*. This file is an extract from the *Blue Book*. While the presentation and layout of the text might be slightly different from the *Blue Book* version, the contents of the file are identical to the *Blue Book* version and copyright conditions remain unchanged (see below).

2 In this Recommendation, the expression “Administration” is used for conciseness to indicate both a telecommunication administration and a recognized operating agency.

© ITU 1988, 1993

All rights reserved. No part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from the ITU.



Documents
Document Sharing Hub

MODELS FOR FORECASTING INTERNATIONAL TRAFFIC

1 Introduction

Econometric and time series model development and forecasting requires familiarity with methods and techniques to deal with a range of different situations. Thus, the purpose of this Recommendation is to present some of the basic ideas and leave the explanation of the details to the publications cited in the reference list. As such, this Recommendation is not intended to be a complete guide to econometric and time series modelling and forecasting.

The Recommendation also gives guidelines for building various forecasting models: identification of the model, inclusion of explanatory variables, adjustment for irregularities, estimation of parameters, diagnostic checks, etc.

In addition the Recommendation describes various methods for evaluation of forecasting models and choice of model.

2 Building the forecasting model

This procedure can conveniently be described as four consecutive steps. The first step consists in finding a useful class of models to describe the actual situation. Examples of such classes are simple models, smoothing models, autoregressive models, autoregressive integrated moving average (ARIMA) models or econometric models. Before choosing the class of models, the influence of external variables should be analyzed. If special external variables have significant impact on the traffic demand, one ought to include them in the forecasting models, provided enough historical data are available.

The next step is to identify one tentative model in the class of models which have been chosen. If the class is too extensive to be conveniently fitted directly to data, rough methods for identifying subclasses can be used. Such methods of model identification employ data and knowledge of the system to suggest an appropriate parsimonious subclass of models. The identification procedure may also, in some occasions, be used to yield rough preliminary estimates of the parameters in the model. Then the tentative model is fitted to data by estimating the parameters. Usually, maximum likelihood estimators or least square estimators are used.

The next step is to check the model. This procedure is often called diagnostic checking. The object is to find out how well the model fits the data and, in case the discrepancy is judged to be too severe, to indicate possible remedies. The outcome of this step may thus be acceptance of the model if the fit is acceptable. If on the other hand it is inadequate, it is an indication that new tentative models may in turn be estimated and subjected to diagnostic checking.

In Figure 1/E.507 the steps in the model building procedure are illustrated.

¹⁾ The old Recommendation E.506 which appeared in the *Red Book* was split into two Recommendations, revised E.506 and new E.507, and considerable new material was added to both.

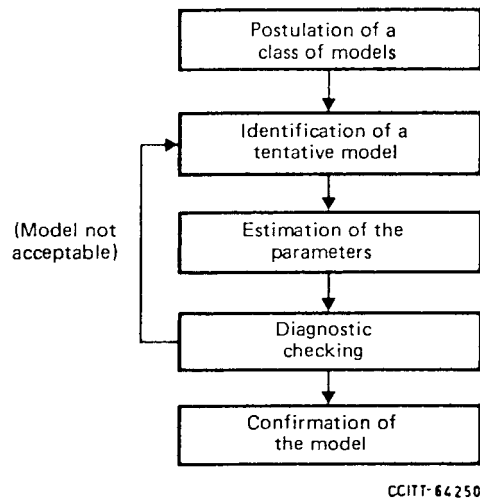


FIGURE 1/E.507
Steps in the model building procedure

3 Various forecasting models

The objective of § 3 is to give a brief overview of the most important forecasting models. In the GAS 10 Manual on planning data and forecasting methods [5], a more detailed description of the models is given.

3.1 Curve fitting models

In curve fitting models the traffic trend is extrapolated by calculating the values of the parameters of some function that is expected to characterize the growth of international traffic over time. The numerical calculations of some curve fitting models can be performed by using the least squares method.

The following are examples of common curve fitting models used for forecasting international traffic:

Linear: $Y_t = a + bt$ (3-1)

Parabolic: $Y_t = a + bt + ct^2$ (3-2)

Exponential: $Y_t = ae^{bt}$ (3-3)

Logistic: $Y_t = \frac{M}{1 + ae^{bt}}$ (3-4)

Gompertz: $Y_t = M(a)^{bt}$ (3-5)

where

Y_t is the traffic at time t ,

a, b, c , are parameters,

M is a parameter describing the saturation level.

The various trend curves are shown in Figures 2/E.507 and 3/E.507.

The logistic and Gompertz curves differ from the linear, parabolic and exponential curves by having saturation or ceiling level. For further study see [10].

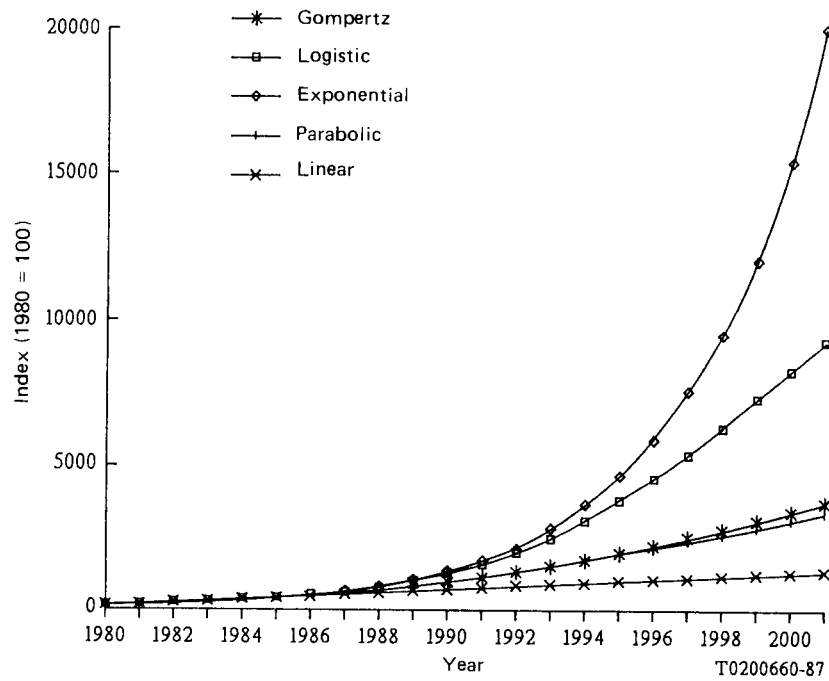


FIGURE 2/E.507

Example of fitting international telephone traffic using different models

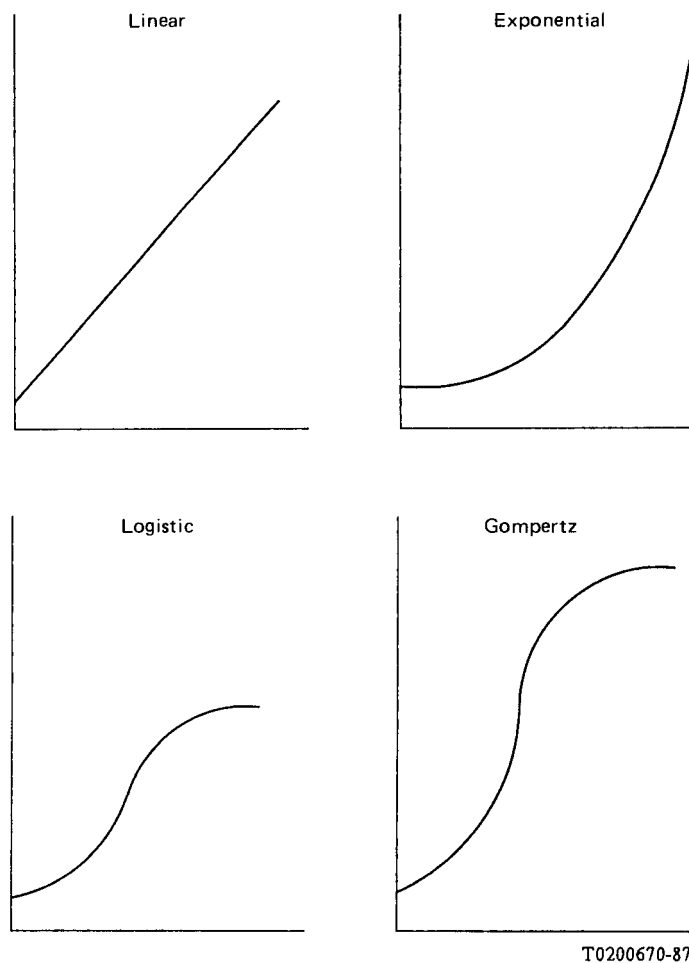


FIGURE 3/E.507

Curve fitting examples

3.2 Smoothing models

By using a smooth process in curve fitting, it is possible to calculate the parameters of the models to fit current data very well but not necessarily the data obtained from the distant past.

The best known smoothing process is that of the moving average. The degree of smoothing is controlled by the number of most recent observations included in the average. All observations included in the average have the same weight.

In addition to moving average models, there exists another group of smoothing models based on weighting the observations. The most common models are:

- simple exponential smoothing,
- double exponential smoothing,
- discounted regression,
- Holt's method, and
- Holt-Winters' seasonal models.

For example, in the method of exponential smoothing the weight given to previous observations decreases geometrically with age according to the following equation:

$$\hat{\mu}_t = (1-a)Y_t + a\hat{\mu}_{t-1} \quad (3-6)$$

Where:

Y_t is the measured traffic at time t ,

μ_t is the estimated level at time t , and

a is the discount factor [and $(1-a)$ is the smoothing parameter].

The impact of past observations on the forecasts is controlled by the magnitude of the discount factor.

Use of smoothing models is especially appropriate for short-term forecasts. For further studies see [1], [5] and [9].

3.3 Autoregressive models

If the traffic demand, X_t , at time t can be expressed as a linear combination of earlier equidistant observations of the past traffic demand, the process is an autoregressive process. Then the model is defined by the expression:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \dots + \Phi_p X_{t-p} + a_t \quad (3-7)$$

where

a_t is white noise at time t ;

Φ_k , $k = 1, \dots, p$ are the autoregressive parameters.

The model is denoted by $AR(p)$ since the order of the model is p .

By use of regression analysis the estimates of the parameters can be found. Because of common trends the exogenous variables (X_{t-1} , X_{t-2} , ... X_{t-p}) are usually strongly correlated. Hence the parameter estimates will be correlated. Furthermore, significance tests of the estimates are somewhat difficult to perform.

Another possibility is to compute the empirical autocorrelation coefficients and then use the Yule-Walker equations to estimate the parameters [Φ_k]. This procedure can be performed when the time series [X_t] are stationary. If, on the other hand, the time series are non stationary, the series can often be transformed to stationarity e.g., by differencing the series. The estimation procedure is given in Annex A, § A.1.

3.4 Autoregressive integrated moving average (ARIMA) models

An extension of the class of autoregressive models which include the moving average models is called autoregressive moving average models (ARIMA models). A moving average model of order q is given by:

$$X_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \dots - \theta_q a_{t-q} \quad (3-8)$$

where

a_t is white noise at time t ;

$[\theta_k]$ are the moving average parameters.

Assuming that the white noise term in the autoregressive models in § 3.3 is described by a moving average model, one obtains the so-called ARIMA (p, q) model:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \dots + \Phi_p X_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \dots - \theta_q a_{t-q} \quad (3-9)$$

The ARIMA model describes a stationary time series. If the time series is non-stationary, it is necessary to difference the series. This is done as follow:

Let Y_t be the time series and B the backwards shift operator, then

$$X_t = (1 - B)^d Y_t \quad (3-10)$$

where

d is the number of differences to have stationarity.

The new model ARIMA (p, d, q) is found by inserting equation (3-10) into equation (3-9).

The method for analyzing such time series was developed by G. E. P. Box and G. M. Jenkins [3]. To analyze and forecast such time series it is usually necessary to use a time series program package.

As indicated in Figure 1/E.507 a tentative model is identified. This is carried out by determination of necessary transformations and number of autoregressive and moving average parameters. The identification is based on the structure of the autocorrelations and partial autocorrelations.

The next step as indicated in Figure 1/E.507 is the estimation procedure. The maximum likelihood estimates are used. Unfortunately, it is difficult to find these estimates because of the necessity to solve a nonlinear system of equations. For practical purposes, a computer program is necessary for these calculations. The forecasting model is based on equation (3-9) and the process of making forecasts l time units ahead is shown in § A.2.

The forecasting models described so far are univariate forecasting models. It is also possible to introduce explanatory variables. In this case the system will be described by a transfer function model. The methods for analyzing the time series in a transfer function model are rather similar to the methods described above.

Detailed descriptions of ARIMA models are given in [1], [2], [3], [5], [11], [15] and [17].

3.5 State space models with Kalman Filtering

State space models are a way to represent discrete-time process by means of difference equations. The state space modelling approach allows the conversion of any general linear model into a form suitable for recursive estimation and forecasting. A more detailed description of ARIMA state space models can be found in [1].

For a stochastic process such a representation may be of the following form:

$$X_{t+1} = \Phi X_t + Z_t + \omega_t \quad (3-11)$$

and

$$Y_t = HX_t + v_t \quad (3-12)$$