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JEDEC STANDARD

**Standard for Lognormal Analysis of
Uncensored Data, and of Singly
Right-Censored Data Utilizing the
Persson and Rootzen Method**

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ELECTRONIC INDUSTRIES ASSOCIATION
ENGINEERING DEPARTMENT



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**Standard for Lognormal Analysis of Uncensored Data, and of Singly
Right-Censored Data Utilizing the Persson and Rootzen Method**

PREFACE

This document is intended for use by those involved with reliability, lifetime, or failure data analysis. This standard enables the user to estimate the parameters of a two parameter lognormal distribution from complete or singly right-censored independent failure data. The methods described were chosen because they can be easily done with a calculator and are as accurate as more complex iterative techniques for the cases described.

Interferences or issues that can complicate and/or invalidate the analysis are discussed. One section discusses analysis of failure data where all units from within the sample have a known failure time. Another section discusses analysis of failure data for the case where the life experiment was stopped and the failure times for the surviving units are not known. Graphical presentation is discussed, and sample calculations are presented in several appendixes.

This material was developed by the Electromigration Task Group in JEDEC JC-14.2 Committee on Wafer-Level Reliability.

**Standard for Lognormal Analysis of Uncensored Data,
and of Singly Right-Censored Data Utilizing
the Persson and Rootzen Method**

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**STANDARD FOR LOGNORMAL ANALYSIS OF
UNCENSORED DATA, AND OF SINGLY RIGHT-CENSORED
DATA UTILIZING THE PERSSON AND ROOTZEN
METHOD**

(From JEDEC Council Ballot JCB-92-19, formulated under the cognizance of JC-14.2 Committee on Wafer Level Reliability)

1. SCOPE

1.1 Intent

This standard enables the user to estimate the parameters of a two-parameter lognormal distribution from complete or singly right-censored independent data samples.

Specifically, this standard is intended for analyzing failure-time (t_f) data obtained from a stress test of a sample of units when the natural logarithms of the failure-time ($\ln t_f$) follow a normal distribution.

This standard is not intended to describe techniques used to determine how well the failure data fits a lognormal distribution. However, if points lie along a straight line for plots generated in section 8 the lognormal distribution estimators will describe the points along the line.

1.2 Results

The results of the analysis provide bias-corrected sample estimates for the median time-to-failure (t_{50}), mean of the $\ln t_f$ values ($\ln t_{50}$), and the standard deviation (σ) of the $\ln t_f$ values of the lognormal distribution. Additionally, confidence intervals are provided for complete data samples (no censoring). These are all obtained from the failure time values (t_f).

1.3 Complete Data Case

This standard may be used to analyze complete data where the failure-time data for the entire sample population is known and used. The analysis uses the most efficient estimators for obtaining estimates of the two primary parameters (t_{50} , σ) of the distribution.

1.4 Right-Censored Case

This standard may be used to analyze singly right-censored (Type II censored) data where the test has been stopped before all the parts have failed. The analysis uses the Persson and Rootzen Estimators [1] corrected for bias [2,3]. These estimators can be calculated with a hand calculator and are as accurate as more complex estimators.

1.5 Other Methods

This standard does not preclude the use of other estimators for determining a lognormal distribution, as long as the method has been shown to provide results similar to those obtained by this standard. For example, Maximum Likelihood Estimators are widely used, have desirable properties, and are acceptable when bias is removed. However, these estimators are difficult to obtain without computerized iterative techniques, are subject to similar, if not identical bias, and many software tools do not remove this bias.

1.6 Normal Distribution

This standard can also be used to estimate the parameters of a normal distribution from complete and right-censored data samples. Raw data is used without taking the natural logarithm. The mean and standard deviation can be used without modification.

2. INTRODUCTION

Analysis of reliability experiments depends extensively on failure statistics, and a commonly used failure distribution is the normal distribution after natural logarithms of the failure times have been calculated. Lognormal distributions and the associated statistics are used by analysts working in semiconductor reliability to interpret and communicate results from failure experiments. Unfortunately, many individuals who lack a sufficient background in statistical methods are using the techniques; errors in usage and interpretation are abundant. Frequently, the number of data points in a distribution is small (i.e., less than 20). Statistical tools for analyzing small samples of data are not available to all who need them. The result is that less than optimum techniques are being used on calculators.

This document was written to provide a very basic set of tools for determining the parameters of the lognormal distribution for cases where sophisticated tools or expertise do not exist. The techniques handle a majority of the experimental cases experienced by the contributors. The step-by-step standard and appendixes are intended to instruct and to help ensure proper analysis.

3. DEFINITIONS

3.1 Terms

3.1.1 Unit

A unit is a single test structure from which one failure time observation is possible.

3.1.2 Sample

A sample is a subset of units from a homogeneous parent population that has undergone stress testing. This subset of the population is assumed to retain the characteristics of the parent population from which it was taken. Failure data from the units in this subset are used to determine distribution parameter estimates of the parent population.

3.1.3 Failure set

The failure set is the subset of the sample that fails the defined test criterion during the stress time.

3.1.4 Complete Data

Complete data indicates that data from all units in the stress test is available; i.e., all units in the sample are also within the failure set.

3.1.5 Censored Data

All nonfailing units removed from the stress test at the same time are considered censored data; i.e., the failure times are known to be greater than the time units were on stress test and the censor time is the removal time. This is also known as singly right-censored data.

3.1.6 Lognormal Distribution

The lognormal distribution is the assumed distribution for the parent population of failure times from which samples are taken. The logarithms of the failure times are assumed to follow a normal distribution.

3.1.7 Bias

The bias of an estimator T for a parameter θ is $E(T) - \theta$; i.e., the difference between the mean (or expectation) of T and the true value of the parameter θ .

3.1.8 Confidence Interval

An interval of the form (A, B) where A and B are the confidence limits calculated from sample statistics such that $P(A \leq \theta \leq B) = 1 - \alpha$ (where α is the probability of error) is a confidence interval. With repeated sampling, at least $100(1-\alpha)\%$ of the similarly constructed intervals will contain the true population parameter θ .

3.2 Symbols

N: the number of units in the sample.

t_f : the failure time observation of a unit from the sample.

$$\begin{aligned} \text{Failtime } (t_f) &= \exp \left[\frac{\ln(t_{\text{last good}}) + \ln(t_{\text{first fail}})}{2} \right] \\ &\quad - \text{ OR } - \\ &= \sqrt{t_{\text{last good}} t_{\text{first fail}}} \end{aligned} \quad (1)$$

where: $t_{\text{last good}}$ is the last time duration the unit was known to be good and

$t_{\text{first fail}}$ is the time duration the unit was known to have failed.

$t_{f\text{-cen}}$: the censor time of a unit from the population. The censor time may or may not be the time that another part is known to have failed.

σ : the real (unknown) standard deviation or shape parameter of the lognormal parent distribution.

S: sample estimate of the standard deviation, σ , of the lognormal parent distribution, calculated from logarithms of the observed failure times.

t_{50} : the real (unknown) median-time-to-fail (MTF) of the parent distribution.

t_{50s} : sample estimate of the MTF of the parent distribution, calculated from the logarithms of observed failure times.

α : The maximum probability of error that is acceptable when making decisions.

4. SUMMARY OF TECHNIQUES

4.1 Complete Data

For complete sample data sets (i.e., sets consisting of the complete population of the sample), S of the parent population distribution is the standard deviation of the $\ln(t_f)$ values, corrected for bias. t_{50s} is determined from the exponential of the mean of the $\ln(t_f)$ values. The confidence intervals for the mean and the standard deviation of the population are obtained through the use of Student's t-distribution and the chi-square (χ^2) distribution, respectively.