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**Leaf springs—Part 2: Design
method**

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Foreword

This Japanese Industrial Standard has been revised by the Minister of Economy, Trade and Industry through deliberations at the Japanese Industrial Standards Committee as the result of proposal for revision of Japanese Industrial Standard submitted by Japan Spring Manufacturers Association (JSMA)/Japanese Standards Association (JSA) with a draft being attached, based on the provision of Article 12, paragraph (1) of the Industrial Standardization Act applied mutatis mutandis pursuant to the provision of Article 16 of the said Act. This edition replaces the previous edition (**JIS B 2710-2: 2008**), which has been technically revised.

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JIS B 2710 series consists of the following 4 parts under the general title *Leaf springs*:

Part 1: Vocabulary

Part 2: Design method

Part 3: Method of testing

Part 4: Specification of products

Leaf springs—Part 2: Design method

1 Scope

This Japanese Industrial Standard specifies the design method of the leaf springs mainly used for automobiles, railway rolling stocks and industrial machines.

This Standard covers the leaf springs composed of only a single leaf.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this Standard. The most recent editions of the standards (including amendments) indicated below shall be applied.

JIS B 0103 *Springs—Vocabulary*

JIS B 0156 *Springs—Symbols*

JIS B 2710-1 *Leaf springs—Part 1: Vocabulary*

JIS B 2710-3 *Leaf springs—Part 3: Method of testing*

JIS B 2710-4 *Leaf springs—Part 4: Specification of products*

3 Terms and definitions

For the purpose of this Standard, the terms and definitions given in **JIS B 0103** and **JIS B 2710-1** apply.

4 Symbols

For the purpose of this Standard, the symbols and definitions shall be as given in Table 1 based on the requirements specified in **JIS B 0156**.

Table 1 Symbols and definitions

Symbol	Definition	Unit
b	Leaf width	mm
C	Camber	mm
E	Young's modulus, $E=206 \times 10^3$	N/mm ²
e	Radius of eye	mm
$2F$	Working force on leaf spring	N
I	Geometrical moment of inertia of leaf	mm ⁴
l	Half span of leaf spring (span: $2l$)	mm
l_A	Half span of fixed side of unsymmetrical leaf spring	mm
l_B	Half span of shackle or slide side of unsymmetrical leaf spring	mm
l_{ST}	Half straight span (straight span: $2l_{ST}$)	mm
$l_{ST,A}$	Half straight span of fixed side of unsymmetrical leaf spring	mm
$l_{ST,B}$	Half straight span of shackle or slide side of unsymmetrical leaf spring	mm

Table 1 (concluded)

Symbol	Definition	Unit
n	Total number of leaves or subscript indicating main leaf	—
n_f	Number of full length leaves	—
R	Spring constant, $R=2F/s$	N/mm
R_T	Wind-up stiffness, $R_T=T/\theta$	N • mm/rad
R_U	Spring constant at U-bolt tightening	N/mm
s	Deflection	mm
T	Wind-up torque	N • mm
t	Leaf thickness	mm
U	Width of U-bolt (see Figure 10)	mm
Z	Section modulus of leaf	mm ³
η	Ratio of the number of full length leaves against the total number of leaves, $\eta=n_f/n$	—
θ	Rotation angle by wind-up	rad
σ	Bending stress of leaf	N/mm ²
σ_R	Bending stress of first leaf by wind-up	N/mm ²

5 Basic concept of design

5.1 Principle of design

Importance shall be placed on spring constant and strength durability in the design of a leaf spring. Each leaf of a multi-leaf spring can be considered dynamically equivalent to the cantilever in which the centre is fixed and both ends are free. Although a general leaf spring has the structure in which multiple leaves with different shapes and dimensions are bundled, as long as the relation between the force which works on whole and the deflection is uniquely determined, the relation between the force which works on an individual leaf and the deflection can be analysed. However, the change of a span by deflection shall be normally ignored. That is, when the deflection on the whole spring is given, the deflection on an individual leaf can be determined from it, the force of the whole spring is determined as the total sum of force which an individual leaf exerts, and the spring constant can be determined. According to the force which works on an individual leaf, the stress distribution in a leaf can be obtained and the strength durability can be studied.

5.2 Design methods

The design methods of a leaf spring shall be as given in Table 2.

Table 2 Design methods

Type of spring	Design method	Summary	Clause
Multi-leaf spring	Developed leaf method	Perform the design calculation assuming that the force is transferred by making neighbouring leaves contact the whole surface and regarding all leaves developed into the same plane as one broad plate.	6.1
	Tip-contact method	Perform the design calculation assuming that the force is transferred by making neighbouring leaves contact the tip of leaf only.	6.2
Tapered leaf spring	Linear taper method	Perform the design by making the leaf thickness of the tapered part decrease gradually and linearly.	7.2.2
	Parabolic taper method	Perform the design by making the leaf thickness decrease gradually and parabolically so that the stress distribution of the tapered part becomes more uniform.	7.2.3
For the design method of an unsymmetrical leaf spring, any one of the methods in this table shall be used for each side of the spring, and the procedure shall be as specified in Clause 8.			

5.3 Procedure of design

In the design of a leaf spring, it is necessary to set the specifications of the spring beforehand in accordance with the requirements of the structure and the equipment incorporating the spring. The specifications, here, include the type, straight span, leaf shape, leaf width, leaf thickness and number of leaves of a spring. On that basis, whether the spring constant and the bending stress of a leaf calculated based on these specifications is in the appropriate range for the requirement or not shall be evaluated. When the result is not enough, a review shall be made by returning to the type and specifications of a spring. That is, the design of a leaf spring includes the repetitive process of the proposal, evaluation and review to the requirement specifications. The relation between nominal bending stress and maximum working bending stress, the fatigue strength and the fatigue strength diagram, which are used for a typical spring material, are given in Annex A. An example of design calculation is given in Annex B.

The product specifications including the detail of shape and dimension, the treatment method and the parts shall be as specified in **JIS B 2710-4**.

The test method for verifying the designed product shall be as specified in **JIS B 2710-3**.

5.4 Points to note

The leaf spring not only can serve a useful purpose of vibrational absorption and cushioning by using the spring force but also can function as the structural member which positions and couples other members. In addition to these, the design concerning characteristics of the leaf spring includes the effect of energy absorption due to friction between plates, the generation of additional stress and deformation due to wind-up torque and the durability degradation due to fretting wear; in particular for the spring for automobiles, importance shall be placed also on the problem of corrosion fatigue. It is also preferable to take measures which sufficiently take these into consideration in the design.

6 Design of multi-leaf spring

6.1 Design by developed leaf method

6.1.1 Principle of developed leaf method

In the developed leaf method, it is assumed that all curvatures of each leaf of the point at an equal distance from a centre hole are equal, and it is considered that neighbouring leaves transfer force through whole-surface contact. In that case, the multi-leaf spring becomes dynamically equivalent to one broad leaf obtained by developing it as the same plane. The developed leaf method is often used for the outline design of a spring and the evaluation of fatigue durability, since the calculations of spring constant and bending stress are relatively easily performed. When a spring has two or more full length leaves, the calculation is normally performed according to the developed leaf method.

The developed leaf method includes the case where a trapezoid model is used and the case where a step model is used.

6.1.2 Applicable object

The analyses of the multi-leaf spring performed according to the developed leaf method shall be applied to the symmetrical leaf spring shown in Figure 1. Although a basic multi-leaf spring is specified here, this concept may be also applied to a two-stage progressive leaf spring and a bolster spring.

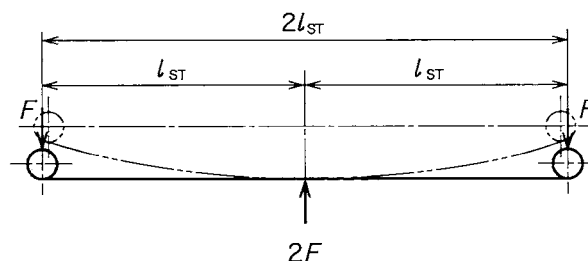


Figure 1 Model of symmetrical leaf spring

6.1.3 Method by trapezoid model

6.1.3.1 Trapezoid model

When the thickness of all leaves is equal, the difference in the step due to the differences in the leaf end shape and the length of a leaf shall be ignored, and approximations shall be made by using the trapezoid model shown in Figure 2.

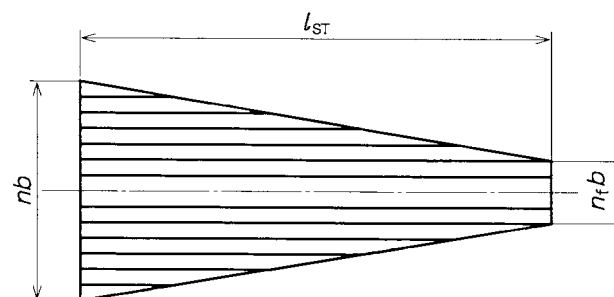


Figure 2 Trapezoid model

The factor K_T based on the trapezoidal shape shall be calculated according to Formula (1). K_T may be obtained by reading the numerical value shown in **C.1**.

$$K_T = \frac{3}{(1-\eta)^3} \left[\frac{1}{2} - 2\eta + \eta^2 \left(\frac{3}{2} - \log_e \eta \right) \right] \dots\dots\dots (1)$$

6.1.3.2 Spring constant and bending stress

The spring constant R shall be calculated according to Formula (2).

$$R = \frac{1}{K_T} \times \frac{6nEI}{l_{ST}^3} \dots\dots\dots (2)$$

I is the geometrical moment of inertia of a leaf, and in the case of a rectangular section leaf, it shall be calculated according to Formula (3).

$$I = \frac{1}{12} bt^3 \dots\dots\dots (3)$$

The bending stress in the centre of length of each leaf (centre hole position) is equal, and shall be calculated according to Formula (4). In the case of a rectangular section, $Z=2I/t$ shall apply.

$$\sigma = \frac{l_{ST}}{nZ} F \dots\dots\dots (4)$$

6.1.4 Method by step model

6.1.4.1 Step model

When the thickness differs for every leaf, approximations shall be made by using the step model shown in Figure 3 irrespective of the leaf end shape. The factor K_S when the step model is used shall be calculated according to Formula (5).

$$K_S = I_n \left[\frac{1}{\sum_{i=1}^n I_i} + \sum_{i=1}^{n-n_k} \frac{I_i(1-\lambda_i)^3}{\left(\sum_{j=i}^n I_j \right) \times \left(\sum_{j=i+1}^n I_j \right)} \right] \dots\dots\dots (5)$$

where, λ_i : relative length ratio of the i th leaf in increasing order of length, which is $\lambda_i = l_i / l_{ST}$

I_i is the geometrical moment of inertia of the i th leaf in the same way, and in the case of a rectangular section leaf it shall be calculated according to Formula (6).

$$I_i = \frac{1}{12} b_i t_i^3 \dots\dots\dots (6)$$

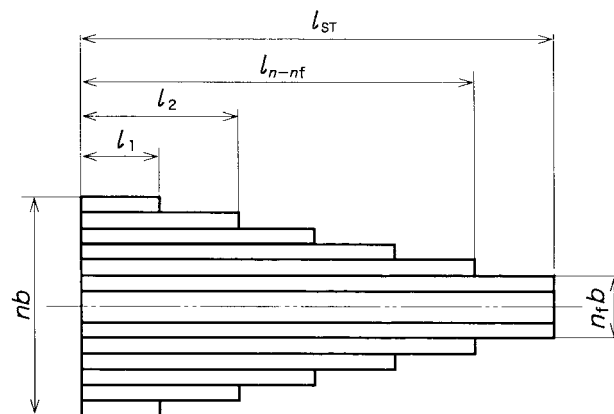


Figure 3 Step model

6.1.4.2 Spring constant and bending stress

The spring constant R shall be calculated according to Formula (7).

$$R = \frac{1}{K_s} \times \frac{6EI_n}{l_{ST}^3} \dots\dots\dots (7)$$

The bending stress σ_i in the centre of length of each leaf (centre hole position) shall be calculated according to Formula (8).

$$\sigma_i = \frac{l_i l_{ST}}{2 \sum_{j=1}^n I_j} F \dots\dots\dots (8)$$

6.2 Design by tip-contact method

6.2.1 Principle of tip-contact method

In the tip-contact method, it is assumed that the force between neighbouring leaves is transferred only at the tip of the leaf. Therefore, this method is applicable to a leaf spring of any leaf thickness, leaf end shape and step for each leaf.

The spring constant obtained by the tip-contact method is often used for the detailed design since not only it is generally in agreement with the measured value but also the stress distribution within each leaf can be easily calculated.

6.2.2 Applicable object

The analysis of the multi-leaf spring by the tip-contact method shall be applied to a symmetrical leaf spring (see Figure 1). Although a basic multi-leaf spring is specified here, this concept may be also applied to a two-stage progressive leaf spring and a bolster spring.

6.2.3 Tip-contact shape factor

The end shape of a leaf shall be approximated by using the model shown in Figure 4. First, for the leaf end of the i th leaf in increasing order of length, the tip-contact shape factor K_i shall be calculated according to Formula (9). K_i may be obtained by reading the numerical value shown in **C.2**.

$$K_i = \frac{3\beta_i^2}{(\xi_i - \beta_i)^3} \log_e \frac{\xi_i}{\beta_i} + \frac{3(\xi_i - 3\beta_i)}{2(\xi_i - \beta_i)^2} - 1 \dots\dots\dots (9)$$

The relative width and the relative thickness of the leaf tip shall be taken as $\beta_i = b_{E,i}/b_i$ and $\xi_i = t_{E,i}/t_i$, respectively (see Figure 4).

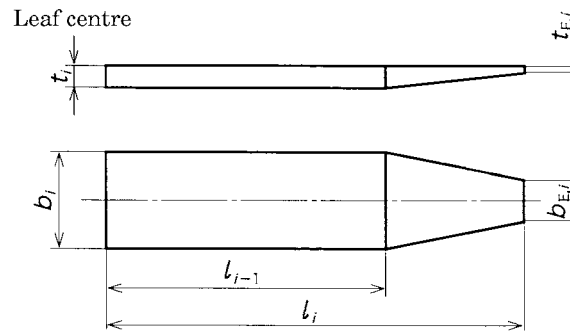


Figure 4 Model of leaf end

The factor η_i for obtaining the spring constant shall be calculated according to Formula (10).

$$\eta_i = B_i - \alpha_{i-1} \times \Gamma_{i-1} \dots\dots\dots (10)$$

$$B_i = 1 + (1 - \mu_{i-1})^3 K_i$$

$$\Gamma_{i-1} = \frac{3 - \mu_{i-1}}{2} \mu_{i-1}^2$$

In the formula, α_{i-1} is the ratio of working force at the $i-1$ th leaf tip to working force at the i th leaf tip, which shall be calculated according to Formula (11) and Formula (12).

$$\alpha_{i-1} = \frac{A_{i-1}}{D_{i-1}} \varphi_{i-1} \dots\dots\dots (11)$$

$$D_{i-1} = \varphi_{i-1} + \eta_{i-1} \dots\dots\dots (12)$$

$$A_{i-1} = \frac{3 - \mu_{i-1}}{2\mu_{i-1}}$$

$$\varphi_{i-1} = \frac{l_{i-1}}{l_i}$$

Furthermore, μ refers to the input position of force, which shall be taken as $\mu_{i-1} = l_{i-1}/l_i$.

Since, as shown in the above calculation, A_{i-1} , B_i , Γ_{i-1} , φ_{i-1} and μ_{i-1} are determined according to the spring specifications, they should be obtained in the beginning. In that case, $A_0 = \Gamma_0 = \varphi_0 = 0$ shall apply.

On that basis, first, η_1 is calculated as $i=1$ according to Formula (10), and if D_1 is calculated by substituting this into Formula (12), α_1 will be calculated from Formula (11). η_2 is calculated by substituting this into Formula (10) again. Next, if repetition is performed in the same way, finally η_n will be calculated.

6.2.4 Spring constant and bending stress

The spring constant R shall be calculated according to Formula (13).

$$R = \frac{1}{\eta_n} \times \frac{6EI_n}{l_{ST}^3} \dots\dots\dots (13)$$

In the i th leaf, the bending stress σ_{io} of the centre part shall be calculated according to Formula (14), and the bending stress σ_{ic} of the contact point with the $i-1$ th leaf according to Formula (15).

$$\sigma_{io} = (1 - \sigma_{i-1} \times \mu_{i-1}) \prod_{j=1}^{n-1} \alpha_j \times \frac{Fl_1}{Z_i} \dots\dots\dots (14)$$

$$\sigma_{ic} = (1 - \mu_{i-1}) \prod_{j=1}^{n-1} \alpha_j \times \frac{Fl_1}{Z_i} \dots\dots\dots (15)$$

7 Design of tapered leaf spring

7.1 Principle of design

The tapered leaf spring is the spring composed of laminated leaves with the tapered shape, of which the thickness gradually decreases from the spring centre to the tip in the longitudinal direction.

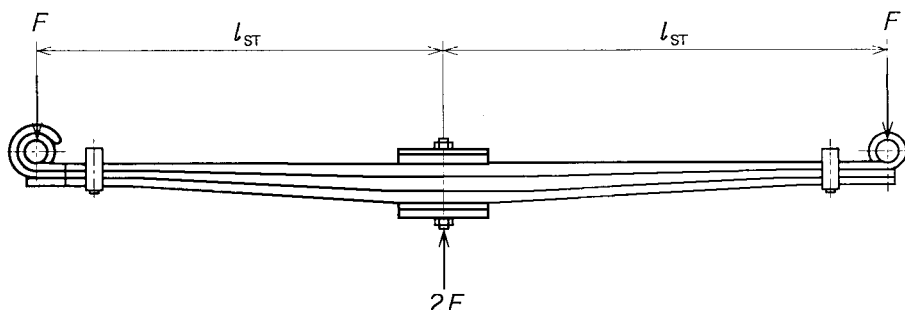


Figure 5 Example of tapered leaf spring

In the symmetrical leaf spring as shown in Figure 5, since the deflection change of the whole spring is equal to the deflection change of each leaf, each leaf shall bear the force according to the spring constant. Therefore, the whole spring constant is equal to the total sum of the spring constant of each leaf. That is, design should be performed with respect to an individual leaf.

NOTE The tapered leaf spring is advantageous to weight saving since the stress distribution in the longitudinal direction of a leaf is easily equalized compared with the multi-leaf spring.

7.2 Model of tapered leaf and calculation formula

7.2.1 Model of tapered leaf

For the symmetric type tapered leaf in which the leaf width b is constant and the leaf thickness t gradually decreases from t_1 to t_2 , the half straight span model shall be as shown in Figure 6.