KES

Significance Test of Difference Between the Two Population Means (Standard Deviations Unknown, Two-Sided)

KES R – A011
(Adaptation KS A 3259)

(EST.) 1989 – 09 – 27

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1. Application Limits

This standard determines whether there is a difference between the 2 populations' population means, by using the test method based on the measurement value from each population (The a lot considered that it represents its set enough) when standard deviations of 2 populations are unknown (Two side test for unknown standard deviations of populations ([1])).

Remark : This test method is for the only case that 2 samples' sizes and the standard deviations are same, and if it is considered not, see the notes. ([2])

2. Technical Condition of Lot

A distribution of characteristic value (X) of two lots shall be a normal distribution or close to a normal distribution for both sides.

This method can be applied when a sample's size measured from a lot is big. ([3])

3. Technical Condition of each sample

The sample shall be the one that is picked up randomly from each lot. Follow KS A 3151 (Random Sampling Method) about random sampling method.

4. Test Procedure and Examples

Procedure : When you pick up each sample, amount of a from 2 lots and measure those characteristic values (X1 or X2), the test procedure whether there is a difference between the 2 populations' population means is as follow. Here, the standard deviations of these two lots are unknown.

(1) The result of the measurement shall be put as Table 1. ([4])

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Table 1

Order of measurement			Set 2	
	Value x_1	x_1^2	Value x_2	x_2^2
1	x_{11}	x_{11}^2	x_{21}	x_{21}^2
2	x_{12}	x_{12}^2	x_{22}	x_{22}^2
3	x_{13}	x_{13}^2	x_{23}	x_{23}^2
4	x_{14}	x_{14}^2	x_{24}	x_{24}^2
5	x_{15}	x_{15}^2	x_{25}	x_{25}^2
6 ⋮ n	x_{16} ⋮ x_{1n}	x_{16}^2 ⋮ x_{1n}^2	x_{26} ⋮ x_{2n}	x_{26}^2 ⋮ x_{2n}^2
Total	T_1	Q_1	T_2	Q_2

(2) Calculate the sum of x's, T_1, T_2 and the sum of x^2's Q_1, Q_2 for each set.

$$T_1 = x_{11} + x_{12} + \cdots\cdots + x_{1n}$$

$$T_2 = x_{21} + x_{22} + \cdots\cdots + x_{2n}$$

$$Q_1 = x_{11}^2 + x_{12}^2 + \cdots\cdots + x_{1n}^2$$

$$Q_2 = x_{21}^2 + x_{22}^2 + \cdots\cdots + x_{2n}^2$$

(3) Calculate each set's mean value, $\overline{x_1}$ or $\overline{x_2}$.

$$\overline{x_1} = \frac{T_1}{n} \qquad \overline{x_2} = \frac{T_2}{n}$$

(4) Calculate the absolute value d, the difference between $\overline{x_1}$ and $\overline{x_2}$.

$$d = |\overline{x_1} - \overline{x_2}|$$

(5) Calculate each set's sum of second power, S_1 and S_2.

$$S_1 = Q_1 - \frac{T_1^2}{n} \qquad S_2 = Q_2 - \frac{T_2^2}{n}$$

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(6) Calculate t0 as bellow.

$$t_o = \sqrt{\frac{n(n-1)}{S_1 + S_2}} \cdot d$$

(7) Determine a level of significance, α, according to a purpose.([5])

(8) $f = 2(n-1)$ and the value α, $t(f, \alpha)$ shall be obtained from the appendix.

(9) Compare t_0 and $t(f, \alpha)$.([6])

If $t_0 \geqq t(f, \alpha)$, it is considered that there is a difference (Level of significance α).

If $t_0 < t(f, \alpha)$, it is considered that there is no difference.

Example. Test whether there is a difference between the two population means of hardness of blend A and blend B's iron plates.

As a result of the measurement by picking up 11 samples of each lot of blend A or blend B, the values are obtained as follow.

(1')

Table 1' Unit : HpB

Order of measurement	A		B	
	Value x_1	x_1^2	Value x_2	x_2^2
1	51	2601	59	3481
2	45	2025	56	3136
3	60	3600	63	3969
4	50	2500	48	2304
5	62	3844	58	3364
	268	14570	284	16254
6	52	2704	49	2401
7	56	3136	64	4096
8	48	2304	44	1936
9	55	3025	58	3364
10	45	2025	53	2809
	256	13194	268	14606
11	49	2401	65	4225
	49	2401	65	4225
Total	573	30165	617	35085

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(2') On the table 1', have subtotals of x's 5 values for each set and put those with underlines.

Have both subtotals 573 and 617, and put those in the box of sum of xs. And then, have subtotals of x^2's 5 values for each set and put those with underlines.

Have both subtotals 30165 and 35085, and put those in the box of sum of x^2's.

(3') $$\bar{x}_1 = \frac{573}{11} = 52.1$$

$$\bar{x}_2 = \frac{617}{11} = 56.1$$

(4') $$d = |52.1 - 56.1| = 4.0$$

(5') $$S_1 = 30165 - \frac{573^2}{11} = 316.91$$

$$S_2 = 35085 - \frac{617^2}{11} = 476.91$$

(6') $$t_0 = \sqrt{\frac{11 \times (11-1)}{316.91 + 476.91}} \times 4.0 = 1.488$$

(7') Here, we determine $\alpha = 0.05$

(8') $$f = 2(11-1) = 20$$

$$\alpha = 0.05$$

$$t(20,\ 0.05) = 2.086$$

(9') Having 1.488 < 2.086 it is not considered that there is a difference between the population means of hardness of blend A and blend B's iron plates.

Note(1) This method shall be applied when you have a little information about the product characteristics of the lots due to a difference of material or manufacturing method etc.

However, this method shall not be applied when the values of two sets correspond each other, such as the case that 1 sample is measured by two kinds

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