

KES

Significance Test of Equality of the Two Population Variance (One-Sided)

KES R – A015
(KS A 3266 adaptation)

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Additional rule

1. Scope of Application

This standard determines the method that tests whether it is considered that there is a difference between the two population variance (The either side that you could say it is bigger or smaller), based on the measurement value from each population (Or a lot considered that it represents its set enough)(One side). And, this test method determines it for the case that the sample sizes of the two sets(Groups) are same, if not, it shall be mentioned for a reference.

2. Technical Condition of Lot

It shall be better that a distribution of characteristic value (X) of two lots is a normal distribution or close to a normal distribution for both sides.

3. Technical Condition of each sample

The sample shall be the one that is picked up randomly from each lot. Follow KS A 3151 (Random Sampling Method) about random sampling method.

4. Test Procedure and Examples

Procedure

When you pick up each sample of amount, n , from two lots and measure its characteristic values (X_1 , X_2), the test procedure whether it is considered that the population variance of one side is different from that of the other side is as follow ⁽¹⁾.

(1) The result of the measurement shall be put as Table 1⁽²⁾⁽³⁾.

Table 1

Oder of measurement	Set(Group) 1		Set(Group) 2	
	Value x_1	x_1^2	Value x_2	x_2^2
1	x_{11}	x_{11}^2	x_{21}	x_{21}^2
2	x_{12}	x_{12}^2	x_{22}	x_{22}^2
3	x_{13}	x_{13}^2	x_{23}	x_{23}^2
4	x_{14}	x_{14}^2	x_{24}	x_{24}^2
5	x_{15}	x_{15}^2	x_{25}	x_{25}^2
6	x_{16}	x_{16}^2	x_{26}	x_{26}^2
⋮	⋮	⋮	⋮	⋮
n	x_{1n}	x_{1n}^2	x_{2n}	x_{2n}^2
Total	T_1	Q_1	T_2	Q_2

(2) Calculate the total of x's, T_1 , T_2 and the total of x^2 's Q_1 , Q_2 for each set (Group).

$$T_1 = x_{11} + x_{12} + + x_{1n}$$

$$T_2 = x_{21} + x_{22} + + x_{2n}$$

$$Q_1 = x_{11}^2 + x_{12}^2 + x_{1n}^2$$

$$Q_2 = x_{21}^2 + x_{22}^2 x_{2n}^2$$

(3) Calculate each set (Group)'s totals of second power, S_1 and S_2 .

$$S_1 = Q_1 - \frac{T_1^2}{n}$$

$$S_2 = Q_2 - \frac{T_2^2}{n}$$

(4) Next, calculate F_0 (⁴).

(a) For the case when you test whether it is considered that X_1 's population variance is bigger than X_2 's,

$$F_0 = \frac{S_1}{S_2}$$

(b) For the case when you test whether it is considered that X_1 's population variance is smaller than X_2 's,

$$F_0 = \frac{S_2}{S_1}$$

(5) Determine a level of significance, α , according to the purpose.(⁵)

(6) Determine that f_1 is the free variable of numerator of F_0 and f_2 is the free variable of denominator (f_1 and f_2 can be indicated by $\emptyset_1 \emptyset_2$).

Have the value $F_{\alpha}^{f_1, f_2}(\alpha)$ corresponding to $f_1 = f_2 = n - 1$ from the appendix [it can be indicated by $F(\emptyset_1, \emptyset_2, \alpha)$].

(7) Compare F_0 and $F_{\alpha}^{f_1, f_2}(\alpha)$.

(a) If $F_0 \geq F_{\alpha}^{f_1, f_2}(\alpha)$,

it is considered that X_1 's population variance is bigger than X_2 's

(Level of significance α).

If $F_0 < F_{\alpha}^{f_1, f_2}(\alpha)$,

it is not considered that X_1 's population variance is bigger than X_2 's(⁶).

(b) If $F_0 \geq F_{\alpha}^{f_1, f_2}(\alpha)$,

it is considered that X_1 's population variance is smaller than X_2 's

(Level of significance, α).

If $F_0 < F_{\alpha}^{f_1, f_2}(\alpha)$,

it is not considered that X_1 's population variance is smaller than X_2 's⁽⁶⁾.

Example : When you thermal process two kinds of material in 800 °C and 850 °C, test whether it is considered that the population variance of a material's tensional strength that is thermal processed in 800 °C is bigger than the one that is thermal processed in 850 °C. The result of randomly picking up 11 samples from the each lot that is thermal processed in 800 °C and 850 °C is obtained as follow.

Table 1'

Unit : %

Oder of measurement	800℃		850℃	
	Value x_1	x_1^2	Value x_2	x_2^2
1	33	1089	35	1225
2	29	841	37	1369
3	42	1764	31	961
4	32	1024	32	1024
5	42	1764	33	1089
	<u>178</u>	<u>6482</u>	<u>168</u>	<u>5668</u>
6	41	1681	29	841
7	42	1764	27	729
8	44	1936	31	961
9	33	1089	39	1521
10	45	2025	37	1369
	<u>205</u>	<u>8495</u>	<u>163</u>	<u>5421</u>
11	38	1444	30	900
	<u>38</u>	<u>1444</u>	<u>30</u>	<u>900</u>
Total	421	16421	361	11989

(2') On the table 1', have subtotals of x 's 5 values for each set (Group) and put those with underlines, and have both subtotals 421 and 361, and put those in the box of sum of x_i 's.

And then, have subtotals of x_i^2 's 5 values for each set and put those with underlines.

Have both subtotals 16421 and 11989, and put those in the box of total of x_i^2 's.

$$(3') S_1 = 16421 - \frac{421^2}{11} = 308.18$$

$$S_2 = 11898 - \frac{361^2}{11} = 141.64$$

(4') Here, it corresponds to the case whether it is considered that the population variance of set (Group) 1 is bigger than that of set (Group) 2,

$$F_0 - \frac{308.18}{141.64} = 2.18$$

(5') Here, we determine $\alpha = 0.05$.

$$(6') f_1 = f_2 = 11 - 1 = 10$$

$$\alpha = 0.05$$

$$F_{20}^{10} (0.05) = 2.98$$

(7') Here, it corresponds to the case whether it is considered that the population variance of set 1 is bigger than that of set 2,

$$2.18 < 2.98$$

therefore, it is not considered that the population mean of a material's tensional strength that is thermal processed in 800 °C is bigger than the one that is thermal processed in 850 °C.

Note⁽¹⁾ In the procedure where methods are separated as (a) and (b), you shall use method

(a) for the case that needs to test whether X_1 's population variance is bigger than

X_2 's, or use the method (b) for the case it is smaller than X_2 's.

⁽²⁾ It is easier for calculation and verification if you group and subtotal for every 5 values.