

Guide to the application of

Addendum modification to involute spur and helical gears

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Foreword

During current work on the overall revision of BS 436¹⁾, the BSI committee concerned with gear standardization gave detailed consideration to the question of addendum modification as applied to the teeth of spur and helical gears generated in accordance with the basic rack defined in that standard. It was realized that this subject could not be presented in the form of a mandatory part of the spur and helical gear standard inasmuch as it is necessary to select addendum modification limits appropriate to specific gearing applications. It was nevertheless considered that the general principles involved in applying addendum modification would provide a useful guide to those concerned with gear design and the application of standards involving the use of this gear tooth form.

The BSI and its committees concerned with gear standardization wish to acknowledge the valuable services and assistance given by Mr. W. O. Davis, M.B.E. in the preparation of this Guide.

Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 18 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

¹⁾ BS 436, "Spur and helical gears" Part 1, "Basic rack form, pitches and accuracy (diametral pitch series)", Part 2, "Basic rack form, modules and accuracy (1 to 50 metric module)".

1 Introduction

This Guide has been prepared under the aegis of Technical Committee MEE/17 — Gears and is concerned with the application of addendum modification to the teeth of involute spur and helical gears with the object of making provision for some design latitude thereby enabling standard cutting tools to be used to their best advantage in covering gear design sizes. The Guide is essentially related to general applications of addendum modification, but particular cases are also considered in the appendix in which the appropriate calculation formulae are given. These formulae, as appropriate, are applicable in terms of both metric and imperial units, it being understood that the units used should be self-consistent.

2 Terminology, definitions and notation

In order to promote international understanding of gear terminology, the terms and definitions given in ISO/R 1122²⁾, “*Glossary of gears — geometrical definitions*” have been applied in this Guide, together with corresponding notation derived from ISO/R 701²⁾, “*International gear notation — symbols for geometrical data*”.

2.1 Definitions

For the purposes of this Guide the following definitions apply:

addendum modification

an enlargement or reduction of gear tooth dimensions produced when the generating basic rack is displaced from its nominal position. It is represented by the coefficient x defined as follows:

addendum modification coefficient (symbol x , x_1 or x_2)

the displacement of the reference plane of the generating basic rack from the reference cylinder of the gear, where x is the equivalent dimension for gears of unit normal module or diametral pitch

NOTE 1 The addendum modification coefficient establishes a datum tooth thickness at the reference circle of the gear, but does not necessarily establish the height of either the reference addendum or the working addendum.

NOTE 2 In a pair of gears the datum tooth thicknesses are those which give zero backlash at the meshing centre distance, in all cases. Allowance for backlash is required in accordance with normal practice for unmodified gears.

2.2 Notation

a	Centre distance (actual)
c	Bottom clearance
d_1	Pinion reference diameter
d_{a1}	Pinion tip diameter
d_2	Wheel reference diameter
d_{a2}	Wheel tip diameter
d_{f1}	Pinion root diameter
d'_{f1}	Pinion effective root diameter
d_{f2}	Wheel root diameter
d_w	Pitch diameter
d_{w1}	Pitch diameter, pinion
d_{w2}	Pitch diameter, wheel
d_R	Diameter of gauge rolls or balls
m_n	Normal module
M_R	Diameter over gauge rolls or balls
N	Number of teeth in gauge span
P_n	Normal diametral pitch
r_b	Base circle radius
r_{b1}	Base circle radius, pinion
r_{b2}	Base circle radius, wheel
s_n	Normal arc tooth thickness at reference diameter
s_{n1}	Normal arc tooth thickness at reference diameter, pinion
s_{n2}	Normal arc tooth thickness at reference diameter, wheel
s_{nw}	Normal arc tooth thickness at pitch diameter
s_{nw1}	Normal arc tooth thickness at pitch diameter, pinion
s_{nw2}	Normal arc tooth thickness at pitch diameter, wheel
s_t	Transverse arc tooth thickness at reference diameter
s_{t1}	Transverse arc tooth thickness at reference diameter, pinion
s_{t2}	Transverse arc tooth thickness at reference diameter, wheel
s_{tw}	Transverse arc tooth thickness at pitch diameter
s_{tw1}	Transverse arc tooth thickness at pitch diameter, pinion
s_{tw2}	Transverse arc tooth thickness at pitch diameter, wheel
u	Gear ratio
W	Base tangent length
x	Addendum modification coefficient, general

²⁾ Information which will eventually be incorporated in BS 2519, “*Glossary of terms for toothed gearing*”.

x_1	Addendum modification coefficient, pinion
x_2	Addendum modification coefficient, wheel
Σx	Sum of x_1 and x_2
y	Centre distance modification coefficient
z	Number of teeth
z_1	Number of teeth in pinion
z_2	Number of teeth in wheel
Σz	Sum of number of teeth
Σz_v	Sum of virtual number of teeth
α_M	Transverse contact angle for gauge roll calculation
α, α_n	Normal pressure angle
α_{nw}	Normal working pressure angle
α_t	Transverse pressure angle
α_{tw}	Transverse working pressure angle
β	Helix angle at reference cylinder
β_b	Helix angle at base cylinder
β_w	Helix angle at pitch cylinder
ϵ_α	Transverse contact ratio
ξ	Auxiliary angle for calculation of internal wheel tip diameter

3 Tooth design (general)

By taking full advantage of the adaptability of the involute system, various tooth design features may be obtained. Addendum modification has application as follows and typical cases are illustrated in Figure 1(a), Figure 1(b) and Figure 1(c).

- 1) To avoid undercut tooth profiles.
- 2) To achieve optimum tooth proportions and control of the proportion of receding to approaching contact.
- 3) To adapt a gear pair to a predetermined centre distance without recourse to non-standard pitches.
- 4) To permit a range of working pressure angles to be achieved using standard geometry tools.

4 Tooth design: value of Σx

4.1 Increase in the value of Σx leads to increase of working pressure angle and reduction of contact ratio. In the case of spur gearing the value of contact stress obtained during the phase of single tooth pair engagement is a minimum where Σx has the greatest value which permits a contact ratio exceeding unity (in practice not less than 1.2). Gears so designed tend to be noisier than those having greater contact ratio. In the case of helical gearing having face width sufficient to ensure maintenance of two tooth pair contact, the total length of the contact lines in simultaneous engagement is reduced by increase of Σx . This offsets the effect of increase of pressure angle on contact stress and values of Σx exceeding + 0.4 are not recommended.

4.2 High values of Σx lead to increase in the bending strength of the individual teeth of either spur or helical gears. This increase, however, is offset almost entirely by reduction of contact ratio.

5 Tooth design: division of Σx between pinion and wheel

5.1 The recommended addendum modification for reduction gear pairs is given by the following formula [see also A.2, Formula (2.1)] and this ensures that slightly more than half of the tooth action takes place during receding contact:

$$x_1 = \frac{1}{3} \left(1 - \frac{1}{u} \right) + \frac{\Sigma x}{1 + u} \text{ where } u = \frac{z_2}{z_1}$$

5.2 For cases covering a balance of beam strength, minimal specific sliding or slide-roll ratios and speed increasing gear pairs, reference should be made to the A.2, Formulae (2.2), (2.3) and (2.4).

6 Limiting values for Σx

Maximum and minimum values for Σx are calculable for certain rational limiting conditions as follows:

- 1) Generation by rack form cutter or hob.
- 2) Tip-to-root clearance should not be reduced by more than $0.1m_n$ or $0.1/P_n$ below its nominal value.
- 3) Extended root fillets should not curtail the path of involute contact.
- 4) The contact ratio of spur gears should not be less than 1.2.

These limits, as set out in Chart 1, may be applied to non-critically loaded gear pairs, provided that the relative values of x_1 and x_2 conform at least approximately to one of the formulae (2.1) to (2.4), (see A.2) as appropriate.

7 Limits to the individual values of x_1 and x_2

7.1 Too low a value of x leads to undercut tooth profiles. Spur pinions having fewer than 18 teeth and having the minimum value of x required to avoid undercutting are known as “standard long addendum pinions”. Dimensions of these pinions are given in Table 1. The subsidiary Table 1A has been included to give centre distances, in terms of unit module or diametral pitch, at which standard long addendum spur pinions and wheels having $x_2 = 0$ engage without backlash.

7.2 Too high a value of x leads to excessively narrow tip width, so that shear strength at the tip is less than the beam strength of the tooth. Curves showing the values of x at which untopped teeth have crest width from zero to $0.4m_n$ or $0.4/P_n$ are given in Chart 2.

7.3 It should be noted that gears having a high value of x require engagement over a long span of generating rack to form the teeth. For example, whilst engagement over a two pitch span of generating rack will form the teeth when $x = 0$, approximately four pitches are required when $x = 1.0$ and approximately five and a half pitches when $x = 2.0$.

7.4 When using pinion type (gear shaping) generating tools, the limits for gears cut by rack form tools are generally applicable, provided that the cutter will fully form the roots and tips of the teeth. This is rarely possible when the cutting tool has less than 35 teeth.

8 Application to internal tooth gearing

8.1 Formulae for evaluating x , y and a for external gears are applicable to internal gears if the following conventions are observed:

Diameters and number of teeth of the wheel and the centre distance to be given the negative sign. The addendum modification coefficient x_2 to be considered negative when its sense is to enlarge the wheel.

It should be noted that negative values of Σx lead to increased pressure angle and vice versa.

8.2 A preferred value for x_2 for internal tooth wheels is:

$$x_2 = -0.4$$

Other values may prove more convenient for designs of internal planetary gear trains, or for designs of internal wheel and pinion pairs of close ratio.

8.3 Satisfactory tooth action can be obtained when the values of x_1 , x_2 and y are such that the working pressure angle is not less than $\approx 14^\circ$ or more than $\approx 30^\circ$, provided that the path of contact crosses the line of centres well within the working depth of the teeth and that the contact ratio is not less than 1.1.

8.4 It should be noted that (1) the wheel tip circle diameter needs to conform with one of the formulae (6.7) to (6.10) (see A.6) as applicable in order that tooth tips of the wheel do not foul pinion root fillets, and that (2) interference between wheel and pinion tooth tips prevents the employment of very close gear ratios if the teeth are of standard proportions.

9 Centre distance modification coefficient (Symbol y)

If the centre distance modification coefficient is made equal to the sum of the addendum modification coefficients of pinion and wheel, and has a positive or negative value other than zero, the following conditions apply:

- 1) Parallel axis external gears have backlash.
- 2) Internal gears have interference.
- 3) Crossed axis helical gears have zero backlash.

Datum values of Σx where y is pre-determined, or of y where Σx is pre-determined, which provide zero backlash for parallel axis gear pairs, may be derived from Table 2.

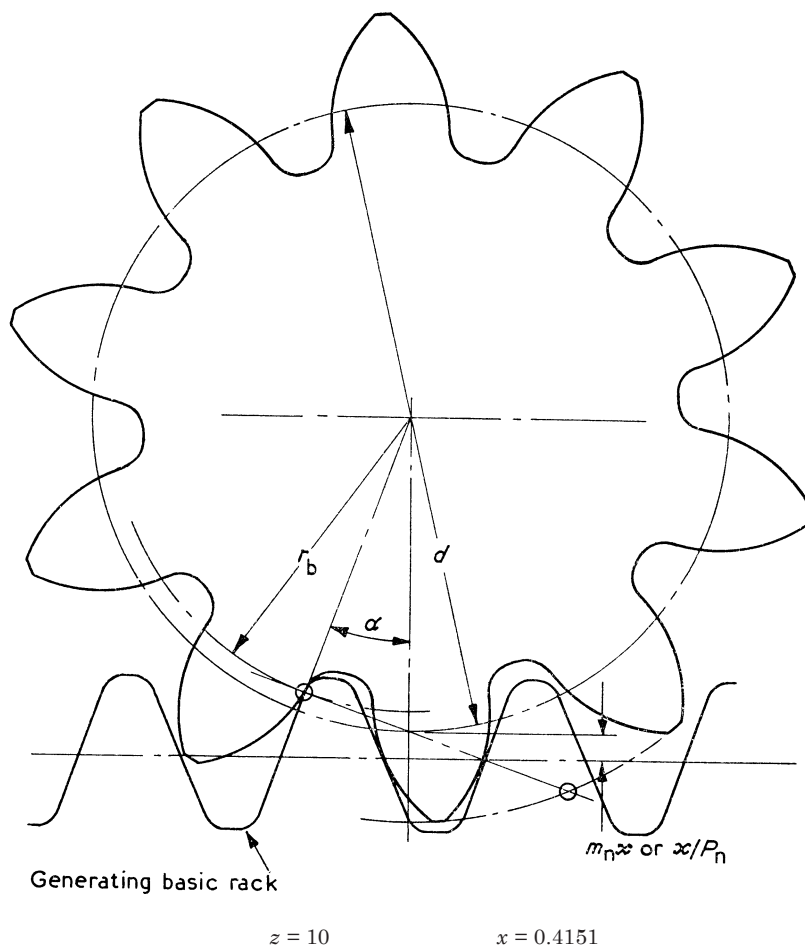


Figure 1(a) — Avoidance of undercut tooth profiles