

General metrology —

Part 4: Practical guide to measurement uncertainty

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Foreword

This Published Document has been prepared by Technical Committee SS/6. This part of PD 6461 fulfils an identified need to provide clear guidance to scientists, engineers and technicians, who might not be fully conversant with detailed statistical techniques, on how to perform an uncertainty evaluation.

Reports on measurement uncertainty have accompanied test results produced by National Standards Laboratories for many years where highly accurate measurements are required. Furthermore, this practice has recently been extended to results produced by accredited calibration and test facilities.

However, as processes become more efficient, performance specifications become tighter and, with the increasing requirement for traceable measurements, more and more engineers and technicians find they are faced with the task of carrying out an uncertainty evaluation on their reported measurement results.

Guidance on measurement uncertainty exists at a very high level in the form of a document entitled *Guide to the expression of uncertainty in measurement* (GUM) and is available from BSI as PD 6461-3:1995.

This document is intended to provide guidance to practising engineers where no other guidance is available. This document is directed at the practising engineer and others who wish to perform a relatively simple evaluation. It does not replace or supersede any published technical or sector-specific standards on the subject, nor does it replace the GUM, which is regarded as the ultimate authority in the area.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii pages 1 to 41 and a back cover.

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1 Scope

This Published Document sets out the procedures to be followed in evaluating the uncertainty in the result of a measurement. It follows the principles set out in the document *Guide to the expression of uncertainty in measurement*, known as the GUM (PD 6461-3). It is intended for use by the practising measurement professional who seeks a practical guide to the application of the GUM's methodology to industrial measurement problems. This Published Document is applicable to situations where a formula or mathematical model is available for the required quantity in terms of input (or influence) quantities. It does not cover circumstances where reproducibility studies are required to provide a full statement of uncertainty.

Where this Published Document stops short of the rigorous approach advocated in the GUM, reference is made to the appropriate clause in the GUM for those requiring details of the full analysis.

Short examples illustrating the individual steps in the procedures are given in the body of the standard and a fuller example of the application of the procedures is given in Annex A.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the cited edition applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

PD 6461-1, *General metrology — Part 1: Basic and general terms* (VIM).

PD 6461-3, *General metrology — Part 3: Guide to the expression of uncertainty in measurement* (GUM).

3 Terms and definitions

For the purposes of this Published Document, the terms and definitions given in PD 6461-1 (VIM) and PD 6461-3 (GUM) and the following apply. References are given to the appropriate clauses of the VIM or the GUM. A small number of additional terms are used in this Published Document and these are defined in 3.2, 3.4 and 3.11 to 3.13.

3.1

uncertainty

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand (VIM 3.9)

3.2

relative uncertainty

uncertainty expressed as a proportion or percentage of the measurement result

3.3

standard uncertainty

$u(x)$

uncertainty of the result of a measurement expressed as a standard deviation (GUM 2.3.1)

3.4

relative standard uncertainty

$u^*(x)$

standard uncertainty expressed as a proportion or percentage of the measurement result

3.5

combined standard uncertainty

$u_c(y)$

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities (GUM 2.3.4)

3.6**expanded uncertainty**

$$U = ku_c(y)$$

quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand (GUM 2.3.5)

NOTE The fraction may be viewed as the coverage probability or the level of confidence of the interval.

3.7**coverage factor** **k**

numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty (GUM 2.3.6)

NOTE k is typically in the range 2 to 3.

3.8**type A evaluation of uncertainty**

method of evaluation of uncertainty by the statistical analysis of a series of observations (GUM 2.3.2)

3.9**type B evaluation of uncertainty**

method of evaluation of uncertainty by means other than the statistical analysis of a series of observations (GUM 2.3.3)

3.10**sensitivity coefficient** **c_i**

equal to the partial derivative, $\partial f / \partial X_i$, of the functional relationship, f , between the input quantities, $x_1 \dots x_N$, and the output quantity, y , evaluated at the nominal values of the input quantities (GUM 5.1.3)

NOTE c_i is used as a weighting factor used to convert uncertainty in an input quantity, x_i , into the resulting uncertainty in the output quantity, y .

3.11**relative sensitivity coefficient** **c_i^***

weighting factor used to convert relative uncertainty in an input quantity, x_i , into the resulting relative uncertainty in the output quantity, y

NOTE c_i^* is equal to $c_i x_i / y$.

3.12**contribution to uncertainty**

product of the standard uncertainty $u(x_i)$ in input quantity x_i and the absolute value of the sensitivity coefficient c_i

3.13**contribution to relative uncertainty**

product of the relative standard uncertainty $u^*(x_i)$ in input quantity x_i and the absolute value of the relative sensitivity coefficient c_i^*

4 Symbols

α_i	estimated half-width of a component of uncertainty associated with input estimate x_i
b_1, b_2	lower and upper bounds respectively of an asymmetric distribution (8.3.6)
c_i	sensitivity coefficient for input quantity x_i
c_i^*	relative sensitivity coefficient for input estimate x_i
f	mathematical relationship between the measurand Y and the input quantities X_i on which Y depends
$\partial f / \partial x_i$	partial derivative with respect to input quantity X_i of the mathematical relationship f between the measurand and the input quantities, given by evaluating $\partial f / \partial X_i$ at $X_1 = x_1, \dots, X_N = x_N$
k	multiplier or coverage factor used to calculate the expanded uncertainty U
n	number of repeat readings or observations
N	number of input quantities X_i on which the measurand depends
$s(x_j)$	experimental standard deviation of a random variable X_j determined from n repeated observations
$s(\bar{x})$	experimental standard deviation of the arithmetic mean $\bar{x}$
U	expanded uncertainty associated with the output estimate y
U^*	relative expanded uncertainty associated with the output estimate y
$u(x_i)$	standard uncertainty associated with the input estimate x_i
$u_c(y)$	combined standard uncertainty associated with the output estimate y
$u^*(x_i)$	relative standard uncertainty associated with the input estimate x_i
$u_c^*(y)$	combined relative standard uncertainty associated with the output estimate y
$x_{i,j}$	j -th observation of random quantity X_i
$\bar{x}_i$	arithmetic mean or average of n repeated observations $x_{i,j}$, $j = 1, \dots, n$ of input quantity X_i or randomly varying quantity X_i
x_i	estimate of the input quantity X_i
y	estimate of the measurand Y
Δx_i	change in x_i used to calculate the sensitivity coefficient numerically
Δy	change in y determined when calculating the sensitivity coefficient numerically
v	degrees of freedom in the experimental standard deviation of a random variable x determined from n repeated observations. v is equal to $n - 1$

5 The evaluation of uncertainty

The analysis of most measurement uncertainties can be reduced to a common, simple and logical, step-by-step procedure. This procedure is equally applicable to a simple measurement such as the length of a machined component and to an evaluation of the uncertainty in the National Standard for length; the difference between the two analyses lies only in the number, nature and magnitude of the uncertainties to be considered.

The procedure comprises the following steps:

- definition of the relationship between all the input measurements and the measurand (Clause 6);
- for each quantity, compilation of a list of all the factors that contribute to uncertainty in that input (Clause 7);
- for each of the input uncertainty sources, estimation of the magnitude of the uncertainty (Clause 8);
- from the relationship defined in step a), estimation of the effect that each quantity has on the measurement result (Clause 9);
- combining the uncertainties in all the input quantities to obtain the uncertainty in the output quantity (Clause 10);
- expressing the expanded uncertainty as an interval about the measurement result within which it is anticipated, with a stated level of confidence, that the measurand will lie (Clause 11).

6 Defining the relationship between the input quantities and the output quantity

6.1 It is rare that the required value of a measurand, Y , is obtained directly and in most cases the result is obtained from a measurement of a number, N , of other measurements, $X_1, X_2, \dots, X_N$, that are combined through a functional relationship, f , to arrive at the required value. Thus

$$Y = f(X_1, X_2, \dots, X_N)$$

In practice the input quantities, $X_1, X_2, \dots, X_N$, can only be estimated and the result of the measurement is an estimate of Y given by

$$y = f(x_1, x_2, \dots, x_N)$$

The uncertainties, $u_{x,1}, u_{x,2}, \dots, u_{x,N}$, in the input estimates then give rise to the uncertainty in the output estimate, y .

6.2 The functional relationship, or model, can vary enormously in its complexity, from that of the simple traditional balance in which the measurand is compared directly with reference weights to that of a multiphase flow meter measuring a flow of oil, water and gas from an oil well and requiring measurements of pressures, temperatures and densities to derive the flow rates. However complex the relationship, the purpose of setting it down at the start of the uncertainty analysis is to draw up a definitive list of the input quantities. In some cases, the relationship will be defined by the theory of the instrument; in others, it will be defined by the calibration process and the resulting calibration equation; in the most complex cases the calibration process may involve computational techniques such as neural networks and the relationship will then be embedded within the resulting software; the input quantities are then, as with any calibration, the inputs to the calibration process.

6.3 In many cases, it will become apparent that the functional relationship involves quantities that themselves are the result of several measurement inputs. At first sight this might appear to complicate the analysis of the measurement process but, in reality, it is easily handled by reviewing each quantity and defining its relationship to its own inputs. In this way, the most complex measurement process can be sub-divided into manageable stages and the result of each sub-analysis can be carried forward to the next stage. An example of this process is given in Annex A.

NOTE Information on reproducibility studies is available in ISO/TS 21748.

7 Listing the factors affecting each input quantity

7.1 General

Modern industrial instrumentation covers the whole range of scientific measurement, from length, mass and time to electrical capacitance and radiation intensity. The presentation here of a definitive list of sources of uncertainty is therefore not possible. However, it is vital to a rigorous uncertainty analysis that it is recognized that the sources extend far beyond the simple reading of an instrument and indeed in a modern well-calibrated instrumentation system the basic instrument uncertainties might well prove to be relatively small in comparison with some of the other sources of uncertainty. The following subclauses (7.2 to 7.10) provide an indication of the sorts of influences that need to be considered in listing the sources of uncertainty in each quantity.

7.2 The measured quantity itself

Some quantities such as mass might well be stable during the measurement process but others can vary considerably; for example, a flowrate might vary as a pump speed varies due to fluctuations in the electrical input to the motor, or a temperature might be affected by changes in some process upstream of the measurement point. In such cases, it is necessary to consider how well the value measured represents the mean of the fluctuating signal. When dealing with periodic signals, it is important to consider the frequency of the variation and ensure that the measurement frequency is significantly greater (typically at least four times as great) to ensure that the measurement is not biased, for example, by repeatedly taking the readings close to the maximum or minimum of the signal. Batch sampling presents special problems and when, for example, drawing a sample of chemical feedstock for a purity analysis, it is important to consider how well the sample taken represents the bulk of the material.

7.3 The instrument itself

An instrument only provides an estimate of the value of its input. The quality of that estimate depends on many aspects of the instrument; for example, non-linearity, hysteresis, resolution, bias, and drift. Some of these, such as bias, drift and non-linearity, can be reduced by calibration (see 7.4). Resolution can be subsumed into the spread of values obtained to define a fluctuating signal. Others, like hysteresis, can be difficult to eliminate. Even when an instrument is carefully calibrated, it is important to recognize that the drift process continues and drift between calibrations should be considered.

7.4 Calibration uncertainties

When an instrument is first brought into service, the relationship between the quantity being measured and the instrument output is defined in a calibration experiment. This may range in rigour and complexity from a simple two-point check, zero and one other point, at the end of the production line to a careful definition of a complex calibration curve through many reference points and with many repeats. However, as noted in 7.3, even when defined by the most rigorous initial calibration, the relationship between input and output will change with time and use and, as a result, uncertainties will increase. These can be reduced by routine periodic calibrations, the purpose of which is to re-establish the relationship between input and output and so replace the unknown uncertainties of a measurement with a quantifiable uncertainty from a defined and controlled calibration experiment. In transferring the uncertainty of the calibration process to the service instrument it should be recognized that, no matter how rigorous the calibration, the uncertainty of the service instrument can never be smaller than the uncertainty of the calibration. It is therefore important that the uncertainty of the calibration is defined, stated on the calibration certificate and included in the uncertainty budget when the instrument is used in normal service.

7.5 Environmental influences

A great many instruments involve electrical measurements and so can be influenced by electromagnetic interference and instabilities in the electrical input. Meters can also be affected by changes in pressure, temperature, humidity and vibration.

7.6 Usage effects

The calibration process is frequently carried out in a test laboratory where the instrument is not necessarily subjected to many of the factors that it will encounter in service. For example, a temperature probe will be calibrated in an indoor environment but, in service, may be employed in a position exposed to the weather and so may be subjected to cooling by wind or heating by solar radiation; or, when being calibrated, a flow meter will be presented with ideal flow conditions, whereas, in service, pipe bends or valves upstream of the meter may introduce asymmetry or swirl into the flow and these may affect its performance. *In situ* calibration may be possible in some situations but, where this is not the case, it is important either to replicate the service conditions in the calibration laboratory or to have a good understanding of the additional effects, so that the effect of the differences between the calibration situation and in-service conditions can be assessed.

7.7 The measurement process

Quality procedures should define the way in which measurements are performed, but there might be circumstances in which measurements have to be taken at a different time or with a different instrument from that defined in the procedures. Such changes in procedure will result in a change in uncertainty.

7.8 Data acquisition and processing

Computer-based data acquisition systems introduce their own sources of uncertainty; analogue-to-digital converters introduce a resolution uncertainty, inter-channel multiplexers have a settling time that can introduce uncertainties into high-speed data logging. Round-off with computer software routines can introduce uncertainty in the processing of data, as can the values of physical constants taken from reference books. When physical property data is taken from published tables or equations, it should be remembered that the uncertainties of the original experiments and of the fitted equations are carried forward into the calculations.

7.9 Operator effects

Although automated data-logging is now common practice, there are still many situations where instruments are read by a human operator. Faced with a flickering needle or a changing digital display every operator will have his own technique for determining the mean reading. Even the differences in parallax resulting from changes in eye level can introduce variations between the results obtained by different operators.

7.10 Additional sources

The classifications of uncertainty sources listed in 7.2 to 7.9 are intended only as a guide to the possible sources of uncertainty within any measurement. In undertaking an uncertainty analysis, it is important to consider whether there are special sources that pertain to the particular measurements being made.

8 Assessing the magnitude of the uncertainty in each source

8.1 General

The sources of uncertainty listed in 7.2 to 7.10 are many and disparate and there is no one method that can be used to assign numerical values to such a wide variety of effects. The GUM recognizes two basic approaches. The first is based on a statistical analysis of a series of readings: an approach referred to in the GUM as a Type A evaluation of uncertainty (GUM 4.2). The second uses past experience and professional judgement to arrive at numerical limits: an approach referred to in the GUM as a Type B evaluation of uncertainty (GUM 4.3). The GUM definitions will be used in the remainder of this Guide when referring to methods of evaluating uncertainties.

8.2 Type A evaluation of uncertainties

8.2.1 Best estimate of a measurand from a series of readings

When the value of a measurand is determined by taking a series of measurements, the best estimate of the measurand is taken to be the arithmetic mean of the values recorded. Thus, if the readings of measurand, X_i , are $x_{i,1}$, $x_{i,2}$, $x_{i,3}$, ..., $x_{i,j}$, ... $x_{i,n}$, the best estimate of X_i is

$$\bar{x}_i = \frac{x_{i,1} + x_{i,2} + \dots + x_{i,n}}{n} = \frac{1}{n} \sum_{j=1}^n x_{i,j} \quad (1)$$

Example 1: The diameter of a cylinder is determined by measuring the diameter across two mutually perpendicular diameters at each of three planes equally spaced along the axis of the cylinder. The values obtained are

83.17 mm, 83.19 mm, 83.16 mm, 83.17 mm, 83.18 mm, 83.17 mm

The best estimate of the mean diameter is obtained from the values recorded as follows:

$$\begin{aligned} \bar{x} &= \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{j=1}^n x_j \\ &= \frac{1}{6} (83.17 + 83.19 + 83.16 + 83.17 + 83.18 + 83.17) \\ &= 83.173 \text{ mm} \end{aligned}$$

Example 2: The temperature within an environmental test chamber is recorded at two-hourly intervals during a 24 h test cycle. The values obtained are

20.9 °C, 19.1 °C, 20.4 °C, 21.2 °C, 20.5 °C, 20.6 °C, 18.6 °C, 19.8 °C, 21.1 °C, 18.8 °C, 19.6 °C, 19.4 °C

The best estimate of the mean temperature during the 24 hour period is obtained from the values recorded as follows:

$$\begin{aligned}\bar{x} &= \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{j=1}^n x_j \\ &= \frac{1}{12} (20.9 + 19.1 + 20.4 + \dots + 19.6 + 19.4) \\ &= 20.00 \text{ } ^\circ\text{C}\end{aligned}$$

8.2.2 Standard deviation of a series of measurements

While the mean of the values recorded can be expected to give a better estimate of the true value than any individual value, the fact that a range of values was obtained raises the question of whether the same mean would have been obtained if more values had been recorded and therefore how well the current mean represents the true value. What is therefore needed is a definition of an interval about the current mean within which the true value might be expected to lie. The range could be defined by the highest and lowest values recorded but this would ignore the majority of the values available and would also be vulnerable to errors, which are likely to result in extreme high or low values. The statistical approach is to calculate the deviation d_i of each value from the mean of the set, $d_i = x_i - \bar{x}$, and from these values to calculate a statistical parameter called the estimated standard deviation, s , of the sample (or set of values).

$$s(x) = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (2)$$

The use of $n - 1$ in the denominator corrects for the fact that the value of s is only an estimate of the true value of the standard deviation and is based on a limited sample of readings.

Note that some spreadsheet packages can use a further form of equation (2) that is unsuitable for use when the scatter is small and can lead to significant errors when the experimental scatter is less than one part per million of the mean. The solution is to calculate the individual deviations from the mean and use the second form of equation (2).

Example 3: The standard deviation of the cylinder diameters of example 1 is obtained from the second form of equation (2) as follows

$$\begin{aligned}s(x) &= \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \\ &= \sqrt{\frac{(83.17 - 83.173)^2 + (83.19 - 83.173)^2 + \dots + (83.17 - 83.173)^2}{6-1}} \\ &= 0.0103 \text{ mm}\end{aligned}$$

Example 4: The standard deviation of the measurements of the temperature in the environmental chamber of example 2 is obtained from the second form of equation (2) as follows

$$\begin{aligned}
 s(x) &= \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \\
 &= \sqrt{\frac{(20.9 - 20.0)^2 + (19.1 - 20.0)^2 + \dots + (19.4 - 20.0)^2}{12-1}} \\
 &= 0.905 \text{ } ^\circ\text{C}
 \end{aligned}$$

In most industrial measurement situations s will be obtained from a special function on a calculator or from a function within a spreadsheet program. Care is needed when applying these functions as the alternative form using n as the denominator is often also available to give the standard deviation assuming that the set of data form the entire population – a parameter required for entirely different statistical purposes. If it is unclear which function is available, a useful check is to calculate the standard deviation of the numbers 1 and 2: if the answer given is 0.707, the function is the correct one, but the answer 0.5 indicates that the n denominator is being used and all calculations of the standard deviation will need to be corrected by multiplying by $\sqrt{\frac{n}{n-1}}$.

In many measurement situations it is easier to envisage the standard deviation when it is expressed as a proportion of the mean. This statistic is called the coefficient of variation CV and is defined as

$$CV = \frac{s(x)}{\bar{x}} \quad (3)$$

It can also be expressed as the percentage deviation or relative deviation RD,

$$RD = \frac{100s(x)}{\bar{x}} \% \quad (4)$$

Although the expression of the standard deviation in relative terms can be of value in many measurements, it should be recognized that some measurements, such as temperatures, have arbitrary zero points and these may lead to nonsensical relative deviations (see example 6).

Example 5: Consider the series of temperature measurements in example 2 with a mean value of 20 °C and a standard deviation of 0.9 °C. The relative deviation would then be $(0.9/20) \times 100 = 4.5 \%$. But if the data had been recorded in Kelvin, the mean value would have been $(20 + 273.15) = 293.15 \text{ K}$ and the standard deviation would have remained at 0.9 °C, giving a relative deviation of $(0.9/293.15) \times 100 = 0.31 \%$.

Example 6: The environmental test chamber of example 2 is used for a low temperature test and the temperature readings are

0.9 °C, −0.4 °C, 1.1 °C, −0.9 °C, 0.5 °C, 0.4 °C, 0.6 °C, 1.2 °C, −1.4 °C, −0.6 °C, −0.2 °C, −1.2 °C

With a mean of 0 °C and a standard deviation of 0.9 °C, the relative deviation would be $0.9/0 \times 100 = \infty \%$.

8.2.3 Standard uncertainty of a mean

The standard deviation of the sample provides a statistical description of the variability of the data values within the sample and, by implication, an estimate of the variability of the data within any other sample drawn from the same overall population. So that, for example, if the cylinder of example 1 were to be re-measured across a different set of diameters one might expect the values to have roughly the same variability as those already recorded. However, the means of the two sets of data might reasonably be expected to vary to a much smaller extent and this is reflected in the statistical calculation of the standard deviation of the mean given by

$$s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \quad (5)$$

The statistical dispersion of likely values for the mean defined by the standard deviation is then taken to be the standard uncertainty of the mean. Thus,

$$u(\bar{x}) = s(\bar{x}) \quad (6)$$

Example 7: The standard uncertainty of the mean diameter of the cylinder of example 1 is calculated as follows

$$\begin{aligned} u(\bar{x}) &= s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \\ &= \frac{0.0103}{\sqrt{6}} \\ &= 0.0042 \text{ mm} \end{aligned}$$

Example 8: The standard uncertainty of the mean test temperature of example 2 is calculated as follows

$$\begin{aligned} u(\bar{x}) &= s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \\ &= \frac{0.905}{\sqrt{12}} \\ &= 0.261 \text{ }^{\circ}\text{C} \end{aligned}$$

8.2.4 Standard uncertainty of a single value

In some measurement situations there might only be a small number of observations of the measurand. This subclause deals with single observations; the treatment of small samples is dealt with in 8.2.8. With a single observation, the evaluation of the uncertainty can be based on measurements taken earlier, provided the conditions of the measurement remain unchanged. It might be, for example, that a sample component is subjected to detailed measurement but that components from the production line are subjected to only a single measurement. In this case, the standard uncertainty is derived from the detailed measurement set as follows: the standard deviation of the set of measurements is obtained from equation (2) using n for the number of measurements in the set. The standard uncertainty of the mean is then obtained from equations (5) and (6) using $n = 1$ as the mean is that of a single value.

Example 9: Having made the detailed set of measurements of cylinder diameter set out in example 1, the inspection manager decides that he will accept a larger uncertainty for an individual cylinder from the production line and orders that all production cylinders will be measured across only one diameter. The standard uncertainty of the diameter of the production cylinders is then calculated as follows:

$s(x)$ is obtained from the data of examples 1 and 3 as 0.0103 mm and

$$\begin{aligned}u(\bar{x}) &= s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \\&= \frac{0.0103}{\sqrt{1}} \\&= 0.0103 \text{ mm}\end{aligned}$$

Example 10: The research manager responsible for the environmental test chamber of example 2 decides that the effort involved in taking temperatures from the chamber at two-hourly intervals throughout the night is unjustified and that it will be sufficient to take a single reading of the temperature at the end of the evening shift. This increases the uncertainty of the test temperature, which is calculated as follows:

$s(x)$ is obtained from the data of examples 2 and 4 as 0.905 and

$$\begin{aligned}u(\bar{x}) &= s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \\&= \frac{0.905}{\sqrt{1}} \\&= 0.905 \text{ }^{\circ}\text{C}\end{aligned}$$

8.2.5 Degrees of freedom in a standard deviation

As described in 8.2.2, the standard deviation is a statistical description of the scatter of the data and, as with any description, the more data the description is based on the more reliable the description will be. The reliability of the standard deviation as a description of the scatter is defined by a statistic called the degrees of freedom, which is one less than the number of data points, thus

$$\nu = n - 1 \tag{7}$$

Example 11: The degrees of freedom in the standard deviation of the cylinder diameter measurements of example 3 are calculated as follows

$$\begin{aligned}\nu &= n - 1 \\&= 6 - 1 \\&= 5\end{aligned}$$

Example 12: The degrees of freedom in the standard deviation of the temperature measurements in the environmental chamber of example 4 are calculated as follows

$$\begin{aligned}\nu &= n - 1 \\&= 12 - 1 \\&= 11\end{aligned}$$