

Stresses in horizontal cylindrical pressure vessels supported on twin saddles: a derivation of the basic equations and constants used in G.3.3 of BS 5500:1982

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Foreword

This Published Document has been prepared under the direction of sub-committee PVE/-/5 the functional committee responsible for advising the Pressure Vessel Standards Committee on design methods.

It provides a commentary on the background to the requirements, for supports and mountings for horizontal pressure vessels specified in **G.3.3** of BS 5500:1982 as well as the derivation of the associated equations, with a view to giving designers a clearer insight into the requirements of this particular clause.

The Committee wishes to acknowledge the fundamental contribution of Dr A S Tooth as the primary author of the text: Dr Tooth (Department of Mechanics of Materials, University of Strathclyde) is presently the Chairman of a Specialist Study Working Party set up by PVE/-/5 to investigate the design of saddle supports.

NOTE The numbers in square brackets used throughout the text of this document relate to the bibliographic references given in Appendix A.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 38, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

0 Introduction

The design of horizontal vessels supported on twin saddles has been dealt with by several authors over the years [1 to 12]. However, the treatment given in BS 5500 is essentially the work of L.P.Zick [13], who used a semi-empirical approach modified beam and ring analysis so that the mathematical model used for the vessel agreed with the experimental results he had available. More recent work [6,10] has indicated that Zick's treatment for the vessel full of fluid predicts stresses which are in reasonable agreement with the experimental values when a very flexible saddle is employed. However, when the saddle is rigid the treatment underestimates the peak stresses which actually occur at the horn.

In the future it is hoped to improve the design approach given in BS 5500 to reflect the more recent theoretical and experimental analyses. At this time, however, this work is not yet complete and therefore such a major revision has to wait. Certain minor modifications and extensions have been carried out by the BSI Specialist Study Working Party SSWP8. To allow these to be made it was necessary to derive the basic equations used in BS 5500 from first principles. The approach used by Zick and presented in BS 5500, is not always wholly acceptable to the committee so it is not to be assumed that this document represents an improvement on the previous work. The aim is merely to document the derivation of the equations so that the essential background to **G.3.3** of BS 5500:1982 is fully appreciated and available to the designer.

BS 5500 has been updated in June 1982 based on experience with large diameter thin walled vessels with diameter to thickness ratios up to the order of 1250 : 1. In these cases the most arduous conditions can occur during filling. However, the methods presented, based on the full condition, produce designs which are satisfactory for the partially full condition.

1 Scope

This Published Document gives a derivation, from first principles, of the equations used in **G.3.3** of BS 5500:1982 describing the stresses in horizontal pressure vessels supported on twin saddles. It does not comment on the design aspects of the requirements of **G.3.3**, since these are considered to need no further clarification.

2 Reference

The title of the publication referred to in this Published Document is given on the inside back cover.

3 Notation

For the purposes of this document the following notation is used.

L	Parallel length of the cylinder
A	Distance from saddle centre to end of parallel length
b	Depth of head
r	Mean radius of the vessel
t	Thickness of vessel
b_1	Width of the saddle
t_e	Thickness of vessel head (or dished ends)
r_2	Radius of the centroid of the ring supported vessel
γ_F	Specific weight of the fluid
γ_C	Specific weight of the dished end or cylinder material
l	Length of cylinder equivalent to hemispherical end
$2W_1$	Total weight of the vessel and contents
w	Uniformly distributed load
α_1, α_2	Angles to element (see Figure 2 and Figure 4)
Δ	Angle to effective part of shell (see Figure 4)
θ	Saddle angle
β	$\left(180 - \frac{\theta}{2}\right)$ or in Figure 31, $\beta = \pi - \phi_1$
ϕ, ψ	Angles to element from top dead centre

σ_x	Longitudinal stress
$\tau_{x\phi}$	Shearing stress on the $x \sim \phi$ plane
V	Vertical shear force
M	Bending moment
I_{00}, I_{NA}, I_{xx}	Second moments of area about axis 00, neutral axis and axis xx, respectively
M_3	Bending moment at mid-span
M_4	Bending moment at the support
f_1	Stress at the highest point of the mid-span position
f_2	Stress at the lowest point of the mid-span position
$\bar{y}$	Distance between the axis 00 and the neutral axis
y_c	Distance to the outermost fibres at the bottom of the vessel
y_T	Distance to the outermost fibres at the top of the Δ portion
f_3	Stress at the top of the effective Δ portion
f_4	Stress at the lowest point

4 Longitudinal bending moments

4.1 General. To determine expressions for the longitudinal bending moments the approach adopted is to consider the vessel as a beam supported at the saddles (see Figure 1). Consideration is given to the additional moment caused by the weight of the dished ends and by the hydraulic pressure on the ends.

4.2 Weight of the dished ends. A cylindrical vessel with dished ends may be treated as an equivalent cylinder having a length equal to $L + (4b)/3$. This approximation assumes that the weight of the head and the fluid contained in the head is equal to two-thirds of the weight of a cylinder of length b and the fluid contained in it. This approximation can be shown to be the case as far as the volume (i.e. the weight of fluid) is concerned, though not for the weight of the end itself. For example in the case of an hemisphere:

$$\text{Enclosed volume} = \frac{2}{3}\pi b^3 = \frac{2}{3}\pi r^3;$$

$$\text{Weight of fluid} = \frac{2}{3}\pi r^3 \gamma_F;$$

$$\text{Surface area} = 2\pi b^2 = 2\pi r^2;$$

$$\text{Weight of end} = 2\pi r^2 \gamma_C$$

For the equivalent cylinder:

$$\text{Enclosed volume} = \pi r^2 l$$

$$\text{Enclosed weight} = \pi r^2 l \gamma_F$$

$$\text{Surface area} = 2\pi r l + \pi r^2$$

$$\text{Weight of cylinder and flat end} = (2\pi r l + \pi r^2) \gamma_C$$

Equating the total weights:

$$\frac{2}{3}\pi r^3 \gamma_F + 2\pi r^2 \gamma_C = (2\pi r l + \pi r^2) \gamma_C + \pi r^2 l \gamma_F$$

$$\frac{2}{3}\pi r^3 \gamma_F + \pi r^2 \gamma_C = l(2\pi r \gamma_C + \pi r^2 \gamma_F)$$

$$l = \frac{\frac{2}{3}r^2 \gamma_F + r \gamma_C}{r \gamma_F + 2\gamma_C}$$

neglecting γ_C

$$l = \frac{2}{3}r.$$

The weight of fluid and vessel may be considered to be a uniform load, w , equal to the total weight $2W_1$ divided by the equivalent length, i.e.,

$$w = \frac{2W_1}{\left(L + \frac{4}{3}b\right)} \quad (1)$$

Thus the weight of the ends is $\frac{2}{3}bw$:

$$= \frac{2}{3}b \left(\frac{2W_1}{L + \frac{4}{3}b} \right)$$

This may be considered to act at the centroid, which is assumed to be $\frac{3}{8}b$ from the end of the parallel length.

4.3 Hydrostatic force on the ends. The hydrostatic pressure acts in a direction normal to the surface of the ends and is equal to the normal pressure on a projected area. In this case a suitable projected area is a cross section of the cylinder normal to the horizontal axis. The distribution of pressure is as shown in Figure 2.

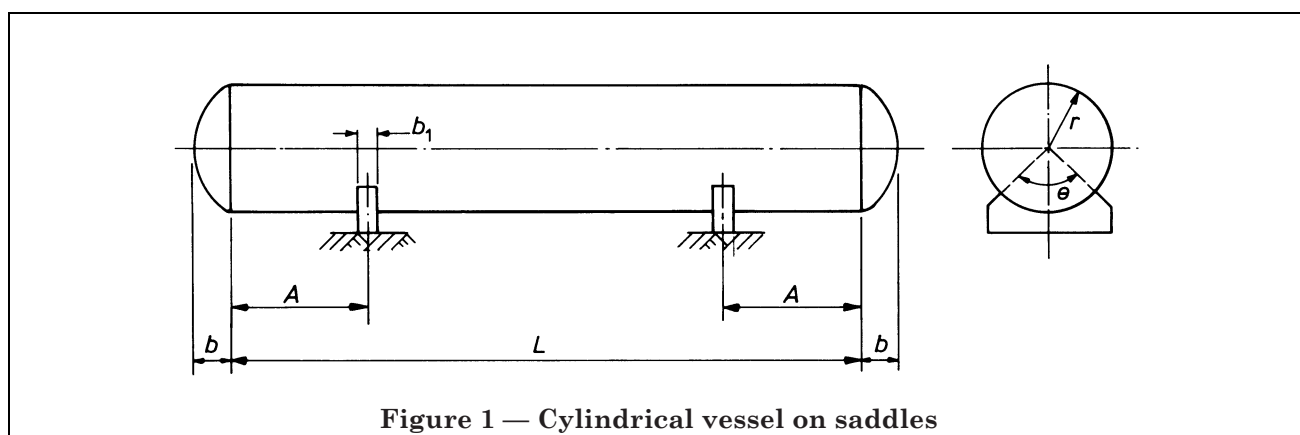


Figure 1 — Cylindrical vessel on saddles

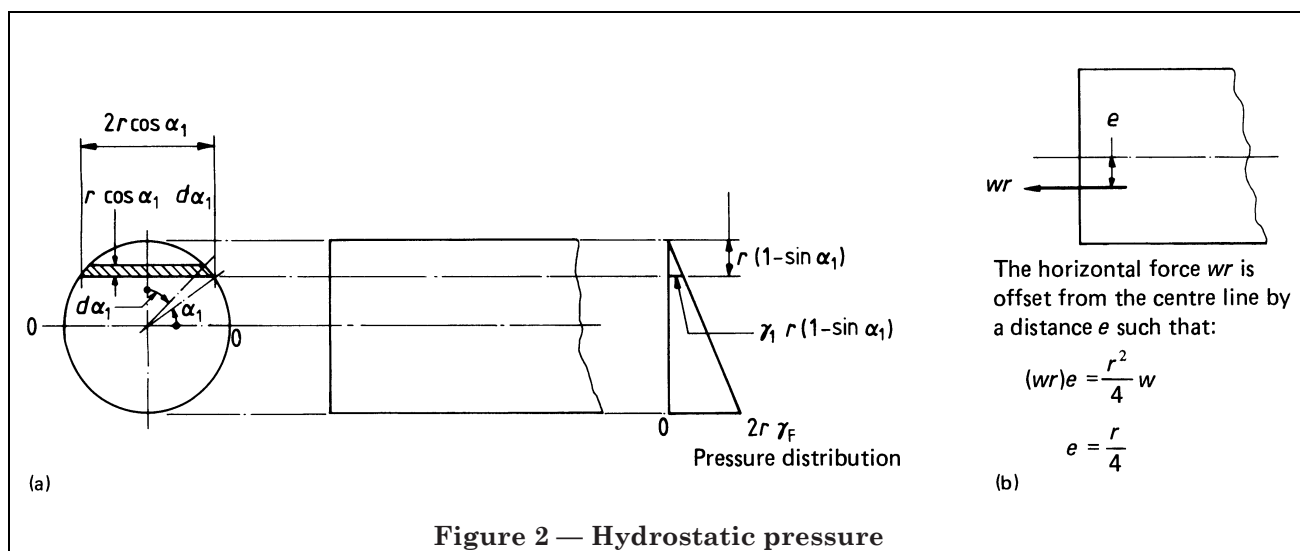


Figure 2 — Hydrostatic pressure

The horizontal force is given by:

$$\begin{aligned} & \int_{-\pi/2}^{\pi/2} 2r \cos \alpha_1 \gamma_F r (1 - \sin \alpha_1) r \cos \alpha_1 d\alpha_1 \\ &= r^3 \gamma_F \pi \\ &= r^3 \pi \left(\frac{w}{\pi r^2} \right) \quad \text{since } w = \pi r^2 \gamma_F \\ &= wr \end{aligned}$$

The moment about the axis 00 is given by:

$$\begin{aligned} & \int_{-\pi/2}^{\pi/2} 2r \cos \alpha_1 \gamma_F r (1 - \sin \alpha_1) r \sin \alpha_1 r \cos \alpha_1 d\alpha_1 \\ &= -\frac{1}{4} r^4 \pi \gamma_F = -\frac{r^2 w}{4} \end{aligned}$$

The system shown in Figure 2(a) is then equivalent to that shown in Figure 2(b).

4.4 Forces on the vessel as a beam

4.4.1 *Bending moment at the support.* The bending moment at the support, M_4 , (see Figure 3) is given by:

$$M_4 = - \left\{ \frac{2}{3} bw \left(A + \frac{3}{8} b \right) + \frac{wA^2}{2} - \frac{r^2 w}{4} \right\}$$

NOTE A positive sign denotes tension on the lower surface.

Substituting $w = \frac{2W_1}{\left(L + \frac{4}{3} b \right)}$ and simplifying gives:

$$M_4 = -W_1 A \left\{ 1 - \frac{1 - \frac{A}{L} + (r^2 - b^2)/2AL}{1 + 4b/3L} \right\} \quad (2)$$

For the purpose of graphical presentation this expression is rearranged in BS 5500:

$$\begin{aligned} M_4 &= \frac{W_1 A}{(1 + 4b/3L)} \left[1 - \frac{A}{L} + \frac{r^2}{2AL} \left\{ 1 - \left(\frac{b}{r} \right)^2 \right\} - \left(1 - \frac{4b}{3L} \right) \right] \\ &= \frac{W_1 A}{C_2} \left(1 - \frac{A}{L} + C_3 \frac{r}{A} - C_2 \right) \end{aligned}$$

where

$$\begin{aligned} C_2 &= (1 + 4b/3L); \\ C_3 &= \frac{r}{2L} \left\{ 1 - \left(\frac{b}{r} \right)^2 \right\} \end{aligned}$$

4.4.2 *Bending moment at mid-span.* The bending moment at mid-span, M_3 , (see Figure 3) is given by:

$$M_3 = -\frac{2}{3} bw \left(\frac{3}{8} b + \frac{L}{2} \right) + \frac{r^2 w}{4} - \frac{wL}{2} \left(\frac{L}{4} \right) + W_1 \left(\frac{L}{2} - A \right)$$

Substituting $w = \frac{2W_1}{\left(L + \frac{4}{3}b\right)}$ and simplifying gives:

$$M_3 = \frac{W_1 L}{4} \left\{ \frac{1 + 2(r^2 - b^2)/L^2}{1 + 4b/3L} - \frac{4A}{L} \right\} \quad (3)$$

As in 4.4.1 the equation is rearranged in BS 5500 as follows:

$$\begin{aligned} M_3 &= W_1 \left[\left\{ \frac{1 + 2(r^2 - b^2)/L^2}{1 + 4b/3L} \right\} \frac{L}{4} - A \right] \\ &= W_1 (C_1 L - A) \end{aligned}$$

where

$$C_1 = \frac{1 + 2(r^2 - b^2)/L^2}{4 + 16b/3L}$$

5 Longitudinal bending stresses due to the bending moments

5.1 Bending stress at mid-span. The longitudinal bending stress at the mid-span position is calculated by assuming that the full vessel section is available and that the section remains circular, i.e. secondary bending is small. This assumption will be adequate for most cases. However, for very thin vessels it is found that the cross section does not remain circular especially so during filling with liquid. Furthermore, the axial membrane compressive forces in the partially full condition are found to be larger than those when the vessel is full. These vessels, therefore, have a tendency to buckle inwards at the location of the liquid height during filling. Despite this, experience has shown that for vessels with diameter : wall thickness ratios up to 1250 : 1, the methods presented herein, based on the full condition and assuming the cross section to remain circular, produce designs which are satisfactory for the partially full condition. Using this assumption the stresses are as follows:

at the highest point the stress, f_1 , is given, by

$$f_1 = -\frac{M_3}{I} r = -\frac{M_3}{\pi r^3 t} r = -\frac{M_3}{\pi r^2 t}; \quad (4)$$

at the lowest point the stress, f_2 , is given by

$$f_2 = +\frac{M_3}{\pi r^2 t} \quad (5)$$

5.2 Bending stress at the saddles. The longitudinal bending stress at the saddles is calculated on the basis that a part of the shell above the saddle is ineffective. As in 5.1 the method has been found applicable for all diameter to wall thickness ratios up to 1250 : 1. The effective part is shown in Figure 4 by $\pm \Delta$. The position of the neutral axis, NA and the second moment of area I_{NA} about this axis must now be found.

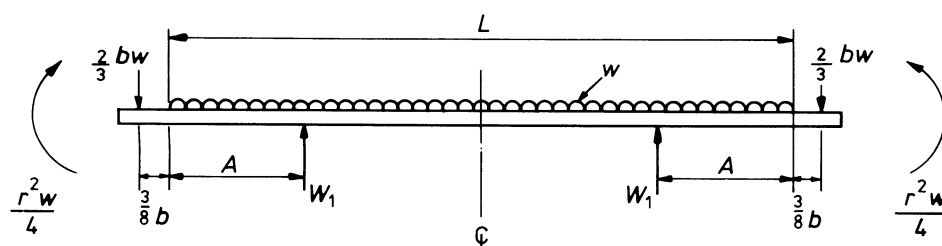


Figure 3 — The vessel as a beam



Taking moments about the axis 00:

$$\bar{y} = \frac{r \sin \Delta}{\Delta}$$

The second moment of area about the axis 00, I_{00} , is given by:

$$I_{00} = \int_{\Delta}^{\Delta} (r \cos \alpha_2)^2 t d\alpha_2 = r^3 t (\Delta + \cos \Delta \sin \Delta)$$

The second moment of area about the neutral axis, I_{NA} , is given by:

$$\begin{aligned} I_{NA} &= I_{00} - 2\Delta r t (\bar{y})^2 = r^3 t (\Delta + \cos \Delta \sin \Delta) - \\ &\quad - 2\Delta r \left(\frac{r \sin \Delta}{\Delta} \right)^2 t \\ &= r^3 t \left(\Delta + \cos \Delta \sin \Delta - \frac{2 \sin^2 \Delta}{\Delta} \right) \end{aligned}$$

5.3 Distances to the outermost fibres. The distance to the outermost fibres at the bottom of the vessel, y_C , is given by:

$$\begin{aligned} y &= y_c \\ &= r - \bar{y} \\ &= r - \frac{r \sin \Delta}{\Delta} \\ &= r \left(1 - \frac{\sin \Delta}{\Delta} \right) \end{aligned}$$

The distance to the outermost fibres at the top of the Δ portion, y_T , is given by:

$$\begin{aligned} y &= y_T \\ &= \frac{r \sin \Delta}{\Delta} - r \cos \Delta \\ &= r \left(\frac{\sin \Delta}{\Delta} - \cos \Delta \right) \end{aligned}$$

Therefore the stress at the lowest point, f_4 , is given by:

$$\begin{aligned} f_4 &= \frac{M_4}{I_{NA}} y_C \\ &= \frac{M_4 r [1 - (\sin \Delta)/\Delta]}{r^3 t \{ \Delta + \cos \Delta \sin \Delta - (2 \sin^2 \Delta)/\Delta \}} \\ &= \frac{M_4}{K_2 \pi r^2 t} \end{aligned} \quad (6)$$

where

$$K_2 = \frac{\Delta + \cos \Delta \sin \Delta - (2 \sin^2 \Delta)/\Delta}{\pi \{1 - (\sin \Delta)/\Delta\}} \quad (7)$$

It should be noted that when the full cross section is available, i.e. $\Delta = \pi$, $K_2 = 1$, equation (6) reduces $f_4 = M_4/(\pi r^2 t)$ consistent with engineer's bending.

The stress at the top of the effective Δ portion, f_3 , is given by:

$$\begin{aligned} f_3 &= \frac{M_4}{I_{NA}} y_T \\ &= - \frac{M_4 r \{ (\sin \Delta)/\Delta - \cos \Delta \}}{r^3 t \{ \Delta + \cos \Delta \sin \Delta - (2 \sin^2 \Delta)/\Delta \}} \\ &= - \frac{M_4}{K_1 \pi r^2 t} \end{aligned} \quad (8)$$

where

$$K_1 = \frac{\Delta + \cos \Delta \sin \Delta - (2 \sin^2 \Delta)/\Delta}{\pi \{ (\sin \Delta)/\Delta - \cos \Delta \}} \quad (9)$$

It should be noted again that if $\Delta = \pi$, $K_1 = 1$, equation (8) reduces to $f_3 = -M_4/(\pi r^2 t)$ consistent with engineer's bending.

From Zick [13], the effective portion is the saddle arc plus $\beta/6$ on each side of the horn, i.e. $\Delta = (\theta/2 + \beta/6)$ where $\beta = 180 - (\theta/2)$, as shown in Figure 5 and the shell is considered effective some distance above the saddle horn.

Using this expression for Δ and equations (7) and (9), it is possible to determine values of K_1 and K_2 for different saddle angles.

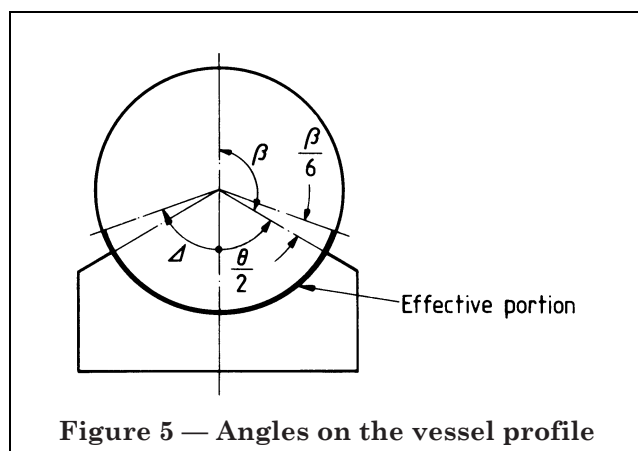


Figure 5 — Angles on the vessel profile

Values of θ , β , Δ , K_1 and K_2 are given in Table 1.

Table 1 — Values of K_1 and K_2

θ	β	Δ	K_2	K_1
150°	105°	92.50°	0.279	0.161
135°	112.5°	86.25°	0.234	0.132
120°	120°	80.00°	0.192	0.107

It can be seen in Table 1 that the top of the “effective” region is near the equator.

When the saddle region is stiffened by rings or by proximity of the ends, then $K_1 = K_2 = 1$.

6 Shearing stresses

6.1 General. Since the bending moment varies along the length of the beam, so also does the longitudinal stress. The effect of this is to introduce longitudinal shear stresses together with complementary shear stresses which occur in the plane of the cross section, as shown in Figure 6.

Considering equilibrium in the x direction:

$$\begin{aligned} & \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} \delta x \right) r d\phi t + \left(\tau_{x\phi} + \frac{\partial \tau_{x\phi}}{\partial \phi} \delta \phi \right) \delta x t - \sigma_x r d\phi t - \\ & - \tau_{x\phi} \delta x t = 0 \\ & r \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{x\phi}}{\partial \phi} = 0 \end{aligned} \quad (10)$$

The normal stress, σ_x , is given by:

$$\sigma_x = \frac{M}{I_{00}} y$$

Differentiating with respect to x :

$$\frac{\partial \sigma_x}{\partial x} = \frac{\partial M}{\partial x} \frac{y}{I_{00}} = V \frac{y}{I_{00}}$$

where

V is the total shear force on the section which is equal to $\frac{\partial M}{\partial x}$.

Equation (10) may then be written:

$$\begin{aligned} \frac{\partial \tau_{x\phi}}{\partial \phi} &= -r \frac{V}{I_{00}} y \\ \tau_{x\phi} &= -\frac{Vr}{I_{00}} \int_0^\phi y d\phi = -\frac{Vr^2}{I_{00}} \sin \phi + \text{constant} \end{aligned}$$

At $\phi = 0$, $\tau_{x\phi} = 0$ and the constant = 0

$$\tau_{x\phi} = -\frac{Vr^2}{I_{00}} \sin \phi \quad (11)$$

Equation (11) is based upon “engineers bending” and neglects secondary effects.

NOTE The negative sign indicates that the shear stress is opposite to that shown in Figure 6(b) i.e. the shear stress on the face nearest to the applied shear force is in the same direction as the force as shown in Figure 6(a).

6.2 Distribution of shear force in the vessel. On the inner saddle side the shear force is given by:

$$W_1 - wA - \frac{2}{3}bw \quad (12)$$