



BSI Standards Publication

Space Engineering — Thermal design handbook

Part 1: View factors

National foreword

This Published Document is the UK implementation of CEN/CLC/TR 17603-31-01:2021.

The UK participation in its preparation was entrusted to Technical Committee ACE/68, Space systems and operations.

A list of organizations represented on this committee can be obtained on request to its committee manager.

Contractual and legal considerations

This publication has been prepared in good faith, however no representation, warranty, assurance or undertaking (express or implied) is or will be made, and no responsibility or liability is or will be accepted by BSI in relation to the adequacy, accuracy, completeness or reasonableness of this publication. All and any such responsibility and liability is expressly disclaimed to the full extent permitted by the law.

This publication is provided as is, and is to be used at the recipient's own risk.

The recipient is advised to consider seeking professional guidance with respect to its use of this publication.

This publication is not intended to constitute a contract. Users are responsible for its correct application.

This publication is not to be regarded as a British Standard.

© The British Standards Institution 2021
Published by BSI Standards Limited 2021

ISBN 978 0 539 16975 1

ICS 49.140

Compliance with a Published Document cannot confer immunity from legal obligations.

This Published Document was published under the authority of the Standards Policy and Strategy Committee on 31 August 2021.

Amendments/corrigenda issued since publication

Date	Text affected
------	---------------

August 2021

ICS 49.140

English version

Space Engineering - Thermal design handbook - Part 1: View factors

Ingénierie spatiale - Manuel de conception thermique -
Partie 1 : Facteurs de vue

Raumfahrttechnik - Handbuch für thermisches Design -
Teil 1: Sichtfaktoren

This Technical Report was approved by CEN on 14 June 2021. It has been drawn up by the Technical Committee CEN/CLC/JTC 5.

CEN and CENELEC members are the national standards bodies and national electrotechnical committees of Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Republic of North Macedonia, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and United Kingdom.



CEN-CENELEC Management Centre:
Rue de la Science 23, B-1040 Brussels

Table of contents

European Foreword.....	7
1 Scope.....	8
2 References	9
3 Terms, definitions and symbols	10
3.1 Terms and definitions	10
3.2 Symbols.....	10
4 Diffuse surfaces.....	12
4.1 General.....	12
4.2 Infinitesimal to finite surfaces	13
4.2.1 Planar to planar.....	13
4.2.2 Planar to spherical	19
4.2.3 Cylindrical to spherical	20
4.2.4 Conical to spherical.....	21
4.2.5 Spherical to spherical.....	23
4.2.6 Ellipsoidal to spherical.....	25
4.2.7 Planar to conical	28
4.3 Finite to finite surface.....	31
4.3.1 Planar to planar. Two-dimensional configurations	31
4.3.2 Planar to planar. Three-dimensional configurations.....	35
4.3.3 Planar to cylindrical. Two-dimensional configurations	46
4.3.4 Planar to cylindrical. three-dimensional configurations	48
4.3.5 Planar to conical	54
4.3.6 Spherical to planar	56
4.3.7 Cylindrical to cylindrical. two-dimensional configurations	62
4.3.8 Cylindrical to cylindrical. axisymmetrical configurations.....	64
4.3.9 Spherical to cylindrical	69
4.3.10 Conical to conical.....	72
4.3.11 Conical to spherical.....	72
4.3.12 Spherical to spherical.....	77

4.4	Additional sources of data.....	80
5	Specular surfaces.....	103
5.1	General.....	103
5.2	Two planar specular surfaces	105
5.2.1	Two-dimensional configurations.....	105
5.2.2	Parallel, directly opposed rectangles of same width and length.....	109
5.2.3	Rectangles of same width and length with one common edge	115
5.3	Planar specular and planar diffuse surface	118
5.3.1	Two dimensional cavities. Cylinders of quadrangular cross section	118
5.4	Non-planar specular surfaces	123
5.4.1	Concentric cylinder or concentric spheres.....	123
	Bibliography.....	125

Figures

Figure 4-1:	Geometric notation for view factors between diffuse surface.	13
Figure 4-2:	Values of F_{12} as a function of x and y . From Hamilton & Morgan (1952) [15].	15
Figure 4-3:	Values of F_{12} as a function of x and y . From Hamilton & Morgan (1952) [15].	17
Figure 4-4:	F_{12} vs. H for different values of dH . Infinitesimal surface to very thin coaxial annulus with finite radius. Calculated by the compiler.....	18
Figure 4-5:	Values of F_{12} vs. λ for different values of H . The analytical expression (case I) is only valid in the shadowed region. Calculated by the compiler.	19
Figure 4-6:	Values of F_{12} as a function of H and λ . Calculated by the compiler.....	20
Figure 4-7:	Values of F_{12} as a function of H and λ , for $\delta = 10^\circ$. Calculated by the compiler.	21
Figure 4-8:	Values of F_{12} as a function of H and λ , for $\delta = 30^\circ$. Calculated by the compiler.	22
Figure 4-9:	Values of F_{12} as a function of H and λ , for $\delta = 50^\circ$. Calculated by the compiler.	22
Figure 4-10:	Values of F_{12} as a function of H and λ , for $\delta = 80^\circ$. Calculated by the compiler.	23
Figure 4-11:	F_{12} as a function of H in the case of an infinitesimal sphere viewing a finite sphere. Calculated by the compiler.	24
Figure 4-12:	F_{12} as a function of angle λ for different values of the dimensionless distance H . Calculated by the compiler.	25
Figure 4-13:	F_{12} as a function of λ and H , for $A = 0,5$. Calculated by the compiler.	26
Figure 4-14:	F_{12} as a function of λ and H , for $A = 1,5$. Calculated by the compiler.	27
Figure 4-15:	F_{12} as a function of λ and H , for $A = 2$. Calculated by the compiler.	27
Figure 4-16:	Values of F_{12} vs. M for different values of L . Configuration 1, $\beta = 10^\circ$. Calculated by the compiler.	29

Figure 4-17: Values of F_{12} vs. M for different values of L . Configuration 1, $\beta = 20^\circ$. Calculated by the compiler.	29
Figure 4-18: Values of F_{12} vs. M for different values of L . Configuration 2, $\beta = 10^\circ$. Calculated by the compiler.	30
Figure 4-19: Values of F_{12} vs. M for different values of L . Configuration 2, $\beta = 20^\circ$. Calculated by the compiler.	30
Figure 4-20: Values of F_{12} as a function of X and Y , for $Z = 0$. Calculated by the compiler.	33
Figure 4-21: Values of F_{12} as a function of X and Y , for $Z = 0,5$. Calculated by the compiler.	33
Figure 4-22: Values of F_{12} as a function of X and Y , for $Z = 1$. Calculated by the compiler.	34
Figure 4-23: Values of F_{12} as a function of X and Y , for $Z = 2$. Calculated by the compiler.	34
Figure 4-24: Values of F_{12} as a function of X and Y , for $Z = 5$. Calculated by the compiler.	35
Figure 4-25: Values of F_{12} as a function of X and Y . Calculated by the compiler.....	36
Figure 4-26: F_{12} as a function of L and N for $\phi = 30^\circ$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].....	39
Figure 4-27: F_{12} as a function of L and N for $\phi = 60^\circ$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].....	39
Figure 4-28: F_{12} as a function of L and N for $\phi = 90^\circ$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].....	40
Figure 4-29: F_{12} as a function of L and N for $\phi = 120^\circ$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].....	40
Figure 4-30: F_{12} as a function of L and N for $\phi = 150^\circ$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].....	41
Figure 4-31: Values of F_{12} as a function of L for different regular polygons. n is the number of sides of the polygon. From Feingold (1966) [11].....	43
Figure 4-32: View factors between different faces of a honeycomb cell as a function of the cell length, L . From Feingold (1966) [11].	44
Figure 4-33: Values of F_{12} as a function of R_1 and R_2 in the case of two parallel coaxial discs. Calculated by the compiler.	46
Figure 4-34: Values of F_{12} and F_{13} as a function of the parameter K . From Jakob (1957) [19].....	48
Figure 4-35: F_{12} as a function of T and R . Calculated by the compiler.	49
Figure 4-36: F_{12} as a function of T and R . Calculated by the compiler.	50
Figure 4-37: F_{12} as a function of T and R . Calculated by the compiler.	50
Figure 4-38: F_{12} as a function of Z , for different values of the dimensionless radius R . Calculated by the compiler.	52
Figure 4-39: F_{12} as a function of R_2 for different values of the sector central angel α . Calculated by the compiler.	57
Figure 4-40: F_{12} as a function of Z for different values of R_2 . Calculated by the compiler.	58

Figure 4-41: F_{12} from a sphere to both sides of a coaxial intersecting disc, vs. H , for different values of R . Calculated by the compiler.....	59
Figure 4-42: F_{12} from a sphere to the upper side of a coaxial intersecting disc, vs. H ($-1 \leq H \leq 1$), for different values of R . Calculated by the compiler.	59
Figure 4-43: Values of F_{12} as a function of Z and R . Calculated by the compiler.....	60
Figure 4-44: F_{12} as a function of x in the case of two infinitely long parallel cylinders of the same diameter. Calculated by the compiler.	64
Figure 4-45: Plot of F_{12} vs. L for different values of R . From Hamilton & Morgan (1952) [15].....	66
Figure 4-46: Plot of F_{22} , vs. R for different values of L . From Hamilton & Morgan (1952) [15].....	67
Figure 4-47: F_{12} as a function of R for different values of Z . Calculated by the compiler.	70
Figure 4-48: Values of F_{12} as a function of H and L_2 for $L_1 = 1$. Calculated by the compiler.	71
Figure 4-49: Values of F_{12} as a function of S and D , for $\delta = 15^\circ$. From Campbell & McConnell (1968) [4].	73
Figure 4-50: Values of F_{12} as a function of S and D , for $\delta = 30^\circ$. From Campbell & McConnell (1968) [4].	74
Figure 4-51: Values of F_{12} as a function of S and D , for $\delta = 45^\circ$. From Campbell & McConnell (1968) [4].	75
Figure 4-52: Values of F_{12} as a function of S and D , for $\delta = 60^\circ$. From Campbell & McConnell (1968) [4].	76
Figure 4-53: Values of F_{12} as a function of S and R . From Jones (1965) [21].	79
Figure 4-54: Values of F_{12} as a function of S and θ . From Campbell & McConnell (1968) [4].	80
Figure 5-1: Values of F_{12} as a function of R and H . Calculated by the compiler.	106
Figure 5-2: Values of F_{11}^s/ρ_2^s as a function of R and H . Calculated by the compiler.....	106
Figure 5-3: Values of F_{12}^s as a function of R for different values of ϕ . Calculated by the compiler.	108
Figure 5-4: Values of F_{11}^s/ρ_2^s as a function of R for different values of ϕ . Calculated by the compiler.	109
Figure 5-5: Values of F_{12}^s as a function of R and X for $Z = 1$. Calculated by the compiler.	110
Figure 5-6: Values of F_{11}^s/ρ_2^s as a function of R and X for $Z = 1$. Calculated by the compiler.	111
Figure 5-7: Values of F_{12}^s as a function of R and X for $Z = 5$. Calculated by the compiler.	111
Figure 5-8: Values of F_{11}^s/ρ_2^s as a function of R and X for $Z = 5$. Calculated by the compiler.	112
Figure 5-9: Values of F_{12}^s as a function of R and X for $Z = 10$. Calculated by the compiler.	112
Figure 5-10: Values of F_{11}^s/ρ_2^s as a function of R and X for $Z = 10$. Calculated by the compiler.	113

Figure 5-11: Values of F_{12}^s as a function of R and X for $Z = 15$. Calculated by the compiler.	113
Figure 5-12: Values of F_{11}^s/ρ_2^s as a function of R and X for $Z = 15$. Calculated by the compiler.	114
Figure 5-13: Values of F_{12}^s as a function of R and X for $Z = 20$. Calculated by the compiler.	114
Figure 5-14: Values of F_{11}^s/ρ_2^s as a function of R and X for $Z = 20$. Calculated by the compiler.	115
Figure 5-15: Values of F_{12}^s vs. aspect ratio, L , for different values of R . $\phi = 30^\circ$. Calculated by the compiler.	116
Figure 5-16: Values of F_{11}^s/ρ_2^s vs. aspect ratio, L , for different values of R . $\phi = 30^\circ$. Calculated by the compiler.	117
Figure 5-17: Values of F_{12}^s vs. aspect ratio, L , for different values of R . $\phi = 45^\circ$. Calculated by the compiler.	117
Figure 5-18: Values of F_{12}^s and F_{11}^s/ρ_2^s vs. aspect ratio, L , for the limiting values of ϕ . Calculated by the compiler.	118
Figure 5-19: Values of F_{11}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	120
Figure 5-20: Values of F_{12}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	121
Figure 5-21: Values of F_{31}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	121
Figure 5-22: Values of F_{32}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	122
Figure 5-23: Values of F_{33}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	122
Figure 5-24: Values of F_{34}^s vs. ϕ for different values of the specular reflectance, ρ^s . Calculated by the compiler.	123

European Foreword

This document (CEN/CLC/TR 17603-31-01:2021) has been prepared by Technical Committee CEN/CLC/JTC 5 "Space", the secretariat of which is held by DIN.

It is highlighted that this technical report does not contain any requirement but only collection of data or descriptions and guidelines about how to organize and perform the work in support of EN 16603-31.

This Technical report (TR 17603-31-01:2021) originates from ECSS-E-HB-31-01 Part 1A .

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association.

This document has been developed to cover specifically space systems and has therefore precedence over any TR covering the same scope but with a wider domain of applicability (e.g.: aerospace).

This document is currently submitted to the CEN CONSULTATION.

1

Scope

In this Part 1 of the spacecraft thermal control and design data handbooks, view factors of diffuse and specular thermal surfaces are discussed.

For diffuse surfaces, calculations are given for radiation emission and absorption between different configurations of planar, cylindrical, conical, spherical and ellipsoidal surfaces for finite and infinite surfaces.

For specular surfaces the affect of reflectance on calculations for view factors is included in the calculations. View factors for specular and diffuse surfaces are also included.

The Thermal design handbook is published in 16 Parts

TR 17603-31-01	Thermal design handbook – Part 1: View factors
TR 17603-31-02	Thermal design handbook – Part 2: Holes, Grooves and Cavities
TR 17603-31-03	Thermal design handbook – Part 3: Spacecraft Surface Temperature
TR 17603-31-04	Thermal design handbook – Part 4: Conductive Heat Transfer
TR 17603-31-05	Thermal design handbook – Part 5: Structural Materials: Metallic and Composite
TR 17603-31-06	Thermal design handbook – Part 6: Thermal Control Surfaces
TR 17603-31-07	Thermal design handbook – Part 7: Insulations
TR 17603-31-08	Thermal design handbook – Part 8: Heat Pipes
TR 17603-31-09	Thermal design handbook – Part 9: Radiators
TR 17603-31-10	Thermal design handbook – Part 10: Phase – Change Capacitors
TR 17603-31-11	Thermal design handbook – Part 11: Electrical Heating
TR 17603-31-12	Thermal design handbook – Part 12: Louvers
TR 17603-31-13	Thermal design handbook – Part 13: Fluid Loops
TR 17603-31-14	Thermal design handbook – Part 14: Cryogenic Cooling
TR 17603-31-15	Thermal design handbook – Part 15: Existing Satellites
TR 17603-31-16	Thermal design handbook – Part 16: Thermal Protection System

2 References

EN Reference	Reference in text	Title
EN 16601-00-01	ECSS-S-ST-00-01	ECSS System - Glossary of terms
TR 17603-31-03	ECSS-E-HB-31-01 Part 3	Thermal design handbook – Part 3: Spacecraft Surface Temperature

All other references made to publications in this Part are listed, alphabetically, in the **Bibliography**.

Terms, definitions and symbols

3.1 Terms and definitions

For the purpose of this Standard, the terms and definitions given in ECSS-S-ST-00-01 apply.

3.2 Symbols

A_i	surface area of the i -th surface, [m ²]
B_i	energy flux leaving surface, i . often called radiosity, [W.m ⁻²]
F_{ij}	view factor from diffuse surface, A_i to diffuse surface, A_j
$F(i_1, i_2, \dots, i_n)(j_1, j_2, \dots, j_n)$	view factor from the ensemble of diffuse surfaces, $A_{i1}, A_{i2}, \dots, A_{in}$ to the ensemble of diffuse surfaces, $A_{j1}, A_{j2}, \dots, A_{jn}$
F_{ij}^s	view factor from specular surface A_i to specular surface A_j
H_i	energy flux incident on surface i , [W.m ⁻²]
K_{i2}	term which appears in the expression for the view factor between elements of parallel plates, $K_{i2} = A_i F_{ii'}$
$K_{mn}(i, j, k, p, q, \dots)$	fraction of the radiative energy leaving A_m which reaches A_n after i perfectly specular reflections from surface A_i , j from surface A_j , k from surface A_k , ...
S	distance between two differential elements, [m]
T	temperature, [K]
β_i	angle from normal to surface i , [angular degrees]
ϵ	hemispherical emittance of a (diffuse-gray) surface
ρ^d	hemispherical diffuse reflectance of a (diffuse-gray) surface

ρ^s specular reflectance of a (gray) surface, it is assumed to be independent of incident angle

σ Stefan-Boltzmann constant, $S = 5,6697 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4}$

Other symbols, mainly used to define the geometry of the configurations, are introduced when required.

Diffuse surfaces

4.1 General

The view factor, F_{12} , between the diffuse surface A_1 and A_2 , is the fraction of the energy leaving the isothermal surface A_1 that arrives at A_2 .

If the receiver surface is infinitesimal, the view factor is infinitesimal for both infinitesimal and finite emitting surfaces, and is given by the expression

$$dF_{12} = \frac{\cos \beta_1 \cos \beta_2}{\pi S^2} dA_2 \quad [4-1]$$

when both surfaces are infinitesimal, and by

$$dF_{12} = \frac{dA_2}{A_1} \int_{A_1} \frac{\cos \beta_1 \cos \beta_2}{\pi S^2} dA_1 \quad [4-2]$$

when A_1 is finite.

If the receiver surface is finite, the view factor is finite for both infinitesimal and finite emitting surfaces, and is given by the expression

$$F_{12} = \int_{A_2} \frac{\cos \beta_1 \cos \beta_2}{\pi S^2} dA_2 \quad [4-3]$$

when A_1 is infinitesimal, and by

$$F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2}{\pi S^2} dA_2 dA_1 \quad [4-4]$$

when A_1 is finite.

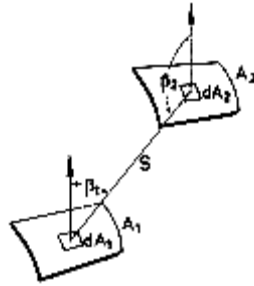


Figure 4-1: Geometric notation for view factors between diffuse surface.

Regardless of which surfaces are considered, their view factors satisfy the following reciprocity relation:

$$A_1 F_{12} = A_2 F_{21}$$

If we consider the diffuse surfaces A_1 , A_2 and A_3 , the view factor between the surfaces A_1 and $A_2 + A_3$ is

$$F_{1(2,3)} = F_{12} + F_{13},$$

when the receiver surface is formed by two surfaces, and

$$F_{(2,3)1} = \frac{A_2 F_{21} + A_3 F_{31}}{A_2 + A_3} \quad [4-5]$$

when the emitting surface is formed by two surfaces. notice that the notation $F_{1(2,3)}$ and $F_{(2,3)1}$ will be used in the following data sheets.

When an enclosure of N surfaces $A_1, A_2, \dots, A_n$ is considered, their view factors satisfy the relation

$$\sum_{j=1}^N F_{ij} = 1 \quad [4-6]$$

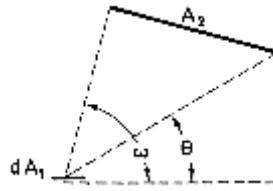
for any surface A_i . This relationship results from the fact that the overall heat transfer in the enclosure should be zero.

4.2 Infinitesimal to finite surfaces

4.2.1 Planar to planar

4.2.1.1 Two-dimensional configurations

A plane point source dA_1 and any surface A_2 generated by an infinitely long line moving parallel to itself and to the plane dA_1 .



Formula:

$$F_{12} = \frac{1}{2}(\cos \theta - \cos \omega) \quad [4-7]$$

References: Hamilton & Morgan (1952) [15], Moon (1961) [26], Kreith (1962) [22]

Comments:

Notice that F_{12} is independent of the shape of A_2 for given values of θ and ω .

View factors for several configurations may be obtained as a particular case of this one. An example is shown in the next page.

A plane point source dA_1 and any infinite plane A_2 with the planes of dA_1 and A_2 intersecting at an angle θ .

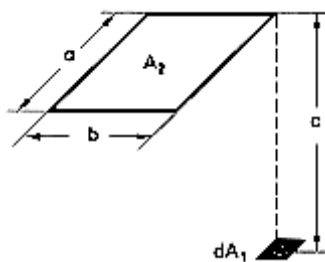
Formula:

$$F_{12} = \frac{1}{2}(1 + \cos \theta) \quad [4-8]$$

References: Hamilton & Morgan (1952) [15], Moon (1961) [26], Kreith (1962) [22]

4.2.1.2 Point source to rectangle

A plane source dA_1 and a plane rectangle A_2 parallel to the plane of dA_1 (see sketch). The normal to dA_1 passes through one corner of A_2 .



$$x = a/c$$

$$y = b/c$$

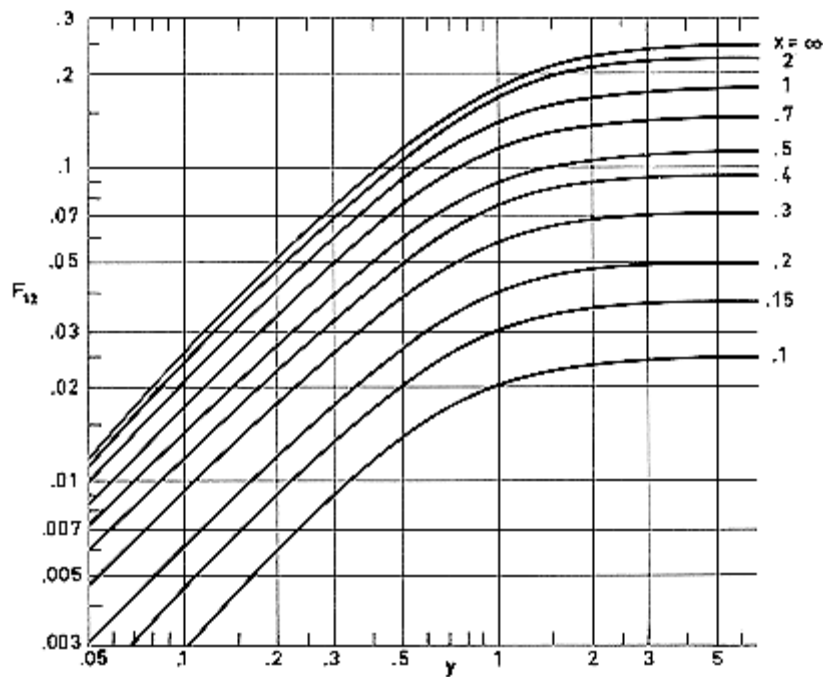
Formula:

$$F_{12} = \frac{1}{2\pi} \left(\frac{x}{\sqrt{1+x^2}} \tan^{-1} \frac{y}{\sqrt{1+x^2}} + \frac{y}{\sqrt{1+y^2}} \tan^{-1} \frac{x}{\sqrt{1+y^2}} \right) \quad [4-9]$$

$$\lim_{y \rightarrow \infty} F_{12} = \frac{x}{4\sqrt{1+x^2}} \quad [4-10]$$

$$\lim_{x \rightarrow \infty} F_{12} = \frac{y}{4\sqrt{1+y^2}} \quad [4-11]$$

References: Hamilton & Morgan (1952) [15], Hottel (1954) [17], Jakob (1957) [19], Moon (1961) [26], Kreith (1962) [22]

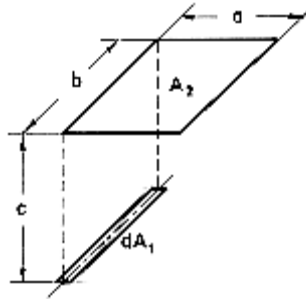


Note: non-si units are used in this figure

Figure 4-2: Values of F_{12} as a function of x and y . From Hamilton & Morgan (1952) [15].

4.2.1.3 Line source to rectangle

A line source dA_1 and a plane rectangle A_2 parallel to the plane of dA_1 with dA_1 opposite one edge of A_2 .



$$x = b/c$$

$$y = a/c$$

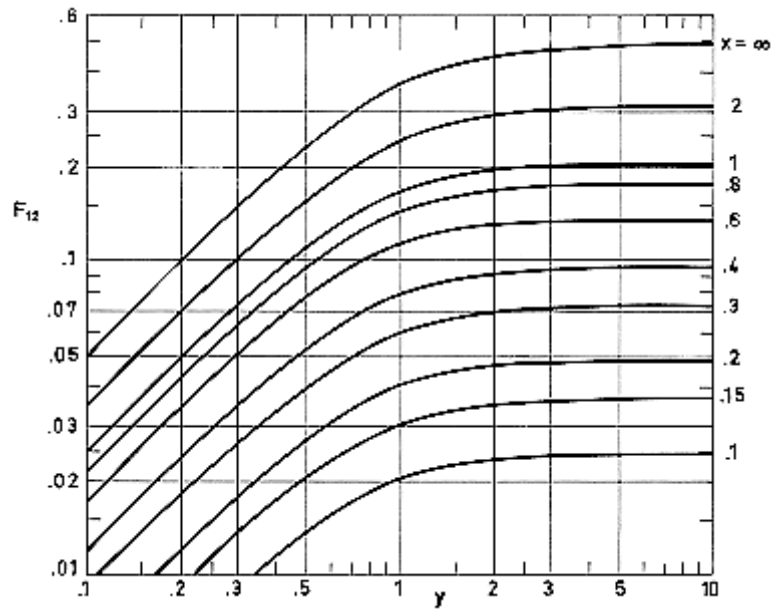
Formula:

$$F_{12} = \frac{1}{\pi x} \left(\sqrt{1+x^2} \tan^{-1} \frac{y}{\sqrt{1+x^2}} - \tan^{-1} y + \frac{xy}{\sqrt{1+y^2}} \tan^{-1} \frac{x}{\sqrt{1+y^2}} \right) \quad [4-12]$$

$$\lim_{x \rightarrow \infty} F_{12} = \frac{y}{2\sqrt{1+y^2}} \quad [4-13]$$

$$\lim_{y \rightarrow \infty} F_{12} = \frac{1}{2} \left(\sqrt{1 + \frac{1}{x^2}} - \frac{1}{x} \right) \quad [4-14]$$

References: Hamilton & Morgan (1952) [15], Kreith (1962) [22]

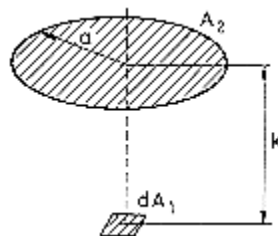


Note: non-si units are used in this figure

Figure 4-3: Values of F_{12} as a function of x and y . From Hamilton & Morgan (1952) [15]

4.2.1.4 Point source to coaxial disc or annulus

Infinitesimal surface to finite coaxial disc.



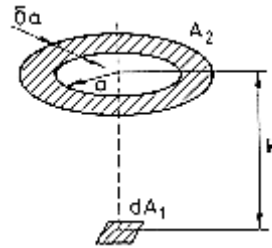
$$H = a/k$$

Formula

$$F_{12} = \frac{H^2}{1 + H^2} \quad [4-15]$$

The following case can be obtained by differentiating the above expression.

Infinitesimal surface to very thin coaxial annulus with finite radius.



$$H = a/k$$

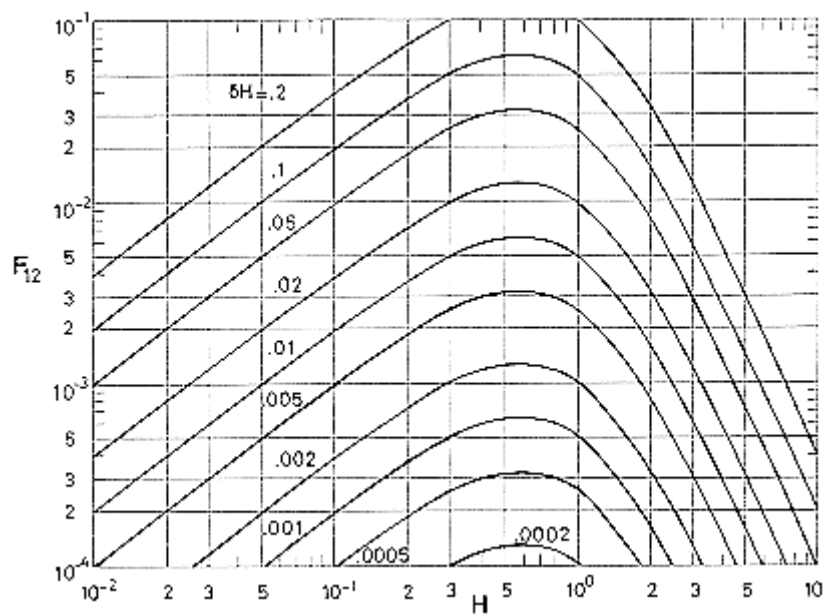
$$\delta H = \delta a/k$$

Formula

$$F_{12} = \frac{2H \cdot \delta H}{(1 + H^2)^2} \quad [4-16]$$

This expression has been represented in the Figure 4-4.

Reference: Chung & Sumitra (1972) [6].

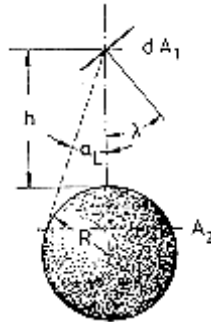


Note: non-si units are used in this figure

Figure 4-4: F_{12} vs. H for different values of δH . Infinitesimal surface to very thin coaxial annulus with finite radius. Calculated by the compiler.

4.2.2 Planar to spherical

One face of an elemental plate to sphere.



$$H = h/R$$

$$\operatorname{cosec} \alpha_L = 1 + H$$

Two cases can be distinguished:

I. The upper cap is fully visible from the plate ($\lambda + \alpha_L \leq \pi/2$).

The analytical expression for the view factor is:

$$F_{12} = \frac{\cos \lambda}{(1 + H)^2} \quad [4-17]$$

II. The upper cap is partially visible from the plate ($\lambda + \alpha_L > \pi/2$).

The self-shielding of the plate destroys the ease of integration of case I. the calculations are carried out numerically, by means of the following finite polynomial:

$$F_{12} = B_0 + B_1 \cos \lambda + B_2 \cos^2 \lambda + B_3 \cos^3 \lambda + B_4 \cos^6 \lambda.$$

The parameters B_i ($i = 0, \dots, 4$) are given in [ECSS-E-HB-31-01 Part 3 Clause 5.1](#)

This expression can be used in both cases although the values in the shadowed area in Figure 4-5 below have been calculated by the use of the analytical expression.

References: Cunningham (1961) [8], Kreith (1962) [22], Bannister (1965) [1], Clark & Anderson (1965) [7].

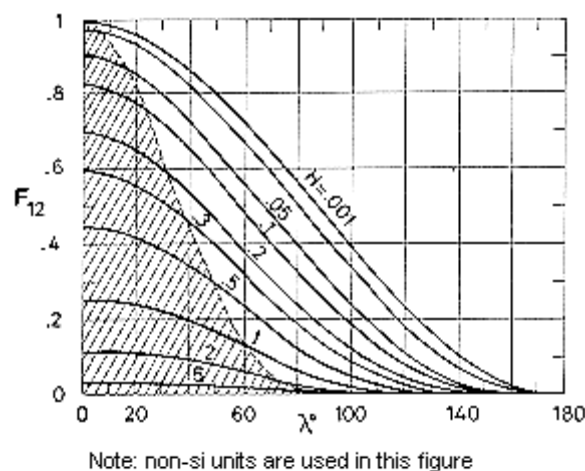
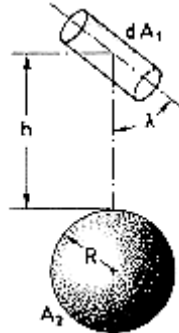


Figure 4-5: Values of F_{12} vs. λ for different values of H . The analytical expression (case I) is only valid in the shadowed region. Calculated by the compiler.

4.2.3 Cylindrical to spherical

Outer area of an infinitesimal cylindrical surface to sphere.

$$H = h/R$$



Formula:

$$F_{12} = \frac{1}{\pi^2} \int_0^{2\pi} d\phi \int_0^{\theta_o} \sqrt{1 - (\cos \theta \cos \lambda + \sin \theta \sin \lambda \cos \phi)^2} \sin \theta d\theta \quad [4-18]$$

where

$$\sin \theta_o = \frac{1}{1 + H} \quad [4-19]$$

References: Watts (1965) [50], Clark & Anderson (1965) [7].

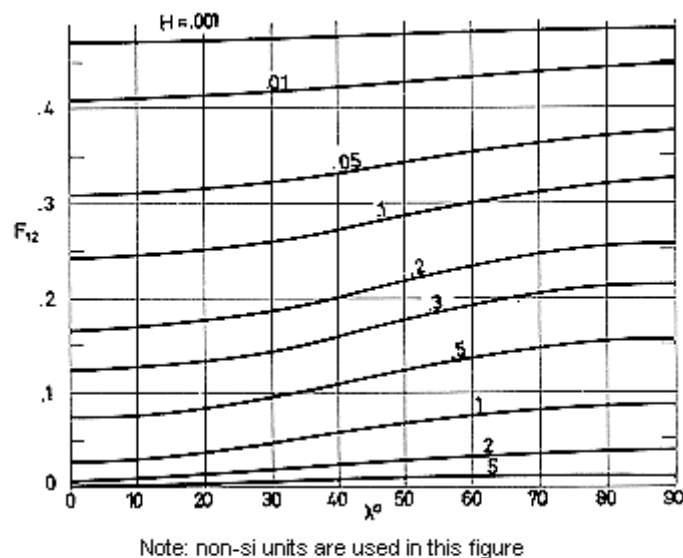
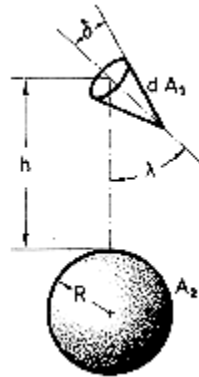


Figure 4-6: Values of F_{12} as a function of H and λ . Calculated by the compiler.

4.2.4 Conical to spherical

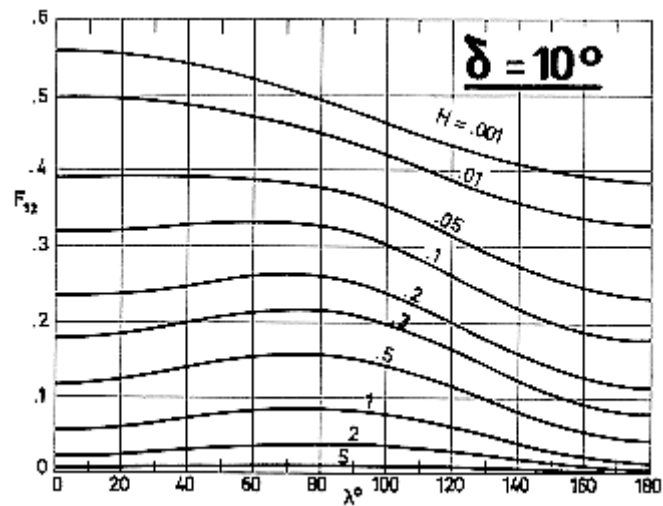
Outer area of an infinitesimal conical surface to sphere.

$$H = h/R$$



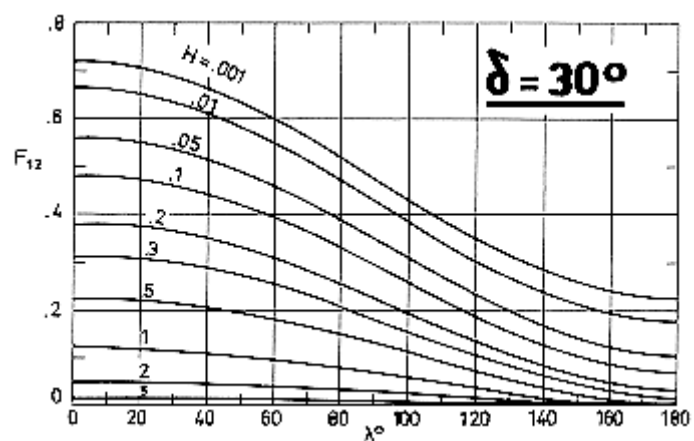
All results presented in the literature are obtained numerically.

References: Clark & Anderson (1965) [7].



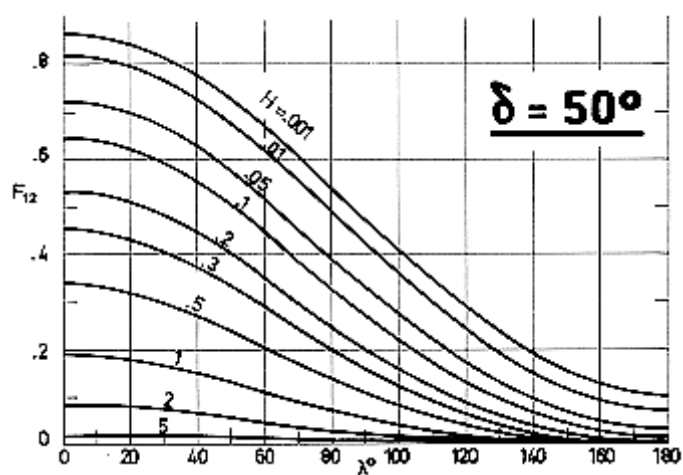
Note: non-si units are used in this figure

Figure 4-7: Values of F_{12} as a function of H and λ , for $\delta = 10^\circ$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-8: Values of F_{12} as a function of H and λ , for $\delta = 30^\circ$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-9: Values of F_{12} as a function of H and λ , for $\delta = 50^\circ$. Calculated by the compiler.

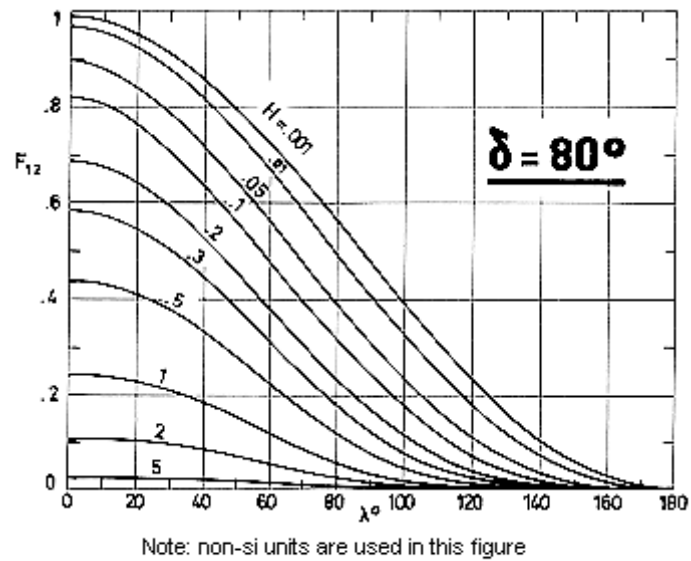


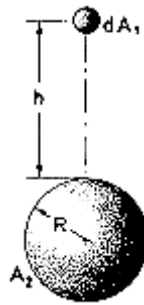
Figure 4-10: Values of F_{12} as a function of H and λ , for $\delta = 80^\circ$. Calculated by the compiler.

4.2.5 Spherical to spherical

4.2.5.1 Sphere to outer sphere

Infinitesimal sphere to finite sphere.

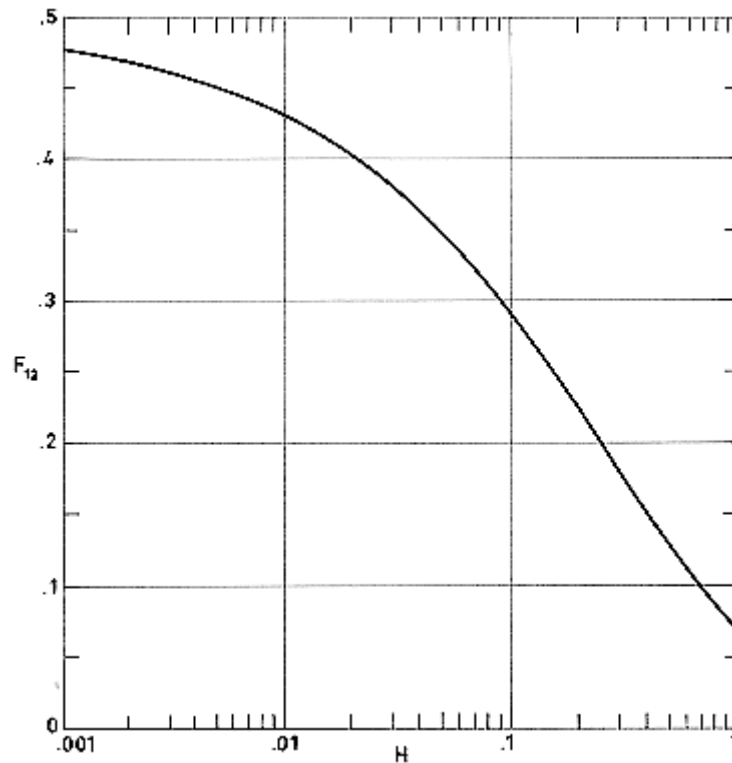
$$H = h/R$$



Formula:

$$F_{12} = \frac{1}{2} \left(1 - \frac{\sqrt{H^2 + 2H}}{1 + H} \right) \quad [4-20]$$

Reference: Watts (1965) [50].



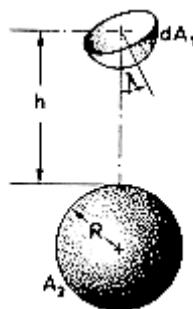
Note: non-si units are used in this figure

Figure 4-11: F_{12} as a function of H in the case of an infinitesimal sphere viewing a finite sphere. Calculated by the compiler.

4.2.5.2 Convex hemispherical surface to outer sphere

Convex surface of an infinitesimal hemisphere to sphere.

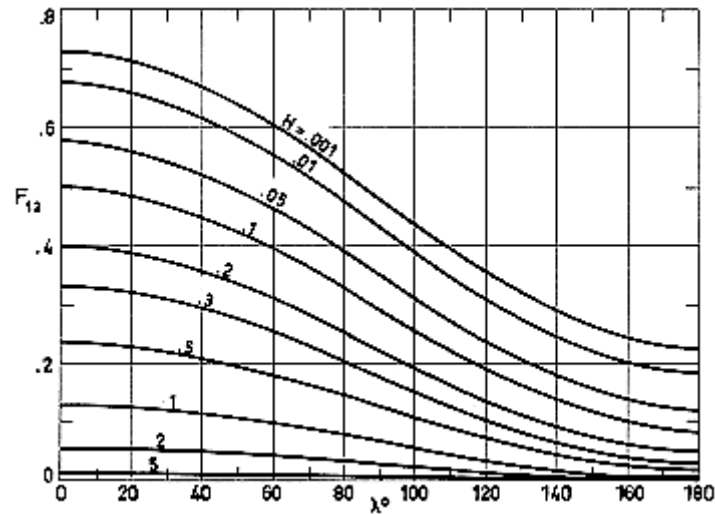
$$H = h/R$$



Formula:

$$F_{12} = \frac{1}{2} \left[1 - \frac{\sqrt{H^2 + 2H}}{1+H} + \frac{\cos \lambda}{2} \left(\frac{1}{1+H} \right)^2 \right] \quad [4-21]$$

Reference: Watts (1965) [50].



Note: non-si units are used in this figure

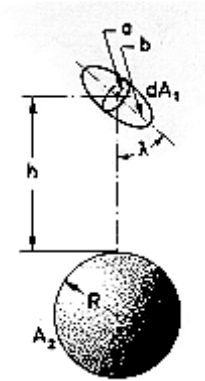
Figure 4-12: F_{12} as a function of angle λ for different values of the dimensionless distance H . Calculated by the compiler.

4.2.6 Ellipsoidal to spherical

Infinitesimal ellipsoid of revolution to sphere.

$$H = h/R$$

$$A = a/b$$



Formula:

$$F_{12} = \frac{1}{S} \int_0^{2\pi} d\phi \int_0^{\theta_0} \sqrt{\frac{A^4 + \tan^2 \Delta}{A^2 + \tan^2 \Delta}} \sin[\Delta + \tan^{-1}(A^2 \cot \Delta)] \sin \theta d\theta \quad [4-22]$$

where

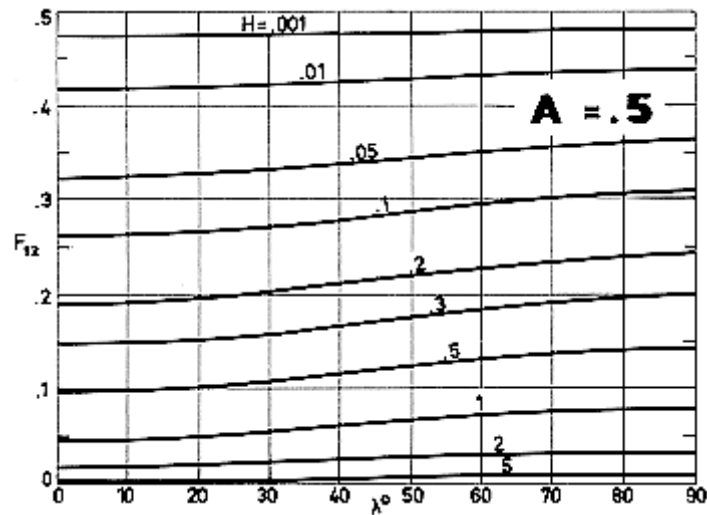
$$\sin \theta_0 = \frac{1}{1+H} \quad [4-23]$$

$$\cos \Delta = \cos \theta \cos \lambda + \sin \theta \sin \lambda \cos \phi \quad [4-24]$$

$$S = 2\pi \left(A + \frac{\sin^{-1} \sqrt{1-A^2}}{\sqrt{1-A^2}} \right) \quad \text{if } A < 1 (\text{prolate ellipsoid}) \quad [4-25]$$

$$S = 2\pi \left[A + \frac{\ln \left(A + \sqrt{A^2 - 1} \right)}{\sqrt{A^2 - 1}} \right] \quad \text{if } A > 1 (\text{oblate ellipsoid}) \quad [4-26]$$

Reference: Watts (1965) [50].



Note: non-si units are used in this figure

Figure 4-13: F_{12} as a function of λ and H , for $A = 0,5$. Calculated by the compiler.

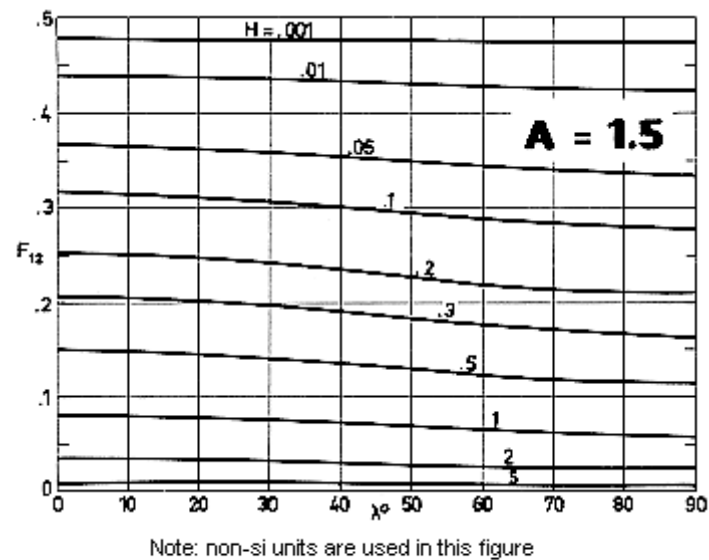


Figure 4-14: F_{12} as a function of λ and H , for $A = 1.5$. Calculated by the compiler.

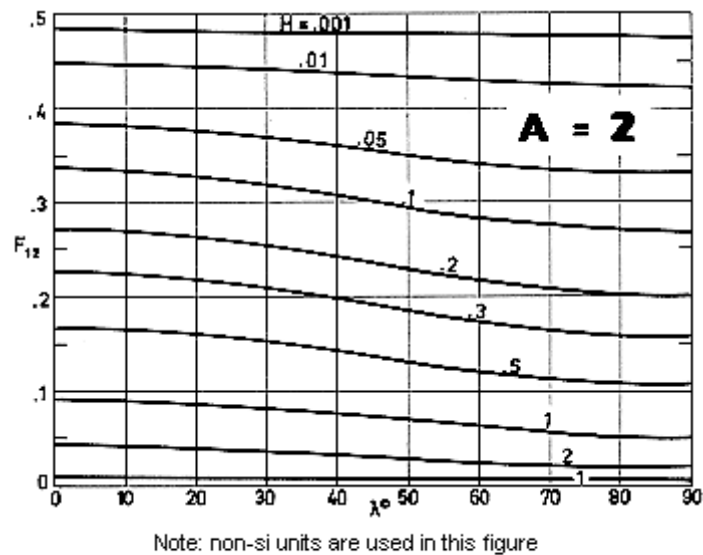
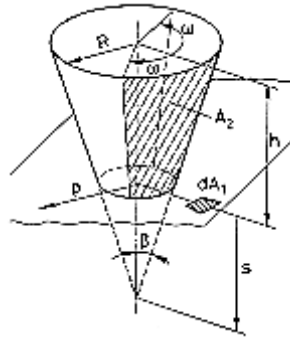


Figure 4-15: F_{12} as a function of λ and H , for $A = 2$. Calculated by the compiler.

4.2.7 Planar to conical

Infinitesimal surface of a ring to an inverted coaxial cone.

Two different configurations could arise depending on whether the cone intersects or not the planar surface.



$$L = s/(h-s)$$

$$M = \rho/R$$

$$\omega = 2 \tan^{-1} \sqrt{\frac{M+L}{M-L}} \quad [4-27]$$

Configuration 1. Cone and plane dA_1 intersect. ($s < 0$, $\rho \geq -s \tan \beta$).

Configuration 2. Cone and plane dA_1 do not intersect. ($s > 0$).

Formula:

Configuration 1:

$$F_{12} = \frac{\sin \beta}{\pi} \tan^{-1} \left(\frac{L+1}{\sin \beta \sqrt{M^2 - L^2}} \right) + \frac{1}{\pi} \tan^{-1} \sqrt{\frac{M-L}{M+L}} + f(M, L, \beta) \quad [4-28]$$

$$f(M, L, \beta) = \frac{(1-M^2) \tan^2 \beta - (L+1)^2}{\pi \sqrt{[(1+M^2) \tan^2 \beta + (L+1)^2]^2 - 4M^2 \tan^4 \beta}} \tan^{-1} \sqrt{\frac{M+L}{M-L} \frac{(1+M^2) \tan^2 \beta + (L+1)^2}{(1-M^2) \tan^2 \beta + (L+1)^2}} \quad [4-29]$$

Configuration 2:

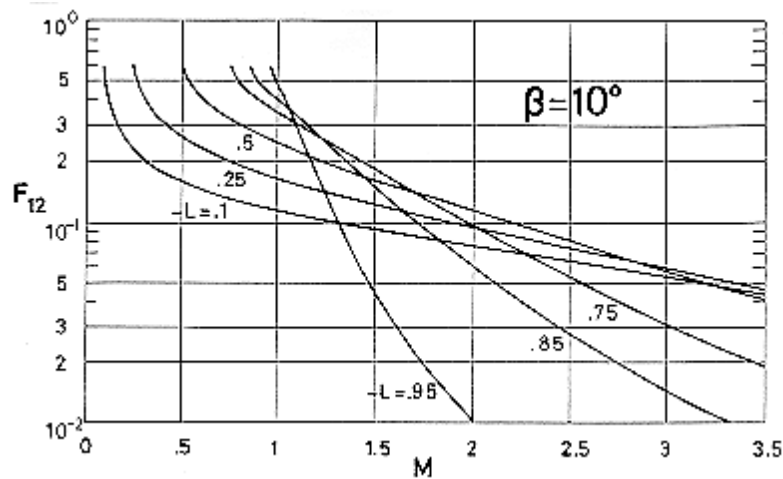
Case 2a: $M > L$

$$F_{12} = \frac{\omega}{2\pi} + \frac{\sin \beta}{\pi} \left[\tan^{-1} \left(\frac{L+1}{\sin \beta \sqrt{M^2 - L^2}} \right) - \tan^{-1} \left(\frac{L}{\sin \beta \sqrt{M^2 - L^2}} \right) \right] + f(M, L, \beta) \quad [4-30]$$

Case 2b: $M \leq L$ (Cone is seen as a disc of radius R . See the sketch).

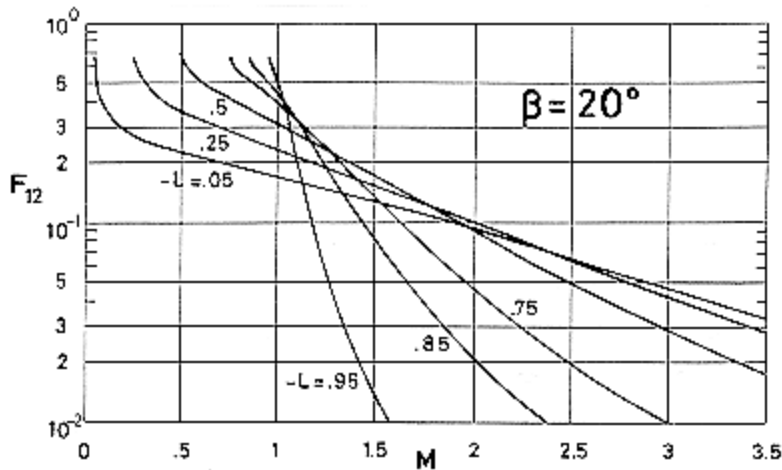
$$F_{12} = \frac{1}{2} \left[1 + \frac{(1 - M^2) \tan^2 \beta - (L + 1)^2}{\pi \sqrt{[(1 + M^2) \tan^2 \beta + (L + 1)^2]^2 - 4M^2 \tan^4 \beta}} \right] \quad [4-31]$$

Reference: Minning (1977) [25].



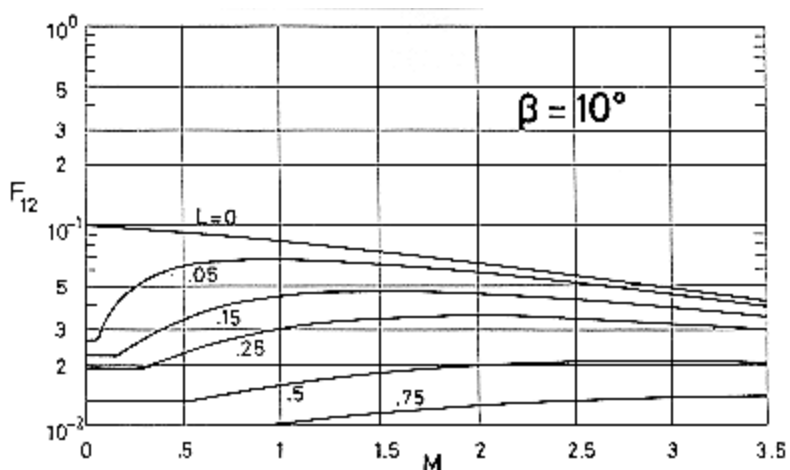
Note: non-si units are used in this figure

**Figure 4-16: Values of F_{12} vs. M for different values of L . Configuration 1, $\beta = 10^\circ$.
Calculated by the compiler.**



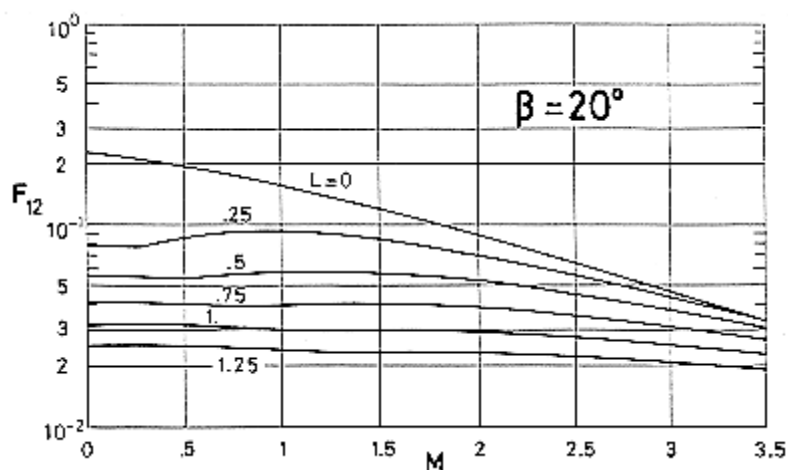
Note: non-si units are used in this figure

**Figure 4-17: Values of F_{12} vs. M for different values of L . Configuration 1, $\beta = 20^\circ$.
Calculated by the compiler.**



Note: non-si units are used in this figure

Figure 4-18: Values of F_{12} vs. M for different values of L . Configuration 2, $\beta = 10^\circ$. Calculated by the compiler.



Note: non-si units are used in this figure

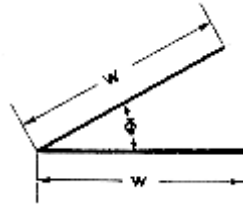
Figure 4-19: Values of F_{12} vs. M for different values of L . Configuration 2, $\beta = 20^\circ$. Calculated by the compiler.

4.3 Finite to finite surface

4.3.1 Planar to planar. Two-dimensional configurations

4.3.1.1 Two strips of equal width at any angle

Two infinitely long plates of equal finite width w , having one common edge, and at an included angle ϕ to each other.



Formula:

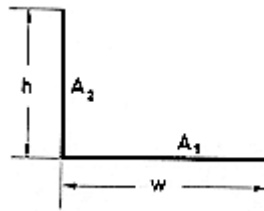
$$F_{12} = F_{21} = 1 - \sin \frac{\phi}{2} \quad [4-32]$$

Reference: Siegel & Howell (1972) [37].

4.3.1.2 Two strips of unequal width normal to each other

Two infinitely long plates of unequal finite width, having one common edge, and at an included angle $\phi = 90^\circ$ to each other.

$$H = h/w$$



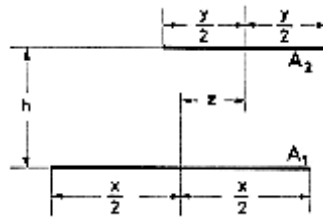
Formula:

$$F_{12} = HF_{21} = \frac{1}{2} \left[1 + H - \sqrt{1 + H^2} \right] \quad [4-33]$$

References: Kreith (1962) [22], Siegel & Howell (1972) [37].

4.3.1.3 Two parallel strips

Two infinitely long parallel strips of unequal width



$$X = x/h$$

$$Y = y/h$$

$$Z = z/h$$

Formula:

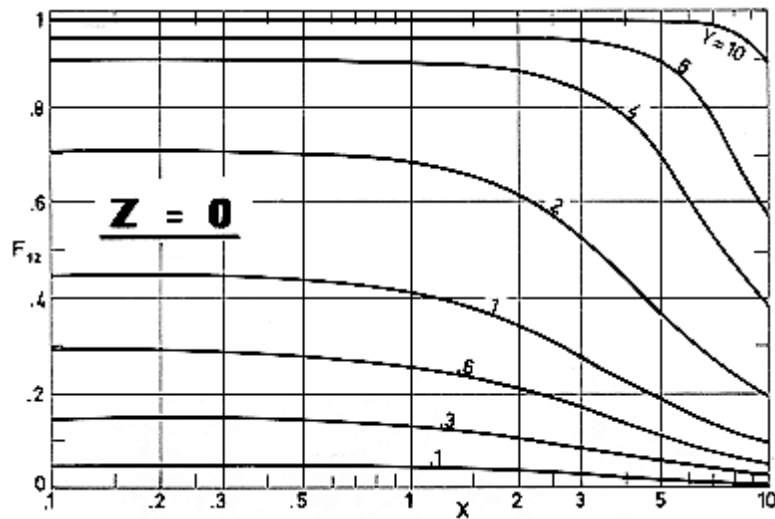
$$F_{12} = \frac{Y}{X} F_{21} = \frac{1}{2X} \left[\sqrt{1 + \left(\frac{Y + X - 2Z}{2} \right)^2} + \sqrt{1 + \left(\frac{Y + X + 2Z}{2} \right)^2} - \sqrt{1 + \left(\frac{X - Y - 2Z}{2} \right)^2} - \sqrt{1 + \left(\frac{X - Y + 2Z}{2} \right)^2} \right] \quad [4-34]$$

Reference: Kutateladze & Borishanskii (1966) [23].

When $X = Y$ and $Z = 0$, one obtains:

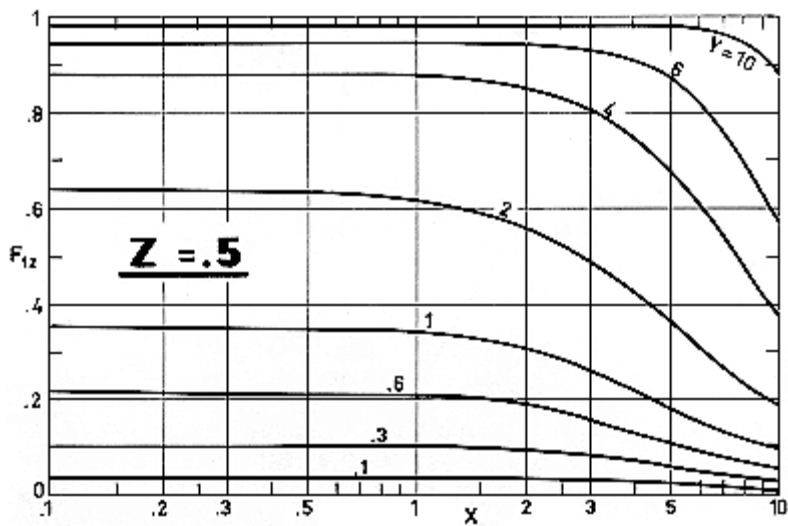
$$F_{12} = F_{21} = \frac{1}{X} \left[\sqrt{1 + X^2} - 1 \right] \quad [4-35]$$

References: Kreith (1962) [22], Siegel & Howell (1972) [37].



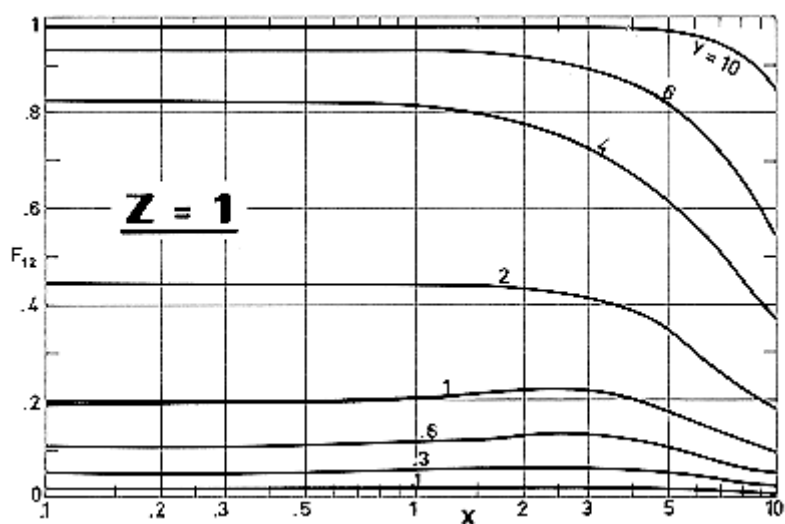
Note: non-si units are used in this figure

Figure 4-20: Values of F_{12} as a function of X and Y , for $Z = 0$. Calculated by the compiler.



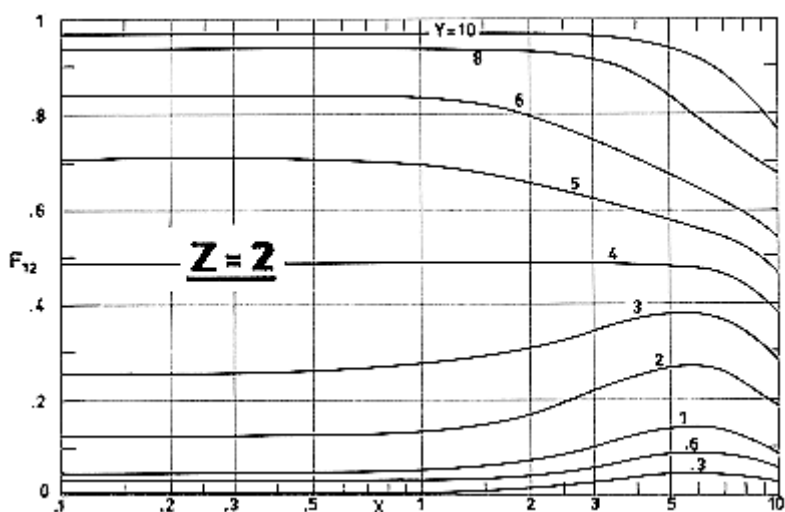
Note: non-si units are used in this figure

Figure 4-21: Values of F_{12} as a function of X and Y , for $Z = 0,5$. Calculated by the compiler.



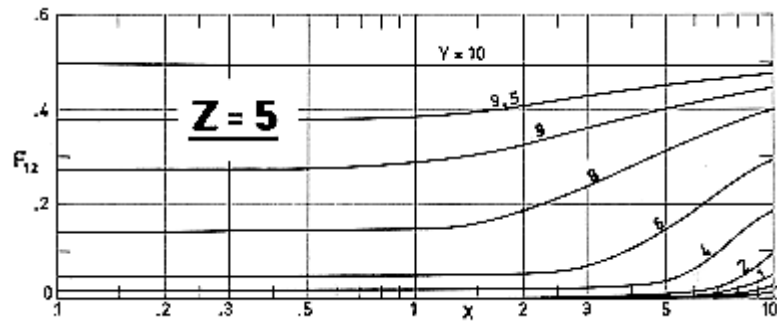
Note: non-si units are used in this figure

Figure 4-22: Values of F_{12} as a function of X and Y , for $Z = 1$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-23: Values of F_{12} as a function of X and Y , for $Z = 2$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-24: Values of F_{12} as a function of X and Y , for $Z = 5$. Calculated by the compiler.

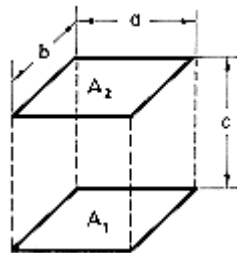
4.3.2 Planar to planar. Three-dimensional configurations

4.3.2.1 Parallel rectangles of the same dimensions

Parallel, directly opposed rectangles of same width and length

$$X = a/c$$

$$Y = b/c$$

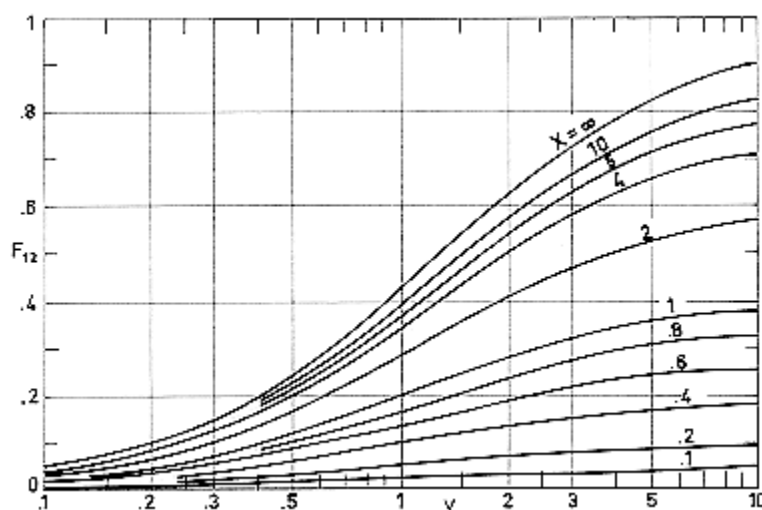


Formulae:

$$F_{12} = \frac{2}{\pi XY} \left\{ \ln \left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2} \right]^{1/2} + X\sqrt{1+Y^2} \tan^{-1} \frac{X}{\sqrt{1+Y^2}} + \right. \\ \left. + Y\sqrt{1+X^2} \tan^{-1} \frac{Y}{\sqrt{1+X^2}} - X \tan^{-1} X - Y \tan^{-1} Y \right\} \quad [4-36]$$

When $X \approx Y \ll 1$, $F_{12} = XY/\pi$.

References: Hamilton & Morgan (1952) [15], Jakob (1957) [19], Kreith (1962) [22], Hsu (1967) [18], Siegel & Howell (1972) [37].



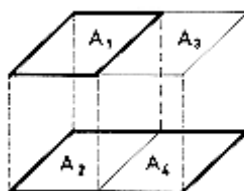
Note: non-si units are used in this figure

Figure 4-25: Values of F_{12} as a function of X and Y . Calculated by the compiler.

4.3.2.2 Parallel rectangle of unequal dimensions

The following view factors can be deduced by using the results for two parallel directly opposed rectangles.

Two rectangles in parallel planes with one rectangle directly opposite to portion of the other.

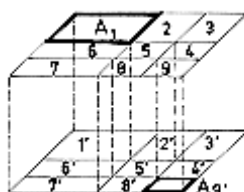


View factor is:

$$F_{1(2,4)} = \frac{1}{2A_1} [A_{(1,3)} F_{(1,3)(2,4)} + A_1 F_{12} - A_3 F_{34}] \quad [4-37]$$

References: Moon (1961) [26], Kreith (1962) [22], Hsu (1967) [18].

Two rectangles of arbitrary size in parallel planes with one edge of a rectangle parallel to one of the other.



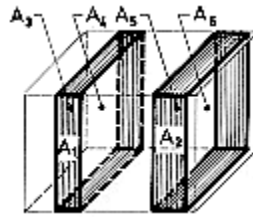
View factor is:

$$A_1 F_{19'} = \frac{1}{4} \left[\begin{aligned} &K_{(1,2,3,4,5,6,7,8,9)^2} - K_{(1,2,5,6,7,8)^2} - K_{(2,3,4,5,8,9)^2} - K_{(1,2,3,4,5,6,7,8,9)^2} - \\ &- K_{(1,2,3,4,5,6)^2} - K_{(1,2,5,6)^2} - K_{(2,3,4,5)^2} - K_{(4,5,8,9)^2} - K_{(4,5)^2} - K_{(5,8)^2} - \\ &- K_{(5,6)^2} - K_{(4,5,6,7,8,9)^2} - K_{(5,6,7,8)^2} - K_{(4,5,6)^2} - K_{(2,5,8)^2} - K_{(2,5)^2} - K_{5^2} \end{aligned} \right] \quad [4-38]$$

where $K_{m2} = A_m F_{mm'}$.

Reference: Hsu (1967) [18].

Finite area on interior of a rectangular channel.



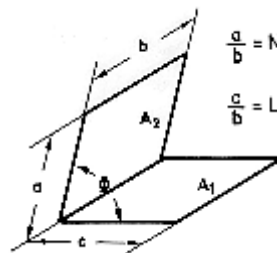
View factor is:

$$F_{12} = \frac{A_4}{A_1} (F_{45} + F_{36} + F_{46} - F_{35}) \quad [4-39]$$

Reference: Siegel & Howell (1972) [37].

4.3.2.3 Rectangles with one common edge

Two rectangles A_1 and A_2 with one common edge and included angle ϕ between the two planes.



Formulae: