



BSI Standards Publication

Flanges and their joints — Design rules for gasketed circular flange connections

Part 6: Background information

National foreword

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The UK participation in its preparation was entrusted to Technical Committee PSE/15, Flanges.

A list of organizations represented on this committee can be obtained on request to its committee manager.

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© The British Standards Institution 2025
Published by BSI Standards Limited 2025

ISBN 978 0 539 32215 6

ICS 23.040.60

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This Published Document was published under the authority of the Standards Policy and Strategy Committee on 31 January 2025.

Amendments/corrigenda issued since publication

Date	Text affected
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English Version

Flanges and their joints - Design rules for gasketed circular flange connections - Part 6: Background information

Brides et leurs assemblages - Règles de calcul des assemblages à brides circulaires avec joint - Partie 6: Document de référence

Flansche und ihre Verbindungen - Regeln für die Auslegung von Flanschverbindungen mit runden Flanschen und Dichtung - Teil 6: Hintergrund-Informationen

This Technical Report was approved by CEN on 22 December 2024. It has been drawn up by the Technical Committee CEN/TC 74.

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European foreword

This document (CEN/TR 1591-6:2025) has been prepared by Technical Committee CEN/TC 74 “Flanges and their joints”, the secretariat of which is held by DIN.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN shall not be held responsible for identifying any or all such patent rights.

This document supersedes CR 13642:1999.

CEN/TR 1591-6:2025 includes the following significant technical changes with respect to CR 13642:1999:

- Update of 6.1.2, 6.1.3, 6.1.4 and 6.2 regarding evolution within EN 1591-1:2024 related to gasket parameters.

NOTE This is not an exhaustive list of all modifications.

This document is part of a series that consists of the following parts:

- EN 1591-1, *Flanges and their joints — Design rules for gasketed circular flange connections — Part 1: Calculation*
- CEN/TR 1591-2, *Flanges and their joints — Design rules for gasketed circular flange connections — Part 2: Gasket parameters*
- CEN/TS 1591-3, *Flanges and their joints — Design rules for gasketed circular flange connections — Part 3: Calculation method for metal-to-metal contact type flanged joint*
- EN 1591-4, *Flanges and their joints — Part 4: Qualification of personnel competency in the assembly of the bolted connections of critical service pressurized systems*
- CEN/TR 1591-5, *Flanges and their joints — Design rules for gasketed circular flange connections — Part 5: Calculation method for full face gasketed joints*
- CEN/TR 1591-6, *Flanges and their joints — Design rules for gasketed circular flange connections — Background information*

Any feedback and questions on this document should be directed to the users' national standards body. A complete listing of these bodies can be found on the CEN website.

1 Scope

This document gives background information for guidance to be used in conjunction with the calculation method for design rules for gasketed circular flange connections as specified in EN 1591-1:2024.

NOTE References to formulae numbered in this document have a decimal format whilst those in EN 1591-1:2024 are indicated by whole numbers.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp/>
- IEC Electropedia: available at <https://www.electropedia.org/>

4 Introduction

Strength assessments in design calculations generally involve a 'proof of load' in the form:

$$\text{actual loads} \leq \text{allowable loads} \quad (4.1)$$

and similarly, when determining wall thicknesses, etc. The classical basis is linear elasticity theory, for which proofs are often written:

$$\text{actual stresses} \leq \text{allowable stresses} \quad (4.2)$$

However, elasticity theory can lead to illogical results such as stress increasing (strength decreasing) with increasing wall thickness. On the other hand, plasticity theory avoids such inconsistencies and Limit Load Analysis gives reliable results. Therefore, in EN 1591-1:2024 Limit Load Analysis is used as the basis for proof of load.

To ensure adequate leak-tightness, it is important that gasket compressive stress Q does not fall below a certain value. For example, in the ASME Boiler and Pressure Vessel Code, Section VIII ('ASME' hereafter) the proof for leak-tightness is:

$$Q \geq m \cdot |P| \quad (4.3)$$

where m is a 'gasket factor' and P is fluid pressure.

NOTE The general form of a leak-tightness proof is equivalent to a load proof of the form:

$$\text{actual load} \geq \text{required load} \quad (4.4)$$

It follows that both upper and lower limits are imposed on gasket and bolt loads, as in:

$$\text{required load} \leq \text{actual load} \leq \text{allowable load} \quad (4.5)$$

Various commonly used design codes (e.g. ASME and related codes) for gasketed joints do not apply this condition but assume:

$$\text{required load} = \text{actual load} \quad (4.6)$$

and neglect interactions between assembly and subsequent test or service conditions, or additional assumptions are introduced (e.g. 'gasket force is constant'). This is a poor model of real joints and leakage problems result.

A code which properly treats all load conditions (assembly, test, and all service conditions) is TGL 32903/13 (1983), a National Standard of the former German Democratic Republic. Variants of this have been in use since 1973 and it has been applied to the design of thousands of gasketed joints without leakage problems. Therefore, when CEN/TC/74/WG 10 "Flanges and their joints - Calculation methods" was requested to produce a design procedure for gasketed joints, the TGL method was chosen as the basis. At the request of CEN/TC 54/WG 53 "Unfired pressure vessels – Design methods" the scope of the TGL version was extended, with basic principles unchanged. The established validity was unaffected by this extension but behaviour in the new domain has yet to be verified. Examples of validation tests for the original domain are given in Annex B.

5 Forces in gasketed joints

5.1 Definition of active and passive forces

Active forces (and moments) are those which can, in principle, cause unlimited deformation. Examples in joints are the axial fluid-pressure force F_Q and an external dead-weight force F_A . If limited plastic deformation is possible without loss of function (as in dished heads) only active forces are required in the proof of load.

Passive forces (and moments) are due to limited elastic deformation and can only cause limited deformation (they do affect fatigue). An example is a force due to differential thermal expansion. If limited plastic deformations can cause loss of function, passive forces are also included in the proof of load.

NOTE This is partially true in the case of leak-tightness. Differential thermal expansion ΔU_I can cause plastic deformation, with loss of bolt and gasket force, and hence of leak-tightness. Therefore ΔU_I is included in the calculation of F_{GI} , F_{BI} and in the proof of load. On the other hand, scatter of assembly bolt-load is not considered in Formula (118). This is because if assembly bolt-load exceeds the minimum (to ensure subsequent leak-tightness) and limited plastic deformation occurs in a subsequent condition, the forces can fall to the minimum required without loss of leak-tightness.

5.2 Coupling of internal forces

Under both assembly conditions and subsequent load conditions the component parts of a gasketed joint are coupled by internal forces. Therefore, the following geometric relation exists between displacements of parts:

$$\begin{aligned} (\theta_F \cdot h_G + \tilde{\theta}_F \cdot \tilde{h}_G + \theta_L \cdot h_L + \tilde{\theta}_L \cdot \tilde{h}_L + U_B + U_G)_{(I=0)} = \\ (\theta_F \cdot h_G + \tilde{\theta}_F \cdot \tilde{h}_G + \theta_L \cdot h_L + \tilde{\theta}_L \cdot \tilde{h}_L + U_B + U_G + \Delta U_I)_{(I \geq 1)} \end{aligned} \quad (5.1)$$

Substituting θ_F , θ_L (see Formulae (C.1), (C.2) and U_B , U_G (Formulae (8.1, 7.27, 7.29)) and the equilibrium condition.

$$F_B = F_G + F_Q + F_R, \text{ for all } I \quad (5.2)$$

gives:

$$F_{G0} \cdot Y_{G0} + F_{Q0} \cdot Y_{Q0} + F_{R0} \cdot Y_{R0} = F_{GI} \cdot Y_{GI} + F_{QI} \cdot Y_{QI} + F_{RI} \cdot Y_{RI} + \Delta U_I \quad (5.3)$$

This is the fundamental formula relating force changes in a joint, subject to $F_{Q0} = 0$ ($P_{I=0} = 0$). The flexibility parameters Y_B , Y_G , Y_Q , Y_R are given in Formulae (97) to (100).

If the required gasket force (Formula 102) is known for subsequent conditions (e.g. from pressure P_I and gasket factor m_I), then from Formula (8.3) the assembly force to ensure leak-tightness is:

$$F_{G0} \geq (F_{GI} \cdot Y_{GI} + F_{QI} \cdot Y_{QI} + F_{RI} \cdot Y_{RI} - F_{R0} \cdot Y_{R0} + \Delta U_I) / Y_{G0} \quad (5.4)$$

which is the basis of Formula (103).

NOTE In Formula (103) of EN 1591-1:2024, an additional term (P_{QR}) is introduced to take the gasket creep phenomena into account (see 6.1.3).

If there is more than one subsequent condition, the largest assembly force is selected in order to be sufficient for all of them (Formula (105)). In the other subsequent conditions (not associated to the largest assembly bolt force) the gasket force is greater than required. Gaskets, flanges and bolts ability to withstand this additional force are checked (Formula (120)).

5.3 Assembly conditions

The bolt-tightening method is selected to produce a bolt load not less than the required minimum; thus, due to scatter, the target bolt load is greater than this minimum. These effects are considered in EN 1591-1:2024, subclause 4.4.2. The scatter parameter can be defined in various ways. In EN 1591-1:2024 the following 'linear definition' is used (as in VDI 2230):

$$\check{F}_{B0} = \bar{F}_{B0} \cdot (1 - \varepsilon); \quad \hat{F}_{B0} = \bar{F}_{B0} \cdot (1 + \varepsilon) \quad (5.5)$$

$$\bar{F}_{B0} = \frac{(\check{F}_{B0} + \hat{F}_{B0})}{2}; \quad 0 < \varepsilon < 1 \quad (5.6)$$

An alternative possibility is the "geometric definition":

$$\check{F}_{B0} = \bar{F}_{B0} / (1 + \varepsilon); \quad \hat{F}_{B0} = \bar{F}_{B0} \cdot (1 + \varepsilon) \quad (5.7)$$

$$\bar{F}_{B0} = \sqrt{\check{F}_{B0} \cdot \hat{F}_{B0}}; \quad 0 < \varepsilon < \infty \quad (5.8)$$

These definitions are nearly the same for $0 < \varepsilon < 0,1 \dots 0,2$, but the alternative definitions account for disagreements between published data. In EN 1591-1:2024, examples of ε are given based on the linear definition.

6 Gasket characteristics

6.1 Mechanical behavior

6.1.1 General

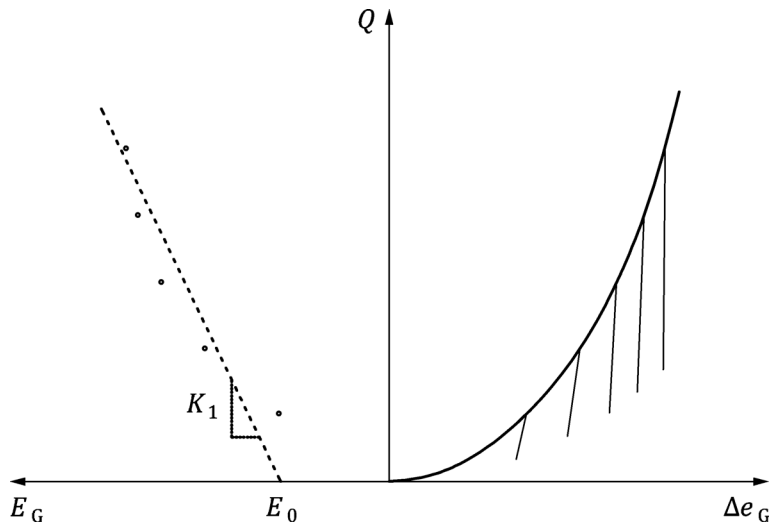


Figure 1 — Gasket mechanical behaviour

When structural deformation was not considered, the details of gasket behaviour were of less concern, but with the present more comprehensive approach they need further consideration. As indicated schematically in Figure 1, the relationship between the load (Q) and the gasket deflection in compression (Δe_G) is very non-linear for most gasket types.

6.1.2 Unloading modulus (E_G)

The initial-loading line (see right side of Figure 1) is strongly non-linear. The unloading-lines (see left side of Figure 1) were initially considered approximately linear with slope increasing with assembly stress. In reality, the unloading line is not strictly linear. Its slope reduces at lower stresses. Therefore, to measure E_G , a detailed definition is given in EN 13555:2021. The corresponding values of unloading modulus E_G , are plotted on the left side of the diagram.

In EN 1591-1:2024, these values were considered varying almost linearly with assembly stress:

$$E_G \approx E_0 + K_1 \cdot Q_0 \quad (6.1)$$

In the following revisions of EN 1591-1:2024, based on EN 13555:2021 test results, it has been decided to use directly tabulated values of $E_G(Q_0)$ without using the linear model given in (6.1).

6.1.3 Creep/relaxation

When a gasket is subjected to compressive stress an immediate elastic (or elastic-plastic) deformation U_G occurs followed by creep, increasing deformation with time at constant load. The Figure 2 shows this diagrammatically.

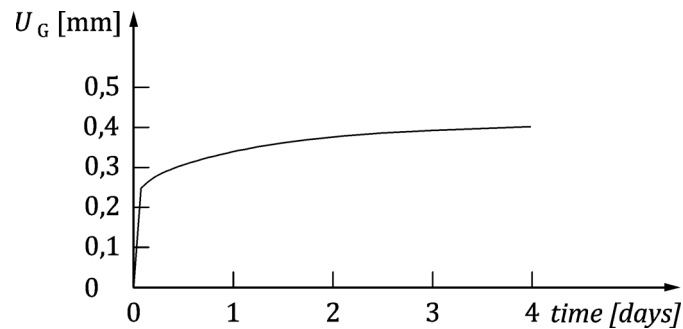


Figure 2 — Creep/relaxation

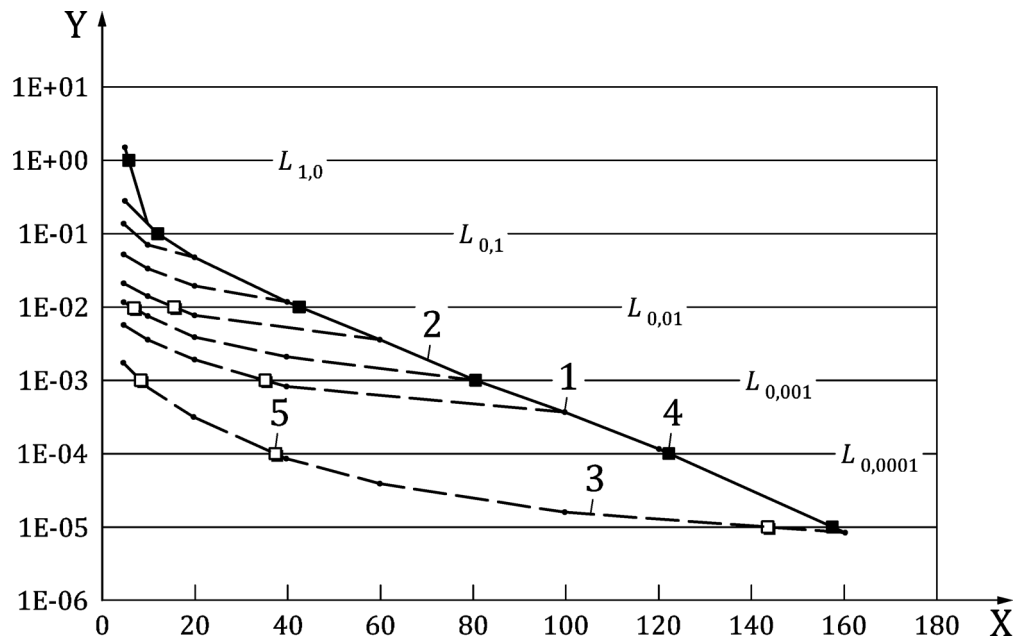
In the case of a bolted connection, the load applied on the gasket will not be held constant. The gasket deflection due to creep will lead to the reduction of the internal forces in the assembly. The amount of force reduction on the gasket will depend on the compliance of the bolted connection components. Therefore, the gasket creep/relaxation behaviour is handled through a creep/relaxation factor called P_{QR} defined as the ratio of the residual and initial surface pressures. EN 13555:2021 details the test procedure enabling to measure this parameter.

6.1.4 Maximum compressive stress

The maximum surface pressure (Q_{smax}) that can be safely imposed upon the gasket at the service temperature without damage is determined through EN 13555:2021 tests. In EN 1591-1:2024, the measured value of Q_{smax} was modified by a factor c_G , (Formula (7.43) and Formula (13.24)) that makes allowance for the gasket width in relation to thickness. For recording purposes, the explanation for this correction is maintained in this document even if it is no longer used in the actual version of EN 1591-1:2024.

6.2 Sealing criteria

$Q_{min(L)}$ is the minimum level of gasket surface pressure required for tightness class L at assembly (on the effective gasket area). $Q_{smin(L)}$ is the minimum level of surface pressure required for leakage rate class L after off-loading. Q_A is the associated required gasket surface pressure at assembly prior to unloading. $Q_{min(L)}$, Q_A and $Q_{smin(L)}$ are variables which are determined in a leakage test according to EN 13555:2021 and which are linked to each other. The lowest acceptable value of Q_A is equal to $Q_{min(L),I}$, in this case $Q_A = Q_{min(L),I} = Q_{smin(L),I}$. The higher Q_A can be chosen, the lower $Q_{smin(L),I}$ can get.



- Key**
- 1 measurement point
 - 2 loading
 - 3 unloading
 - 4 $Q_{\min}(L)$
 - 5 $Q_{\min}(L)$
 - X effective gasket surface pressure [in MPa]
 - Y leakage rate [in mg/(m s)]

Figure 3 — Leakage rate as a function of gasket surface pressure (for one specific internal pressure level of the fluid)

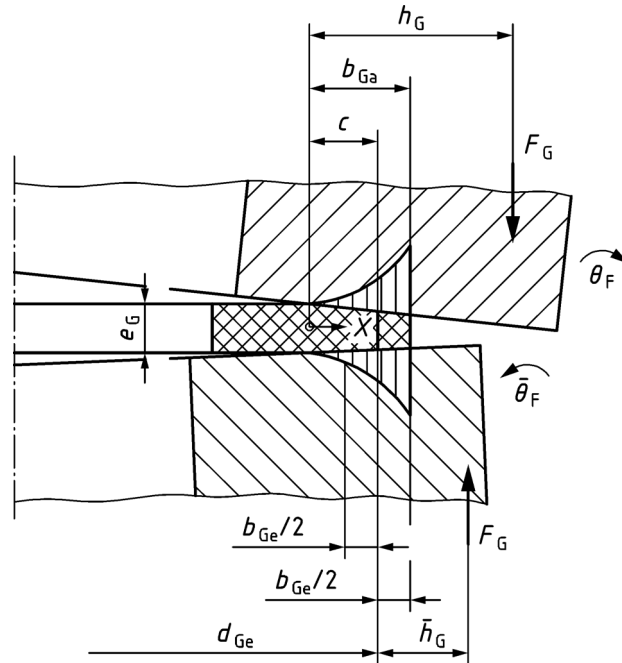
6.3 Effective width

The effective width of a gasket varies with flange rotation, which also causes a radial variation of compressive stress. Strictly, an iterative calculation is needed to reconcile the changing width, gasket stresses and bolt load. However, the approach adopted in EN 1591-1:2024 is to calculate gasket-width for the assembly condition and then assumed this to be unchanged for subsequent conditions. This simplifying assumption is strictly correct only if gasket-force F_G and flange rotations do not change. However, the assumption is conservative if the effective width for subsequent conditions is actually smaller than in the assembly condition, which is often the case.

7 Calculations for gaskets

7.1 Effective width of gaskets

7.1.1 Flat gaskets



Key

b_{Ga} = contact width

b_{Ge} = effective width

b_{Gc} = calculated width

$Q(x)$ = compressive stress

Figure 4 — Flat gaskets

Elastic rotation of flanges (see Formula (10.36)):

$$\theta_F + \tilde{\theta}_F = F_G \cdot (Z_F \cdot h_G / E_F + \tilde{Z}_F \cdot \tilde{h}_G / \tilde{E}_F) \quad (7.1)$$

Elastic deformation of gasket (for $0 \leq x \leq b_{Ga}$):

$$\varepsilon = (\theta_F + \tilde{\theta}_F) \cdot x / e_\varepsilon = k \cdot x \quad (7.2)$$

$$E_G = E_0 + K_1 \cdot Q = \frac{dQ}{d\varepsilon} \quad (7.3)$$

$$Q = \frac{E_0}{K_1} \cdot (\exp(K_1 \cdot \varepsilon) - 1) \approx E_0 \cdot \varepsilon \cdot \left(1 + \frac{1}{2} \cdot K_1 \cdot \varepsilon\right) \quad (7.4)$$

The resultant gasket force F_G is:

$$F_G = \pi \cdot d_{Ge} \cdot \int_0^{b_{Ga}} Q(x) \cdot dx \quad (7.5)$$

acting at $x = c$, given by:

$$c \cdot \int_0^{b_{Ga}} Q(x) \cdot dx = \int_0^{b_{Ga}} Q(x) \cdot x \cdot dx \quad (7.6)$$

From this follow, step by step:

$$F_G = \pi \cdot d_{Ge} \cdot E_0 \cdot k \cdot \frac{1}{2} \cdot b_{Ga}^2 \cdot \left(1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}\right) \quad (7.7)$$

$$b_{Ga} = \sqrt{\frac{F_G \cdot e_G \cdot 2}{\pi \cdot d_{Ge} \cdot E_0 \cdot (\Theta_F + \bar{\Theta}_F) \cdot \left(1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}\right)}} \quad (7.8)$$

$$c = \frac{2}{3} \cdot b_{Ga} \cdot \frac{1 + \frac{3}{8} \cdot K_1 \cdot k \cdot b_{Ga}}{1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}} \quad (7.9)$$

$$b_{Gc} = 2 \cdot (b_{Ga} - c) = \frac{2}{3} \cdot b_{Ga} \cdot \frac{1 + \frac{1}{4} \cdot K_1 \cdot k \cdot b_{Ga}}{1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}} \quad (7.10)$$

$$b_{Gc} = \sqrt{\frac{F_G \cdot e_G \cdot \frac{8}{9}}{\pi \cdot d_{Ge} \cdot (\Theta_F + \bar{\Theta}_F) \cdot E_0}} \cdot \frac{\left(1 + \frac{1}{4} \cdot K_1 \cdot k \cdot b_{Ga}\right)^2}{\left(1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}\right)^3} \quad (7.11)$$

The remaining elimination of $k \cdot b_{Ga}$ is simplified by assuming $K_1 \cdot k \cdot b_{Ga} \cdot \frac{1}{3} \ll 1$, then Formula (7.11) becomes:

$$b_{Gc} = \sqrt{\frac{F_G \cdot e_G \cdot \frac{8}{9}}{\pi \cdot d_{Ge} \cdot (\Theta_F + \bar{\Theta}_F) \cdot E_0 \cdot \left(1 + \frac{1}{2} \cdot K_1 \cdot k \cdot b_{Ga}\right)}} \quad (7.12)$$

[This approximation gives an error on $b_{Gc} \leq 5\%$ for $K_1 \cdot k \cdot b_{Ga} \leq 10$, almost always true!]

F_Q will be at least:

$$F_Q = \pi \cdot d_{Ge} \cdot b_{Ge} \cdot \bar{Q} \text{ where } \bar{Q} \text{ is an average value,} \quad (7.13)$$

and with Formulae (7.7) and (7.10) this gives:

$$E_0 \cdot k \cdot b_{Ga} = Q \cdot \frac{2 \cdot b_{Ge}}{b_{Ga} \cdot \left(1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}\right)} = Q \cdot \frac{4 \cdot b_{Ge}}{3 \cdot b_{Gc}} \cdot \frac{1 + \frac{1}{4} \cdot K_1 \cdot k \cdot b_{Ga}}{\left(1 + \frac{1}{3} \cdot K_1 \cdot k \cdot b_{Ga}\right)^2} \quad (7.14)$$

$$\text{With the approximation: } E_0 \cdot k \cdot b_{Ga} \approx Q \quad (7.15)$$

and omitting the factor $8/9$, Formula (7.12) becomes:

$$b_{Gc} = \sqrt{\frac{F_G \cdot e_G}{\pi \cdot d_{Ge} \cdot (\Theta_F + \bar{\Theta}_F) \cdot \left(E_0 + K_1 \cdot \frac{Q}{2}\right)}} = b_{Gc(el)} \quad (7.16)$$

for elastic behaviour of a gasket

For plastic behaviour:

$$b_{Gc(pl)} = \frac{F_G}{\pi \cdot d_{Ge} \cdot Q_{smax}} \quad (7.17)$$

True elasto-plastic deformation gives an effective-width greater than for pure-elastic and pure-plastic deformation, approximately:

$$b_{Gc} \approx \sqrt{b_{Gc(el)}^2 + b_{Gc(pl)}^2} \quad (7.18)$$

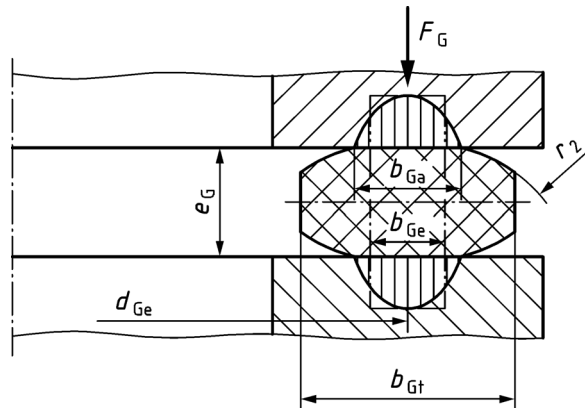
From Formulae (7.1), (7.16) and (7.17):

$$b_{Gc} = \sqrt{\frac{e_G}{\pi \cdot d_{Ge} \cdot \left(h_G \cdot \frac{Z_F}{E_F} + \tilde{h}_G \cdot \frac{\tilde{Z}_F}{E_F} \right) \cdot \left(E_0 + K_1 \cdot \frac{Q}{2} \right)} + \left(\frac{F_G}{\pi \cdot d_{Ge} \cdot Q_{smax}} \right)^2} \quad (7.19)$$

This is the formula in EN 1591-1:2024, Table 1 ('Type 1'). The auxiliary notation b_{Gc} is introduced because Formula (7.19) can give $b_{Gc} > b_{Gt}$, but it is necessary that $b_{Ge} \leq b_{Gt}$. (Later the denomination b_{Gc} was changed into b_{Gi} , because b_{Gc} and b_{Ge} are too similar.)

NOTE As introduced in 6.1.2, the model proposed in Formula (6.1) for E_G is no longer used in EN 1591-1:2024 and the expression " $E_0 + K_1 \cdot \frac{Q}{2}$ " in Formula (7.19) has been replaced by the parameter " E_{Gm} " in Formula (63) of EN 1591-1:2024. Nevertheless, it can be observed that the model proposed in Formula (6.1) is still used (see Formula (7.3)) to obtain Formula (7.19). In EN 1591-1:2024, for metallic gaskets where the value of E_G is considered nearly independent of the applied load ($K_1 \cdot Q \ll E_0$), EN 1591-1:2024, Table 1 gives $E_{Gm} = E_{G0} \sim E_0$. For non-metallic gaskets, for which the E_G values are considered to depend highly on the applied load, EN 1591-1:2024, Table 1 gives the approximation of $E_{Gm} = 0,5 \cdot E_{G0}$.

7.1.2 Effective width of gaskets with curved surfaces



Key

b_{Ga} = contact width

b_{Ge} = effective width

b_{Gc} = calculated width

Figure 5 — Effective width of gaskets with curved surfaces

For elastic deformation of the gasket (Hertzian contact) the contact width is:

$$b_{Ga}^2 = \frac{32}{\pi} \cdot \frac{F_G \cdot r_2 \cdot (1-\nu^2)}{E_G \cdot \pi \cdot d_{Ge}} \quad (7.19)$$

and the maximum contact pressure, assumed equal to the mean pressure in b_{Ge} , is:

$$Q_{smax} = \frac{4}{\pi} \cdot \frac{F_G}{\pi \cdot d_{Ge} \cdot b_{Ga}} \quad (7.20)$$

and $Q_{smax} = Q$ where:

$$Q = \frac{F_G}{\pi \cdot d_{Ge} \cdot b_{Gc}} \quad (7.21)$$

therefore

$$b_{Gc} = b_{Ga} \cdot \frac{\pi}{4} \quad (7.22)$$

i.e.

$$b_{Gc} = \sqrt{\frac{F_G \cdot r_2 \cdot 2 \cdot \pi \cdot (1 - \nu^2)}{E_G \cdot \pi \cdot d_{Ge}}} \quad (7.23)$$

If the contacting surfaces are inclined at angle φ_G to the reference plane as drawn, the normal forces on the curved surfaces are $F_G / \cos \varphi_G$ (instead of F_G), the real contact width increases with $\sqrt{1 / \cos \varphi_G}$, but the calculated width is the projection on the reference plane (projection factor $\cos \varphi_G$). Therefore:

$$b_{Gc} = \sqrt{\frac{F_G \cdot \cos \varphi_G \cdot r_2 \cdot 2 \cdot \pi \cdot (1 - \nu^2)}{E_G \cdot \pi \cdot d_{Ge}}} = b_{Gc(el)} \quad (7.24)$$

The plastic width $b_{Gc(pl)}$, and approximation for elasto-plastic behaviour, correspond to Formulae (7.17) and (7.18). FpEN 1591-1:2024, Table 2 is simplified by assuming $2 \cdot \pi \cdot (1 - \nu^2) \approx 6$ (similar to $8/9 \approx 1$ for flat gaskets).

Double contact:

Each surface sees half the force hence the calculated width is double that for one surface with half the force, which is the basis of FpEN 1591-1:2024, Table 2.

7.2 Elastic stiffness of gaskets

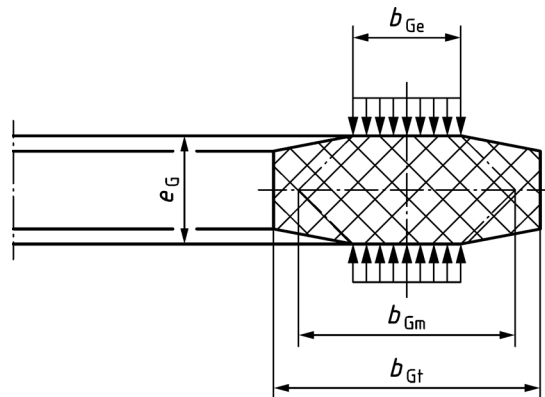


Figure 6 — Elastic stiffness of gaskets

A simple model gives:

$$\Delta U_G = \frac{\Delta F_G \cdot e_G}{E_G \cdot \pi \cdot d_{Ge} \cdot \bar{b}_G}; b_{Ge} \leq \bar{b}_G \leq b_{Gt} \quad (7.25)$$

The active (deformed) width assumed is indicated by the dashed line in the diagram (defined by the angle $\pi/4$) which gives a maximum 'active width' (mid-plane):

$$b_{Gm} = b_{Ge} + 2 \cdot \frac{e_G}{2}, \quad (b_{Gm} \leq b_{Gt}) \quad (7.26)$$

and average active width:

$$\bar{b}_G = b_{Ge} + \frac{e_G}{2}, \quad (\bar{b}_G \leq b_{Gt}) \quad (7.27)$$

The following simple formula gives the values for flat gaskets ($e_G \ll b_{Ge}, b_{Gt}$) as well as for thick gaskets ($e_G \gg b_{Ge}, b_{Gt}$):

$$\bar{b}_G = \frac{b_{Ge} + e_G/2}{b_{Gt} + e_G/2} \cdot b_{Gt} \quad (7.28)$$

writing

$$\Delta U_G = \Delta F_G \cdot X_G / E_G \quad (7.29)$$

gives

$$X_G = \frac{e_G}{A_{Gt}} \cdot \frac{b_{Ge} + e_G/2}{b_{Gt} + e_G/2} \cdot b_{Gt} \quad (7.30)$$

7.3 Load carrying capability of gaskets

Assumptions:

The gasket is a plastic deformable strip (yield or design stress S_G) between two rigid planes. Stresses $S_{xx} = S_{yy}$ and $S_{zz} = -Q$ are functions of x only, independent of y and z .

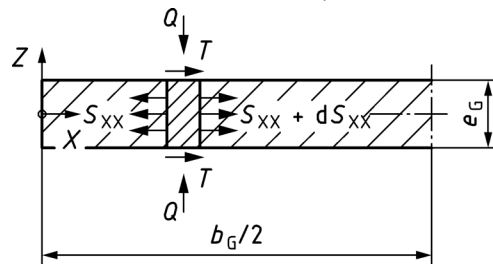


Figure 7 — Load carrying capability of gaskets

$$\text{Limit load formula: } \Psi = 1 - (n_{xx} + p)^2 \geq 0 \quad (7.31)$$

where

$$n_{xx} = \frac{S_{xx}}{S_G} = n_{yy}; p = \frac{Q}{S_G} \leq 1 - n_{xx} \quad (7.32)$$

Equilibrium and Coulomb friction give:

$$e_G \cdot \frac{dS_{xx}}{dx} + 2T = 0 \quad (7.33)$$

$$T \leq \mu_G \cdot Q \quad (7.34)$$

From Formulae (7.31) to (7.34) it follows that:

$$\frac{dn_{xx}}{dx} + \frac{2 \cdot \mu_G}{e_G} \cdot (1 - n_{xx}) = 0 \quad (7.35)$$

with boundary condition $n_{xx}(x=0) = 0$ this solution is:

$$n_{xx} = 1 - \exp\left(2 \cdot \mu_G \cdot \frac{x}{e_G}\right); p = \exp(2 \cdot \mu_G \cdot x/e_G) \quad (7.36)$$

Validity of Formulae (7.34) to (7.36) extends to $x = x_1$ with shear limit:

$$T = \mu_G \cdot Q \leq S_G/\sqrt{3} \quad (7.37)$$

$$p = \frac{1}{\mu_G \sqrt{3}} = \exp\left(2 \cdot \mu_G \cdot \frac{x_1}{e_G}\right); \frac{x_1}{e_G} = \frac{1}{2 \cdot \mu_G} \cdot \ln\left(\frac{1}{\mu_G \sqrt{3}}\right) \quad (7.38)$$

For $x > x_1$, Formulae (7.35), (7.36) are replaced by:

$$\frac{dn_{xx}}{dx} + \frac{2}{e_G \cdot \sqrt{3}} = 0 \quad (7.39)$$

$$n_{xx} = n_{xx1} - \frac{2 \cdot (x - x_1)}{e_G \cdot \sqrt{3}} = 1 - p$$

$$p = \frac{1}{\sqrt{3}} \cdot \left\{ \frac{1}{\mu_G} \cdot \left[1 - \ln\left(\frac{1}{\mu_G \sqrt{3}}\right) \right] + 2 \cdot \frac{x}{e_G} \right\} \quad (7.40)$$

From Formula (7.38) the limits are:

μ_G	0,05	0,10	0,20	0,30	0,40	0,50
x_1/e_G	24,46	8,77	2,65	1,09	0,46	0,14

The average pressure is:

$$\bar{p} = \frac{2}{b_G} \cdot \int_0^{b_G/2} p \cdot dx \quad (7.41)$$

Some results from Formulae (7.36), (7.40) are shown in Figure 8 a), and corresponding results from Formula (7.41) in Figure 8 b). The corresponding polynomial series for small $\mu_G \cdot b_G/e_G$ is:

$$\bar{p} = 1 + \frac{1}{2} \cdot \left(\frac{\mu_G \cdot b_G}{e_G}\right) + \frac{1}{6} \cdot \left(\frac{\mu_G \cdot b_G}{e_G}\right)^2 + .. \quad (7.42)$$

Therefore, the approximation (broken lines in Figure 8 b))

$$\bar{p} = 1 + \frac{\mu_G \cdot b_G}{2 \cdot e_G} (= c_G \text{ in EN 1591-1:2024}) \quad (7.43)$$

is conservative whilst $0 \leq \mu_G < 1/\sqrt{3}$.

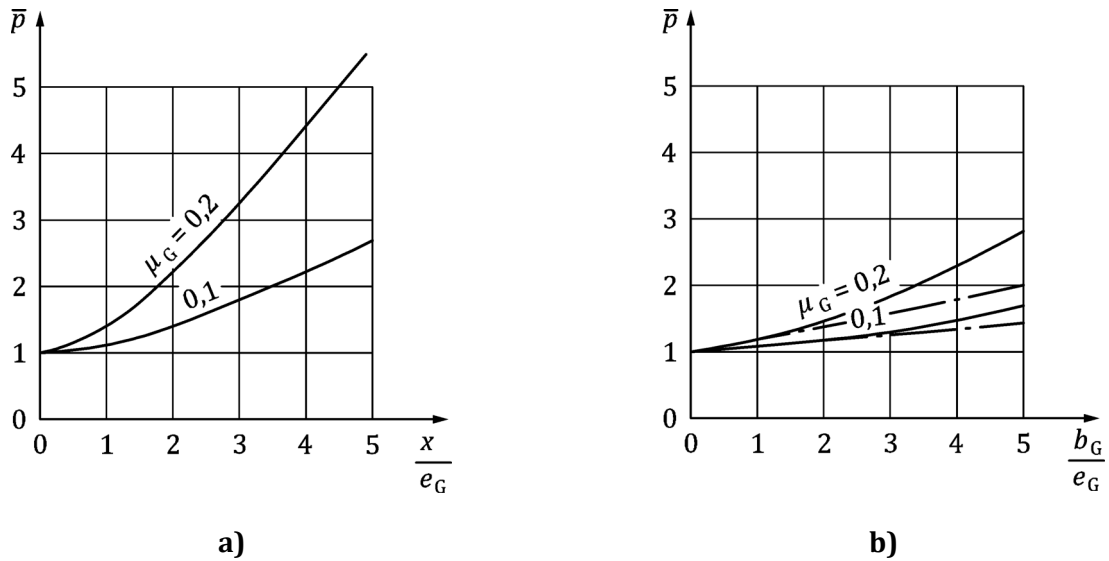


Figure 8 — Load carrying capability of gaskets

8 Calculations for bolts

8.1 Elastic stiffness of bolts

The axial elongation of the bolts is:

$$\Delta U_B = \Delta F_B \cdot \frac{X_B}{E_B} \quad (8.1)$$

where

$$X_B = \frac{4}{\pi \cdot n_B} \cdot \left(\frac{l_s}{d_{Bs}^2} + \frac{l_e}{d_{Be}^2} + \frac{0,8}{d_{B0}} \right) \quad (8.2)$$

The last term in brackets is an approximation for the elastic deformation of the two nuts (or nut and bolt head).

8.2 Load carrying capacity of bolts

Maximum permissible tensile force (for sum of n_B bolts) is:

$$F_B \leq S_B \cdot A_B \quad (8.3)$$

$$A_B = n_B \cdot \frac{\pi}{4} \cdot (\min(d_{Be}, d_{Bs}))^2 \quad (8.4)$$

$$d_{Be} = (d_{B2} + d_{B3})/2 \quad (8.5)$$

Formula (8.5) represents ISO effective diameter.

8.3 Bolt tightening by torque-wrench

To obtain bolt load F_{B0} (for one bolt) using a torque-wrench, given thread pitch p_t and thread friction coefficient

μ_t , requires torsional moment:

$$M_{T0} = F_{B0} \cdot \frac{d_{B2}}{2} \cdot \left(\frac{p_t}{\pi \cdot d_{B2}} + \frac{\mu_t}{\cos \alpha} \right) \quad (8.6)$$

(α = thread angle, commonly 60° or 55°)

Assuming $d_{Be} \leq d_B$ (see Formula (8.4)) the limit load formula is:

$$\left(\frac{F_{B0}}{\pi/4 \cdot d_{Be}^2} \right)^2 + 3 \cdot \left(\frac{M_{T0}}{\pi/12 \cdot d_{Be}^2} \right)^2 \leq S_B^2 \quad (8.7)$$

$$F_{B0} \leq S_B \cdot \frac{\pi}{4} \cdot d_{Be}^2 / \sqrt{1 + 3 \cdot \left[\frac{3 \cdot d_{B2}}{2 \cdot d_{Be}} \cdot \left(\frac{p_t}{\pi \cdot d_{B2}} + \frac{\mu_t}{\cos \alpha} \right) \right]^2} \quad (8.8)$$

Bolt threads commonly $p_t/(\pi \cdot d_{B2}) \approx 0,05 \dots 0,03$ have and $\mu_t \approx 0,1 \dots 0,2$, hence:

$$F_{B0} \leq S_B \cdot \frac{\pi}{4} \cdot d_{Be}^2 / (1,1 \dots 1,2) \quad (8.9)$$

This scatter is less than that of most tightening methods (EN 1591-1:2024, Annex C), further the torsional moment M_{T0} is active only while the wrench is moving. After tightening, the torsional moment M_{T0} is passive and can relax by passive deformation or creep. It is therefore neglected in this document, this is established practice in Germany.

9 Elasticity of shells

9.1 Conical and spherical shells with uniform wall-thickness

Equilibrium conditions are:

$$(r \cdot N_{uu})' + \sin \varphi \cdot N_{vv} + \frac{1}{r_k} \cdot (r \cdot Q_u) = 0 \quad (9.1)$$

$$(r \cdot Q_u)' - \frac{1}{r_k} \cdot (r \cdot N_{uu}) - \cos \varphi \cdot N_{vv} + r \cdot P = 0 \quad (9.2)$$

$$(r \cdot M_{uu})' + \sin \varphi \cdot M_{vv} - (r \cdot Q_u) = 0 \quad (9.3)$$

(N_{vv} , M_{vv} correspond to the circumferential direction v)

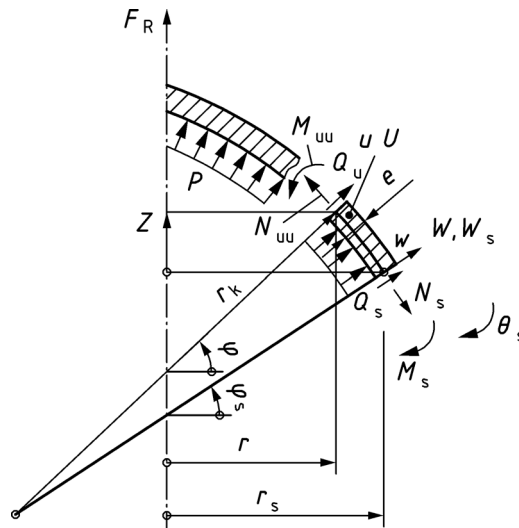
Elastic relations are:

$$N_{uu} = \frac{E \cdot e}{1 - \nu^2} \cdot \left[U' - \nu \cdot \frac{\sin \varphi}{r} \cdot U + \left(\frac{1}{r_k} + \frac{\cos \varphi}{r} \right) \cdot W \right] \quad (9.4)$$

$$N_{vv} = \frac{E \cdot e}{1 - \nu^2} \cdot \left[\nu \cdot U' - \frac{\sin \varphi}{r} \cdot U + \left(\frac{1}{r_k} + \frac{\cos \varphi}{r} \right) \cdot W \right] \quad (9.5)$$

$$M_{uu} = \frac{E \cdot e^3}{12 \cdot (1 - \nu^2)} \cdot \left[-W'' + \nu \cdot \frac{\sin \varphi}{r} \cdot W' \right] \quad (9.6)$$

$$M_{vv} = \frac{E \cdot e^3}{12 \cdot (1 - \nu^2)} \cdot \left[-\nu \cdot W'' + \frac{\sin \varphi}{r} \cdot W' \right] \quad (9.7)$$



Key

u, w : coordinates, mm

U, W : displacements, mm

 N_{uw}, Q_u : forces/length, N/mm

N_s, Q_s : same forces at $u=0$, N/mm

 M_{yy} : moment/length, Nmm/mm

M_s : same moment at $u=0$, Nmm/mm

$$\Theta = W' \quad \text{inclination}$$

Figure 9 — Conical and spherical shells with uniform wall-thickness

$$(\quad)' = \frac{d(\quad)}{du} = \frac{1}{r_k} \cdot \frac{d(\quad)}{d\varphi}; \quad r' = -\sin \varphi; \quad z' = +\cos \varphi$$

$$r_k = \text{const}$$

From Formulae (9.3), (9.6), (9.7) the shear force is:

$$Q_u = \frac{E \cdot e^3}{12 \cdot (1 - \nu^2)} \cdot \left[-W'''' + \nu \cdot \frac{\sin \varphi}{r} \cdot W''' + \left(\nu \cdot \frac{\cos \varphi}{r \cdot r} + \frac{\sin^2 \varphi}{r^2} \right) \cdot W' \right] \quad (9.8)$$

Transformed and simplified equilibrium conditions:

$$(r \cdot N_{uu})' + \sin \varphi \cdot N_{vv} \approx 0, \quad (r \cdot Q_u / r_k \text{ relatively small}) \quad (9.9)$$

$$-(r \cdot M_{uu})'' - (\sin \varphi \cdot M_{vv})' + \frac{1}{r_k} \cdot (r \cdot N_{uu}) + \cos \varphi \cdot N_{vv} = P \cdot r \quad (9.10)$$

Substitution of Formulae (9.4) to (9.7) in Formulae (9.9), (9.10) and division by $E \cdot e \cdot r / (1 - \nu^2)$ gives two differential formulae:

$$U'' - \frac{\sin \varphi}{r} \cdot U' + \dots + \left(\frac{1}{r^k} + \frac{v \cdot \cos \varphi}{r} \right) \cdot W' + \dots = 0 \quad (9.11)$$

$$\left(\frac{1}{r_k} + \frac{v \cdot \cos \varphi}{r}\right) \cdot U' + \dots + \frac{e^2}{12} \cdot \left[W'''' - \frac{2 \cdot \sin \varphi}{r} \cdot W''' + \dots\right] + \left(\frac{1}{r_k^2} + \frac{2 \cdot v \cdot \cos \varphi}{r_k \cdot r} + \frac{\cos^2 \varphi}{r^2}\right) \cdot W = \frac{P \cdot (1 - v^2)}{E \cdot e} \quad (9.12)$$

lower derivatives are neglected (indicated by '...')

Approximate solution for $P = 0$:

Putting $U = A \cdot \exp(\chi \cdot u)$ and $W = C \cdot \exp(\chi \cdot u)$ gives:

$$\left\{ \chi^2 - \frac{\sin \varphi}{r} \cdot \chi + \dots \right\} \cdot A + \left\{ \left(\frac{1}{r_k} + \frac{v \cdot \cos \varphi}{r} \right) \cdot \chi + \dots \right\} \cdot \left\{ \left(\frac{1}{r_k} + \frac{v \cdot \cos \varphi}{r} \right) \cdot \chi + \dots \right\} \cdot A \\ + \left\{ \frac{e^2}{12} \cdot \left[\chi^4 - \frac{2 \cdot \sin \varphi}{r} \cdot \chi^3 + \dots \right] + \left(\frac{1}{r_k^2} + \frac{2 \cdot v \cdot \cos \varphi}{r_k \cdot r} + \frac{\cos^2 \varphi}{r^2} \right) \right\}$$

The determinant of those two formulae is zero if the following relation is fulfilled.

$$\frac{e^2}{12} \cdot \left[\chi^6 - \frac{3 \cdot \sin \varphi}{r} \cdot \chi^5 + \dots \right] + \chi^2 \cdot (1 - v^2) \cdot \frac{\cos^2 \varphi}{r^2} + \dots = 0 \quad (9.13)$$

The roots of this are $\chi^2 = 0$

$$\text{and } \chi^4 = -\frac{12 \cdot (1 - v^2) \cdot \cos^2 \varphi}{e^2 \cdot r^2} + \chi^3 \cdot \frac{3 \cdot \sin \varphi}{r}$$

A first approximation for the latter is:

$$\chi^2 = \pm 2 \cdot i \cdot \sqrt{\frac{12 \cdot (1 - v^2) \cdot \cos^2 \varphi}{4 \cdot e^2 \cdot r^2}}; \\ \chi = (\pm 1 \pm i) \cdot \sqrt[4]{\frac{12 \cdot (1 - v^2) \cdot \cos^2 \varphi}{4 \cdot e^2 \cdot r^2}}$$

where $i^2 = -1$.

A second approximation is:

$$\chi^2 = \pm 2 \cdot i \cdot \sqrt{\frac{12 \cdot (1 - v^2) \cdot \cos^2 \varphi}{4 \cdot e^2 \cdot r^2} \cdot \left\{ 1 + (\pm 1 \pm i) \cdot \frac{3 \cdot \sin \varphi}{2 \cdot r} \cdot \sqrt[4]{\frac{4 \cdot e^2 \cdot r^2}{12 \cdot (1 - v^2) \cdot \cos^2 \varphi}} \right\}}$$

and the ratio of this to the first approximation is:

$$\frac{\chi_{second}}{\chi_{first}} = \sqrt[4]{1 + (1 \pm i) \cdot 3 \cdot \sin \varphi \cdot \sqrt{\frac{e}{d \cdot \cos \varphi \cdot \sqrt{12 \cdot (1 - v^2)}}}}; \quad d = 2r$$

It can be assumed that the first approximation is sufficient, if

$$3 \cdot |\sin \varphi| \cdot \sqrt{\frac{e}{d \cdot \cos \varphi \cdot \sqrt{12 \cdot (1 - v^2)}}} = \Delta \leq 0,15 \dots 0,25,$$

i.e.

$$|\sin \varphi| \leq \frac{\Delta}{3} \cdot \sqrt{\frac{d}{e} \cdot \cos \varphi \cdot \sqrt{12 \cdot (1 - v^2)}},$$

which can be simplified to:

$$\cos \varphi_s \geq 1/[1 + (0,005 \dots 0,015) \cdot d_s/e_s] \quad (9.14)$$

Hence the first approximation is valid for $\varphi_s \geq 60^\circ$ with $d_s/e_s \geq 100 \dots 200$. Numerical justification is given in Annex A.

Solution of Formulae (9.11), (9.12) as first approximation (in (9.11) U' and in (9.12) W'''' neglected):

$$U = A_0 + A_1 \cdot \frac{u}{l_s} + \exp\left(-\frac{u}{l_s}\right) \left[A_2 \cdot \cos \frac{u}{l_s} + A_3 \cdot \sin \frac{u}{l_s} \right] \quad (9.15)$$

$$W = C_0 + C_1 \cdot \frac{u}{l_s} + \exp\left(-\frac{u}{l_s}\right) \left[C_2 \cdot \cos \frac{u}{l_s} + C_3 \cdot \sin \frac{u}{l_s} \right] \quad (9.16)$$

$$l_s = \sqrt{\frac{4 \cdot e^2 \cdot r^2}{12 \cdot (1-\nu^2) \cdot \cos^2 \varphi}}, \quad (r \approx r_s, \varphi \approx \varphi_s) \quad (9.17)$$

Substitution in (9.11), (9.12) gets:

$$C_1 = 0; A_1 \cdot \frac{1}{l_s} \cdot \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) + C_0 \cdot \left(\frac{1}{r_k^2} + \frac{2 \cdot \nu \cdot \cos \varphi}{r_k \cdot r} + \frac{\cos^2 \varphi}{r^2} \right) = + \frac{P \cdot (1-\nu^2)}{E \cdot e} \quad (9.18)$$

$$A_2 = \frac{l_s}{2} \cdot \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) \cdot (C_3 + C_2); \quad A_3 = \frac{l_s}{2} \cdot \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) \cdot (C_3 - C_2) \quad (9.19)$$

Boundary conditions at $u = 0$:

$$W_{(0)} = W_s = C_0 + C_2 \rightarrow C_2 = W_s - C_0 \quad (9.20)$$

$$W'_{(0)} = \Theta_s = \frac{1}{l_s} \cdot (C_1 - C_2 + C_3) \rightarrow C_3 = \Theta_s \cdot l_s + (W_s - C_0) \quad (9.21)$$

$$N_{uu(0)} \cdot \cos \varphi + Q_{u(0)} \cdot \sin \varphi \equiv N_s \cdot \cos \varphi_s - Q_s \cdot \sin \varphi_s = \frac{P \cdot r_s}{2} + \frac{F_R}{2\pi r_s} \quad (9.22)$$

$$N_{uu(0)} \approx \frac{E \cdot e}{1-\nu^2} \cdot \left[U' + \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) \cdot W \right]_{(0)} = \frac{E \cdot e}{1-\nu^2} \cdot \left\{ A_1 \cdot \frac{1}{l_s} + C_0 \cdot \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) \right\} \quad (9.23)$$

$$Q_{u(0)} \approx - \frac{E \cdot e_3}{12 \cdot (1-\nu^2)} \cdot W'''_{(0)} = - \frac{E \cdot e^3}{12 \cdot (1-\nu^2)} \cdot \frac{2}{l_s^3} \cdot [2 \cdot (W_s - C_0) + \Theta_s \cdot l_s] \quad (9.24)$$

$$M_{uu(0)} \approx - \frac{E \cdot e_3}{12 \cdot (1-\nu^2)} \cdot W''_{(0)} = + \frac{E \cdot e^3}{12 \cdot (1-\nu^2)} \cdot \frac{2}{l_s^2} \cdot [1 \cdot (W_s - C_0) + \Theta_s \cdot l_s] \quad (9.25)$$

Formulae (9.23) and (9.24) show $Q_U \ll N_{UU}$; therefore (9.22) gives:

$$A_1 \cdot \frac{1}{l_s} + C_0 \cdot \left(\frac{1}{r_k} + \frac{\nu \cdot \cos \varphi}{r} \right) \approx \frac{r_s}{2 \cdot \cos \varphi_s} \cdot \left(P + \frac{F_R}{\pi \cdot r_s^2} \right) \cdot \frac{1-\nu^2}{E \cdot e} \quad (9.26)$$

Formulae (9.26) and (9.18) determine A_1 and C_0 :

$$C_0 = \frac{r_s^2}{E \cdot e \cdot \cos^2 \varphi_s} \cdot \left\{ P - \frac{1}{2} \cdot \left(\frac{r_s}{r_k \cdot \cos \varphi_s} + \nu \right) \cdot \left(P + \frac{F_R}{\pi \cdot r_s^2} \right) \right\} \quad (9.27)$$

A_1 and A_0 need not be calculated explicitly.

Finally (with $E = E_s$, $e = e_s$, $r = r_s = d_s / 2$) the results for the elastic case are as follows:

$$-Q_{u(0)} = Q_S = \frac{E_S \cdot e_S \cdot 2 \cdot \cos^2 \varphi_S}{d_S^2} \cdot \{(W_S - W_0) \cdot k_1 \cdot 2 \cdot l_S + (\Theta_S - \Theta_0) \cdot k_2 \cdot l_S^2\} \quad (9.28)$$

$$+M_{uu(0)} = M_S = \frac{E_S \cdot e_S \cdot 2 \cdot \cos^2 \varphi_S}{d_S^2} \cdot \{(W_S - W_0) \cdot k_2 \cdot l_S^2 + (\Theta_S - \Theta_0) \cdot k_3 \cdot l_S^3\} \quad (9.29)$$

$$+N_{uu(0)} = N_S = \frac{1}{\cos \varphi_S} \cdot \left\{ Q_S \cdot \sin \varphi_S + \frac{1}{\pi \cdot d_S} \cdot \left(P \cdot \frac{\pi}{4} \cdot d_S^2 + F_R \right) \right\} \quad (9.30)$$

$$C_0 = W_0 = \frac{P \cdot d_S^2 \cdot k_4}{E_S \cdot e_S \cdot 4 \cdot \cos^2 \varphi_S} + \frac{F_R \cdot k_6}{\pi \cdot E_S \cdot e_S \cdot \cos^2 \varphi_S} + \frac{\alpha_S \cdot \Delta T_S \cdot d_S}{2 \cdot \cos \varphi_S} \quad (9.31)$$

$$\Theta_0 = \frac{P \cdot d_S^2 \cdot k_5}{E_S \cdot e_S \cdot l_S \cdot 4 \cdot \cos^2 \varphi_S} + \frac{F_R \cdot k_7}{\pi \cdot E_S \cdot e_S \cdot l_S \cdot \cos^2 \varphi_S} \quad (9.32)$$

Additional coefficients k_1 to k_7 and Θ_0 are included to facilitate numerical comparison with the analytical solution, for which:

$$k_1 = k_2 = k_3 = 1 ; \quad k_5 = k_7 = 0 ; \quad \text{and/or } \Theta_0 = 0 \quad (9.33)$$

$$k_4 = 1 - \frac{1}{2} \cdot \left(\frac{r_S}{r_k \cdot \cos \varphi_S} + v \right); \quad k_6 = -\frac{1}{2} \cdot \left(\frac{r_S}{r_k \cdot \cos \varphi_S} + v \right) \quad (9.34)$$

subject to:

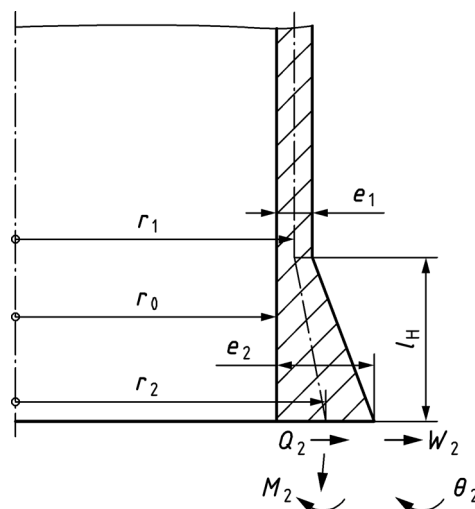
$$\cos \varphi_S \geq 1/[1 + (0,005 \dots 0,015) \cdot d_S/e_S] \quad (9.35)$$

9.2 Conical hub with cylindrical shell

The elastic stiffness of the system in Figure 10 has been calculated numerically (using computer program 'ROSCHA', TU Dresden).

For simplicity it was assumed that the system could be represented by an equivalent cylindrical shell as follows:

$$e_1 \leq e_E \leq e_2; \quad r_1 = r_0 + e_1/2; \quad r_E = r_0 + e_E/2$$



From Formulae (9.28) and (9.29):

$$Q_2 = \frac{E_S \cdot e_1}{2 \cdot r_1^2} \cdot [W_2 \cdot 2 \cdot l_1 \cdot k_1 + \Theta_2 \cdot l_1^2 \cdot k_2] = \frac{E_S \cdot e_E}{2 \cdot r_E^2} \cdot [W_2 \cdot 2 \cdot l_E + \Theta_2 \cdot l_E^2] \quad (9.36)$$

$$M_2 = \frac{E_S \cdot e_1}{2 \cdot r_1^2} \cdot [W_2 \cdot l_1^2 \cdot k_3 + \Theta_2 \cdot l_1^3 \cdot k_3] = \frac{E_S \cdot e_E}{2 \cdot r_E^2} \cdot [W_2 \cdot l_E^2 + \Theta_2 \cdot l_E^3] \quad (9.37)$$

$$l_1 = \sqrt{\frac{2 \cdot r_1 \cdot e_1}{\sqrt{12 \cdot (1 - \nu^2)}}} l_E = \sqrt{\frac{2 \cdot r_E \cdot e_E}{\sqrt{12 \cdot (1 - \nu^2)}}} l_E \quad (9.38)$$

Calculations were performed for the following value:

$$d_1/e_1 = 10 \dots 1000; \beta = e_2/e_1 = 1,5; 2; 3; 4; 6$$

$$\nu = 0,30; \chi = l_H / \sqrt{2 \cdot r_1 \cdot e_1} = 0,55; 1,10; 2,20; 4,40$$

Figure 10 — Conical hub with cylindrical shell

Values of factors k_1, k_2, k_3 were then obtained by comparison of coefficients in Formulae (9.36) and (9.37)

$$\frac{e_E}{e_1} = \frac{r_E}{r_1} \cdot k_1^{2/3}; \quad \frac{e_E}{e_1} = \left(\frac{r_E}{r_1} \cdot k_2 \right)^{1/2}; \quad \frac{e_E}{e_1} = \left(\sqrt{\frac{r_E}{r_1}} \cdot k_3 \right)^{2/5} \quad (9.39)$$

Calculated results are plotted in Figure 11, vertical bars indicate scatter, and the fitted curves are given by:

$$\frac{e_E}{e_1} \approx 1 + \frac{(\beta - 1) \cdot \chi}{\beta/3 + \chi} \quad (9.40)$$

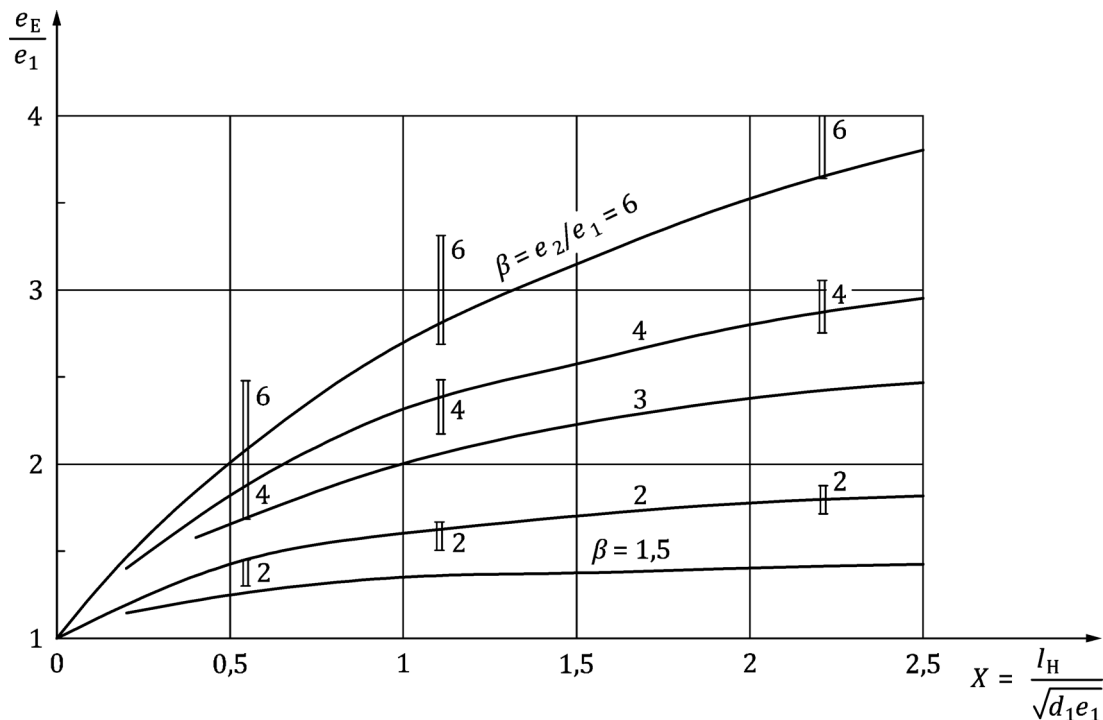


Figure 11 — Calculated results

Effective diameters d_E for different cases are defined in Figure 12.

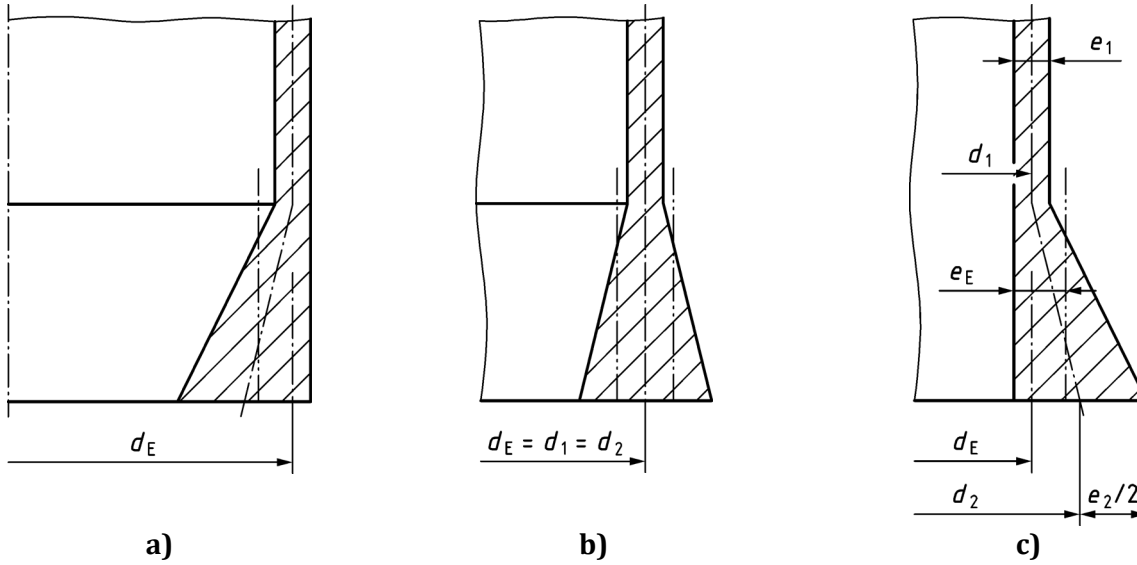


Figure 12 — Effective diameters d_E for different cases

Maximum and minimum values are:

$$d_{E\max} = \min(d_1 - e_1 + e_E; d_2 + e_2 - e_E) \quad (9.41)$$

$$d_{E\min} = \min(d_1 + e_1 - e_E; d_2 - e_2 + e_E) \quad (9.42)$$

If the outer surface is cylindrical (Figure 12 a)):

$$d_{E\max} = d_2 + e_2 - e_E = d_{E\min} = d_1 + e_1 - e_E \quad (9.43)$$

and if the inner surface is cylindrical (Figure 12 c)):

$$d_{E\max} = d_1 - e_1 + e_2 = d_{E\min} = d_2 - e_2 + e_E \quad (9.44)$$

these two extreme cases are exact.

For intermediate cases assume:

$$d_E = (d_{E\max} + d_{E\min})/2 \quad (9.45)$$

with this assumption the symmetric case (Figure 12 b)) is also exact:

$$\text{Putting } e_E = \frac{1+\varepsilon}{2} \cdot (e_1 + e_2) \quad (9.46)$$

and $d_2 = d_1$ Formulae (9.41) and (9.42) give:

$$d_{E\max} = d_1 + \frac{1}{2} \cdot (e_2 - e_1) - \frac{|\varepsilon|}{2} \cdot (e_1 + e_2) \quad (9.47)$$

$$d_{E\min} = d_1 - \frac{1}{2} \cdot (e_2 - e_1) + \frac{|\varepsilon|}{2} \cdot (e_1 + e_2) \quad (9.48)$$

and so, for all values of ε , Formula (9.45) gives $d_E = d_1 = d_2$.