



BSI Standards Publication

Estimation of uncertainty in the single burning item test

National foreword

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TECHNICAL REPORT

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Estimation of uncertainty in the single burning item test

Messunsicherheit - Thermische Beanspruchung durch
einen einzelnen brennenden Gegenstand (SBI)

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European foreword

This document (CEN/TR 16988:2016) has been prepared by Technical Committee CEN/TC 127 “Fire Safety in Buildings”, the secretariat of which is held by BSI.

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1 Scope

1.1 General

The measuring technique of the SBI (single burning item) test instrument is based on the observation that, in general, the heats of combustion per unit mass of oxygen consumed are approximately the same for most fuels commonly encountered in fires [Huggett [12]]. The mass flow, together with the oxygen concentration in the extraction system, suffices to continuously calculate the amount of heat released. Some corrections can be introduced if CO₂, CO and/or H₂O are additionally measured.

1.2 Calculation procedure

1.2.1 Introduction

The main calculation procedures for obtaining the HRR and its derived parameters are summarized here for convenience. The formulas will be used in the following clauses and especially in the clause on uncertainty.

The calculations and procedures can be found in full detail in the SBI standard [1].

1.2.2 Synchronization of data

The measured data are synchronized making use of the dips and peaks that occur in the data due to the switch from 'primary' to 'main' burner around $t = 300$ s, i.e. at the start of the thermal attack to the test specimen. Synchronization is necessary due to the delayed response of the oxygen and carbon dioxide analysers. The filters, long transport lines, the cooler, etc. in between the gas sample probe and the analyser unit, cause this shift in time.

After synchronization, all data are shifted so that the 'main' burner ignites – by definition – at time $t = 300$ s.

1.2.3 Heat output

1.2.3.1 Average heat release rate of the specimen (HRR_{30s})

A first step in the calculation of the HRR contribution of the specimen is the calculation of the global HRR. The global HRR is constituted of the HRR contribution of both the specimen and the burner and is defined as

$$\text{HRR}_{\text{total}}(t) = E' \dot{V}_{D298}(t) x_{a_O_2} \left(\frac{\phi(t)}{1 + 0,105\phi(t)} \right) \quad (1)$$

where

- $\text{HRR}_{\text{total}}(t)$ is the total heat release rate of the specimen and burner (kW);
- E' is the heat release per unit volume of oxygen consumed at 298 K, = 17 200 (kJ/m³);
- $\dot{V}_{D298}(t)$ is the volume flow rate of the exhaust system, normalized at 298 K (m³/s);
- $x_{a_O_2}$ is the mole fraction of oxygen in the ambient air including water vapour;
- $\phi(t)$ is the oxygen depletion factor.

The last two terms $x_{a_O_2}$ and $\left(\frac{\phi(t)}{1 + 0,105\phi(t)} \right)$ express the amount of moles of oxygen, per unit volume, that have chemically reacted into some combustion gases. Multiplication with the volume flow gives the

amount of moles of oxygen that have reacted away. Finally this value is multiplied with the 'Huggett' factor. Huggett stated that regardless of the fuel burnt roughly a same amount of heat is released.

The volume flow of the exhaust system, normalized at 298 K, $V_{D298}(t)$ is given by

$$V_{D298}(t) = cA \frac{k_t}{k_\rho} \sqrt{\frac{\Delta p(t)}{T_{ms}(t)}} \quad (2)$$

where

$$c = \sqrt{(2T_0 / \rho_0)} = 22,4 \text{ [K}^{0,5} \cdot \text{m}^{1,5} \cdot \text{kg}^{-0,5} \text{]}$$

A is the area of the exhaust duct at the general measurement section (m²);

k_t is the flow profile correction factor; converts the velocity at the height of the bi-directional probe in the axis of the duct to the mean velocity over the cross section of the duct;

k_ρ is the Reynolds number correction for the bidirectional probe, taken as 1,08;

$\Delta p(t)$ is the pressure difference over the bi-directional probe (Pa);

$T_{ms}(t)$ is the temperature in the measurement section (K).

The oxygen depletion factor $\phi(t)$ is defined as

$$\phi(t) = \frac{\bar{x}O_2(30s...90s)\{1 - xCO_2(t)\} - xO_2(t)\{1 - \bar{x}CO_2(30s...90s)\}}{\bar{x}O_2(30s...90s)\{1 - xCO_2(t) - xO_2(t)\}} \quad (3)$$

where

$xO_2(t)$ is the oxygen concentration in mole fraction;

$xCO_2(t)$ is the carbon dioxide concentration in mole fraction;

Ys...Zs mean taken over interval Y s to Z s.

The mole fraction of oxygen in ambient air, taking into account the moisture content, is given by

$$x_{a_O_2} = \bar{x}O_2(30s...90s) \left[1 - \frac{H}{100p} \exp \left\{ 23,2 - \frac{3816}{\bar{T}_{ms}(30s...90s) - 46} \right\} \right] \quad (4)$$

where

$xO_2(t)$ is the oxygen concentration in mole fraction;

H is the relative humidity (%);

p is the ambient pressure (Pa);

$T_{ms}(t)$ is the temperature in the general measurement section (K).

Since we are interested in the HRR contribution of the specimen only, the HRR contribution of the burner should be subtracted. An estimate of the burner contribution $HRR_{burner}(t)$ is taken as the $HRR_{total}(t)$ during the base line period preceding the thermal attack to the specimen. A mass flow controller ensures an identical HRR through the burners before and after switching from primary to the main burner. The average HRR of the burner is calculated as the average $HRR_{total}(t)$ during the base line period with the primary burner on ($210 \text{ s} \leq t \leq 270 \text{ s}$):

$$\text{HRR}_{\text{av_burner}} = \overline{\text{HRR}}_{\text{total}}(210\text{ s} \dots 270\text{ s}) \quad (5)$$

where

HRR_{av_burner} is the average heat release rate of the burner (kW);

HRR_{total}(t) is the total heat release rate of specimen and burner (kW).

HRR of the specimen

In general, the HRR of the specimen is taken as the global HRR, $\text{HRR}_{\text{total}}(t)$, minus the average HRR of the burner, $\text{HRR}_{\text{av_burner}}$:

For $t > 312\text{ s}$:

$$\text{HRR}(t) = \text{HRR}_{\text{total}}(t) - \text{HRR}_{\text{av_burner}} \quad (6)$$

where:

HRR(t) is the heat release rate of the specimen (kW);

HRR_{total}(t) is the global heat release rate of specimen and burner (kW);

HRR_{av_burner} is the average heat release rate of the burner (kW).

During the switch from the primary to the main burner at the start of the exposure period, the total heat output of the two burners is less than $\text{HRR}_{\text{av_burner}}$ (it takes some time for the gas to be directed from one burner to the other). Formula (24) gives negative values for $\text{HRR}(t)$ for at most 12 s (burner switch response time). Such negative values and the value for $t = 300\text{ s}$ are set to zero, as follows:

For $t = 300\text{ s}$:

$$\text{HRR}(300) = 0\text{ kW} \quad (7)$$

For $300\text{ s} < t \leq 312\text{ s}$:

$$\text{HRR}(t) = \max\{0\text{ kW}, \text{HRR}_{\text{total}}(t) - \text{HRR}_{\text{av_burner}}\} \quad (8)$$

where

max.[a, b] is the maximum of two values a and b.

Calculation of $\text{HRR}_{30\text{s}}$

In view of the calculation of the FIGRA index, the HRR data are smoothened with a 'flat' 30 s running average filter using 11 consecutive measurements:

$$\text{HRR}_{30\text{s}}(t) = \frac{0,5\text{HRR}(t-15) + \text{HRR}(t-12) + \dots + \text{HRR}(t+12) + 0,5\text{HRR}(t+15)}{10} \quad (9)$$

where

HRR_{30s}(t) is the average of $\text{HRR}(t)$ over 30 s (kW);

HRR(t) is the heat release rate at time t (kW).

1.2.3.2 Calculation of THR(t) and THR_{600s}

The total heat release of the specimen $\text{THR}(t)$ and the total heat release of the specimen in the first 600 s of the exposure period ($300\text{ s} \leq t \leq 900\text{ s}$), $\text{THR}_{600\text{s}}$, are calculated as follows:

$$\text{THR}(t_a) = \frac{1}{1000} \sum_{300s}^{t_a} \text{HRR}(t) \times 3 \quad (10)$$

$$\text{THR}_{600s} = \frac{1}{1000} \sum_{300s}^{900s} \text{HRR}(t) \times 3 \quad (11)$$

whereby the factor 1 000 is introduced to convert the result from kJ into MJ and the factor 3 stands for the time interval in-between 2 consecutive measurements,

and where

$\text{THR}(t_a)$ is the total heat release of the specimen during the period $300 \text{ s} \leq t \leq t_a$ (MJ);

$\text{HRR}(t)$ is the heat release rate of the specimen (kW);

THR_{600s} is the total heat release of the specimen during the period $300 \text{ s} \leq t \leq 900 \text{ s}$ (MJ); (equal to $\text{THR}(900)$).

1.2.3.3 Calculation of FIGRA_{0,2MJ} and FIGRA_{0,4MJ} (Fire growth rate indices)

The FIGRA is defined as the maximum of the ratio $\text{HRR}_{av}(t)/(t - 300)$, multiplied by 1 000. The ratio is calculated only for that part of the exposure period in which the threshold levels for HRR_{av} and THR have been exceeded. If one or both threshold values are not exceeded during the exposure period, FIGRA is equal to zero. Two combinations of threshold values are used, resulting in FIGRA_{0,2MJ} and FIGRA_{0,4MJ}.

- a) The average of HRR, HRR_{av} , used to calculate the FIGRA is equal to HRR_{30s} , with the exception of the first 12 s of the exposure period. For data points in the first 12 s, the average is taken only over the widest possible symmetrical range of data points within the exposure period:

$$\text{For } t = 300 \text{ s: } \text{HRR}_{av}(300 \text{ s}) = 0 \quad (12)$$

$$\text{For } t = 303 \text{ s: } \text{HRR}_{av}(303 \text{ s}) = \overline{\text{HRR}}(300 \text{ s} \dots 306 \text{ s}) \quad (13)$$

$$\text{For } t = 306 \text{ s: } \text{HRR}_{av}(306 \text{ s}) = \overline{\text{HRR}}(300 \text{ s} \dots 312 \text{ s}) \quad (14)$$

$$\text{For } t = 309 \text{ s: } \text{HRR}_{av}(309 \text{ s}) = \overline{\text{HRR}}(300 \text{ s} \dots 318 \text{ s}) \quad (15)$$

$$\text{For } t = 312 \text{ s: } \text{HRR}_{av}(312 \text{ s}) = \overline{\text{HRR}}(300 \text{ s} \dots 324 \text{ s}) \quad (16)$$

$$\text{For } t \geq 315 \text{ s: } \text{HRR}_{av}(t) = \text{HRR}_{30s}(t) \quad (17)$$

- b) Calculate FIGRA_{0,2MJ} for all t where:

$$(\text{HRR}_{av}(t) > 3 \text{ kW}) \text{ and } (\text{THR}(t) > 0,2 \text{ MJ}) \text{ and } (300 \text{ s} < t \leq 1\,500 \text{ s});$$

and calculate FIGRA_{0,4MJ} for all t where:

$$(\text{HRR}_{av}(t) > 3 \text{ kW}) \text{ and } (\text{THR}(t) > 0,4 \text{ MJ}) \text{ and } (300 \text{ s} < t \leq 1\,500 \text{ s});$$

both using:

$$\text{FIGRA} = 1\,000 \times \max \left(\frac{\text{HRR}_{\text{av}}(t)}{t - 300} \right) \quad (18)$$

where:

FIGRA is the fire growth rate index

HRR_{av}(t) is the average of HRR(t) as specified in a) (kW);

As a consequence, specimens with a HRR_{av} not exceeding 3 kW during the total test have FIGRA values FIGRA_{0,2MJ} and FIGRA_{0,4MJ} equal to zero. Specimens with a THR not exceeding 0,2 MJ over the total test period have a FIGRA_{0,2MJ} equal to zero and specimen with a THR not exceeding 0,4 MJ over the total test period have a FIGRA_{0,4MJ} equal to zero.

2 Uncertainty

2.1 Introduction

According to EN ISO/IEC 17025 [3], which sets out the general requirements for the competence of testing and calibration laboratories, and EN ISO 10012 [7], which sets out the requirements for assuring the quality of measuring equipment, uncertainties shall be reported in both testing and calibration reports.

The general principles for evaluating and reporting uncertainties are given in the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [6], but need to be applied to the specific case of fire testing. Due to the harmonization of fire testing in the European Community (EUROCLASSES; EN 13501-1 [21]) and the pressure on testing laboratories to operate under accreditation, this is becoming even more important.

It is of common knowledge that measurement results are never perfectly accurate. In practice the sources of systematic and random errors which can affect the results of measurement are numerous, even for the most careful operators. To describe this lack of perfection, the term 'uncertainty' is used. Although the concept of uncertainty may be related to a 'doubt', in the real sense the knowledge of uncertainty implies increased confidence in the validity of results.

The qualitative concept of accuracy is quantified by the uncertainty which varies inversely 'proportioned' to it. Accuracy consists of both trueness and precision as shown in Figure 1. A numerical measure for precision is the standard deviation, while trueness is expressed numerically by the systematic error or the bias.

It is considered good practice to eliminate any systematic errors. However, if the value of a systematic error is unknown it may be regarded as a random error. Random errors result in a spread of the values and can usually be reduced by increasing the number of observations. Its expectation or expected value is zero.

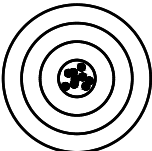
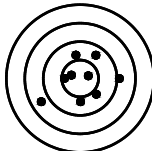
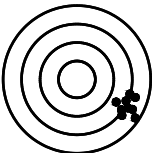
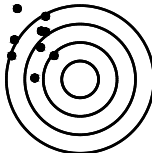
	high precision	low precision
high trueness	 (high accuracy)	
low trueness		

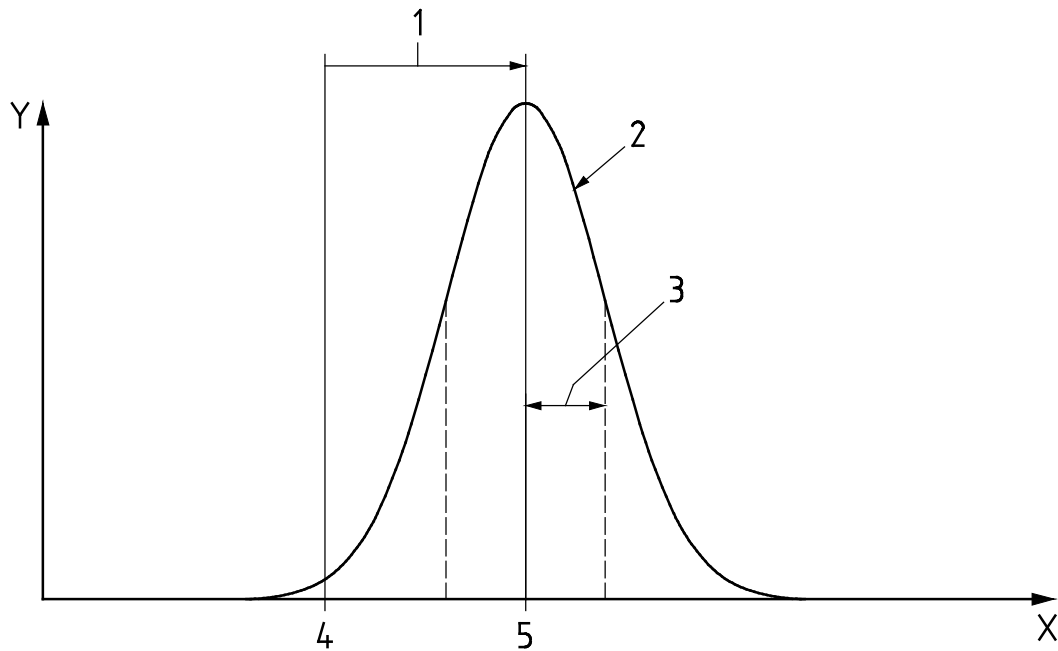
Figure 1 — Concepts of accuracy (uncertainty), precision (standard deviation) and trueness (bias)

In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement, that is, the measurand, and so the result is complete only when accompanied by a quantitative statement of its uncertainty.

Without knowledge of the accuracy (trueness and precision) of measurement methods and/or the uncertainty of measurement results, it can appear very easy to make decisions. But, in practice, these decisions might be incorrect and sometimes lead to serious consequences, if the measurement uncertainty is not taken into account.

For example, in fire testing, when rejecting instead of accepting a good product during a certification process or, conversely, when accepting a bad product by error. So, it is vital to quantify the reliability of the measurement results to greatly reduce any disputes and adverse consequences of legal proceedings. This is of particular importance if the growing number of cases of litigation in Europe and the liability problems of manufacturers in case of accidents are considered.

The difference between error and uncertainty should always be borne in mind. For example, the result of a measurement after correction can unknowably be very close to the unknown value of the measurand, and thus have negligible error, even though it might have a large uncertainty.



Key

- X value
- Y frequency
- 1 bias
- 2 repeated measurements would give values with this frequency curve
- 3 standard deviation (σ)
- 4 true value
- 5 expected value

Figure 2 — Concepts of accuracy (uncertainty), precision (standard deviation) and trueness (bias)

2.2 Elaboration of terms and concepts

2.2.1 Mean and variance

A population with a 'normal' probability density function is characterized by its mean value μ and its variance σ^2 : $N(\mu, \sigma^2)$. When both μ and σ^2 are unknown, they can be estimated by taking a number n of samples and by calculating the estimated mean $\bar{x}$, the estimated variance s^2 and the estimated standard deviation s .

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (19)$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (20)$$

If a covariance exists between two variables x and y , it is given by

$$s_{ij}^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) \quad (21)$$

2.2.2 Estimation of the confidence interval for the population mean

Often the standard deviation σ is unknown. To evaluate the confidence interval, some estimate of σ shall be made. The most obvious candidate is the sample standard deviation s . But the use of s introduces an additional source of unreliability, especially if the sample is small. To retain the confidence interval, the interval shall therefore be widened. This is done by using the t distribution instead of the standard normal distribution. For a sample size larger than 100, the t -distribution approaches the normal distribution. For a 95 % (two tails of 2,5 %) confidence interval – which we strive for – the uncertainty is estimated by

$$t_{0.025} \frac{s}{\sqrt{n}} \quad (22)$$

The value $t_{0.025}$ depends on the amount of information used in calculating s^2 , i.e. on the degrees of freedom. For large sample sizes, $t_{0.025}$ approaches 1,96 which is the value for a normal distribution. For a normal distribution, a coverage factor 2 (1,96) corresponds to a 95 % confidence interval (see 2.2.6).

2.2.3 Sources of uncertainty

According to GUM [6] any detailed report of the uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value. The components may be grouped into two categories based on their method of evaluation:

Type A **The components in category A are characterized by the estimated variances s_i^2 or by the estimated standard deviation s_i derived from data by statistical methods. Where appropriate the covariance s_{ij}^2 should be given.**

For such a component, the standard uncertainty is $u_i = s_i$.

Type B **The standard uncertainty of a Type B evaluation is approximated based on specifications, calibrations, handbooks, experience, judgements etc. and is represented by a quantity u_j . It is obtained from an assumed probability distribution based on all the available information.**

Where appropriate the covariance should be given and should be treated in a similar way.

The 'type' classification does not indicate any difference in the nature of the components resulting from the two types of evaluation. Both are based on probability distributions, and the uncertainty components resulting from either type are quantified by standard deviations. It should be recognized that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation.

The standard deviation of a Type B evaluation is based on the shape of the distribution. Distributions used in this document are the rectangular, the triangular, the trapezoidal and the normal distribution. For the rectangular and triangular also asymmetric distributions are discussed.

2.2.4 Standard uncertainties for different distributions

Normal distribution

Often calibration certificates, handbooks, manufacturer's specifications, etc. state a particular multiple of a standard deviation. In this case, a normal distribution is assumed to obtain the standard uncertainty.

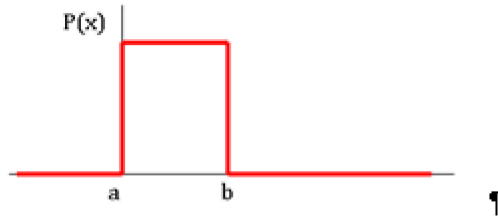
Rectangular distribution

In other cases the probability that the value of X_i lies within the interval a^- to a^+ for all practical purposes is equal to one and the probability that X_i lies outside this interval is essentially zero. If there is no specific knowledge about the possible values of X_i within the interval, a uniform or rectangular

distribution of values is assumed. The associated standard deviation is function of the width of the distribution as:

$$u_{rect} = \frac{a}{\sqrt{3}} \quad (23)$$

Indeed, for a rectangular distribution, the variance is obtained as in 24. Given the probability function of the rectangular distribution



$$P(x) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a < x < b \\ 0 & x > b \end{cases} \quad (24)$$

This can be written in terms of the Heaviside step function $H(x)$ as

$$P(x) = \frac{H(x-a) - H(x-b)}{b-a} \quad (25)$$

This makes that the variance σ^2 with population mean μ for an asymmetric distribution becomes

$$\mu = \int_{-\infty}^{\infty} P(x)x dx = \int_a^b \frac{x}{(b-a)} dx = \frac{b+a}{2} \quad (26)$$

$$\sigma^2 = \int_{-\infty}^{\infty} P(x)(x-\mu)^2 dx \quad (27)$$

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{H(x-a) - H(x-b)}{b-a} (x - \frac{a+b}{2})^2 dx \quad (28)$$

$$\sigma^2 = \int_a^b \frac{(x - \frac{a+b}{2})^2}{b-a} dx = \frac{(a+b)^2}{12} \quad (29)$$

So for a symmetric rectangular interval $a-$ to $a+$, the variance reduces to

$$\sigma^2 = \frac{a^2}{3}. \quad (30)$$

The sample estimate of the standard deviation thus is:

$$u_{rect} = \frac{a}{\sqrt{3}}. \quad (31)$$

The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the centre of the limits are more likely than values close to the limits, a triangular or a normal distribution might be a better model.

Triangular and trapezoidal distribution

In many cases it is more realistic to expect that values near the bounds are less likely than those near the midpoint. It is then reasonable to replace the symmetric rectangular distribution by a symmetric trapezoidal distribution having equal sloping sides, a base of width $2a$ and a top of width $2a\beta$ where $0 \leq \beta \leq 1$. Similar as for a rectangular distribution, for a trapezoidal distribution the standard deviation becomes:

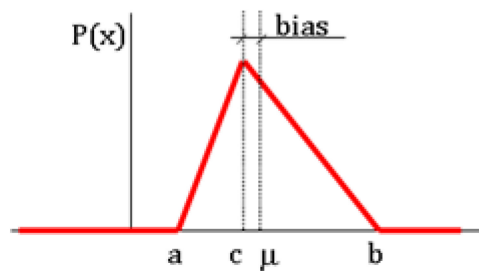
$$u_{trap} = \frac{a\sqrt{(1+\beta^2)}}{\sqrt{6}} \quad (32)$$

As β goes to 1 this trapezoidal distribution approaches the rectangular distribution while for $\beta = 0$ it is a triangular distribution.

$$u_{trian} = \frac{a}{\sqrt{6}} \quad (33)$$

Asymmetric distributions

For an asymmetric triangular distribution the mean value and standard deviation become



$$\mu = \frac{1}{3}(a + b + c) \quad (34)$$

$$u_{asymmetric_triangular} = \sqrt{\frac{1}{18}(a^2 + b^2 + c^2 - ab - ac - bc)} \quad (35)$$

For the limit case of a 'one sided triangular distribution ($a = c = 0$; $b = b$) this reduces to

$$\mu = \frac{b}{3} \quad (36)$$

$$u_{asymmetric_triangular} = \frac{b}{3\sqrt{2}} \quad (37)$$

The limit case of a 'one sided rectangular distribution ($a = 0$; $b = b$) reduces to

$$\mu = \frac{b}{2} \quad (38)$$

$$u_{\text{asymmetric_rectangular}} = \frac{b}{\sqrt{12}} \quad (39)$$

For asymmetric distributions the difference between the mean value μ and c will be considered as a bias and, as recommended by the GUM [6], it will be corrected for.

2.2.5 Combined uncertainty

2.2.5.1 Uncorrelated input quantities

The standard uncertainty of y , where y is the estimate of the measurand Y and thus the measurement, is obtained by appropriately combining the standard uncertainties of the input estimates x_i . In case all input quantities are independent, this combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$ and is given by

$$u_c = \sqrt{\sum_{i=1}^N [c_i u(x_i)]^2} \quad (40)$$

where the sensitivity coefficient is given by

$$c_i = \frac{\partial y}{\partial x_i} \quad (41)$$

2.2.5.2 Correlated input quantities

Formula (41) is only valid when the input quantities X_i are independent or uncorrelated. When the input quantities are correlated, the appropriate expression for the combined variance $u_c^2(y)$ associated with the result of the measurement is

$$u_c = \sqrt{\sum_{i=1}^N [c_i u(x_i)]^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N c_i c_j u(x_i) u(x_j) r(x_i, x_j)} \quad (42)$$

whereby the degree of correlation between x_i and x_j is characterized by the estimated correlation coefficient (Pearson)

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i) u(x_j)} \quad -1 \leq r(x_i, x_j) \leq +1 \quad (43)$$

If the estimates x_i and x_j are independent, $r(x_i, x_j) = 0$, and a change in one does not imply an expected change in the other. Pearson's coefficient reflects the degree of linear relationship between two data sets. Its value is between -1 and $+1$. A value of $+1$ means that there is a perfect positive linear relationship between the two data sets. A value of -1 means that there is a perfect negative linear relationship, and a value of 0 means there is no linear relationship at all between the data sets.

The measurands E' , p_{atm} , T_{room} , ϕ , α , c , A , k_t and k_p , are constant or can be considered constant throughout the test. They are treated as independent input quantities.

The oxygen concentration (XO_2), the carbon dioxide concentration (XCO_2), the exhaust gas temperature in the measurement section (T_{ms}) and the differential pressure over the velocity probe (Δp) are significantly correlated to each other and the correlation coefficients will be chosen as indicated in Table 1.

Table 1 — Assumed correlation coefficients between XO_2 , XCO_2 , T_{ms} and Δp

	O_2	CO_2	Δp	T_{ms}
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O₂	1	-1	-1	-1
CO₂	-1	1	1	1
Δp	-1	1	1	1
T_{ms}	-1	1	1	1

2.2.6 Expanded uncertainty

Although the combined standard uncertainty u_c is used to express the uncertainty of a wide variety of applications, what is often required is a measure of uncertainty that defines an interval about the measurement result y within which the value of the measurand Y can be confidently asserted to lie. The measure of uncertainty intended to meet this requirement is termed expanded uncertainty, U , and is obtained by multiplying $u_c(y)$ by a coverage factor k . So $U = k u_c(y)$ and it is confidently believed that Y is greater than or equal to $y - U$, and is less than or equal to $y + U$, which is commonly written as $Y = y \pm U$.

In general, the value of the coverage factor k is chosen on the basis of the desired level of confidence to be associated with the interval defined by $U = k u_c$. Typically, k is in the range 2 to 3. When the normal distribution applies and u_c is a reliable estimate of the standard deviation of y , $U = 2 u_c$ (i.e. $k = 2$) defines an interval having a level of confidence of approximately 95 % (95,44 %), and $U = 3 u_c$ (i.e. $k = 3$) defines an interval having a level of confidence greater than 99 % (99,73 %).

In this document, we will work with standard deviations to express the uncertainty of individual measurands and combined standard uncertainties, and with a coverage factor of 2 to express the confidence interval on the estimate of the overall uncertainty.

On occasion, a self-defined apparent standard deviation defined as the uncertainty for a 95 % interval divided by 1,96 will be used. This is useful when for example using t-distributions since for this distribution the expanded uncertainty $U(k = 1,96) \neq 1,96 * U(k = 1)$. Since we know that we finally want to end up with a 95 % confidence interval but we are working with standard deviations, t-distributions will be calculated based on a 95 % confidence interval divided by 1,96.

This allows us to work with standard deviations all the time and to, at the end, multiply with the coverage factor of $k = 1,96$.

$$U_{95\%}(x) = 1,96 \sqrt{\left(c_{x1} \frac{U(x_1)}{1,96} \right)^2 + \left(c_{x2} \frac{U(x_2)}{1,96} \right)^2 + \dots} \quad (44)$$

2.2.7 Uncorrected bias

Although it is recommended (and strongly preferred) practice of correcting for all known bias effects, in view of backwards compatibility of test results for example, an increased uncertainty interval may be the preferred option.

Unfortunately, the GUM [6] does not deal directly with the situation where a known measurement bias is present but is uncorrected.

Several proposed methods of treating uncorrected bias are available. We propose to follow the guidelines of Phillips et al. [14] because of its conservative approach. This method algebraically sums the signed bias δ with the expanded uncertainty, unless the bias is larger:

$$Y = y_{-U}^{+U} \quad (45)$$

Where