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Space engineering — Structural materials handbook

Part 2: Design calculation methods and general design aspects

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European Foreword

This document (CEN/TR 17603-32-02:2022) has been prepared by Technical Committee CEN/CLC/JTC 5 “Space”, the secretariat of which is held by DIN.

It is highlighted that this technical report does not contain any requirement but only collection of data or descriptions and guidelines about how to organize and perform the work in support of EN 16603-32.

This Technical report (CEN/TR 17603-32-02:2022) originates from ECSS-E-HB-32-20 Part 2A.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association.

This document has been developed to cover specifically space systems and has therefore precedence over any TR covering the same scope but with a wider domain of applicability (e.g.: aerospace).

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Introduction

The Structural materials handbook is published in 8 Parts.

A glossary of terms, definitions and abbreviated terms for these handbooks is contained in Part 8.

The parts are as follows:

TR 17603-32-01	Part 1	Overview and material properties and applications	Clauses 1 - 9
TR 17603-32-02	Part 2	Design calculation methods and general design aspects	Clauses 10 - 22
TR 17603-32-03	Part 3	Load transfer and design of joints and design of structures	Clauses 23 - 32
TR 17603-32-04	Part 4	Integrity control, verification guidelines and manufacturing	Clauses 33 - 45
TR 17603-32-05	Part 5	New advanced materials, advanced metallic materials, general design aspects and load transfer and design of joints	Clauses 46 - 63
TR 17603-32-06	Part 6	Fracture and material modelling, case studies and design and integrity control and inspection	Clauses 64 - 81
TR 17603-32-07	Part 7	Thermal and environmental integrity, manufacturing aspects, in-orbit and health monitoring, soft materials, hybrid materials and nanotechnoligies	Clauses 82 - 107
TR 17603-32-08	Part 8	Glossary	

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10 Stress-strain relationships

10.1 Introduction

The many analytical and practically derived methods developed for predicting the stress-strain behaviour of composite materials are reviewed.

Calculation methods are given for:

- Behaviour prediction from constituent properties, [See:10.2].
- The calculation of intra-ply, [See: 10.10].
- Interlaminar stress-strains, [See: 16.16].

10.2 Elastic property prediction for UD ply from constituent properties

UD ply properties can be predicted by the procedures of micro-mechanics and be measured by physical means. These can then be used in a macro-mechanical analysis of the structure.

Micro-mechanical analysis has inherent limitations. Fibre properties cannot easily be measured, so they are determined from measurements of UD by applying inverse homoginisation formulae. The transverse and shear properties are particularly inaccurate.

In addition, a perfect bond between fibres and matrix is a common assumption for analysis, which is not always true for some composites. Imperfect bonds give a material with poorer properties than those of the micro-mechanical analysis. So, the analysis should be proven by careful experimental work. The derivation of macro-mechanical properties is summarised in Figure 10.2-1.

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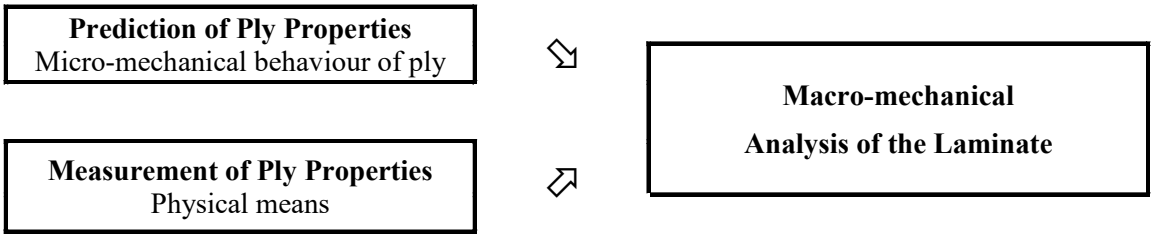


Figure 10.2-1 - Derivation of macro-mechanical properties for the analysis of laminates

The objective of the micro-mechanical approach is to determine the elastic moduli of a composite material in terms of the moduli of the constituent materials. This is also often referred to as the stiffness or compliance.

The additional and complementary objective of the micro-mechanical approach is to determine the strengths of the composite material in terms of the strengths of the constituent materials.

The basic assumptions for the analytical treatment of a composite material are:

- UD ply:
  - Macroscopically homogeneous
  - Linearly elastic
  - Macroscopically orthotropic
  - Initially stress-free
  - Perfect bond between fibres and matrix
- Fibre:
  - Homogeneous
  - Linearly elastic
  - Orthotropic
  - Regularly spaced
  - Perfectly aligned
- Matrix:
  - Homogeneous
  - Linearly elastic
  - Isotropic

Material characteristics of matrices and fibres, along with the fibre volume fraction, can significantly affect the macro-mechanical properties and therefore need careful evaluation.

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10.3 Analytical notation for elastic constant methods

Formulae:

E	Young's modulus
ν	Poisson's ratio
G	Shear modulus
V	Volume ratio
E_1	Young's modulus of a ply in the fibre direction
E_2	Young's modulus of a ply in the direction transverse to the fibre
ν_x	Poisson's ratio of the ply when loaded in x-direction
G_{12}	In-plane shear modulus of the ply
η	Stress partitioning parameter

Subscripts:

f	Fibre
m	Matrix
1	Fibre direction
2	Transverse to fibre
12	Shear (in-plane)

10.4 Calculation methods for elastic constants of UD ply

Various authors have expressed analytically the elastic constants of an orthotropic layer dependent upon the Young's moduli, Poisson's ratio and shear moduli of the fibre and matrix according to their volume ratio.

The equations presented have been developed in each case for one specific material system, i.e. glass/epoxy or carbon/epoxy. To verify each equation with respect to commonly used composites, they are depicted together graphically, with test results. Where no test results are given, the diagrams can be used for preliminary design purposes only.

Data used for comparison are given in Table 10.4-1. The basis of the approach used by each author, as referenced, is shown in Table 10.4-2.

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Table 10.4-1 - Elastic property data for comparison

	E_1 (kN/mm²)	E_2 (kN/mm²)	G_{12} (kN/mm²)	ν_{12} (%)	Ref. [X] (Key)
Epoxy (1)	3.5	3.5	1.3	0.35	[10-3]
Cyanate Ester (4)	-	-	-	-	-
Glass, E	73	73	30	0.22	[10-2]
Carbon, HT	230	24	50 9.4(2)	0.28	[10-3]
Carbon, HM	550	6.0	18.0 2.0(2)	0.36	(3)
Aramid	130	7.0	12.0 2.5(2)	0.38	(3)
Key: (1) Epoxy: CY209/HT 972, Fibredux 914 (2) G_{12} defined, Ref. [10-5] (3) Estimated (4) ESA-study					

Table 10.4-2 - Basis of calculation methods for different authors

Author	Ref. [X]	Basis
Jones	[10-1]	Based on "representative volume element" with isotropic fibres.
Förster/Knappe	[10-2]	Semi-empirical approach for glass/epoxy.
Schneider	[10-3]	Semi-empirical approach for carbon (HT)/epoxy.
Puck	[10-4]	Semi-empirical approach for glass/epoxy.
Tsai	[10-5]	Semi-empirical approach including transverse and shear modulus.
HSB	[11-6]	Model of circular fibres with "square packing".

The approach used by each referenced author is presented for:

- Longitudinal modulus, [See:10.5].
- Longitudinal Poisson's ratio, [See: 10.6].
- Transverse modulus, [See: 10.7].
- Transverse Poisson's ratio, [See: 10.8].
- Transverse shear modulus, [See: 10.9].

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10.5 Longitudinal modulus

Each of the referenced authors, [See: Table 10.4.2], uses the same approach to derive a relationship for longitudinal modulus. This is the 'method of mixtures', i.e.:

$$E_1 = E_f V_f + E_m (1 - V_f) \qquad [10.5\text{-}1]$$

[See: 10.3 for notation]

The predicted relationship between fibre volume fraction and longitudinal modulus along with test results, where available, are shown for:

- glass/epoxy in Figure 10.5-1.
- carbon high strength HT/epoxy in Figure 10.5-2.
- carbon high modulus HM/epoxy in Figure 10.5-3.
- aramid/epoxy in Figure 10.5-4.

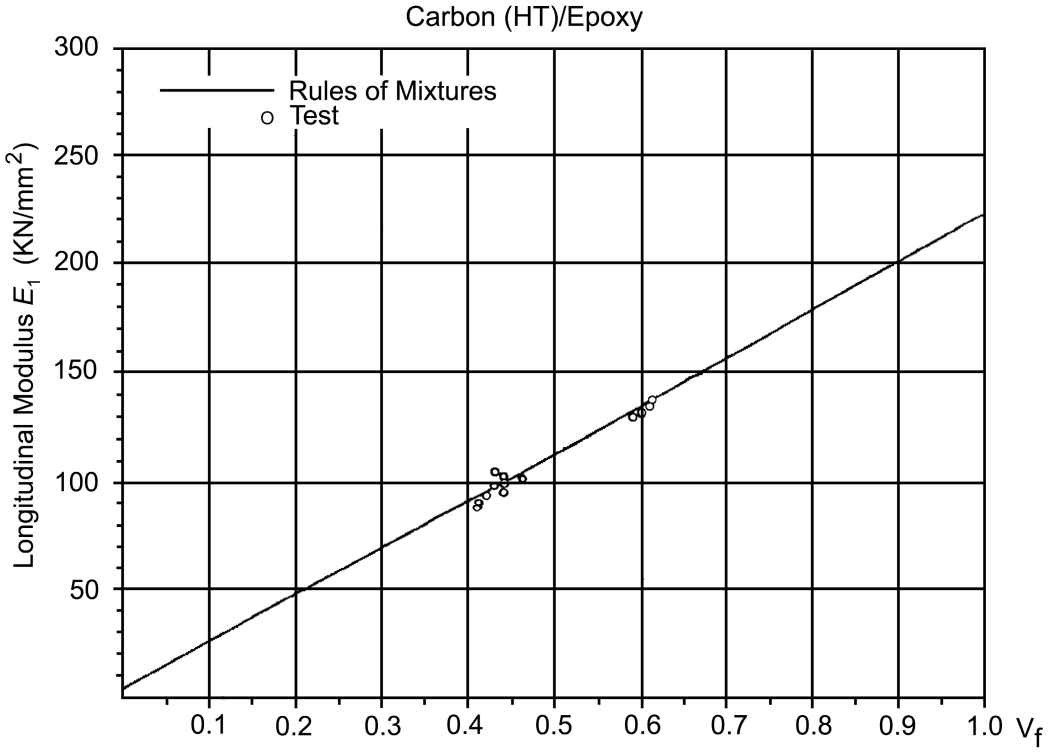


Figure 10.5-1 - Longitudinal modulus (E_1) of glass/epoxy

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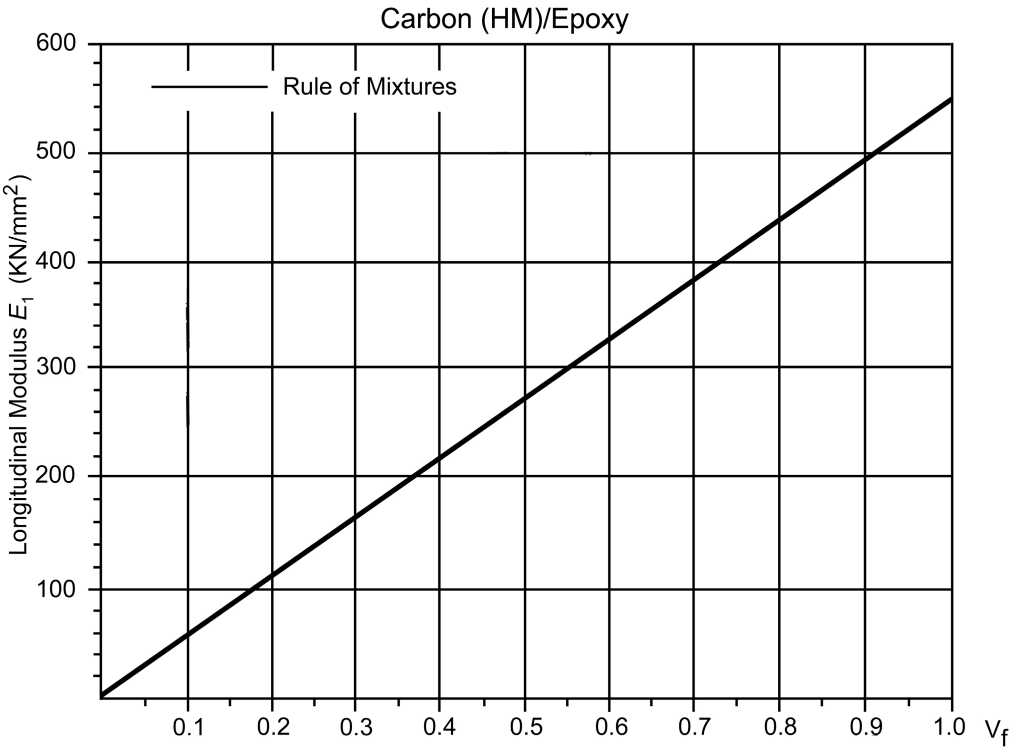


Figure 10.5-2 - Longitudinal modulus (E_1) of carbon (HT)/epoxy

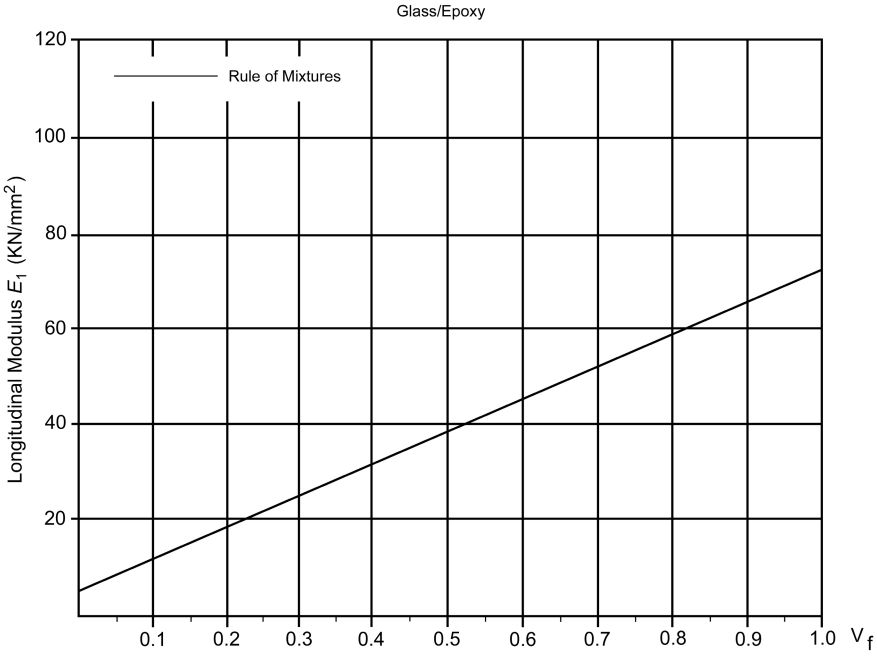


Figure 10.5-3 - Longitudinal modulus (E_1) of carbon (HM)/epoxy

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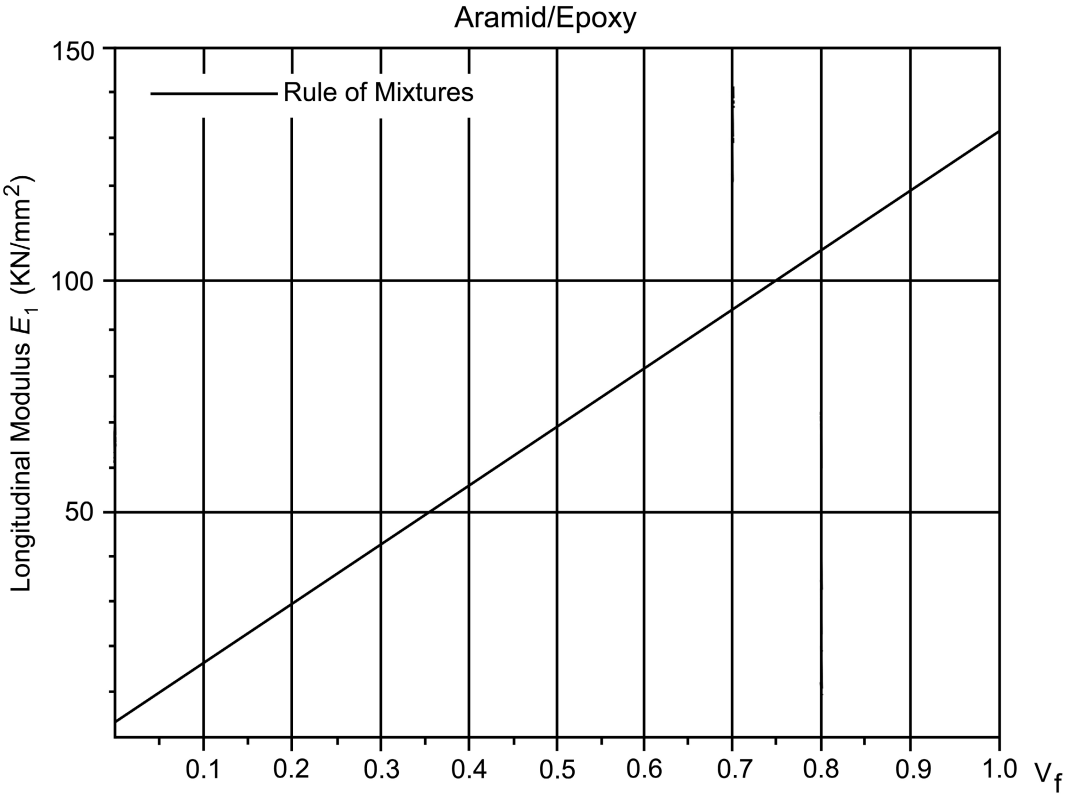


Figure 10.5-4 - Longitudinal modulus (E_1) of aramid/epoxy

10.6 Longitudinal Poisson's ratio

Each of the referenced authors, [See: Table 10.4-2], uses the same 'method of mixtures' to derive a relationship for longitudinal Poisson's ratio:

$$\nu_{12} = \nu_f V_f + \nu_m (1 - V_f) \qquad [10.6\text{-}1]$$

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10.7 Transverse modulus

10.7.1 General

The methods used by each referenced author, [See: Table 10.4-2] are given along with their predictions for transverse modulus:

- Jones method.
- Förster/Knappe method.
- Schneider method.
- Puck method.
- Tsai method.
- HSB method.

[See: 10.3 for notation]

The homogenisation formulas of Jones, Förster/Knappe, Schneider and Puck are suitable for isotropic fibres only and cannot be applied to carbon or aramid fibres, which are not isotropic.

10.7.2 Jones method

$$E_2 = \frac{E_m}{(1-V_f)+E_m V_f / E_f} \qquad [10.7\text{-}1]$$

10.7.3 Förster/Knappe method

$$E_2 = \frac{E_m^0}{(1-V_f)^{1.45}+V_f E_m^0 / E_f} \qquad [10.7\text{-}2]$$

Where: $E_m^0 = \frac{E_m}{(1-\nu_m^{\,2})}$

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10.7.4 Schneider method

$$E_2 = \frac{E_m^0 (1 - V_f^{\ 3})}{(1 - V_f)^{0.75} + 6 V_f E_m^0 / E_f} \qquad [10.7\text{-}3]$$

Where: $E_m^0 = \frac{E_m}{(1 - \nu_m^{\ 2})}$

10.7.5 Puck method

$$E_2 = \frac{E_m^0 (1 + 0.85 V_f^{\ 2})}{(1 - V_f)^{1.25} + V_f E_m^0 / E_f} \qquad [10.7\text{-}4]$$

Where: $E_m^0 = \frac{E_m}{(1 - \nu_m^{\ 2})}$

10.7.6 Tsai method

$$E_2 = \frac{4 k_y G_y}{k_y + m G_y} \qquad [10.7\text{-}5]$$

Where: $m = 1 + \frac{4 \nu_x^{\ 2} k_y}{E_x}$

$$\frac{1}{k_y} = \frac{1}{V_f + \eta_k V_m} \left[\frac{V_f}{k_{fy}} + \eta_k \frac{V_m}{k_m} \right]$$

$$V_m = 1 - V_f$$

$$\frac{1}{G_y} = \frac{1}{V_f + \eta_G V_m} \left[\frac{V_f}{G_{fy}} + \eta_G \frac{V_m}{G_m} \right]$$

$$\eta_k = \frac{1}{2(1 - \nu_m)} \left[2 - 4\nu_m + \frac{G_m}{G_{fy}} \right]$$

$$\eta_G = \frac{1}{4(1 - \nu_m)} \left[3 - 4\nu_m + \frac{G_m}{G_{fy}} \right]$$

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$$k_{fy} = \frac{G_{fy}}{1-2\nu_f}$$

$$k_m = \frac{G_m}{1-2\nu_m}$$

10.7.7 HSB method

$$E_2 = k_y E_m \left[\left(1-2\sqrt{\frac{V_f}{\pi}}\right) - \frac{\pi}{2(1-E_m/E_{fy})} + \ldots \right]$$

$$\left[\ldots \frac{2}{(1-E_m/E_{fy})\sqrt{1-4V_f/\pi(1-E_m/E_{fy})^2}} \ldots \right]$$

$$\left[\ldots \arctan \sqrt{\frac{1+2\sqrt{V_f/\pi}(1-E_m/E_{fy})}{1-2\sqrt{V_f/\pi}(1-E_m/E_{fy})}} \right] \qquad [10.7\text{-}6]$$

k_y = Correction factor; Ideal laminate $k_y = 1$

10.7.8 Graphs

The predicted relationship between fibre volume fraction and transverse modulus along with test results, where available, are shown for:

- Isotropic fibre: Glass/epoxy in Figure 10.7-1.
- Anisotropic fibres:
- Carbon high strength HT/epoxy in Figure 10.7-2.
  - — Carbon high modulus HM/epoxy in Figure 10.7-3.
  - — Aramid/epoxy in Figure 10.7-4.

The data presented for anisotropic fibres, derived from the homogenisation formulas of Jones, Förster/Knappe, Schneider and Puck can be ignored in Figure 10.7-1 to Figure 10.7-4, inclusive.

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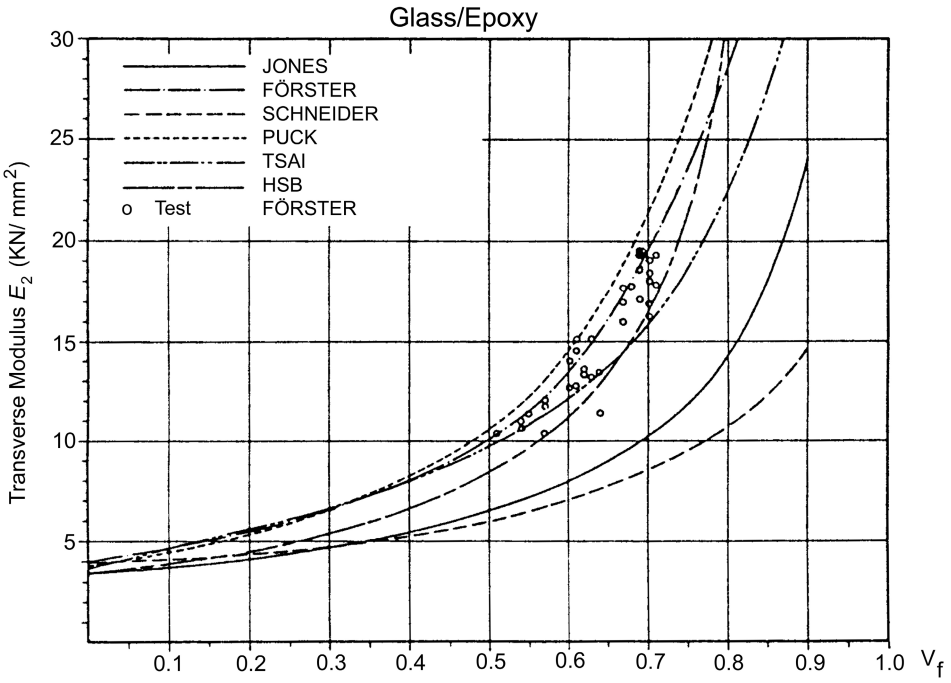
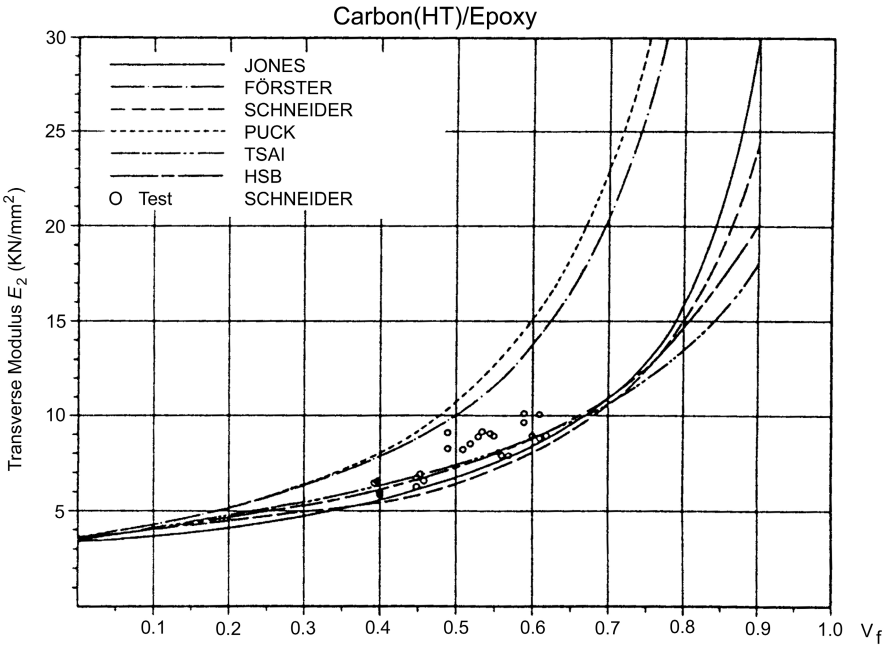


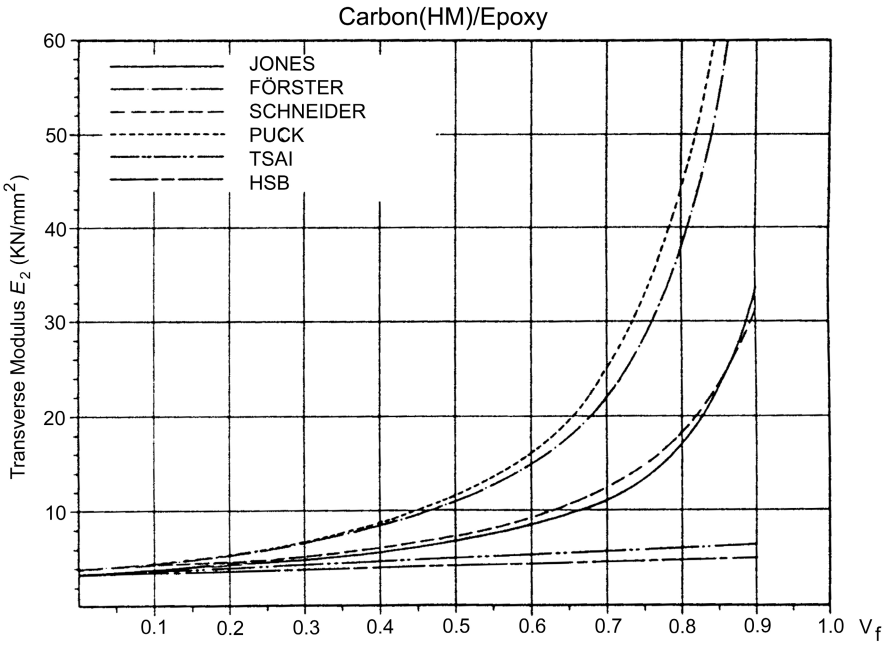
Figure 10.7-1 - Transverse modulus (E_2) of glass/epoxy composites



Carbon fibres are anisotropic. Data derived from the homogenisation formulas of Jones, Förster/Knappe, Schneider and Puck can be ignored.

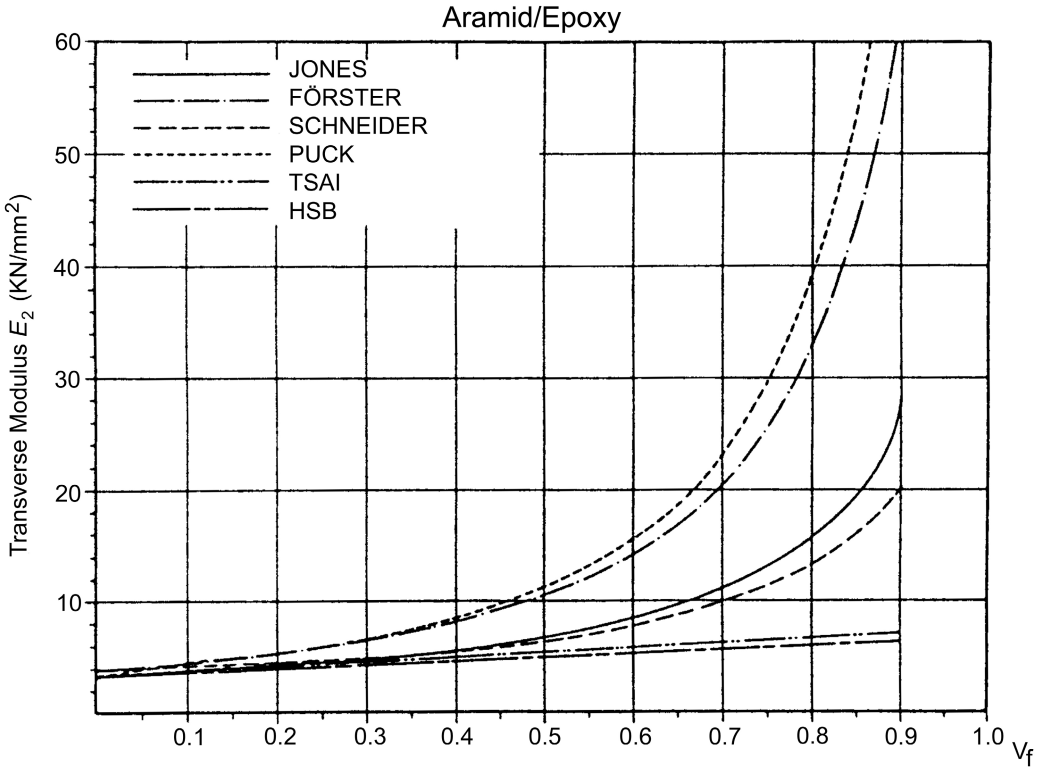
Figure 10.7-2 - Transverse modulus (E_2) of carbon (HT)/epoxy composites

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Carbon fibres are anisotropic. Data derived from the homogenisation formulas of Jones, Förster/Knappe, Schneider and Puck can be ignored.

Figure 10.7-3 - Transverse modulus (E_2) of carbon (HM)/epoxy composites



Aramid fibres are anisotropic. Data derived from the homogenisation formulas of Jones, Förster/Knappe, Schneider and Puck can be ignored.

Figure 10.7-4 - Transverse modulus (E_2) of aramid/epoxy composites

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10.8 Transverse Poisson's ratio

Each of the referenced authors, [See: Table 10.4-2] uses the same method to derive a relationship for transverse Poisson's ratio:

$$\nu_{21} = \nu_{12}\frac{E_2}{E_1} \qquad [10.8\text{-}1]$$

[See: 10.3 for notation]

The transverse Poisson's ratio is the Poisson's ratio of the ply when loaded in y-direction.

10.9 Transverse shear modulus

10.9.1 General

The methods used by each referenced author, [See: Table 10.4.2] are given along with their predictions for transverse shear modulus.

[See: 10.3 for notation]

10.9.2 Jones method

$$G_{12} = \frac{G_m}{(1-V_f)+V_f G_m/G_f} \qquad [10.9\text{-}1]$$

10.9.3 Förster/Knappe method

$$G_{12} = G_m\frac{1+0.4\sqrt{V_f}}{(1-V_f)^{1.45}+V_f G_m/G_f} \qquad [10.9\text{-}2]$$

10.9.4 Schneider method

$$G_{12} = \frac{G_m(1+0.25\sqrt{V_f})}{(1-V_f)^{1.25}+1.25V_f G_m/G_f} \qquad [10.9\text{-}3]$$

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10.9.5 Puck method

$$G_{12} = G_m \frac{1 + 0.6\sqrt{V_f}}{(1 - V_f)^{1.25} + V_f G_m / G_f} \quad [10.9\text{-}4]$$

10.9.6 Tsai method

$$G_{12} = G_m \frac{1 + 0.6\sqrt{V_f}}{(1 - V_f)^{1.25} + V_f G_m / G_f} \quad [10.9\text{-}5]$$

Where: $\eta_{12} = {}^{1}\!/_{2}\left[1 + \frac{V_m}{G_f}\right]$

10.9.7 HSB method

$$G_{12} = k_{12} G_m \left[\left(1 - 2\sqrt{\frac{V_f}{\pi}}\right) - \frac{\pi}{2(1 - G_m / G_f)} + \ldots \right]$$

$$\left[\ldots \frac{2}{(1 - G_m / G_f)\sqrt{1 - 4V_f / \pi (1 - G_m / G_f)^2}} \ldots \right]$$

$$\left[\ldots \arctan \sqrt{\frac{1 + 2\sqrt{V_f / \pi}(1 - G_m / G_f)}{1 - 2\sqrt{V_f / \pi}(1 - G_m / G_f)}} \right] \quad [10.9\text{-}6]$$

k_{12} = Correction factor. For an ideal laminate $k_{12} = 1$

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10.9.8 Graphs

The predicted relationship between fibre volume fraction and transverse shear modulus along with test results, where available, are shown for:

- Glass/epoxy in Figure 10.9.1.
- Carbon high strength HT/epoxy in Figure 10.9.2.
- Carbon high modulus HM/epoxy in Figure 10.9.3.
- Aramid/epoxy in Figure 10.9.4.

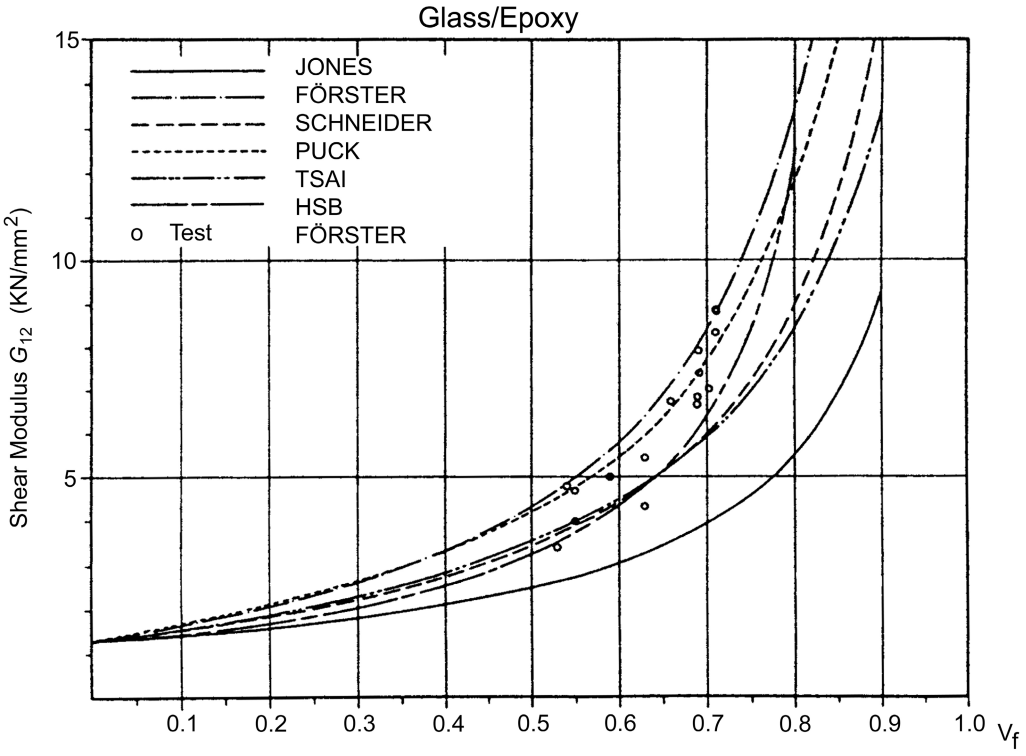


Figure 10.9-1 - Shear modulus (G_{12}) of glass/epoxy composites

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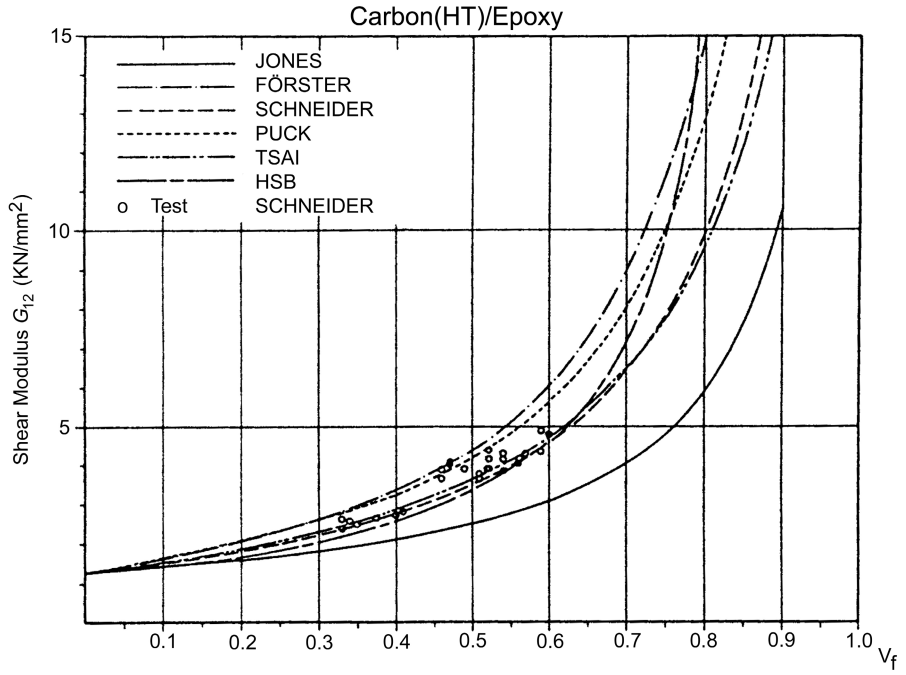


Figure 10.9-2 - Shear modulus (G_{12}) of carbon (HT)/epoxy composites

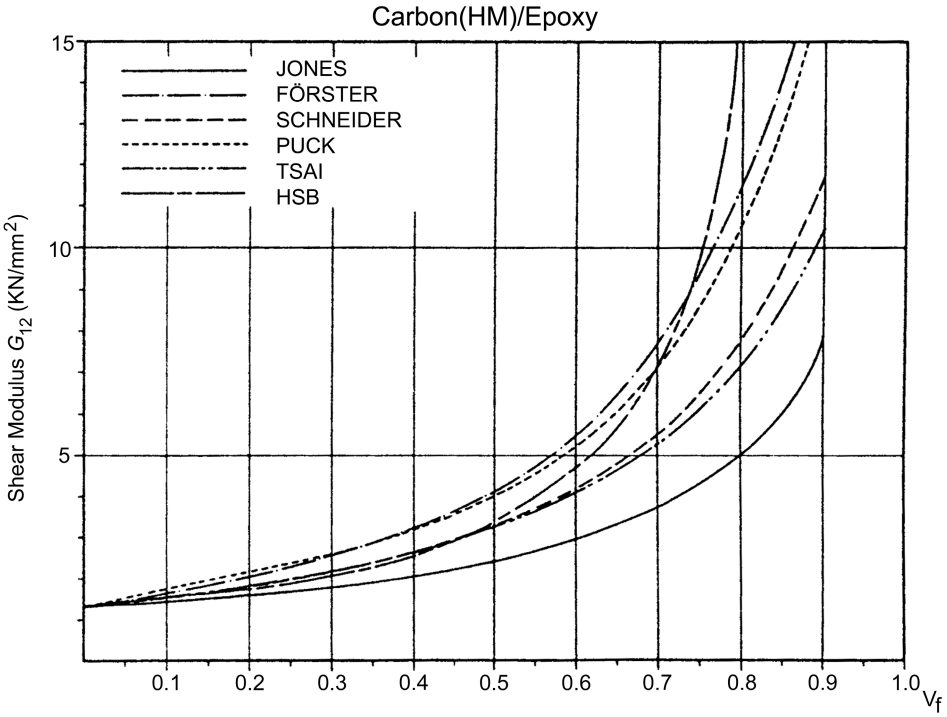


Figure 10.9-3 - Shear modulus (G_{12}) of carbon (HM)/epoxy composites

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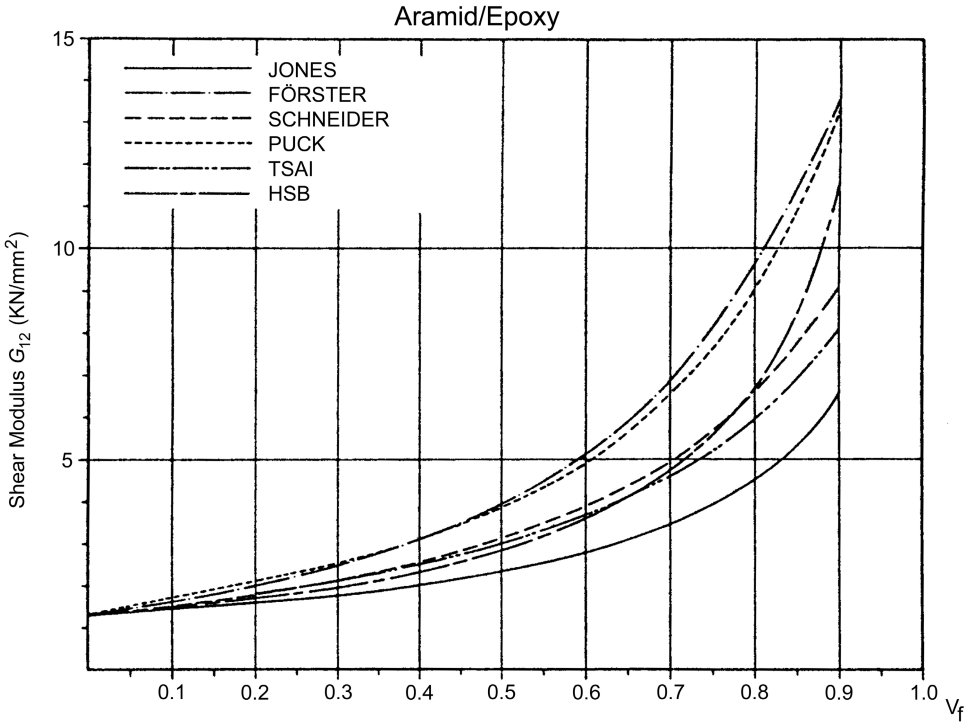


Figure 10.9-4 - Shear modulus (G_{12}) of aramid/epoxy composites

10.10 In-plane stress calculation methods

The stiffness of a unidirectional fibre reinforced layer, like any other structural material, is defined by appropriate stress-strain relations and is coupled to the stress-strain relation of a thin laminate plate according to the arbitrary orientation and thickness of each ply.

[See: 10.12 for unidirectional plies; 10.13 for on-axis relationship; 10.14 for ply orientation]

10.11 Analytical notation for in-plane methods

System of reference axes:

$(1,2,3)$ Ply reference axes (on-axis). Principal axes of orthotropy of a ply.

(x,y,z) (x,y,z) Structural axes (off-axis). Plate reference axes.

Strains:

$\varepsilon_k = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_k$ Column matrix of direct and shear strains referred to (*1*,*2*) co-ordinates for *k*-th layer.

$\bar{\varepsilon}_k = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k$ Column matrix of direct and shear strains referred to (x,y) co-ordinates for *k*-th layer.

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System of reference axes:

Stresses:

$= \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_k$ Column matrix of direct and shear stresses referred to (*1,2*) co-ordinates for *k*-th layer.

$\overline{\sigma}_k = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k$ Column matrix of direct and shear stresses referred to (*x,y*) co-ordinates for *k*-th layer.

Plate curvatures:

W Deflection of plate in *z*-direction.

$K = \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}_k$ Column matrix of curvature components of plate.

ε Column matrix of strains components of plate.

Plate loading intensities:

$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}$ Column matrix of direct and shear load intensities referred to (*x,y*) co-ordinates.

$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$ Column matrix of bending moment intensities acting in *xz* and *yz* planes and twisting moment intensity acting about *x* and *y* axes, respectively.

Plate stiffnesses:

A In plane stiffness matrix for plate.

B Coupled in plane and flexural stiffness matrix for plate.

D Flexural stiffness matrix for plate.

Q_k Matrix relating stress and strain in (*1,2*) co-ordinates for *k*-th layer.

$\overline{Q}_k$ Matrix relating stress and strain in (*x,y*) co-ordinates for *k*-th layer.

Ply properties:

E_1, E_2 Moduli of elasticity of a ply in 1- and 2-directions, respectively.

G_{12} Shear stiffness of a ply in (1, 2) direction.

p $1/\left(1-\nu_1 \times \nu_1 \times E_2/E_1\right)$ for a ply.

Geometry:

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n	Number of plies in plate.
h_k	Thickness of k-th layer.
θ_k	Angle of principal axis of orthotropy, x, of k-th layer measured from plate reference axis, 1.
m_n	Number of groups of layers having identical orientations.

10.12 Stress-strain relations for unidirectional plies

10.12.1 Fibre-oriented co-ordinate system

As the thickness of layers is small, they are assumed to be thin plates in a plane stress state.

The co-ordinate system is defined by the principal material directions, as shown in Figure 10.12.1.

Axes, 1-2, coincide with the longitudinal and transverse directions. This mate-rial symmetry is called 'orthotropic' and 'on axis'.

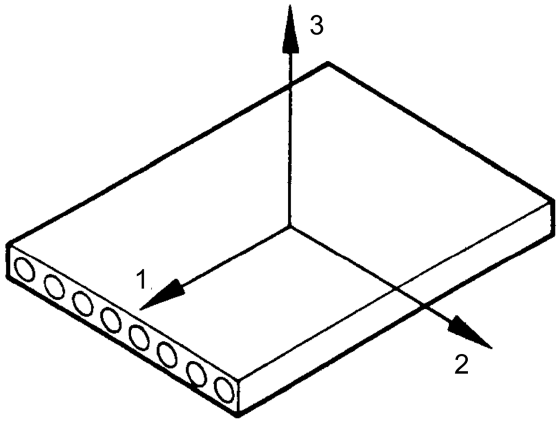


Figure 10.12-1 - Definition of the co-ordinate system for the analysis of unidirectional plies

10.13 On axis stress strain relations

10.13.1 General

The elastic properties of an orthotropic ply are defined, using matrix notation, by equations giving the compliance and modulus matrices.

[See also: 10.11 for notation]

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10.13.2 Compliance matrix

$$\{\varepsilon_k\} = [S_k]\{\sigma\}_k \rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_k = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_k \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_k \quad [10.13\text{-}1]$$

[Compliance matrix]

Where:

γ_{12} (shear strain) is determined by $(\partial u/\partial y + \partial v/\partial x)$.

$$S_{11} = \frac{1}{E_1}; S_{22} = \frac{1}{E_2}; S_{66} = \frac{1}{G_{12}}; S_{12} = -\frac{\nu_{12}}{E_1}$$

10.13.3 Modulus matrix

$$\{\sigma_k\} = [Q_k]\{\varepsilon\}_k \rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_k \quad [10.13\text{-}2]$$

[Modulus matrix]

Where:

$$Q_{11} = pE_1 \; ; \; Q_{22} = pE_2 \; ; \; Q_{66} = G_{12} \; ; \; Q_{12} = p\nu_{12}E_2$$

10.13.4 Symmetry of compliance and modulus matrices

There are four independent elastic constants used to describe the behaviour of a thin UD layer. These constants are determined by experiment or from fibre and matrix properties using micromechanical formulae, [See: 10.4].

10.14 Stress-strain relations for a ply of arbitrary orientation

10.14.1 General

A UD unidirectional ply with principal material axes X and Y is defined with respect to an arbitrary system of axes 1 and 2 by the angle 1:X, as shown in Figure 10.14.1.

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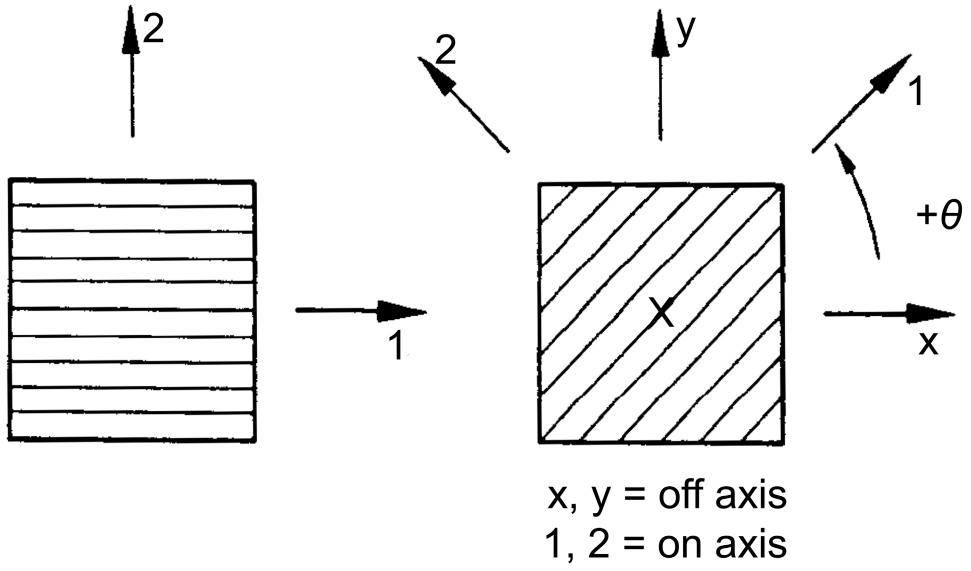


Figure 10.14-1 - Definition of axes

From elementary mechanics of materials, the transformation equations for stresses and strains are known to be:

$$\{\bar{\sigma}\}_k = [M]_k \{\sigma\}_k$$

$$\{\bar{\varepsilon}\}_k = \left([M]^{-1}\right)_k^T \{\varepsilon\}_k$$

$$[\bar{Q}]_k = [M]_k [Q]_k [M]_k^T$$

Where:

$$[M] = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix}$$

$$\left([M]^{-1}\right)^T = \begin{bmatrix} m^2 & n^2 & -mn \\ n^2 & m^2 & mn \\ 2mn & -2mn & m^2 - n^2 \end{bmatrix}$$

With: $m = \cos\theta; \quad n = \sin\theta$

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10.14.2 Off axis stiffness of a unidirectional ply

The off axis stress strain relation is:

$$\{\bar{\sigma}\}_k = [\bar{Q}]_k \{\bar{\varepsilon}\}_k \rightarrow \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xS} \\ Q_{xy} & Q_{yy} & Q_{yS} \\ Q_{xS} & Q_{yS} & Q_{SS} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad [10.14\text{-}1]$$

The relationship between the components of the on axis and off axis stiffness matrix is:

$$\begin{bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{SS} \\ Q_{xS} \\ Q_{yS} \end{bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2)^2 \\ m^3n & -mn^3 & mn^3-m^3n & 2(mn^3-m^3n) \\ mn^3 & -m^3n & m^3n-mn^3 & 2(m^3n-mn^3) \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \end{bmatrix} \quad [10.14\text{-}2]$$

These relationships are limited to transformation from the on axis orthotropic where shear coupling components are zero.

The off axis stress strain relation in terms of compliance is:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{1S} \\ S_{12} & S_{22} & S_{2S} \\ S_{1S} & S_{2S} & S_{SS} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} \quad [10.14\text{-}3]$$

Engineering constants are determined for on-axis load-displacement relationships only and not for off-axis.

In terms of compliance, applying the same steps gives:

$$\begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \\ S_{ss} \\ S_{xs} \\ S_{ys} \end{bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & m^2n^2 \\ m^2n^2 & m^2n^2 & m^4+n^4 & -m^2n^2 \\ 4m^2n^2 & 4m^2n^2 & -8m^2n^2 & (m^2-n^2)^2 \\ 2m^3n & -2mn^3 & 2(mn^3-m^3n) & mn^3-m^3n \\ 2mn^3 & -2m^3n & 2(m^3n-mn^3) & m^3n-mn^3 \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{22} \\ S_{12} \\ S_{66} \end{bmatrix} \quad [10.14\text{-}4]$$

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