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Space engineering — Buckling of structures

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National foreword

This Published Document is the UK implementation of CEN/TR 17603-32-24:2022.

The UK participation in its preparation was entrusted to Technical Committee ACE/68, Space systems and operations.

A list of organizations represented on this committee can be obtained on request to its committee manager.

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© The British Standards Institution 2022
Published by BSI Standards Limited 2022

ISBN 978 0 539 20507 7

ICS 49.035; 49.140

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This Published Document was published under the authority of the Standards Policy and Strategy Committee on 31 August 2022.

Amendments/corrigenda issued since publication

Date	Text affected

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TECHNICAL REPORT

RAPPORT TECHNIQUE

TECHNISCHER BERICHT

CEN/TR 17603-32-24

June 2022

ICS 49.035; 49.140

English version

Space engineering - Buckling of structures

Ingénierie spatiale - Flambement des structures

Raumfahrttechnik - Handbuch Knicken und Beulen von Strukturen

This Technical Report was approved by CEN on 13 April 2022. It has been drawn up by the Technical Committee CEN/CLC/JTC 5.

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European Foreword

This document (CEN/TR 17603-32-24:2022) has been prepared by Technical Committee CEN/CLC/JTC 5 “Space”, the secretariat of which is held by DIN.

It is highlighted that this technical report does not contain any requirement but only collection of data or descriptions and guidelines about how to organize and perform the work in support of EN 16603-32.

This Technical report (CEN/TR 17603-32-24:2022) originates from ECSS-E-HB-32-24A.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association.

This document has been developed to cover specifically space systems and has therefore precedence over any TR covering the same scope but with a wider domain of applicability (e.g.: aerospace).

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Introduction

The handbook is organised in three parts:

- Part 1: General.

  It includes chapters 4 to 13, and provides general information on the structural stability.

  — Chapter 4 addresses the history of stability problems starting from Euler (1757), who investigated buckling of beams as well as of plates. More recently, during the past 50 years, the discrepancy between experiments and linear buckling analysis as well as the large scatter of the tests stimulated a large amount of research work. The focus was especially on the influence of boundary conditions, of the nonlinear pre-buckling behaviour, of the postbuckling behaviour of the perfect shell and of the buckling and postbuckling behaviour of the imperfect shell.

  — In chapter 5 the occurrence of different types of instabilities are discussed on hand of relatively simple examples. The following topics are addressed: elastic buckling of columns; buckling of thin plates; instability of axially compressed cylindrical panels; structural behaviour of thin shells.

  — The first scope of chapter 6 is to give a survey of all the types of analysis which could be used for the study of stability. A simple descriptive presentation is provided to fix a common language and a clear understanding. For rigorous derivation and comprehensive details, proper reference is indicated. A second scope is to categorize the phenomena of loss of stability, by indicating for each type the appropriate analysis to be performed.

  — Chapter 7 addresses the following topics: linear elasticity and elastic properties; strength properties and hygro-thermal properties; elastic and inelastic material behaviour; plasticity and damage; material testing methods, test data, and evaluation.

  — Chapter 8 describes the design to stability and its verification by analysis. The following topics are included: the design development process; analysis pre-work and load input data; safety concept and factors of safety; dimensioning load cases selection; stability design allowable and knock-down-factors; analyses, modelling, and design aspects; procedure of design verification, including the determination of the margin of safety.

  — Chapter 9 describes the influence of manufacturing and assembly processes on the buckling load.

  — In chapter 10 the use of the finite element method and the numerical procedures applied to perform a buckling or post-buckling analysis are described with a focus on space applications. A guide to select the most adequate solution scheme is provided.

  — This handbook promotes the “Hierarchical High Fidelity Analysis” approach, which is reported in chapter 11. In practice, in order to arrive at a reliable prediction of the critical buckling load and to make an estimate of its imperfection sensitivity which can be used with confidence, the structural analyst should proceed step by step from simple analytical solutions (Level-1) to more complex models and solution procedures (Level-3).

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This chapter provides a detailed description of the hierarchical high fidelity analysis approach including a flow chart establishing a strategy to handle buckling phenomena.

— Chapter 12 is dedicated to the buckling experimental methods and design verification by tests. Experiments can explore the physical behaviour near buckling, at buckling and in the post-buckling range and they also yield empirical data upon which design guidelines can be based. The realization of the tests can be done at several levels: tests on elementary sample, tests on structural elements and test on full scale structure.

— The test instrumentation is described in chapter 13. It deals with measurement techniques in structural testing and measurement systems.

- Part 2: Structural elements with examples

  It includes chapters 14 to 17, and it is devoted to structural elements with examples. It reports the mathematical formulae for calculating the buckling loads for the most common structural elements, loads and boundary conditions.

  — Chapter 14 is an introductory chapter covering the following topics: static versus kinematic approach; problems requiring nonlinear analysis; approximate solutions of bifurcation problems; computational tools for bifurcation problems.

  — The remaining chapters 15, 16 and 17, present the solutions with examples for the following structural elements: columns, beams, arches, and rings; flat and curved panels; closed shells.

- Part 3: Structures

  It includes chapters 18 to 23, and addresses the “real world” structures.

  — Chapter 18 is an introductory chapter describing the design, analysis and testing aspects of large aerospace structures, including the approach for the mathematical model correlation and validation. Relevant flow-charts and examples are provided, mainly related to the structure of the European launcher ARIANE 5.

  The remaining chapters 19 to 23 report various “large examples” provided by various European aerospace companies.

  — The buckling analysis and test activities performed on the liquid hydrogen (LH2) tank of the ESC-A ARIANE 5 stage are presented in chapter 19.

  — Chapter 20 describes the Inner Dome of the ARIANE 5 Upper Stage ESC-A. This structure is subjected to compressive loads in case of internal pressure. For this reason the Inner Dome shell has been designed carefully against buckling.

  — The results of the non-linear stability analysis performed on the ARIANE 5 Front Skirt (JAVE) and its adjacent structures are summarized in chapter 21. The chapter presents the main characteristics of the mathematical model, including the definition of material models, loadings, geometric and physical imperfections and analysis technique.

  — Chapter 22 presents the results of the buckling analyses and static strength tests of the Interstage 1/2 for the Vega launch vehicle. The Interstage 1/2 is constructed as monocoque and is buckling critical, hence the need to obtain accurate predictions for buckling strength, including the influence of initial imperfections. The Interstage 1/2 was qualified by a static load test, and then was further loaded to final failure in a rupture test.

  — Finally, chapter 23 reports the results of the stability analysis of the 3rd-stage skirts of the European launcher ELDO-A.

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1 Scope

The document recommends engineering practices for European programs and projects. It may be cited in contracts and program documents as a reference for guidance to meet specific program/project needs and constraints.

The target users of this handbook are engineers involved in design, analysis and verification of launchers and spacecraft in relation to structural stability issues. The current know-how is documented in this handbook in order to make this expertise available to all European developers of space systems.

It is a guidelines document; therefore it includes advisory information rather than requirements.

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2 References

Due to the structure of the document, each chapter includes at its end the references called in it.

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3 Terms, definitions and abbreviated terms

3.1 Terms from other documents

For the purpose of this document, the terms and definitions from ECSS-S-ST-00-01 apply.

For the purpose of this document, the following terms and definitions from ECSS-E-ST-32 apply:

buckling
design allowable
design factor
design limit load (DLL)
design load (DL)
design parameters
design ultimate load (DUL)
design ultimate stress
design yield load (DYL)
design yield stress
detrimental deformation
factor of safety (FOS)
failure
limit load (LL)
relieving loads
residual stress
stiffness
structural design
structure
ultimate strength
yield strength

For the purpose of this document, the following terms and definitions from ECSS-E-ST-32-10 apply:

local design factor (KLD)
margin policy factor (KMP)
model factor (KM)
project factor (KP)
test factors (KA and KQ)
ultimate design factor of safety (FOSU)
yield design factor of safety (FOSY)

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3.2 Terms specific to the present document

3.2.1 buckling analysis

method to study the equilibrium stability of a structure, based on eigenvalue problem analysis

NOTE The most general approach for the analysis of equilibrium stability requires the study of non-linear equilibrium and stability equations; the buckling analysis approach is usually based on specific assumptions and formulation which lead to solve eigenvalue problems.

3.2.2 buckling mode or shape (with reference to buckling)

changed deformed configuration of a structure, due to occurrence of buckling

NOTE 1 Buckling mode and shape is defined in shape and amplitude, depending from load level.

NOTE 2 Buckling mode can be a new equilibrium or the failed structure configuration

3.2.3 buckling modes or shapes (with reference to buckling analysis)

eigenvectors associated to the eigenvalues of a buckling analysis

NOTE Buckling modes and shapes are defined only in shape, not in amplitude.

3.2.4 buckling resistance

limit state caused by loss of stability under compressive and shear membrane loads

3.2.5 design buckling load

buckling load multiplied by a design factor

3.2.6 global buckling

buckling whose buckling shape is affecting quite all the structure

NOTE Usually the global buckling causes structure collapse.

3.2.7 local buckling

buckling whose buckling shape is affecting only a minor part of the structure

NOTE 1 Examples of local buckling are intracel buckling of sandwich, inter rivet buckling, and beam leg buckling

NOTE 2 Usually the local buckling causes load path changes and not structure collapse.

3.2.8 margin of safety (MOS)

ratio of the buckling resistance and the design buckling load minus 1

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3.3 Abbreviated terms

The following abbreviated terms are defined and used within this handbook:

Abbreviation	Meaning
AIT	assembly, integration and tests
CAD	computer aided design
CAE	computer aided engineering
CAM	computer aided manufacturing
COG	centre of gravity
DL	design load
DLL	design limit load
DOF	degree of freedom
DRD	document requirement definition
DUL	design ultimate load
DYL	design yield load
FCI	fracture critical item
FEA	finite element analysis
FE	finite element
FM	flight model
FMECA	failure mode, effects and criticality analysis
FOS	factor(s) of safety
FOSU	factor(s) of safety at ultimate
FOSY	factor(s) of safety at yield
LL	limit load
KDF	knock down factor
MOS	margin of safety
NDT	nondestructive test
NDI	nondestructive inspection
PFCI	potential fracture critical item

NOTE Some additional abbreviated terms which are used in a specific chapter are defined at the end of the relevant chapter.

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Part 1: General

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4 Overview

4.1 Instability of structures: concepts and definitions

The stability is a property of the equilibrium configurations of a given structure, subjected to static and/or dynamic loads.

The equilibrium is said stable if small "pertubations" do not cause significant, unexpected and unwished changes of equilibrium configuration, otherwise the equilibrium is said unstable (it can change "abruptly").

Although the above sentences are qualitative, nevertheless they clearly describe the experimental evidence and they can highlight many key aspects related to the concept and study of stability of structures.

Three facts deserve special attention.

- First of all, the possibility of finding additional equilibrium configurations is strictly related to the nonlinear nature of the stability problem. In the case of linear systems, in fact, only one equilibrium configuration exists, which can be changed only by linear relationships of configuration parameters (e.g. loads and displacements).
- Secondly, to study the stability of the equilibrium, it is necessary not only to find an equilibrium configuration, but also to study possible changes of these configurations, and stability equations need to be formulated for this purpose. These stability equations are also known as incremental equilibrium equations.
- Finally, the study of changed configurations may or may not be performed in the time domain, i.e. stability may be investigated dynamically or statically. There are cases where only dynamic approach is appropriate (e.g. non-conservative systems); however the static approach is adequate for many stability problems of aerospace structures. Nevertheless, the most general method relies on Liapunov's dynamic stability definition.

Various methods and techniques, which have been proposed and are necessary to properly study the stability of the equilibrium state, are discussed in detail in subsequent chapters (see in particular Chapter 6: "Analysis methods"). These concepts can be subdivided by the three criteria for defining or detecting instability of structures, which differ in complexity and generality [1]:

a. The criterion of non-trivial equilibrium state (see Chapter 6.5)

b. The dynamical criterion (see Chapter 6.8)

c. The total potential energy criterion (see Chapter 6.3)

For conservative elastic systems all three criteria give identical results.

Using the criterion of non-trivial equilibrium state means to ask if there is an adjacent configuration where equilibrium is achieved. This is the case, if the stability determinant of a system is equal to zero for a non-trivial solution. The stability determinant can be obtained from the equilibrium conditions or by the second derivative of the energy function. The method does not work for problems of stability where there is no adjacent equilibrium state and for dynamical buckling in general.

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The dynamical criterion is the most general approach. The idea is to determine if a system returns to the initial configuration after applying a deflection or if the deflections increase or converge to a different configuration. Mathematically, this can be described by the frequency determinant and the so-called Burchart determinant, which both have to be equal to zero.

The most widely used criterion is the total potential energy criterion, which uses the fact that for stability the energy function must be a minimum and for instability it must be a maximum. Thus, the critical state is reached, when the second derivative of the energy function is equal to zero. The disadvantage of this criterion is that it is restricted to conservative systems.

All these concepts are now well established in today's mechanics of structures, but an enormous effort has been dedicated to understand and describe the phenomenon. The history of stability problems dates back to Euler (1757), who investigated buckling of beams as well as plates. Prominent researchers like Bernoulli, Lagrange, Navier and Kirchoff further developed the theory of plate buckling. The historical overview in this chapter (and the bibliography) gives an idea on this development, by focussing efforts and different concepts.

4.2 Stability of columns, beams and plates

4.2.1 Buckling of beams

Euler [2] was the first one who solved the buckling problem of a cantilever beam in 1744 leading to his famous formulas. The corresponding eigenvalue problem can be solved easily for different boundary conditions (see e.g. [3], [4]). While Euler regarded the perfect column, Rivello [5] considered an initial geometric imperfection and concluded that for columns with measurable imperfections the buckling load of the perfect column is sufficiently accurate. Further, Rivello determined the yielding stress limits considering linearly elastic and idealized plastic material behaviour. Simitses [6] showed that Rivello's curves in [5] can also be obtained if the geometric imperfection is replaced by an eccentricity of the applied load.

Experimental evidence indicates that for a slenderness ration $L\sqrt{A/I} > 80$ Euler's formulas are quite accurate, but for $80 > L\sqrt{A/I} > 20$ to get reasonably accurate predictions of the buckling load the use of Shanley's tangent modulus theory [7] is recommended. For smaller slenderness ratios failure occurs mainly by plastic crushing (see chapter 4.9).

4.2.2 Lateral torsional buckling

For beams that are bent with respect to their "strong" axis, further cases of instability can occur, if the torsional rigidity I_T and the moment of inertia I_{zz} are relatively small compared to the moment of inertia I_{yy} of the strong axis. Then, lateral buckling can occur, which means that the beam twists and deflects out of the loading plane. The special case, that the beam is strained only with bending moments was first solved by Prandtl [8] and almost at the same time by Michell [9] in 1899. Further solutions were given by Timoshenko [10], Nylander [11] and Kindem [12]. Chawalla [13] showed some possibilities for practical applications of these solutions.

The more general problem of lateral torsional buckling under bending and compression was first mentioned by Wagner [14] and solved by Kappus [15],[16] in 1937. Thus, design rules for beams which tend to lateral buckling could be adopted in the German guideline DIN 4114 [17] in 1952. A multitude of solutions of the lateral buckling load for different load cases and boundary conditions are listed by Petersen [18].

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4.2.3 Buckling of plates

If one regards a column as an assemblage of thin plates, the question of local stability of the assembly arises. This leads to the stability of in-plane loaded thin-walled shells and plates.

First experimental tests on plates were executed by Fairbairn and Hodgkinson for the bridge projects of Stephenson in 1845, the Britannia and Conway Tubular Bridges [19],[20]. The first one who determined the buckling load of a simply supported plate analytically was Bryan [21] in 1891. He derived the critical stress, which depends on the buckling coefficient k_c. Gerhard and Becker [22] calculated buckling coefficients for several combinations of boundary conditions and load cases. Coan [23] studied rectangular plates under uniformly displaced loaded edges including a half-wave sine initial imperfection and using a large deflection theory. He found that the axial stress of the compressed plate is uniformly distributed over the cross section, but after buckling has occurred, the stresses increase almost only close to the edges of the plate whereas in the middle of the plate the stresses maintain almost a constant magnitude equal to the buckling stress, if the plate is compressed further. If one assumes that the total load is carried by the stripes at the edges, one can determine the "effective width". This concept was already described by Schuman and Back [24], then defined and theoretically derived by von Kármán et al [25] and later improved by Sechler, Cox, Maguerre and others [26]-[30] with experiments and theory. The concept of the effective width, which was proposed by aeronautic engineers, was adapted to civil and marine engineering. Several theoretical and experimental studies were added from all branches over the century (see e.g. [31]-[36]).

Because for plates it is possible to increase the sustained load after buckling, the post buckling behaviour plays an important role, if one wants to exploit this load carrying reserve. Increasing the load after exceeding the buckling load leads to a change of the buckled form. There, one can distinguish between minor and major changes in buckled form. A minor change is a gradual smooth change like the flattening of the transverse half-wave, whereas a major change consists of an abrupt snap from one buckled form to another, like the increase in the number of waves along the direction of compression in a rectangular plate. The major changes in postbuckling pattern were already noted by Sechler [29], [30] and Stein [37],[38] who studied the change in wavelength with the aid of a simplified mathematical model. This phenomenon, which is often called "secondary buckling" in the literature, was subject of further investigations on plate buckling (see e.g. [39]).

4.3 History of shell buckling

4.3.1 Overview

For solving the buckling problem of circular cylindrical shells first analytical approaches were published in 1908 by Lorenz [40], in 1910 by Timoshenko [41] and in 1914 by Southwell [42]. Improved methods were developed in the 1930ies, e.g. the well-known checkerboard approach of Flügge [43]. Batdorf in 1947 [44] presented this curvature parameter $Z = \left(L^2 / Rt\right)\sqrt{1-\nu^2}$, which made it possible to combine the cylinder dimensions and the material properties in such a way that the results of a series of buckling load calculations could be presented by a single curve. All papers mentioned are focussing on linear stability analysis methods for the bifurcation load of perfect cylinders.

First tests of thin-walled elastic cylindrical shells were performed in 1933 by Lundquist [45] and in 1934 by Donnell [46]. During the 1950ies and 1960ies a large number of tests followed. The axially compressed cylindrical shell represents one of the best known examples of the very complicated stability behaviour which can occur with thin-walled constructions. The nature of the problem is well illustrated in Figure 4-1 [47], where some of the experimental results for isotropic shells have been plotted as a function of the R/t (radius over wall thickness) ratio. As can be seen in Figure 4-1, the

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tests reveal a wide scatter in the experimental results, with experimental buckling loads for very thin shells ($R/t > 1000$, say) as low as 20% of the theoretical values.

In the absence of a better solution, for practical applications all current shell design manuals rely on the so-called "Lower Bound Design Philosophy" that has been in use for over 50 years. These references recommend the use of an "empirical knockdown factor" which is chosen such that when it is multiplied by the perfect shell buckling load, a "lower bound" to the appropriate part of the existing test data is obtained. The lower bound curve shown in Figure 4-1 was proposed in 1965 for isotropic shells by Weingarten et al [47] as

$$\frac{P}{P_{cl}} = 1 - 0.901\left(1 - e^{-\phi}\right) \text{ for } \frac{R}{t} < 1500 \qquad 4\text{-}1$$

where $\phi = \frac{1}{16}\sqrt{\frac{R}{t}}$, $P_{cl} = 2\pi R N_{cl}$, $N_{cl} = \frac{Et^2}{cR}$, $c = \sqrt{3\left(1-\nu^2\right)}$.

Introducing the empirical knockdown factor γ as

$$\gamma = 1 - 0.901\left(1 - e^{-\phi}\right) \qquad 4\text{-}2$$

one obtains the well-known critical buckling load formula for an axially compressed isotropic shell

$$P_c = \gamma P_{cl} \qquad 4\text{-}3$$

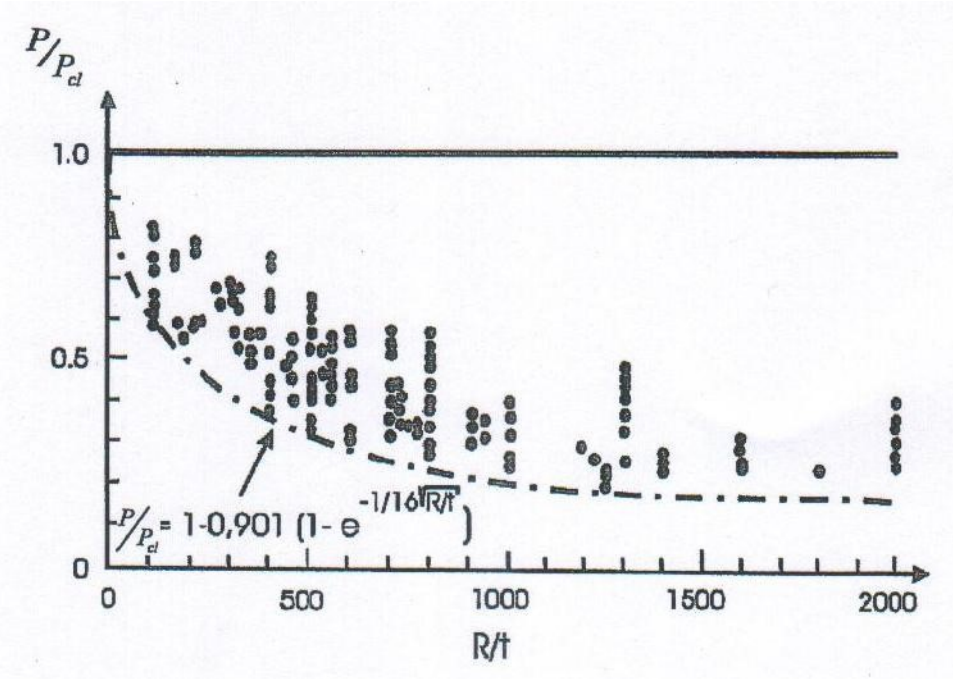


Figure 4-1: Test results of axially loaded cylinders with different slenderness ratio (from [47])

The discrepancy between experiments and linear buckling analysis as well as the large scatter of the tests stimulated a large amount of research work during the past 50 years. The focus was especially on the influence of boundary conditions, of the nonlinear pre-buckling behaviour, of the postbuckling behaviour of the perfect shell and of the buckling and postbuckling behaviour of the imperfect shell.

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4.3.2 Boundary conditions and nonlinear pre-buckling behaviour

Initially, it was tried to explain the discrepancy by different boundary conditions or by the nonlinear pre-buckling state which is not covered by linear analyses. Historically, Ohira [48] and Hoff [49] in 1961 were the first to publish papers where for simply supported boundary conditions ($w = M_x = 0$, $N_x=-N_0$) and using $N_{xy} = 0$ instead of $v = 0$ they obtained solutions of the linearized stability equations, which were about 50 percent lower than the classical solutions published by Lorenz, Timoshenko and Southwell.

For free boundaries Nachbar and Hoff [50] in 1962 published a paper where they obtained a critical buckling load of 37 percent of the classical solution. As stated all theses solutions were based on assuming a membrane prebuckling state. Hoff and Soong [51] in 1965 published the results of a very extensive investigation based on membrane prebuckling for eight possible combinations of edge support conditions. In the same year Almroth [52] published the results of a similar investigation for the same eight combinations of edge support but he used a rigorous solution of the prebuckling problem. His results were in most cases 10-15 percent lower.

Thus, the conclusion from these results is that boundary conditions can have a significant influence on the buckling load obtained, but they cannot account for the large discrepancies between the theoretical predictions and the available experimental results by themselves.

4.3.3 Postbuckling of the perfect shell

The development of nonlinear buckling theories was an important progress towards interpretation of the test results. The basic equations were formulated in 1934 by Donnell [46] and were not restricted to small deformations. The quadratic terms of the radial displacement w were taken into account. For solving the nonlinear equations the energy method was used and approximations for w in terms of trigonometric series were made. Based on these equations von Kármán and Tsien performed first postbuckling analyses of axially loaded cylinders already in 1941 [53]. The solution of the nonlinear Donnell type shell equations based on a Ritz technique showed for the first time that for certain values of the loading parameter λ (= P/P_{cl}) there exist three possible equilibrium configurations [53](see points G-A-C in Figure 4-10a). The form of the assumed radial displacement w was based on experimental evidence consisting of three parts

$$w(x,y) = f_0 + f_1 w_1(x,y) + f_2 w_2(x,y) \qquad 4\text{-}4$$

where f_0 is introduced in order to allow the shell to expand radially, w_1 represents the diamond shaped stable postbuckling pattern and w_2 the asymmetric chess-board like bifurcation buckling mode of the classical theory.

Leggett and Jones [54], Michielsen [55], Kempner [56] and Almroth [57] calculated the load shortening curve with an increasing number of coefficients. The potential energy was minimized not only with respect to these coefficients but also with respect to the ℓ and ℓ , the lengths of the buckles in axial and circumferential direction, respectively. The buckling load decreased from 34 percent of the linear buckling load (according to von Kármán and Tsien) to only 10,8 percent (according to Almroth) (see Figure 4-2). An overview of the results was given by Hoff [58]. The search for improved solutions came to an end in 1965 by the work of Hoff, Madsen und Mayers [59]. They revealed that the postbuckling load tends towards zero when the number of coefficients is increased more and more.

Thielemann and Esslinger confirmed by theoretical and experimental investigations, that the buckling load decreases with increasing length of the cylinder [60]. For the infinitely long cylinders investigated so far, the length ℓ of the buckles in axial direction could become even larger than the cylinder length due to the missing boundary conditions. Realistic postbuckling loads could only be achieved by taking into account the finite length of the cylinder. Moreover, so far only regular postbuckling

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patterns were assumed, that spread over the whole cylinder surface. Test of cylinders with finite length revealed a locally concentrated pattern, usually with two rows of buckles along the length. (see Figure 4-3). In 1955 Yoshimura [61] confirmed with simple approximate calculations that a finite cylinder tends to buckle with a concentrated pattern. The deformation mode with concentrated buckles has a lower energy content than the regular, so-called von Kármán postbuckling pattern. Esslinger [62] developed a theory for describing the concentrated pattern by taking into account the boundary conditions. The postbuckling curve with minimum shortening was assumed to be the appropriate one.

Investigations on axially loaded cylindrical shells showed that instable equilibrium modes exist in the postbuckling regime showing small radial displacements and loads below the buckling load. Therefore the shells were sensitive to perturbations. The large scatter of results can also be traced back to this phenomenon. The structural behaviour was explained by Esslinger [63] using the idea of perturbations and restoring forces. According to that in a axially loaded beam only the bending in axial direction creates back tracing forces, whereas in a plate an additional force is present due to bending in transverse direction. In cylindrical shells the circumferential membrane force generates an additional back tracing force and thus leads to very high buckling stresses. However, the support by the membrane forces is strongly influenced by small manufacturing imperfections. The support is very much weakened in the postbuckling regime and finally vanishes fully, thus leading to a load carrying capability below the theoretical buckling load.

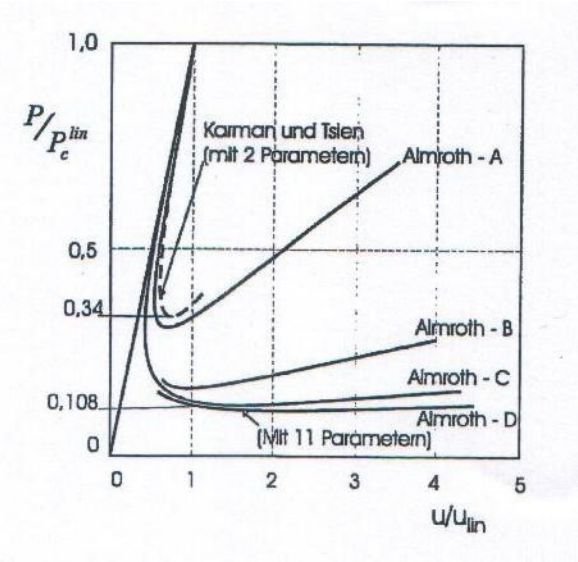


Figure 4-2: Calculated postbuckling curve (from [117])

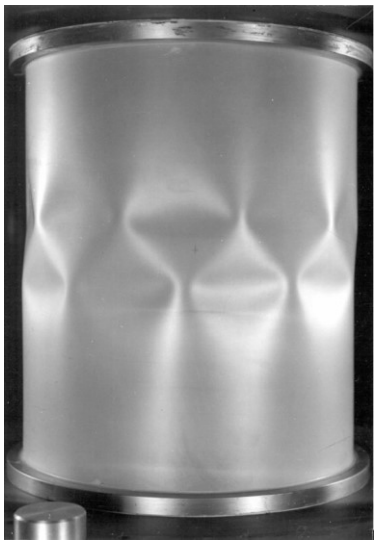


Figure 4-3: Postbuckling pattern of axially loaded mylar cylinder (from [60])

In the theoretical work either load or displacement was applied to the cylinder, whereas the experiments were displacement driven in order to not damage the structure. When the buckling strain was reached the load decreased momentarily at constant strain. The cylinder achieved a new stable equilibrium mode at a load well below the buckling load. The highly dynamic transition from the stable prebuckling to another stable postbuckling mode was visualized by Almroth et al. in 1964 [64] and Esslinger in 1970 [65] using a high speed camera. Starting from an initial single buckle the buckles spread over the whole surface of the cylinder. Several unstable deformation patterns were passed before the stable postbuckling mode, as depicted in Figure 4-3, was reached.

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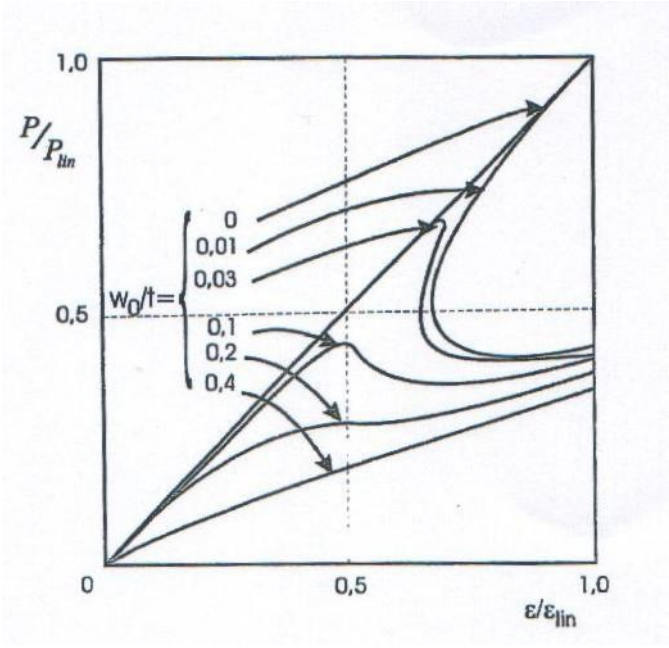


Figure 4-4: Postbuckling curves ([69])

4.3.4 Buckling and postbuckling behaviour of imperfect shells

Koiter was the first, who in 1945 recognized the close relation between low buckling loads and geometrical imperfections [66]. Geometrical imperfections were defined as deviations from the perfect cylindrical geometry. The thesis, written in Dutch language, was not recognized by the scientific community until Riks translated it to English [67]. Independently, and based on [46] and [68], Donnell and Wan in 1950 [69] investigated the load shortening behaviour of imperfect shells up to the deep postbuckling regime. They detected that already at small imperfection amplitudes w_0 the actual buckling load can be well below the linear one. (see Figure 4-4). Donnell and Wan assumed an imperfection pattern affine to von Kármán's postbuckling pattern. The work was thoroughly evaluated by Arbocz [70]. Within the deep postbuckling range the postbuckling paths of imperfect and perfect shells approached each other. The subsequent table shows publications which use different boundary conditions and imperfection patterns.

In contrast to Donnell and Wan, Koiter did not focus on the deep postbuckling regime but on the initial postbuckling behaviour. A comprehensive presentation of Koiter's theory was given by Budiansky and Hutchinson [77] and by Budiansky [78], [79]. Very detailed descriptions of the theory were provided by Seide [80] and Masur [81], short overviews are given by Almroth and Brush [82] and by Tvergaard [83]. A chronological overview of papers using the asymptotic approach is given in Table 4-2.

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Table 4-1: Chronological overview of papers providing full load shortening curves for imperfect cylindrical shells (from [71])

Authors	Literature	Year	Imperfection pattern	Amplitude
Donnell, Wan	[69]	1950	Affine to postbuckling pattern	variable
Pflüger	[72]	1962	Affine to critical eigenmode	R/200
Arbocz et al.	[73], [74]	1969/74	Measured imperfection pattern	measured
Esslinger, Geier	[75]	1972	Affine to mode of characteristic postbuckling curve	variabel
Yamaki	[76]	1972-84	Different meridian shapes	variabel

Table 4-2: Chronological overview of papers describing the asymptotic postbuckling behaviour (from [78]

Authors	Literature	Year	Imperfection pattern	Amplitude
Koiter	[66], [84]	1945	Eigenmode affine imperfections	varying
Koiter	[85]	1963	Axisymmetric imperfections	varying
Tennyson, Muggeridge	[86]	1969	Local axisymmetric imperfection	varying
Yamaki	[76]	1984	Affine to characteristic postbuckling shape	varying
Jürcke	[87]	1985	Affine to periodic eigenmode	varying

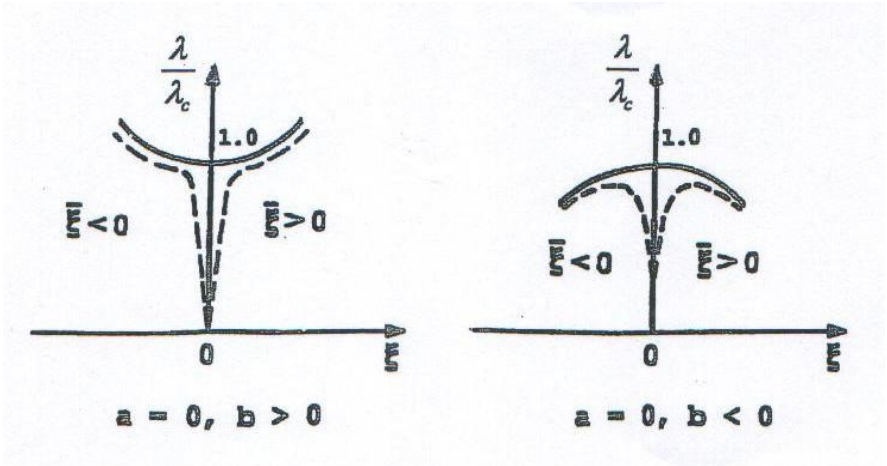


Figure 4-5: Equilibrium paths for symmetric postbuckling behaviour

KOITER introduced for his asymptotic approach a series expansion near the lowest (critical) eigenvalue $\lambda = \lambda_c$ of the structure. The initial postbuckling path is described using the load parameter λ and the normalized amplitude ξ of the first buckling mode by

$$\lambda / \lambda_c = 1 + a\xi + b\xi^2 + \ldots \qquad 4\text{-}5$$

where a and b are called the first and second postbuckling coefficient, respectively. For the symmetric case a = 0 follows, whereas the coefficient b, the so-called b-factor, determines the structural behaviour

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in the initial postbuckling regime. In case of $b > 0$ the load can be further increased after having reached the branching load. Thus the structure is insensitive to imperfections. For the unstiffened cylindrical shells $b < 0$ applies, which means that the structure collapses when reaching the load λ_c and instable equilibrium modes exist at lower load. (see Figure 4-5). The asymptotic approach has the attractive feature that the postbuckling coefficients a and b only depend on the perfect shell. Without knowing shape and amplitude of the imperfections a qualitative prediction can be made of the imperfection sensitivity of the structure in the initial postbuckling regime.

4.4 Design load for thin-walled isotropic shells

4.4.1 Overview

In the design process the influence of, usually unknown, imperfections has to be taken into account properly so that a safe structure is constructed. Typical guidelines like NASA SP-8007 [88] or DASt-Richtlinie 017 [89] recommend very conservative knock-down factors, however, they do not require knowledge about pattern or even amplitude of the imperfections. Measuring imperfections and introducing them into the numerical analysis is very costly, but promises less conservative design loads. A definition would be highly desirable of a physics based design load, which quantitatively describes imperfection sensitivity without requiring knowledge of the real imperfections.

4.4.2 Empirical knock-down factors

Empirical methods were used for designing axially loaded cylindrical shells, since adequate theoretical analysis methods were missing, which could cover the influence of realistic imperfections. The first attempt to describe the loading capacity by evaluating existing test results using statistical methods was made in 1957 by Harris et al. [90]. Almroth et al. [91] determined the knock-down factor as a function of the radius to wall-thickness ratio for different safety levels P (see Figure 4-6). A disadvantage of these statistical methods is the data basis, i.e. also those test results influence the design, which are not relevant for the structure under consideration due to its specific manufacturing procedure or its very different size [91].

Very conservative guidelines were established based on such statistical investigations. In aerospace engineering NASA SP-8007 (see [92]) is still used for designing axially loaded cylinders, the knock-down factors are depicted in Figure 4-1. The guideline DASt-Richtlinie 017 is applicable to steel structures in all engineering applications. The knock-down factors in both guidelines are based on test results [94], the statistical evaluation of which is not specified in detail [47]. Grimmelt et al [93] drew the conclusion, that the guidelines are at least partially based on the same test results, and the knock-down factors of NASA SP-8007 are 90%-fractiles, the ones in DASt-Richtlinie 017 are 99,9 fractiles. A summary of guidelines used by NASA for different shell geometries is given by Nemeth and Starnes [92].

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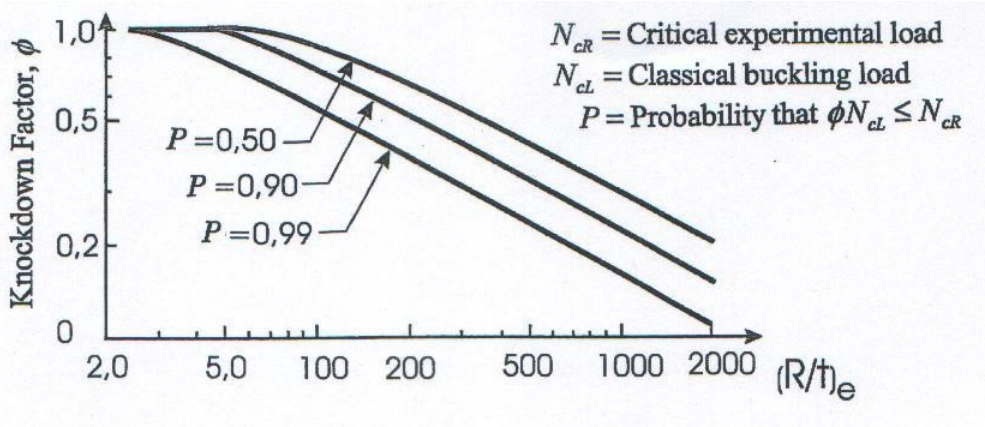


Figure 4-6: Empirical knock-down factors for axially loaded cylinders (from [91])

4.4.3 The Southwell method

In 1932 Southwell published a method which allows determining the buckling load of a perfect column from experimental tests [95]. He found a linear relation of the lateral displacement δ and the ratio of δ and the axial force P

$$\frac{\delta}{P} = \frac{1}{P_E}\left(\delta + \delta_0\right) \qquad 4\text{-}6$$

where δ_0 is the amplitude of the initial geometric imperfection and P_E the buckling load of the perfect column. The plot δ/P versus δ (see Figure 4-7) is called the Southwell plot and its inverse slope corresponds to P_E. Hence, with the measurement of δ and P in tests it is possible to determine P_E, the buckling load of the perfect column.

For further details the interested reader should consult chapter 4 of [96].

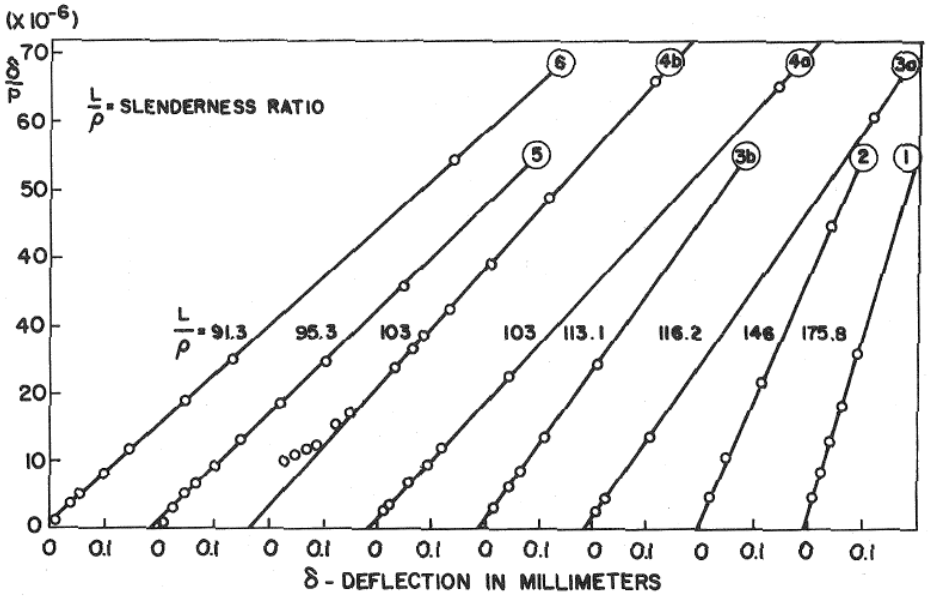


Figure 4-7: Von Kármán's data on compressed columns plotted in the linear form by Southwell (from [95])

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Roorda showed that the extension of Southwell's method to plates does not work in general, but depends on the post buckling behavior of the regarded structure [97]. He noted that considering nonlinear effects the lines in the Southwell plot are not straight. Donnell [98], [99] was the first who applied the Southwell method to cylindrical shells, followed by other investigators, who applied the method to several types of shells (see e.g. [100]-[101]).

The extension of Southwell's method to the inelastic region seems to be impossible a priori, but Wang showed in 1948 that the application to plastic buckling of columns is possible providing that E_r is constant [102] where E_r is a combination of the elastic modulus and the tangent modulus in the plastic region. This was confirmed by Singer [103]. Without theoretical evidence, the Southwell method has also been applied to plastic buckling of shells (see e.g. [104]-[106]).

4.4.4 Numerical analysis with realistic geometrical imperfections

The main emphasis in the literature was on geometrical imperfections since they were made responsible for the large discrepancy between analysis and test. However, without a priori knowledge of the shape and amplitude of the imperfections they cannot be taken into account in a numerical analysis. Winterstetter and Schmidt [107] define three different approaches for choosing geometrical imperfections: stimulating, worst and realistic imperfections.

Realistic geometrical imperfections in a numerical analysis are such, that they describe the existing prebuckling deformations as good as possible [107]. This approach assumes that specific manufacturing processes statistically result in basically the same imperfections [73], [108]. Methods for accurately measuring imperfections of cylindrical shells and for formulating them as two-dimensional Fourier series were developed by Arbocz, Babcock, Sechler [70]. For proving practical applicability a cylindrical shell with ten feet diameter and stiffened in circumferential and axial direction was measured by Arbocz and Williams [109]. All imperfections were stored in data banks [110]. Assuming a certain safety level and using statistical methods the load capacity of imperfect shells was predicted. While Velds et al. [111] used the „Monte Carlo" method, Elishakoff and Arbocz [112] applied the First Order Second Moment (FOSM) method (see also [113]-[115]). Coupling of stochastic methods with the finite element methods was first demonstrated by Chryssanthopoulos [116].

4.4.5 Numerical analysis with worst geometrical imperfection

The objective of this approach was to find the imperfection pattern, which for a certain imperfection amplitude caused the lowest buckling load [107]. Initially, eigenform affine patterns were assumed to be worst imperfections, later axisymmetric imperfections were believed to show this behaviour [117].

In Koiter's general theory the imperfection pattern was assumed to be the critical buckling mode. With given b-factor and imperfection amplitude $\bar{\xi} = \bar{\delta} / t$ the knock-down factor could be calculated from

$$\left(1 - \frac{\lambda_s}{\lambda_c}\right)^{3/2} = \frac{3(3)^{1/2}}{2}(-b)^{1/2}\,\bar{\xi}\,\frac{\lambda_s}{\lambda_c} \qquad b < 0 \qquad \text{4-7}$$

(see Figure 4-8, where $\lambda_S = P_S / P_C$ and $\bar{\xi} = \bar{\delta} / t$). This equation was first presented by Koiter [66], for its derivation most authors refer to Cohen [118]. This approach was used by Hutchinson and Amazigo [119] and by Hutchinson and Frauenthal [120] in their work on eccentrically stiffened shells.

The influence of axisymmetric imperfections was investigated by Koiter using a specific theory [85]. It was further developed by Tennyson and Muggeridge [86] for local axisymmetric imperfections. A good overview of the many publications regarding the influence of geometrical imperfections on the buckling behaviour of axially loaded cylindrical shells is given by Hutchinson and Koiter [121], Tvergaard [83], Budiansky and Hutchinson [122] and Pignataro [123]. Deml [124] developed a FE

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code, which calculated by rigorous mathematical methods the worst imperfection pattern for each specific shell geometry. For axially loaded cylindrical shells the result was a single buckle (see [124]-[126]).

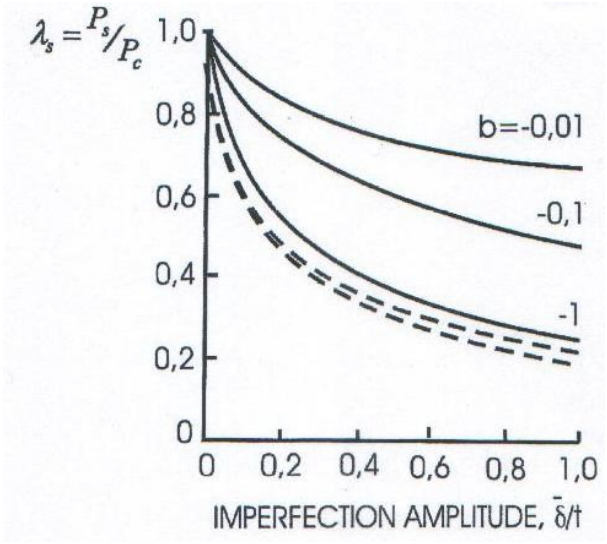


Figure 4-8: Imperfection sensitivity as a function of b-factor and imperfection amplitude [119]

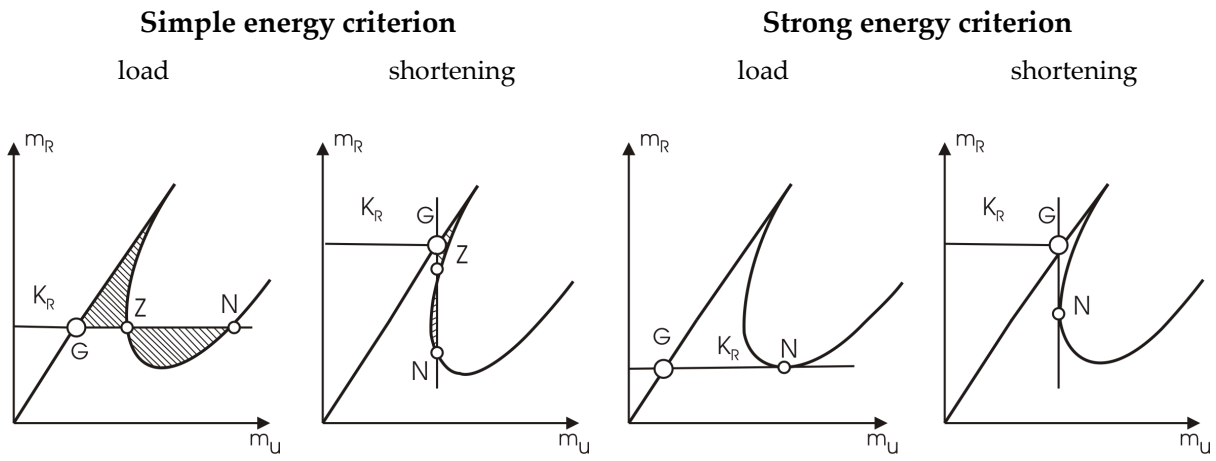
4.4.6 Numerical analysis with stimulating imperfections

While the scope of realistic geometric imperfections was to introduce existing imperfections into the numerical model, stimulating imperfections were meant to provide simple geometric patterns, which stimulate the real physical behaviour of the shell and thus provide realistic results for the knock-down factor [107]. Often axisymmetric or eigenform affine imperfections were selected for this approach. Recommendations for the amplitude of imperfections are given in the guidelines DASt-Richtlinie 017 [89] and ENV 1993-1.6 [127]. However, this often results in a very conservative design [128]. The influence of local axisymmetric imperfections resulting from welding seams were investigated by Pircher et al. [129]-[131], Jürcke et al. [132], Teng and Rotter [133], Hautala and Schmidt [134] and Bornscheuer et al. [135].

4.4.7 Physics based design load

As an alternative to geometric imperfections it was attempted to establish a physics based lower bound for the load capacity. Starting from a closed form solution for the load-shortening-curve Tsien [136] formulated a simple energy criterion. Failure should occur at a load where the whole potential energy before and after buckling is the same (see Figure 4-9 a-b). The simple criterion assumed that energy from an outside perturbation initializes buckling, but cannot be transferred to the system. Tsien [137] in 1947 released this assumption and developed a stronger energy criterion stating, that under outside energetic perturbation failure can occur at the earliest, if the load-shortening-curve begins to show multiple equilibrium configurations (see Figure 4-9c-d). Brendel compared these energetic criteria with many experimental investigations and concluded, that this approach had only limited success in providing bounds for the test results [138].

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Source: [136] [136], [61] [139], [137] [137], [75]

(a) (b) (c) (d)

Figure 4-9: Lower bound for the buckling load derived from load-shortening-curve (from [138])

In general Kármán and Tsien [139] defined failure as the transition from the stable prebuckling to the stable postbuckling configuration (point G-C,Figure 4-10). To overcome the "energy barrier" represented by the unstable portion of the postbuckling path, they relied on the external disturbances always present around the test set-up. Since it is difficult to determine the magnitudes of the disturbances in advance, it has been suggested to use the minimum of the postbuckling curve as a safe design value in practical engineering application. Motivated by own experimental results, Esslinger [63] came to the same conclusion, that the load capacity is equal to the minimum postbuckling load (load in point N, Figure 4-9c). Later Esslinger and Geier suggested the postbuckling load at the minimum shortening as conservative load capacity (load in point N, Figure 4-9d) and, together with Garkisch, the maximum load at minimum post critical shortening as progressive load capacity (load in point G) [75]. The design load derived from tests by Esslinger is equal to the predictions of the strong energy criterion by Tsien.

Continuing the failure definition of Kármán and Tsien, in the perturbation energy concept of Duddeck et al. [140] using a nonlinear eigenvalue analysis the minimum deformation energy Π_{St} (perturbation energy) and the related deformation pattern v_{St} (perturbation vector) is calculated which is required for transition from a defined stable prebuckling reference configuration (point G, Figure 4-10a) to an adjacent equilibrium configuration. Thus, if there is sufficient energy Π_{St} available the transition is from the stable prebuckling to the stable postbuckling configuration (point C) passing the unstable postbuckling configuration (point B). The relation between perturbation energy Π_{St} and knock-down factor ρ results from a large number of calculations (see Figure 4-10b). The perturbation energy level determines the imperfection sensitivity of a certain structure and is derived from experiments. Using this value the knock-down factor and the design load can be evaluated using the diagram in Figure 4-10b. The perturbation vector can be interpreted as worst imperfection pattern.

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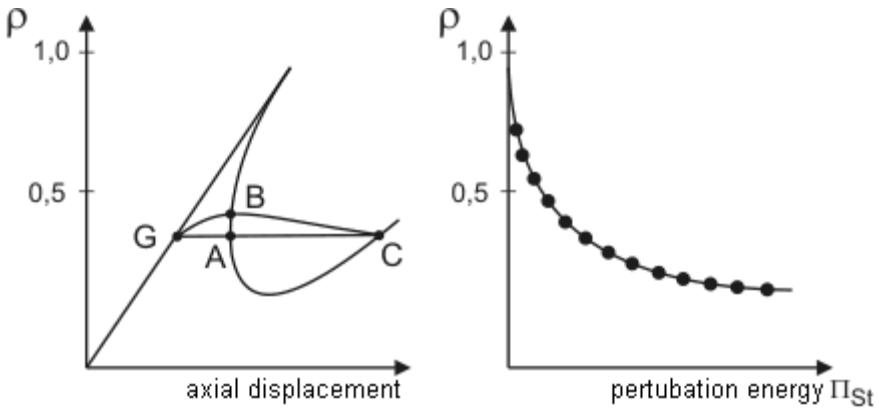


Figure 4-10: (a) Load shortening curve with related perturbation energy; (b) relation between perturbation energy and knock-down factor

Basic contributions to the perturbation energy concept were provided by Hillmann [141], Wagenhuber [142] and Dinkler [143]. Based on this, Tranel carried out many numerical investigations on elastic cylindrical shells [144]. Schäfer [145] and Spohr [146] extended the scope of the concept to cylinders with plasticity, Knoke to cylinder cone configurations [147]. According to Knebel [148] the question remains open, whether always the closest neighbouring configuration is found.

Rottner et al. [149], [150] applied a dynamic start deflection to the reference configuration. A large number of simulations was run to detect the critical energy, which is required to stimulate transition to a neighbouring equilibrium configuration. Similar to the perturbation energy concept the load capacity can be determined by an energy threshold.

Further concepts tried to explain the influence of imperfections with a reduction of the membrane stiffness [151]-[153]. The concept of quasi strain free bending by Fritz und Wittek [154] regards the energy distribution at the critical buckling point and from that draws conclusions to the postbuckling behaviour. According to Fritz and Wittek the whole membrane effect disappears at the buckling point, and the energy balance comprises only bending parts in the postbuckling regime. Then, the design load is lower than the minimum postbuckling load and therefore by far too conservative (Brendel [138]). The method of direction dependent reduced membrane buckling is a further development of the quasi strain free bending. A part of the membrane energy remains in the structure, which results in a somewhat higher minimum postbuckling load. Wittek [153] states that this method provides a good approximation of the minimum postbuckling load for such shell structures, that do not have multiple mode switches before reaching the postbuckling minimum and that have almost linear prebuckling behaviour. However, these prerequisites are not fulfilled by the unstiffened cylindrical shell.

The basic idea of the lower bound theory developed by Batista and Croll [155], [156] is comparable to the direction dependent reduced membrane buckling method. The approaches only differ in the parts of membrane energy taken into account.

4.4.8 Overview of different approaches for modelling imperfections

The knock-down factors for shells of isotropic material recommended in existing guidelines lead to a very conservative design and do not take into account pattern or amplitude of the existing imperfections. Geometrical imperfections can be covered in the numerical analysis when pattern and amplitude are measured and a probabilistic evaluation is performed. However, this approach is costly as it should be adopted to each manufacturing process anew. For the approach with worst imperfection the pattern is computed with high numerical effort. Amplitude or perturbation energy is specified or derived from tests. For using imperfection perturbations, pattern and amplitude should

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be specified such that a realistic load capacity is predicted. As before, this specification is very difficult, since often sufficient test data for validation are missing. Some approaches for predicting physics based design loads were derived from observing the structural behaviour. The significance of the results cannot be fully assessed so far. It can be stated that for isotropic shells so far no alternative to the traditional knock-down factors has been established.

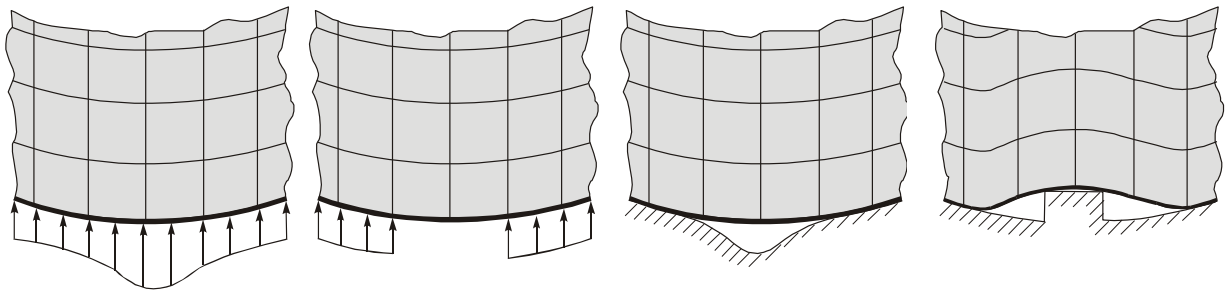
4.5 Influence of non-traditional imperfections

In literature „imperfections“ are mostly understood as geometrical imperfections. Beside deviations from perfect geometry also deviations from ideal, homogeneous load introduction occur in reality. These non-traditional imperfections can also significantly reduce the buckling load.

Already in 1932 Flügge [43] investigated the influence of inhomogeneous loads $p = p_0 + p_1 \cdot \cos\varphi$ acting on both boundaries on the buckling of isotropic cylindrical shells (see Figure 4-11a). He concluded, that assuming constant loading $p=p_0+p_1$ leads to a safe design. 1959 Abir und Nardo [157] published analytical investigations on cylindrical shells with axial thermal stress varying in circumferential direction. For the same problem Bijlaard and Gallagher [158] presented one year later their results of numerical calculations. Analytical solutions for cylinders with general axial loading, which was developed in a Fourier series in circumferential direction, were developed in 1974 by Peter [159]. All mentioned publications drew the conclusion that the maximum ordinate of the load distribution determines the buckling load rather than the integral loading.

Uniform loading, interrupted by unloaded areas (see Figure 4-11b) was numerically investigated by Guggenberger [160]. The ratio μ of the length of the loaded boundary to the circumference was varied and its influence on the buckling load was determined. Limit states for μ are uniform loading ($\mu = 1$) and a single load ($\mu \to 0$). Even small unloaded areas resulted in significant reduction of the buckling load compared to uniform loading. Boundary imperfections due to uneven support were investigated numerically and experimentally by Ummenhofer [161] and Knebel [148] (see Figure 4-11c). Also in this case significant reductions were observed.

Thielemann, Esslinger and Geier already in the 1960ies ran tests on cylinders from mylar foil (see Figure 4-11d). The boundary perturbation was realized by placing pieces of thin cigarette paper between cylinder and load introduction ring at different positions along the circumference. Surprised by drastic load reduction, the authors carried out a large number of further tests, which confirmed the initial result.



(a) (b) (c) (d)

Figure 4-11: Different methods of applying non-uniform loading

In the 1980ies Geier, Zimmermann and Klein tested the structural behaviour of anisotropic composite shells first under uniform loading [162]. Then they introduced load imperfections using thin metallic plates (shims) [163]. For composite shells they detected imperfection sensitivity with respect to fibre orientations and stacking sequence, similar to shells under uniform loading [164]. A comprehensive

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evaluation and numerical analysis of these so-called shim tests was initiated by Hühne et al. [165]. Practial relevance of non-uniform loading for the ARIANE 5 design was highlighted by Albus et al. [166].

4.6 External pressure and torsion on cylindrical shells

Subsequent to his studies on plate buckling Fairbairn was the first who carried out extensive and well documented experiments on cylindrical shells under external pressure in 1858 [167]. His experiments and formulas were the basis of boiler design and further research for the following decades. In the last decades of the 19th century Bach carried out a multitude of buckling experiments on several structures including spherical and torispherical shells [168]. Further comprehensive test on tubes under external pressure were executed by Carman [169] and Steward [170]. With the results of the empirical investigations Steward developed design rules, while Carman showed that for thin shells analytical determination of the buckling load gives passable results. The first theoretical treatment of shell under external pressure was published by Southwell [171], [172] and almost at the same time by von Mises [173]. Their predictions of buckling loads gave good results for a set of thin-walled steel shells tested by von Mises [174], but did not agree with the various tests on thicker shells executed at that time. In 1932 Saunders and Windenburg [175] published a report about the problems and necessities in modeling and testing shells. Later studies on shells under external pressure are the experimental and theoretical works of Tokugawa [176] and Sturm [177], which included stiffened shells. For an extensive summary of the history of shell buckling under external pressure the reader should consult chapter 9 of [178].

The problem of cylindrical shells under torsion was first solved by Donnell [179], who obtained the critical stress using his famous Donnell equations. Batdorf et al. [180] determined a more accurate solution by applying a Galerkin approach to Donnell's equations. Further solutions using the Donnell as well as the Flügge equations are given by Yamaki [181]. The corresponding post buckling problem was treated theoretically by Loo [182], Nash [183] and Hayashi [184] and experimentaly by Nash [185], Weingarten [186] and Yamaki [187].

4.7 Dynamic buckling

The first notion of "dynamic buckling" can be traced to the investigation of Koning and Taub [188]in the early 30's. Further monographs, which treat the various aspects of dynamic buckling were published by Bolotin [189], Lindberg and Florence [190], Simitses [191] and Ari-Gur, Weller and Singer [192]-[194].

There are two different phenomena, which are assigned to the term "dynamic buckling" in the literature in general [178]. The first one is the response of a structure to oscillating loads, which is also associated to the term "vibration buckling" and "parametric resonance". It deals with the effect that the transverse vibration of a beam becomes unacceptably large at a critical combination of load amplitude, load frequency and structural damping. This phenomenon was studied by Bolotin [189], Simitses [191] and as well by Evan-Iwanowski [195]. More attention was paid to the phenomenon of the transient response to a pulse load i.e. dynamic buckling under impact loads. It was found that, unlike under static loading "dynamic buckling" depends on both magnitude of loading and its duration. Depending on duration, usually a structure will buckle under loads the magnitudes of which are higher than its static buckling load. The shorter the duration the higher the magnitude. However, within s range of loading duration corresponding to the structure's first lateral frequency a structure can buckle under loads that are smaller than its static buckling load. This topic was already investigated by Koning and Taub [188], but only in the last three decades a basic understanding of

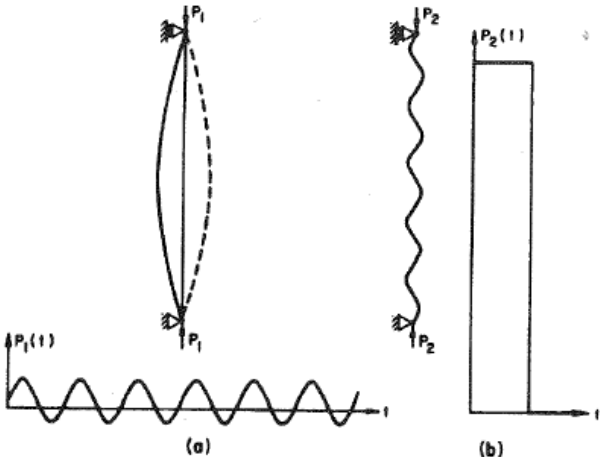
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buckling under impulse loads has been developed, supported by the development of high-speed electronics and photographic instrumentation [191], [192]

It has already been recognized by Koning and Taub [188] that the definition of dynamic buckling is only possible under consideration of small lateral geometric imperfections. Based on simple imperfection sensitive models, Budiansky and Hutchinson [196], [197] proposed some theoretical criteria and estimates for dynamic buckling loads. Further definitions of dynamic buckling are given by Hoff [198], Lindberg and Florence [190] and Maymon and Libai [199] and in Chapter 6.8. An energy based criterion is given by Humphreys and Bodner [200], which, amongst others, was discussed and compared in detail by Simitses [191].

For more information on dynamic stability analysis the interested reader should consult Section 6.8.



(a) column supporting oscillating load,
(b) a long column subjected to a sudden applied axial load; from [190]

Figure 4-12: column subjected to dynamic load

4.8 Stiffened shells

4.8.1 Overview

Since stiffeners or ribs very significantly increase the buckling loads of plates, stiffened plates have been extensively studied, starting with Timoshenko's investigations in the first decades of the 20th century (see [201], [202] and [203]). In the thirties, forties and fifties many studies were primarily motivated by aircraft design (for example [204], [205]) as well as by civil engineering problems (for example [206], [207]) Marine engineering stimulated much of the research on stiffened panels in the sixties and seventies (for example [208]-[210]), and the 1970-71 bridge disasters in Europe and Australia added their inducement to studies that continued into the nineties and beyond.

Stiffening cylindrical shells by stringers in axial direction or by frames in circumferential direction not only increases the theoretical buckling load but also decreases imperfection sensitivity [120], [211]. The stiffening influence mainly depends on geometrical parameters (cross section and distance of stringers, skin thickness, see Weller und Singer [212]) , on eccentricity (inside or outside stiffeners) and on the stiffening direction (axial or circumferential). Already in 1949, Shanley [213] gave a purely empirical design method for stiffened cylindrical shells, based on very few test results. In the following, Peterson and Dow [214], Burns and Almroth [215] and Smith and Spier [216] enhanced Shanley's method, but still these methods were not applicable to general use.

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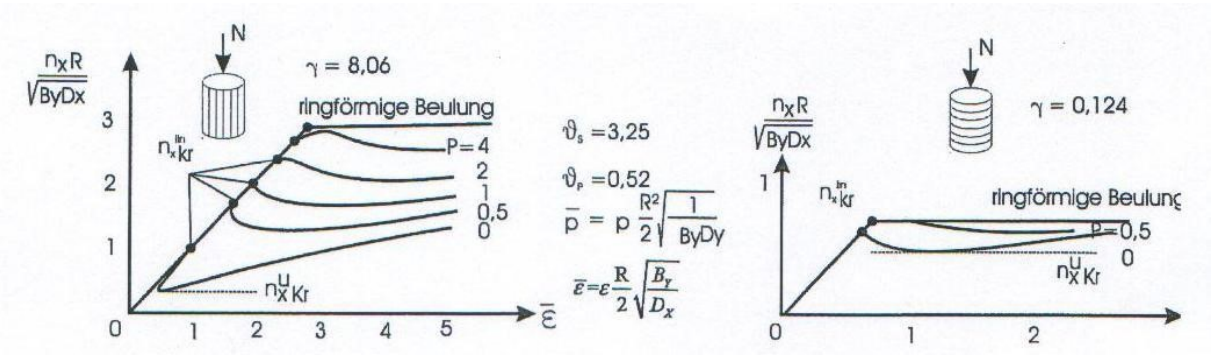
4.8.2 Imperfection sensitivity

In 1956 Koiter [217] performed an asymptotic postbuckling analysis of narrow, long cylindrical panels. Stephens [218] extended Koiter's analysis to include internal pressure and finite torsional stiffness of the stringers in 1971. Koiter and Skaloud [219] found that the load-carrying capability of structures with simultaneous local plate buckling and Euler-type column buckling may be especially sensitive to initial imperfections. Hutchinson and Amazigo [220] concluded that the major effect of axial stiffening (of cylindrical shells) is to reduce imperfection sensitivity. Using Koiter's general theory in 1973 Tvergaard [221] , [222] obtained asymptotic estimates of the imperfection-sensitivity of plates reinforced by axial stiffeners on one side. A panel designed so that local buckling coincides with buckling as an Euler column displays a high sensitivity to initial imperfections due to mode interaction. But the optimum usually corresponds to a design with Euler load lower than the critical load associated with local buckling of the skin between the stringers [223]. In the following, plates with several kind of stiffeners were investigated experimentally and analytically by Koiter and Pignataro [224], van der Neut [225], Thompson, Tulk and Walker [226] and Tvergaard and Needleman [227], [228]. Byskov and Hutchinson [229] found that while a single column or a single plate is not sensitive to initial imperfections, the general buckling load of axially stiffened cylindrical shells is always sensitive to initial imperfections, whereas the effect of interaction between local and general instability is to increase the sensitivity. For stiffened cylinders Weller and Singer [230] determined that the imperfection sensitivity depends on the geometry of stiffeners and on the ratio A_s/bt, respectively, where A_s: cross-section area of the stringer, b: circumferential distance between stringers, t: wall-thickness of the shell.

Imperfection sensitivity was either described by Koiter's b-factor or by the ratio of buckling to lowest postbuckling load. Using the latter one for orthotropic shells under axial loading March [231] and Schnell and Brühl [232] found, that in some cases the imperfection sensitivity was similar to isotropic cylinders, in other cases it was far lower. This stimulated Almroth [57] to perform comprehensive calculations for determining the minimum postbuckling load of axially loaded orthotropic cylinders (without imperfections) with parameters $0 \le \vartheta_p \le 2$ and $0.5 \le \vartheta_s \le 8.0$; ϑ_p is a plate parameter depending on the bending stiffness whereas ϑ_s is a shell parameter depending on the membrane stiffness. Almroth showed that the ratio of buckling to postbuckling load only weakly depends on the parameters ϑ_p and ϑ_s, but is dominated by the **main stiffness ratio** γ depending on membrane and bending stiffness. Cylinders with $1/\gamma \ge 1$ were denoted as circumferentially stiffened, such with $1/\gamma < 1$ as axially stiffened. Test results of Gerard, Schulz and Singer et al. [233]-[235] as well as analytical results of Thielemann [236] (see Figure 4-13) showed, that axially stiffened cylinders are more sensitive to imperfections than circumferentially stiffened ones. Among these, weakly stiffened shells were more sensitive than strongly stiffened ones. However, for given stiffener cross section and eccentricity the linear buckling load of the perfect shell is usually higher for axial than for circumferential stiffening, since axial stiffeners carry a significant part of the load (see Figure 4-13). Regarding the load carrying capability of imperfect shells (this is according to Thielemann the minimum postbuckling load) no a priori selection of axial or circumferential stiffening could be made, since this would require knowledge about imperfection sensitivity. Neglecting optimization aspects, Öry [211] recommended to use short circumferentially stiffened shells, since these structures have low imperfection sensitivity and can therefore be designed safely without problems.

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(a) (b)

(From [236])

Figure 4-13: Postbuckling behaviour of axially and circumferentially stiffened cylinders under axial loading n_x and internal pressure p

4.8.3 Orthotropic shell approach

Even though the effect of eccentricity of stiffeners has already been observed in the thirties and forties the orthotropic shell approach for cylindrical shells was generally used [237]. Advanced by work at NASA in the fifties and sixties also axially stiffened panels were analyzed as equivalent orthotropic plates until about 10 years ago [223].

In 1963 Baruch and Singer [238] developed equations using "smeared out" stiffeners and in the following, it has been shown that the smeared stiffeners theory is satisfactory for closely stiffened (cylindrical) shells that fail by general instability [239]-[248].

Analytical methods for calculating the buckling load of perfect shells based on a linear pre-buckling configuration were developed by Geier [249], Seggelke and Geier [250] as well as Hutchinson and Amazigo [119]. Hutchinson and Frauenthal took into account also nonlinear pre-buckling deformations [120]. In all these approaches the influence of discrete stiffeners was smeared [249] and orthotropic material behaviour was assumed [250]. An explanation of the complicated load carrying behaviour as given by Geier [249] was displayed by Kollár and Dulácska [117]. Shells were classified by Thielemann [236] and Schnell and Brühl [232] by the shell parameter ϑ_s, by the plate parameter ϑ_p and by the **main stiffness ratio** γ. For isotropic, unstiffened shells $\vartheta_s = \vartheta_p = \gamma = 1$ applies.

Wittrick [251] published an analysis of prismatic structures composed of flat plates in 1968, which predicts the three types of instability: general instability/overall buckling, local buckling and crippling. Viswanathan et al [252] extended Wittrick's analysis to allow orthotropic wall properties and intermittent elastic beam-type support. Williams [253] extended Wittrick's analysis to include vibration and incorporated substructuring techniques and in 1974 Wittrick and Williams [254] implemented their methods in the program VIPASA.

In [178] Singer, Arbocz and Weller summarized the **three approaches** for the analysis of stiffened plates:

a. "Smearing" the stiffeners and consider an orthotropic plate

   This approach has some constraints and is only applicable to analyze general instability.

b. Considering the stiffeners as linear discontinuities (Dirac function). This approach was developed for cylindrical shells and considers local buckling.

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c. Dividing the cross sections into its elements, which leads to a series of elements with rigidly connected edges

This approach is implemented in VIPASA by Wittrick and Williams and is more efficient than general-purpose finite element codes.

4.8.4 Eccentricity of stringers

The important influence of the eccentricity of stiffeners was detected by van der Neut [255] in 1947. Van der Neut [225] and Seggelke and Geier [250] showed, that outside stiffeners lead to a higher buckling load than inside stiffeners, independent from their orientation.

In the sixties the eccentricity effect of stringer stiffened cylinders has been analyzed by Singer et al [247], [256] and Thielemann and Esslinger [248]. Also Esslinger determined the important influence of the eccentricity of stringers and that on the outside the stringers increase the buckling load more than on the inside [257]. Further, Esslinger concluded that the longer a shell is the smaller is the influence of the stringers on the buckling load. Singer et al. [240], [247], [258] found that depending on loading and shell geometry exceptions are possible where inside stiffeners stiffen the shell more than outside stiffeners.

Imperfection sensitivity of stiffened shells was investigated by Hutchinson and Amazigo [119] as well as by Budiansky and Hutchinson [259] using Koiter's b-factor. They revealed that outside stiffened shells have a higher imperfection sensitivity than inside stiffened ones.

4.8.5 Boundary conditions

For stringer-stiffened cylindrical shells the effect of boundary conditions differs significantly from isotropic and ring-stiffened shells [119], [212], [260]-[262]. Contrary to isotropic cylinders, where geometric imperfections have the main influence, for stiffened shells the effect of boundary conditions is predominant [263]. In 1964 Esslinger and Thielemann [264] found that axial and circumferential stringer stiffened cylindrical shells are insensitive to the shear softness of the boundary condition which was confirmed by Singer et al [265] and Weller [266]. In the early eighties Singer and co-workers [267], [268], [269] developed the non-destructive vibration correlation technique (VCT) to determine boundary conditions, load eccentricities and buckling loads on a cylindrical shell.

4.8.6 Load cases

The effect of load eccentricities on cylindrical shells was investigated by Stuhlman et al in 1966 [270], followed by Weller et al [261] and Singer [268] who found that the differences in buckling loads due to the effect of load eccentricity can be up to 50 % for some practical configurations.

Tennyson [271] tested ring-stiffened and stringer-stiffened cylindrical shells under axial compression and external pressure and found that the influence of external pressure as well as the influence of geometric imperfection on the buckling load is much lower for ring-stiffened cylinders than for stringer-stiffened cylinders. Hutchinson and Amazigo [220] emphasized in 1967 that under certain circumstances, axial stiffening may be a more efficient means of strengthening against buckling under hydrostatic pressure than ring stiffening.

For isotropic cylinders Libai und Durban [272] concluded that a cosine distributed load does not decrease the buckling load of cylindrical shells significantly compared to a constant load distribution.

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4.8.7 Optimization

Because of the additional parameters, optimization is an important topic for stiffened structures. Further, it turned out that mode interaction has a significant influence on the optimum design [222], [223], [225], [228], [273]-[275]. In 1969 Van der Neut [276] analyzed the two-flange column model. He determined that if the local plate buckling and the general column load are close, the structure collapses suddenly associated with high imperfection-sensitivity. This model was used for several optimization procedures on stiffened panels in the seventies [277].

4.9 Plastic buckling

Only for sufficiently slender structures is it valid to assume that yield stresses are not reached before instability occurs. For thicker components it is necessary to consider the effect of plasticity. For this case Engesser [278] proposed in 1889 to determine the Euler buckling loads with tangent modulus E_t instead of the Young's modulus E. Considère [279] and von Kármán [280] established the "reduced modulus" or "double-modulus" E_r, which is a function of the Young's modulus E and E_t, where $E_t < E_r < E$. Hence, this approach provides a less conservative buckling load. Shanley [7] showed by analytical and experimental investigations that the two buckling loads, determined with E_r and with E_t, can be considered as an upper and a lower bound for the critical inelastic buckling load. An extensive summary of this development is given by Sewell [281]. Hill [282] developed a generalized tangent modulus approach leading to a bifurcation criterion for three-dimensional elastic-plastic solids.

A simple model of elastic-strain hardening plastic material behaviour is given by Ramberg and Osgood [283]. It describes a stress-strain curve with three material parameters, which have to be determined experimentally and hence, it allows calculating the tangent modulus E_t, which was already done and tabulated for numerous materials.

Concerning plastic buckling of plates and shells there is one paradox, which is still not resolved. It is called "flow theory versus deformation theory paradox" or "plastic buckling paradox" and has been comprehensively discussed by Hutchinson [284] and Tvergaard and Needleman [285]. It turned out that calculations with the flow theory overestimate the buckling load. By contrast, the corresponding deformation theory, which is not physically acceptable, gives much better agreements with test results. One attempt to resolve this contradiction is the unavoidable imperfections approach by Onat and Drucker [286]. They showed that already extremely small initial imperfections decrease the buckling load significantly, but their theory does not explain the experimentally determined scattering of the buckling load. Hutchinson and Budiansky [287] confirmed Onat and Drucker, but only for materials with relatively low strain hardening. For high strain hardening the imperfections must be much larger in order to obtain the required decrease of the buckling load. The corner theories are another direction of approaches, which were initiated by Batdorf [288]. Christoffersen and Hutchinson [289] developed the "J_2 corner theory" which was employed by Needleman and Tvergaard [290], who showed that the predicted buckling load of this theory always falls between the J_2 deformation theory and the J_2 flow theory. But this theory cannot be used as a design method, because one cannot estimate the appropriate corner a priori.

Though further investigations lead to more accurate predictions of the buckling load (see e.g. [291]), the plastic buckling paradox cannot yet be considered as settled (see chapter 16 of [178]).

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4.10 Composite shells

In section 4.8 it was discussed, that buckling of axially loaded stiffened shells is decisively influenced by eccentricity, orientation and size of stiffeners [117], [250], [292]-[294]. When coming to fibre composite shells, the question of optimal fibre orientations arises. Zimmermann showed with analytical calculations, that the buckling load can be significantly influenced by fibre orientations and stacking sequence [295]. The results were confirmed by the DEVILS project [296]. Geier et al. [297] revealed that the buckling load of composite cylinders can be doubled by only changing the stacking sequence. In order to find a physical explanation, the authors referred to stiffened shells. The composite shell with higher buckling load was compared to an outside stiffened shell, the one with lower buckling load to an inside stiffened shell.

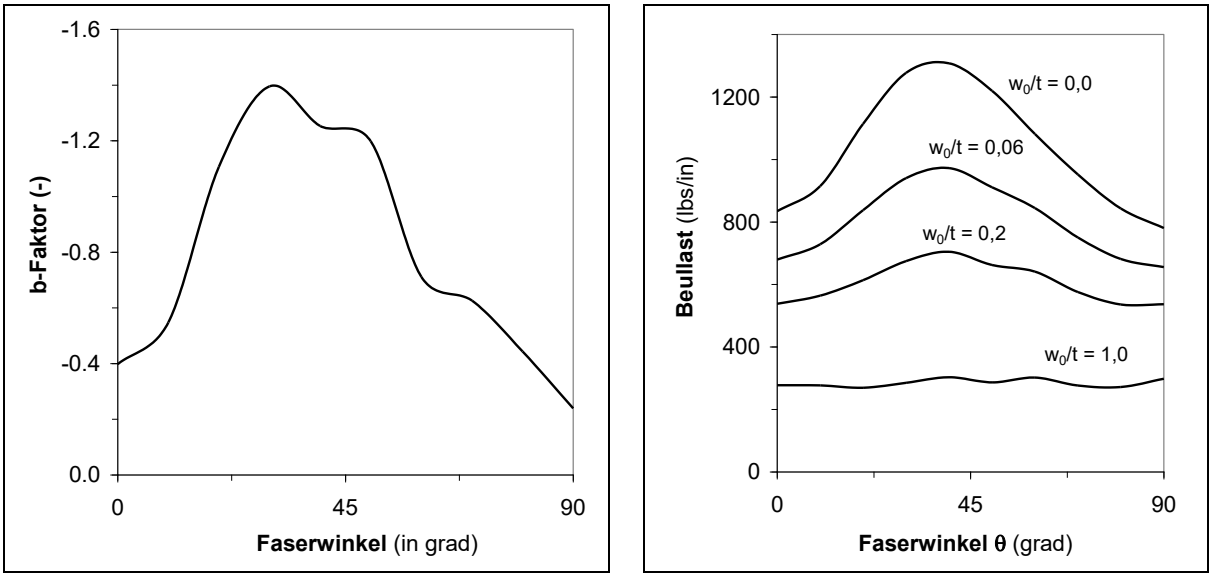
For designing shells the guideline NASA SP-8007 [88] is often used in aerospace engineering. It can be used for calculating the load carrying capability of isotropic and orthotropic shells. Knock-down factors for orthotropic shells hardly differ from those for isotropic shells.

The ESA handbook [298] recommends the method of Almroth et al. [64]. There, the linear buckling load of perfect shells is determined using a semi-analytical approach. The effective ratio of radius to wall thickness $(R/t)_e$ is calculated from (R/t) and from the stiffeners. Using this, the knock-down factor can be extracted from Figure 4-6 when assuming a certain probability of failure. Applying the 99 percent probability curve was recommended in [91]. This method turned out to be only a little more conservative than the direct determination of the knock-down factor for equivalent isotropic shells. Therefore, Bushnell [91] designed anisotropic shells using the empirical knock-down factors for isotropic shells (see Figure 4-6).

Alternatively, the ESA handbook describes the calculation of a knock-down factor according to Tennyson et al. using the b-factor [299]-[301]. The amplitude of axisymmetric imperfection perturbations was derived from measured imperfections. Tennyson et al. found good comparison between their analytical results and test results for a small number of shells [300]-[302]. Parameteric studies were performed for orthotropic laminates of type (90°, ± θ, 90°) in [301] and for type (90°, θ_2, 90°) in [302]. Khot and Venkayya [303]-[306] also carried out parametric studies for anisotropic laminates ((θ, -θ, 0°) and (θ, -θ, 90°)) from different materials. It was observed that the imperfection sensitivity depends on the fibre angle. (see Figure 4-14a). Shells that were close to the optimum showed the largest sensitivity. However, under very large imperfections the dependency of the buckling load on the fibre angle disappeared (Figure 4-14b). Similar to the investigations of Tennyson et al. it turned out that the sequence of buckling loads was the same for perfect and imperfect shells. The optimal design for the perfect shell was still valid for the imperfect shell. Thus, imperfections did not have to be included in the optimization process.

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(a) (b)

Figure 4-14: b-factor and influence of geometric imperfections as a function of fibre orientation θ (stacking sequence (θ, -θ, 0), from [307])

In [298] the results using the approaches of Almroth et al. and Tennyson et al. were compared. Since large deviations were revealed in some cases, further investigations were recommended.

Almroth [307] discussed the question, whether or not cylinders from composite material are more sensitive to imperfections than those from isotropic material. By evaluating many publications it turned out, that unstiffened composite shells are less sensitive than isotropic shells. Therefore, using the knock-down factors for isotropic shells would lead to a safe design [91].

Hilburger and Starnes [308] raised the problem, that the existing test results for isotropic shells did not comprise any information about the structural behaviour and the characteristic imperfections of composite shells. Therefore, Hilburger and Starnes as well as Almroth stimulated a detailed investigation of characteristic imperfections [307], [308]. Arbocz and Hol [309], [310] and Esong et al. [311] showed that also for composite shells characteristic imperfection patterns are associated with certain fabrication processes. Accordingly, imperfection data were stored in a data bank [312]. Almroth [307] recommended to investigate, whether imperfections that are specific for composite materials, e.g. delaminations, reduce the buckling load. Large delaminations can be avoided by detection with non-destructive testing. Small delaminations, according to [91], reduce the stiffness of the shell. In the 1990ies accurate measurements revealed that a strong influence on the buckling load is given not only by the traditional geometric imperfections [308] but also by non-traditional load imperfections caused by unevenness of edges or inhomogeneous stiffness distributions [308], [313], [314].

As a prerequisite for safe design based on numerical analysis, a validation of numerical models is necessary. Hilburger and Starnes took into account traditional and non-traditional imperfections [308], while Elghazouli et al. [315]-[317], Spagnoli et al. [318] and Chryssanthopoulos et al. [319] mainly investigated geometrical imperfections. Stochastic approaches were applied by Arbocz und Starnes [320], Chryssanthopoulos and Poggi. [321] and Elishakoff et al. [322]. A simplified probabilistic approach was suggested by Hilburger et al. [323]. However, the large number of test data required and the exact knowledge of shape and magnitude of imperfections needed make the method costly. Zimmermann [324] mentioned, that geometrical imperfections may arise not only from fabrication but

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also from the mounting processes. These and other types of imperfections should also be covered by this approach.

It can be concluded that for designing anisotropic cylindrical shells very conservative knock-down factors are applied which are based on test results for isotropic shells and therefore are not suited for optimization. Almroth suggested to account for orthotropic material behaviour by a modified ratio $(R/t)_e$, for which the knock-down factor is evaluated. Tennyson recommended to use Koiter's b-factor for estimating the imperfection sensitivity and to determine the knock-down factor for an assumed imperfection amplitude. Both methods, however, are not sufficiently validated so far. Numerical methods like FEM are presently validated by test results. Stochastic approaches are very costly and not suited for design purposes. Since test data for composite shells are very rare as compared to isotropic shells, no practically applicable design concept is available as yet.

In order to be able to handle the nonlinearities in the material behaviour of fibre composites, Hühne developed an improved approach for determining the material properties of thin walled composite structures [325].

A comprehensive explanation of the structural behaviour of cylindrical composite shells under compression is given by Hühne et al. [326]. By numerical and experimental investigations Hühne showed that the collapse of cylindrical shells always starts with a single buckle, as it was already detected by Esslinger for isotropic shells [65]. Using this fact, Hühne developed a new deterministic approach for a lower bound of the buckling load.

If one regards stiffened composite structures, further considerations about cases of failure have to be made. Thus, in the first years of this century several approaches for the modelling and simulation of delamination were developed by Jarlas [327], Nilson et al [328], Wagner et al [329], [330] and Thomson et al [331], [332]. Sjögren et al [333] determined the stiffness reduction caused by impact, which is controlled by fibre breakage, and Yap, Scott, Thompson and Hachenberg [334] analyzed skin stiffener debonding. The durability of stiffened composite structures was already investigated by Weller et al [335], [336] and Frostig et al [337] in the early nineties. In 1996 Hambley [338] tested damaged cylindrical shells to investigate the influence of damage on the first buckling load.

In 2001 the POSICOSS project [339] developed fast and reliable procedures and a final design guideline for stiffened fibre composite curved panels. The following project COCOMAT [340] focused on a reliable simulation of postbuckling and collapse considering structural degradation in order to determine less conservative buckling loads at the same safety.

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