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Worm gears — Worm profiles and gear mesh geometry

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TECHNICAL REPORT

PD ISO/TR 10828:2015

**ISO/TR
10828**

Second edition
2015-08-15

Worm gears — Worm profiles and gear mesh geometry

*Engrenages à vis cylindriques — Géométrie des profils de vis et
des engrenements*



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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This second edition cancels and replaces the first edition (ISO/TR 10828:1997), which has been technically revised.

This edition includes the formulation for the geometrical dimensions of the worm and worm wheel, and that for the determination of gear mesh geometry (path of contact, zone and lines of contact) with the details to determine the non-dimensional parameters used to apply load capacity calculations (radius of curvature, sliding velocities).

Introduction

Thread forms of the worms of worm gear pairs are commonly related to the following machining processes:

- the type of machining process (turning, milling, grinding);
- the shapes of edges or surfaces of the cutting tools used;
- the tool position relative to an axial plane of the worm;
- where relevant, the diameters of disc type tools (grinding wheel diameter).

This Technical Report introduces all the aspects concerning the gear mesh geometry to define conjugate worm wheel, path of contact, lines of contact and other associated geometrical characteristics.

Worm gears — Worm profiles and gear mesh geometry

1 Scope

In this Technical Report, thread profiles of the five most common types of worms at the date of publication are described and formulae of their axial profiles are given.

The five worm types covered in this technical report are designated by the letters A, C, I, K and N.

The formulae to calculate the path of contact, the conjugate profile of the worm wheel, the lines of contact, the radius of curvature and the velocities at points of contact are provided. At the end the application of those formulae to calculate parameters used in load capacity calculations are provided.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1122-2, *Vocabulary — Worm gears*

ISO 701, *International gear notations — Symbols for geometrical data*

ISO/TR 14521, *Worm gears — Load capacity of worm gears*

3 Symbols and abbreviated terms

For the purposes of this document, Tables 1 to 3 give the symbols the indices and the description.

Table 1 — Symbols for worm gears from Clause 4 of this document

Symbols	Description	Units	Figures	Formula number
A	distance from the worm axis to virtual point of the cutter (see ref.[1])	mm	Fig. A.4	
a	centre distance	mm	Fig. 3	41/42
a_0	refers to the worm/tool centre distance (length of the common perpendicular to the worm/tool axes)	mm	Fig. 18	54
a_1 to a_4	coefficient for A, I and N profile			
b_1	facewidth of worm	mm		24
b_{2H}	effective wheel facewidth	mm	Fig. 4	39
b_{2R}	wheel rim width	mm	Fig. 4	
c_1, c_2	tip clearance	mm		46/47

Symbols	Description	Units	Figures	Formula number
d_{a1}	worm tip diameter	mm		14
d_{a2}	worm wheel throat diameter	mm		35
d_{b1}	base diameter of involute helicoid (for I profile)	mm		22
d_{e2}	worm wheel outside diameter	mm		36
d_{f1}	worm root diameter	mm		15
d_{f2}	worm wheel root diameter	mm		34
d_{m1}	worm reference diameter	mm	Fig. 1/3	10
d_{m2}	worm wheel reference diameter	mm	Fig 2/3	25
d_{w1}	worm pitch diameter	mm		43
d_{w2}	worm wheel pitch diameter	mm	Fig 5	44
e_{mx1}	worm reference tooth space width in axial section	mm	Fig. 1	17
e_{n1}	worm normal tooth space width in normal section	mm		19
e_{m2}	worm wheel tooth space width in mid-plane section	mm		28
h_1	worm tooth depth	mm		11
h_2	worm wheel tooth depth	mm		32
h_{am1}	worm tooth reference addendum in axial section	mm	Fig. 3	12
h_{am2}	worm wheel tooth reference addendum in mid-plane section	mm	Fig. 3	30
h_{am1}^*	worm tooth reference addendum coefficient in axial section	-		
h_{am2}^*	worm wheel tooth reference addendum coefficient in mid-plane section	-		
h_{e2}	worm wheel tooth external addendum	mm		33
h_{fm1}	worm tooth reference dedendum in axial section	mm		13
h_{fm2}	worm wheel tooth reference dedendum in mid-plane section	mm		31
h_{fm1}^*	worm tooth reference dedendum coefficient in axial section	-		
h_{fm2}^*	worm wheel tooth reference dedendum coefficient in mid-plane section	-		
j_x	axial backlash	mm		
m_n	normal module	mm		9
m_{x1}	axial module	mm		2/G.1
p_{bn1}	normal pitch on base cylinder	mm		23
p_{n1}	normal pitch	mm		8
p_{t2}	transverse pitch	mm		26
p_{x1}	axial pitch	mm	Fig. 1	1
p_{z1}	lead (of worm)	mm		3
p_{zu1}	unit lead (lead of worm per radian)	mm/rad		4
q_1	diameter quotient	mm		5
r_{g2}	worm wheel throat form radius	mm		40

Symbols	Description	Units	Figures	Formula number
r_{b1}	base radius for involute profile	mm	Fig. A.4 and A.5	
r'_{b1}	base radius of a notional base circle	mm	Fig. A.4 and A.5	
r_t	radius at cusp	mm	Fig. 29	
s_{m2}	tooth thickness at the reference diameter of the worm wheel	mm	Fig. 2	27
s_K	rim thickness	mm	Fig. 12	
s_{mx1}	worm thread thickness in axial section	mm	Fig. 1	16
s_{mx1}^*	worm thread thickness in axial section coefficient	-		
s_{n1}	normal worm thread thickness in normal section	mm		18
u	gear ratio			45
x_2	worm wheel profile shift coefficient	-		29
z_1	number of threads in worm	-		
z_2	number of teeth in worm wheel	-		
α_{0n}	tool normal pressure angle	°		
α_{0t}	tool transverse pressure angle for A and I profiles	°	Fig. 7	
α_n	normal pressure angle	°		20
β_{m1}	reference helix angle of worm	°		7
γ_{m1}	reference lead angle of worm	°		6
γ_{b1}	base lead angle of worm thread (for I profile)	°	Fig. A.1	21
γ'_{b1}	base lead angle of the notional base helix	°	Fig. A.4 and A.5	
ρ_{Gm}	Radius of curvature of grinding wheel (C profile)			

In calculation, when a radius is derived, the symbol d for diameter shall be replaced by r for radius.

Table 2 — Subscripts for worm gears

Symbols	Description
0	cutting tool
1	worm
2	Wheel
G	grinding wheel

Table 3 — Coordinate of remarkable points

Symbols	Description
$x_G(y_G), y_G, \alpha_G(y_G)$	Coordinates of a point on the tool flank when the origin is at the point of intersection of the tool axis and the tool median plane, with the x-axis as the tool spindle axis and the

	abscissa on the trace of the median plane;
$x_x(y_r), y_x(y_r), \alpha_x(y_r)$	Coordinates of axial profile and axial pressure angle for A, I, N worm profiles
$x_x(y_G), y_x(y_G), \alpha_x(y_G)$	Coordinates of axial profile and axial pressure angle for K and C worm profiles
$x_D(y_p, D), y_D(y_p, D), \alpha_D(y_p, D)$	Coordinates of worm profile and pressure angle of worm profile in an offset plane
$x'_D(y_p, D), y_D(y_p, D), \alpha_D(y_p, D)$	Coordinates of worm profile and pressure angle of worm profile in an offset plane with origin on pitch axis
$x_{ID}(y_p, D), y_{ID}(y_p, D)$	Coordinates of path of contact in an offset plane with origin on pitch axis
$xR_D(y_p, D), yR_D(y_p, D)$	Coordinates of conjugate worm wheel profile of the worm in an offset plane with origin on worm wheel axis
$xT_D(r_{t2D}, D), yT_D(r_{t2D}, D)$	Coordinates of trochoid profile of the worm wheel profile in an offset plane with origin on worm wheel axis
$x_D(ycusp, D), y_D(ycusp, D)$	Coordinates of cusp point in an offset plane with origin on pitch axis
$C_{eq1D}(y_p, D)$	Curvature for the worm at a point in an offset plane
$C_{eq2D}(y_p, D)$	Curvature for the worm wheel at a point in an offset plane
$R_{eqD}(y_p, D)$	Equivalent radius of curvature in an offset plane
$r_{e2D}(D)$	outside radius of the worm wheel in the offset plane D
r_{t2D}	root radius of the worm wheel in the offset plane D
$\overline{M}_1(y_p, D)$	Coordinate of a point of contact for the worm (Eq 118)
$M_2(y_p, D)$	Coordinate of a point of contact for the worm wheel (Eq 119)
$\overline{TN1}_{cont}(y_p, D)$	Tangent unit vector to a line of contact (Eq 128)
$\overline{NormalNxy}(y_p, D)$	Normal unit vector to the lines of contact (Eq 114)
$\overline{NORMAL}(y_p, D)$	Normal unit vector to the lines of contact (Eq 116)
$Req(y_p, D)$	Radius of curvature along the line the contact (Eq 135)
$\vec{V}_1(y_p, D)$	Velocity of a point of the thread of the worm (Eq 138)
$\vec{V}_2(y_p, D)$	Velocity of a point of the tooth flank of the worm (Eq 140)
$V_{cDn}(y_p, D)$	Velocity at the contact point along the path of contact (Eq 150)
$V_{SUMn}(y_p, D)$	Sum of velocities at the point of contact (Eq 153) for method B in ISO/TR 14521

4 Formulae for calculation of dimensions

4.1 Parameters for a cylindrical worm

4.1.1 Axial pitch

$$p_{x1} = \pi \cdot m_{x1} \quad (1)$$

4.1.2 Axial module

$$m_{x1} = \frac{p_{x1}}{\pi} \quad (2)$$

4.1.3 Lead

$$p_{z1} = z_1 \cdot p_{x1} \quad (3)$$

4.1.4 Unit lead

$$p_{zu1} = \frac{p_{z1}}{2 \cdot \pi} \quad (4)$$

4.1.5 Diameter quotient

$$q_1 = \frac{d_{m1}}{m_{x1}} \quad (5)$$

4.1.6 Reference lead angle

$$\tan \gamma_{m1} = \frac{m_{x1} \cdot z_1}{d_{m1}} = \frac{z_1}{q_1} \quad (6)$$

4.1.7 Reference helix angle

$$\beta_{m1} = 90^\circ - \gamma_{m1} \quad (7)$$

4.1.8 Normal pitch on reference cylinder

$$p_{n1} = p_{x1} \cdot \cos \gamma_{m1} \quad (8)$$

4.1.9 Normal module

$$m_n = m_{x1} \cdot \cos \gamma_{m1} \quad (9)$$

4.1.10 Reference diameter

$$d_{m1} = q_1 \cdot m_{x1} \quad (10)$$

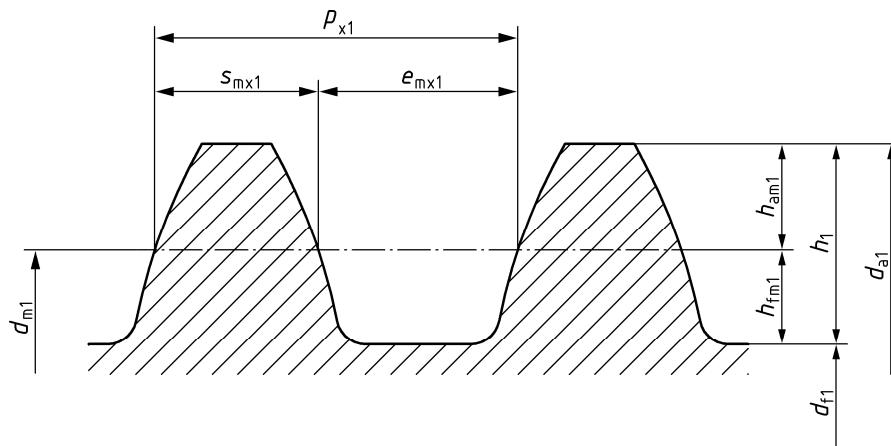


Figure 1 — Axial parameters for worm

4.1.11 Reference tooth depth

$$h_1 = h_{am1} + h_{fm1} = \frac{1}{2} \cdot (d_{a1} - d_{f1}) \quad (11)$$

4.1.12 Reference addendum

$$h_{am1} = h_{am1}^* \cdot m_{x1} = \frac{1}{2} \cdot (d_{a1} - d_{m1}) \quad (12)$$

where h_{am1}^* is the addendum coefficient; normally $h_{am1}^* = 1$

4.1.13 Reference dedendum

$$h_{fm1} = h_{fm1}^* \cdot m_{x1} = \frac{1}{2} \cdot (d_{m1} - d_{f1}) \quad (13)$$

where h_{fm1}^* = dedendum coefficient; generally $1,1 < h_{fm1}^* < 1,3$, the recommended value is 1,2

4.1.14 Tip diameter

$$d_{a1} = d_{m1} + 2 \cdot h_{am1} \quad (14)$$

4.1.15 Root diameter

$$d_{f1} = d_{m1} - 2 \cdot h_{fm1} \quad (15)$$

4.1.16 Thread thickness coefficient s_{mx1}^*

A recommended value is $s_{mx1}^* = 0,5$

In general practice, this coefficient is very often less than 0,5 when there is a wish to increase the worm wheel thread thickness to extend durability against wear of worm wheel.

See Figure 1.

4.1.17 Reference thread thickness in the axial section

$$s_{mx1} = s_{mx1}^* \cdot p_{x1} \quad (16)$$

4.1.18 Reference space width in the axial section

$$e_{mx1} = p_{x1} - s_{mx1} \quad (17)$$

4.1.19 Normal thread thickness

$$s_{n1} = s_{mx1} \cdot \cos \gamma_{m1} \quad (18)$$

4.1.20 Normal space width

$$e_{n1} = e_{mx1} \cdot \cos \gamma_{m1} \quad (19)$$

4.1.21 Profile flank form

It is specified by a letter:

- A is the envelope of straight line in the axial section;
- N is the envelope of straight line in the normal section of the space width;
- I is the involute helicoid (the envelope of straight line in a plane tangent to the base cylinder);
- K is a milled helicoid by double cone form;
- C is a milled helicoid by circular convex form.

4.1.22 Normal pressure angle

For type-A

$$\tan \alpha_n = \tan \alpha_{0t} \cdot \cos \gamma_{m1} \quad (20)$$

For other type

$$\alpha_n = \alpha_{0n}$$

where α_{0n} is defined in 6.3, 6.4 and 6.5 for I and N and in 6.6 and 6.7 for K and C.

4.1.23 Base lead angle for I profile

$$\cos \gamma_{b1} = \cos \gamma_{m1} \cdot \cos \alpha_{0n} \quad (21)$$

4.1.24 Base diameter for I profile

$$d_{b1} = d_{m1} \cdot \frac{\tan \gamma_{m1}}{\tan \gamma_{b1}} = \frac{m_{x1} \cdot z_1}{\tan \gamma_{b1}} \quad (22)$$

NOTE For I profile if the root diameter is less than the base diameter attention should be taken in order that the diameter of start of active profile (SAP) is greater than d_{b1} .

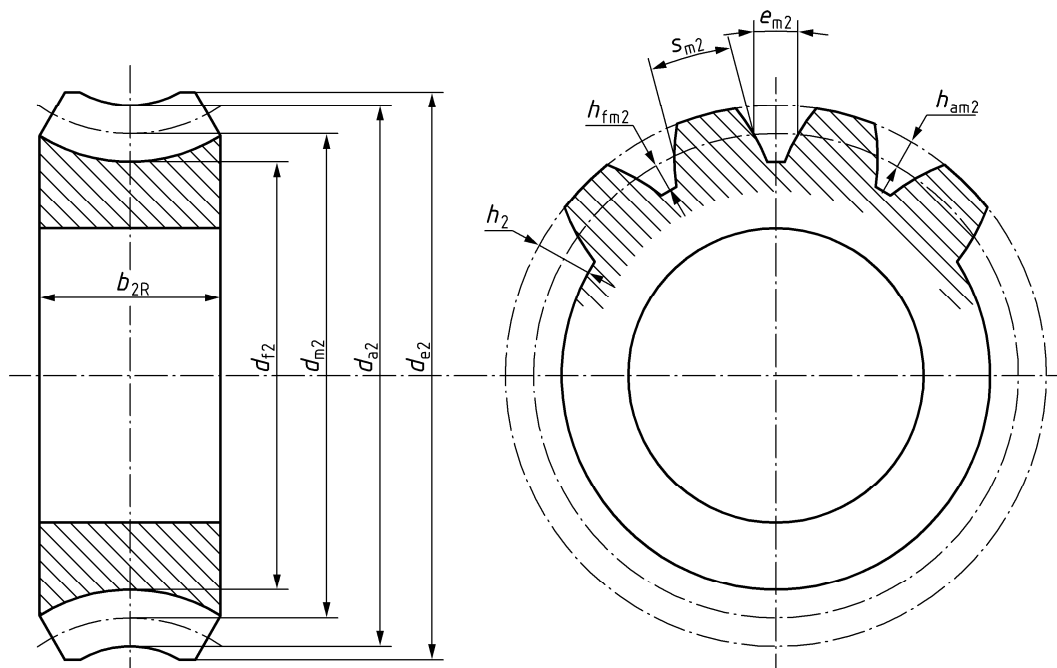
4.1.25 Normal pitch on base cylinder

$$p_{bn1} = p_{x1} \cdot \cos \gamma_{b1} \quad (23)$$

4.1.26 Worm face width

$$b_1 \geq \sqrt{(d_{e2})^2 - (2 \cdot a - d_{a1})^2} \quad (24)$$

4.2 Parameters for a worm wheel



NOTE On Figure 2 the profile shift coefficient x_2 is negative.

Figure 2 — Parameters for worm wheel

4.2.1 Reference diameter

$$d_{m2} = d_{w2} + 2 \cdot x_2 \cdot m_{x1} \quad \text{or} \quad d_{m2} = 2 \cdot a - d_{m1} \quad (25)$$

4.2.2 Transverse pitch

$$p_{t2} = p_{x1} \quad (26)$$

4.2.3 Transverse tooth thickness at reference diameter

This value can be calculated only for a worm wheel without profile shift as follows:

$$s_{m2} = e_{mx1} - j_x \quad (27)$$

where j_x = axial backlash.

See Figure 2.

4.2.4 Space width at reference diameter

This value can be calculated only for a worm wheel without profile shift as follows:

$$e_{m2} = p_{x1} - s_{m2} \quad (28)$$

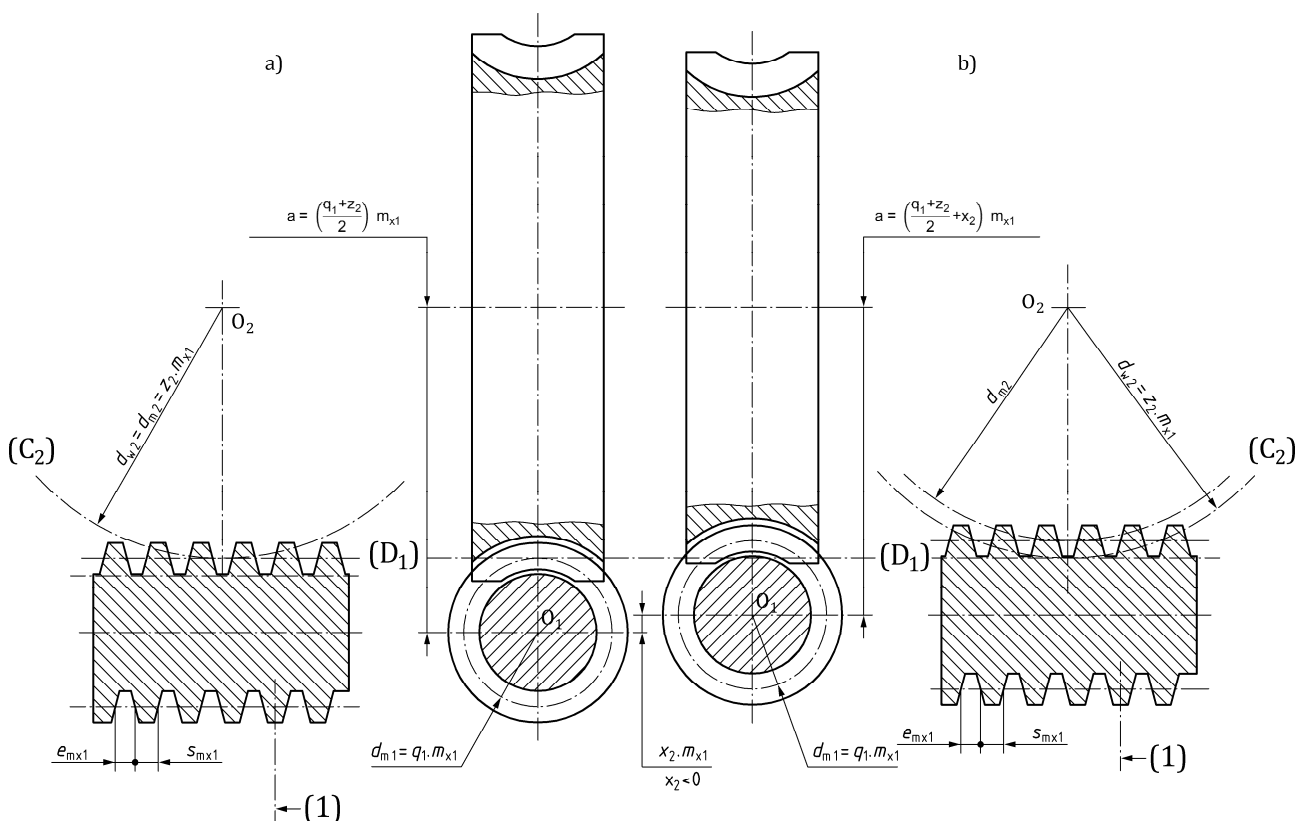
4.2.5 Profile shift coefficient

$$x_2 = \frac{2 \cdot a - d_{m1} - m_{x1} \cdot z_2}{2 \cdot m_{x1}} \quad (29)$$

4.2.6 Addendum

$$h_{am2} = m_{x1} \cdot h_{am2}^* = \frac{1}{2} \cdot (d_{a2} - d_{m2}) \quad (30)$$

where h_{am2}^* is the addendum coefficient; $h_{am2}^* = 1$ (normally).



NOTE On both parts a) and b), the worm is the same. On part a) (left), the profile shift coefficient equal zero and on part b) (right) the profile shift coefficient is negative.

Key

1 symmetrical axis of the thread

Figure 3 — Pitch and reference diameters for worm gear set

4.2.7 Dedendum

$$h_{fm2} = m_{x1} \cdot h_{fm2}^* = \frac{1}{2} \cdot (d_{m2} - d_{f2}) \quad (31)$$

where h_{fm2}^* is the dedendum coefficient; generally $1,1 < h_{fm2}^* < 1,3$, the recommended value is 1,2.

4.2.8 Tooth depth

$$h_2 = h_{am2} + h_{fm2} \quad (32)$$

4.2.9 Outside addendum

$$h_{e2} = \frac{1}{2} \cdot (d_{e2} - d_{a2}) \quad (33)$$

Generally: $0,4 \leq \frac{h_{e2}}{m_{x1}} \leq 1,5$ Normally $h_{e2} / m_{x1} = 0,5$

4.2.10 Root diameter

$$d_{f2} = d_{m2} - 2h_{fm2} \quad (34)$$

4.2.11 Throat diameter

$$d_{a2} = d_{m2} + 2h_{am2} \quad (35)$$

4.2.12 Outside diameter

$$d_{e2} = d_{a2} + 2 \cdot h_{e2} \quad (36)$$

NOTE For min/max values see 4.2.9.

4.2.13 Minimum and maximum outside diameter

Generally

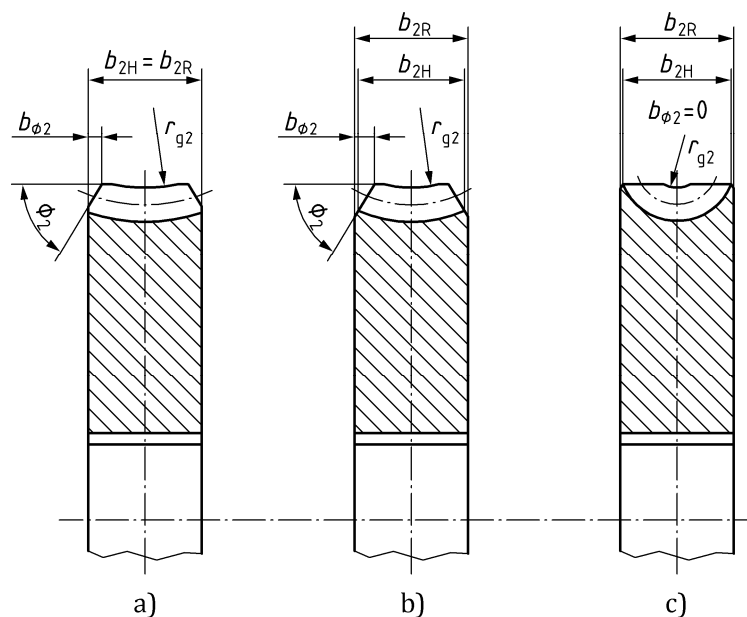
$$d_{e2min} = d_{a2} + 0,8 \cdot m_{x1} \quad (37)$$

$$d_{e2max} = d_{a2} + 3 \cdot m_{x1} \quad (38)$$

4.2.14 Worm wheel face width

$$b_{2H} \leq \sqrt{(2 \cdot a - d_{f2})^2 - (2 \cdot a - d_{e2})^2} \quad (39)$$

See Figure 4.



NOTE Case c) represents the geometrical conditions where b_{2H} reaches its maximum value.

Figure 4 — Worm wheel face width

4.2.15 Throat form radius

$$r_{g2} \geq a - \frac{d_{a2}}{2} \quad (40)$$

4.3 Meshing parameters

4.3.1 Centre distance

$$a = 0,5 \cdot (d_{m1} + d_{m2}) = 0,5 \cdot (d_{w1} + d_{w2}) \quad (41)$$

or

$$a = m_{x1} \cdot [0,5 \cdot (q_1 + z_2) + x_2] \quad (42)$$

See Figure 3.

4.3.2 Pitch diameter of worm wheel

$$d_{w2} = z_2 \cdot m_{x1} \quad (43)$$

4.3.3 Pitch diameter of worm

$$d_{w1} = 2 \cdot a - d_{w2} \quad (44)$$

4.3.4 Worm gear ratio

$$u = \frac{z_2}{z_1} \quad (45)$$

4.3.5 Contact ratio

The calculation of the contact ratio is defined in 10.6.

4.3.6 Tip clearance

$$c_1 = a - 0,5 \cdot (d_{a2} + d_{f1}) \quad (46)$$

$$c_2 = a - 0,5 \cdot (d_{a1} + d_{f2}) \quad (47)$$

5 Generalities on worm profiles

5.1 Definitions

Type A	straight sided axial profile;
Type C	concave axial profile formed by machining with a convex circular profile disc type cutter or grinding wheel;
Type I	involute helicoid, straight generatrix in base tangent planes;
Type N	straight profiles in normal plane of thread space helix;
Type K	milled helicoid generated by biconical grinding wheel or milling cutter, convex profiles in axial planes.

5.2 Conventions relative to the formulae of this document

5.2.1 The worm threads are right-handed.

The formulae in this Technical Report define the coordinates of the left flank of the axial profile of worm, i.e. in the plane XOY of Figure 5.

To obtain the right flanks, it is necessary to draw a symmetric profile to the left flank relative to a perpendicular axis to the worm axis.

5.2.2 The worm and wheel pairs operate as speed reducing gears with directions of rotation as shown in Figure 5, thus the worm thread left flanks contact the wheel teeth. These are the flanks studied in this report.

5.2.3 The worm wheel is above the worm.

5.2.4 With the origin O, the reference axes X Y Z, are mutually perpendicular (see Figure 5):

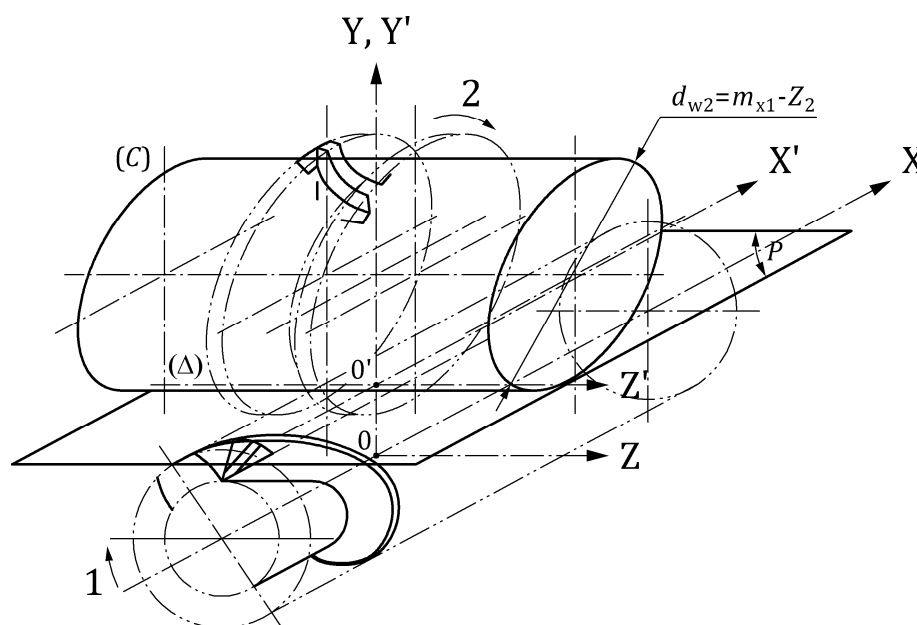
- OX the worm axis coincides with the X axis;
- OY the common perpendicular to the worm and wheel axes coincides with the Y axis;

— OZ to complete the direct coordinate system.

A point is defined by its x, y, z coordinates. The following subscripts are used:

- x refers to the X-Y axial plane;
- D refers to an offset plane;
- n refers to the normal plane;
- t for any point refers to a transverse plane.

5.2.5 If the worm is driving, the worm gear set is a reducer. If the worm wheel is driving the worm gear set is an increaser.



Key

- | | | | |
|-----|---|-----|--|
| 1 | direction of rotation of the worm | (C) | pitch cylinder of the worm wheel – diameter d_{w2} |
| 2 | direction of rotation of the worm wheel | P | pitch plane of the worm – distance from the axis of the worm equal to $d_{w1}/2$ |
| (Δ) | pitch line | | |

NOTE For the clarity of drawing only one thread for worm is represented.

Figure 5 — Conventions used in formulae

6 Definition of profiles

6.1 Introduction

There are 2 types of worm profiles:

- profiles A, I, N are generated by a helical movement of a straight line in the space. The formulae of the axial profile are a direct function according to the radius of the worm. In that case gear mesh in an

offset plane uses two parameters: y_r which is the radius of the worm and D which is the distance of the offset plane

- profiles K and C are generated by a helical movement of a grinding wheel with a certain profile (see Figures 18) in the space. The generated flanks on the worm are the envelope of the grinding wheel. Each point of the worm profile is generated by a point of the grinding wheel. The formulae of the axial profile IS NOT a direct function according to the radius of the grinding wheel profile. In that case gear mesh in an offset plane uses two parameters: y_r which is the radius of the grinding wheel and D which is the distance of the offset plane. In those cases the calculations are more complex.

6.2 Type A

6.2.1 Geometrical definition

The thread flanks of type A are generated as envelopes of straight lines in axial planes which are inclined at a constant angle: $\frac{\pi}{2} - \alpha_{ot}$ to the axis (see Figure 6). This line as it is moved with simultaneous rotation about and translation along the axis X, defines the worm thread flank (Figure 6). The form of which is commonly described as an Archimedean helix.

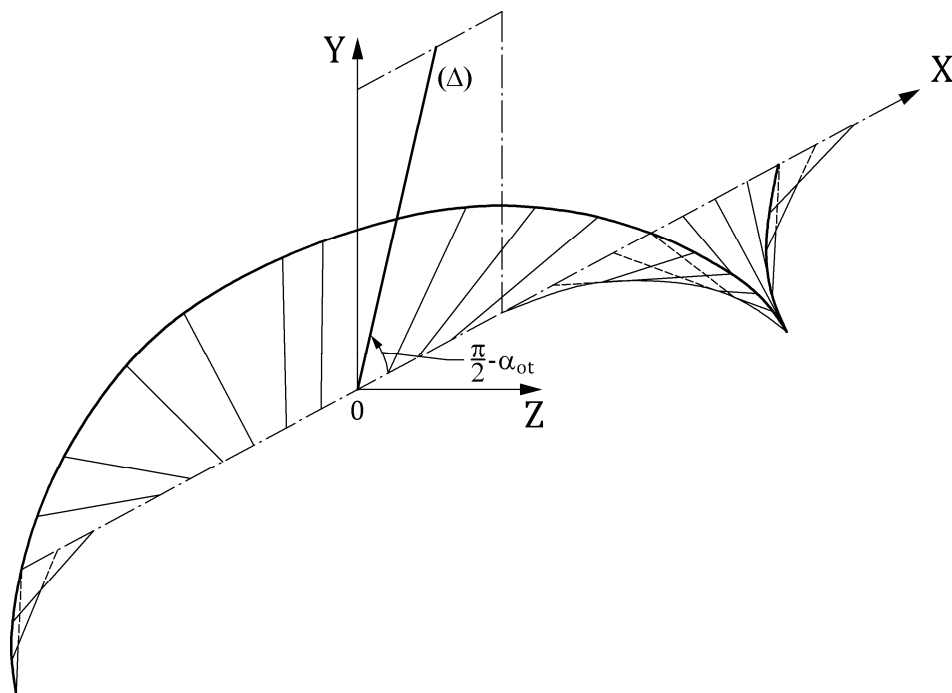


Figure 6 — Profile A: Theoretical Generation

6.2.2 Machining methods

The straight generatrix is always crossing the worm axis; the flank of thread in an axial plane is always a straight line; so machining methods should ensure to generate this straight axial flank.

The threads may be cut on a lathe with a tool having straight edges, the cutting plane of which lies in an axial plane of the worm (Figure 7 a)).

Both flanks of a thread space may be machined simultaneously by using a tool of trapezoidal form.

Another method which is an inversion of the process of cutting a helical gear with a rack cutter, involves the use of an involute shaper to produce the desired rectilinear rack profile in an axial plane of the worm. The cutting face must lie in that axial plane (Figure 7 b)).

It is also necessary that the pitch circle of the shaper should roll without slip on the datum line of the rack profile. This coincides with a straight line generatrix of the worm pitch cylinder.

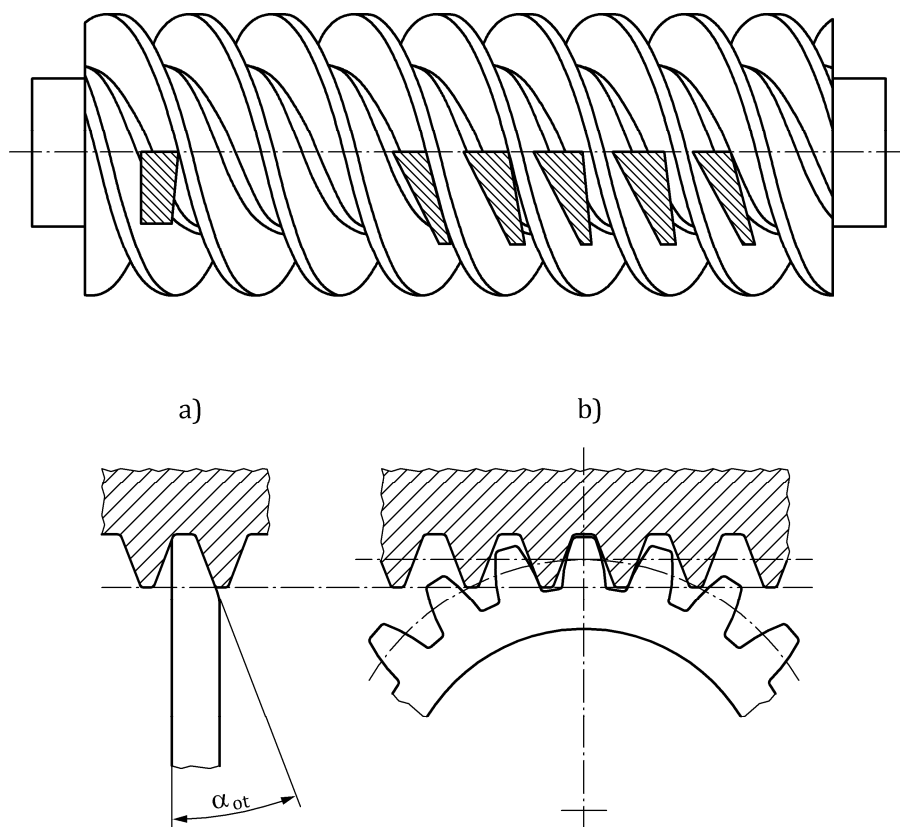


Figure 7 — Profile A - Machining Methods

Type A profile is a straight line in an axial plane.

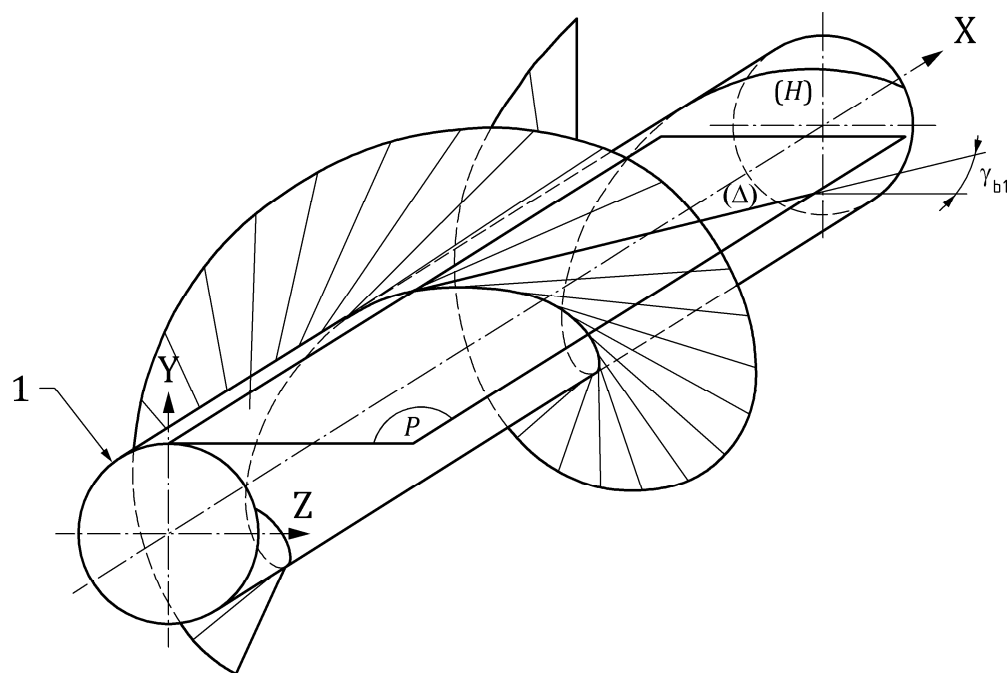
The development of the formulae for the axial profile of type A worm is in Annex A, A.2.

6.3 Type I

6.3.1 Geometrical definition

A flank of a type I worm is an involute helicoidal surface. The form of which may be generated by a base tangent (Δ) to a helix (H), which moves along this - the base helix lying on the base cylinder of the worm, which is concentric with the worm axis (Figure 8).

A transverse profile (in a normal plane to the worm axis) of a flank is an involute to a circle.



Key

1 base cylinder of the worm

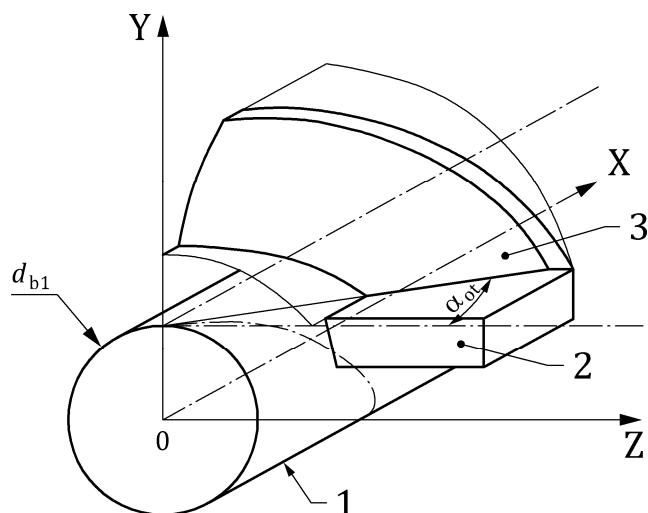
Figure 8 — Profile I - Theoretical generation

6.3.2 Machining methods

The straight generatrix is always tangential to the base helix in a plane which is tangential to the base cylinder, so the flank of the worm is a straight line in an offset plane which is tangential to the base cylinder. Machining methods should ensure this straight offset profile.

The involute helicoidal flanks of the threads can be generated by turning on a lathe using a knife tool with its straight edge aligned with the base tangent generatrix in a plane tangential to the base cylinder.

In order to machine both flanks of a thread simultaneously, it is necessary to set one left hand tool in one plane and one right hand tool in another plane as described after (Figure 9).



Key

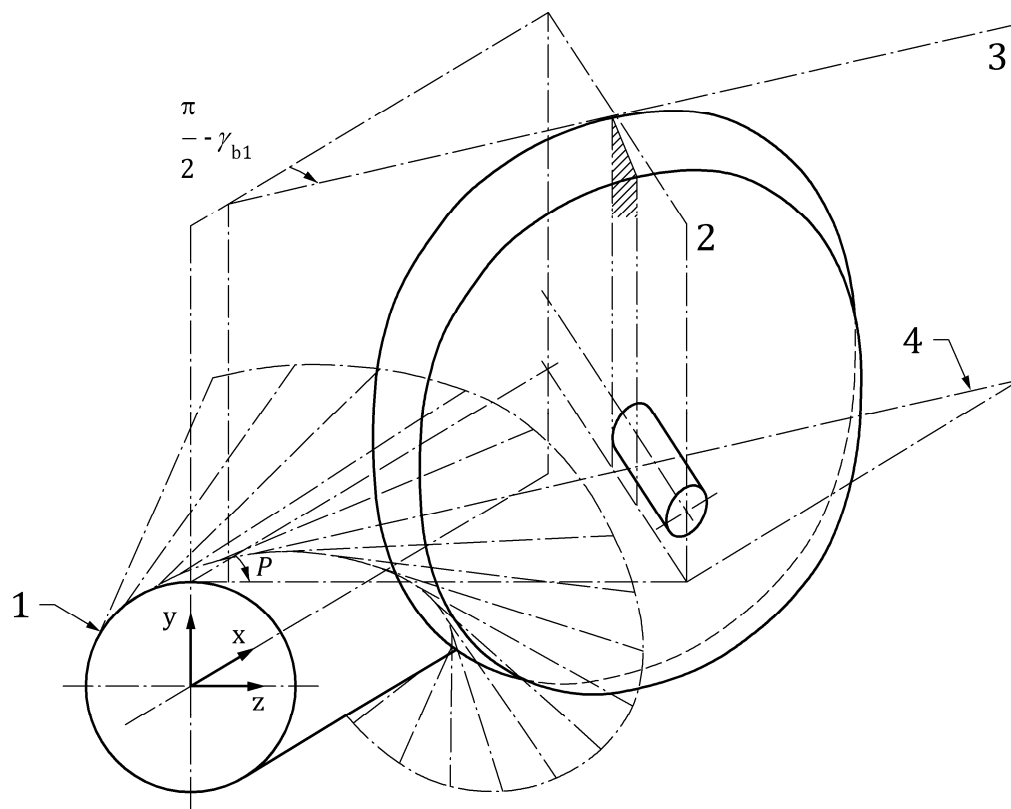
- 1 base cylinder
- 2 cutting tool
- 3 generatrix

NOTE $\alpha_{0t} = \gamma_{b1}$

Figure 9 — Profile I - Machining method with a lathe

The worm flanks can be machined by milling or grinding using the plane side face of a disc type milling cutter or grinding wheel. The cutting face has to be aligned either

- with its axis parallel to the X-Z plane and the base tangent generatrix of the flank in the cutting face (Figure 10), or
- with the reference helix of the worm and in a plane perpendicular to the reference helix set to the normal pressure angle α_{0n} (Figure 11).



Key

- 1 base cylinder
- 2 grinding wheel (with flat active face)
- 3 action plane of the grinding wheel
- 4 generatrix
- P plane tangent to the base circle containing the generatrix

Figure 10 — Profile I - Machining method by grinding (solution 1)

The latter method of alignment has the advantage that the cutting face extends to near the thread root whereas in order to do so by the previous method it will be necessary raise the cutter/grinding-wheel spindle so that the cutter/wheel periphery is tangential to the point of intersection of the base tangent generatrix with the root cylinder of the worm.

Both methods require that the mounting of the worm in the milling/grinding machine must be reversed between machining right and left flanks.

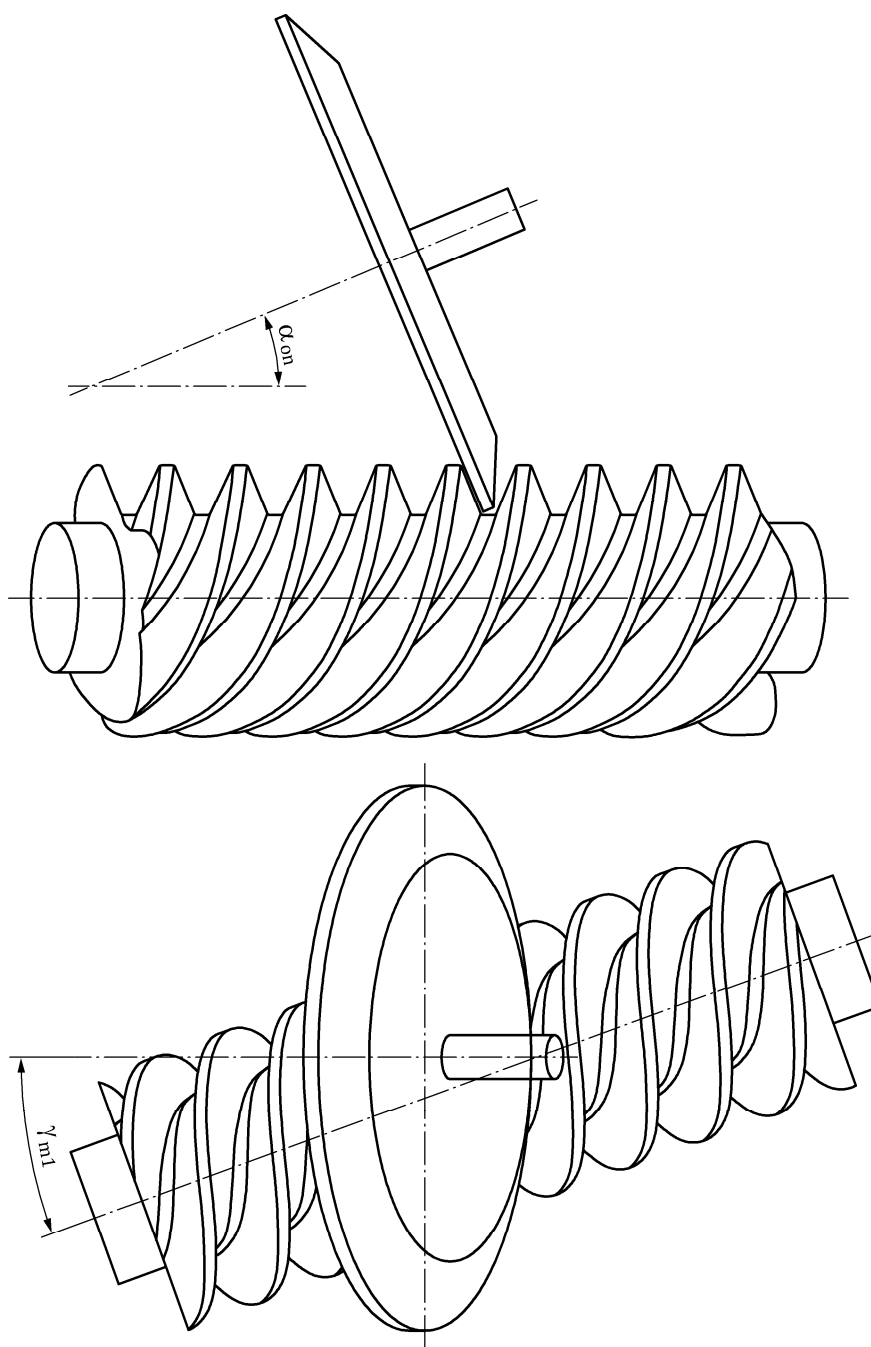


Figure 11 — Profile I - Machining method by grinding (solution 2)

Type I profile is slightly convex in an axial plane.

The development of the formulae for the axial profile of type I worm is in Annex A, A.3.

6.4 Type N

6.4.1 Geometrical definition

Each flank of a type N worm is formed by a straight line generatrix (Δ) which lies in a plane normal to the reference helix (H_1); crossing (M) which is a common point of intersection of a vector radius, the generatrix (Δ) and the reference helix H. The angle α between (Δ) and the radius vector at M point is constant.

The flank envelope is formed by the generatrix (Δ), due to the helicoidal movement of the vector radius carrying the point M which describes the reference helix (Figure 12).

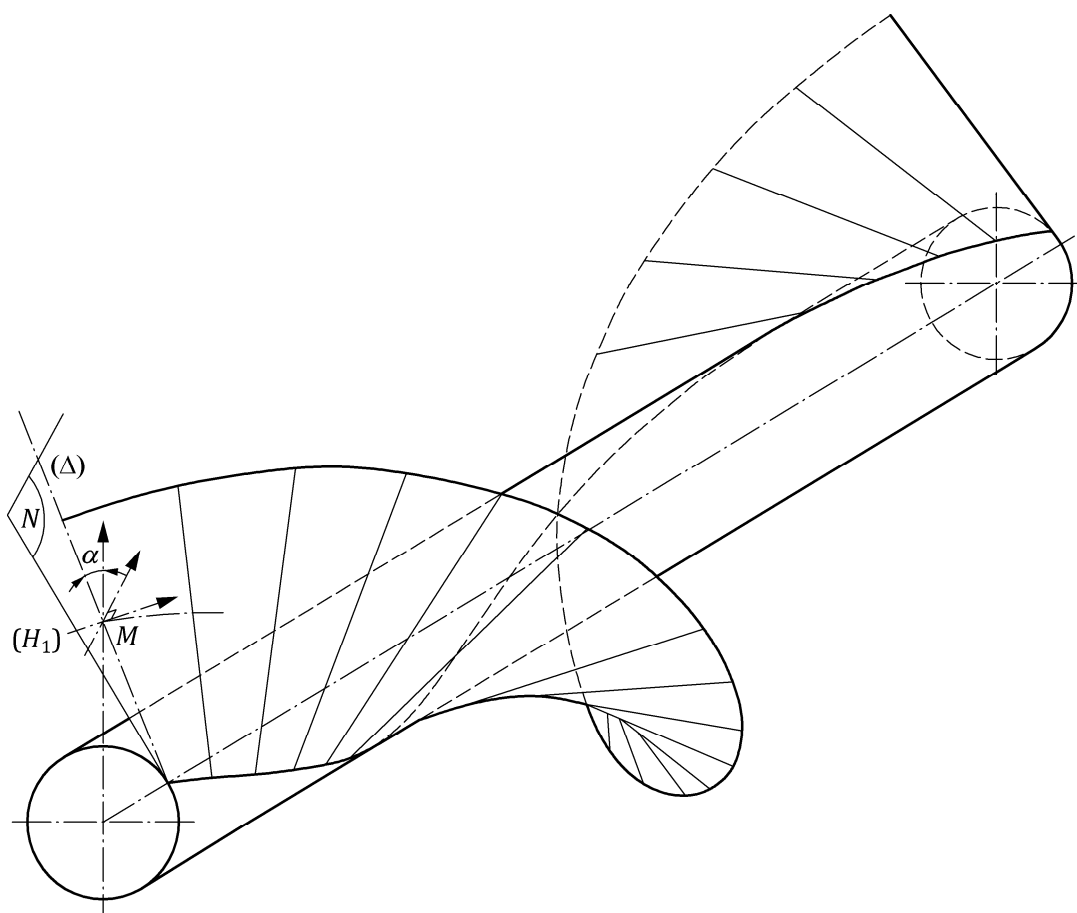


Figure 12 — Profile N - Theoretical generation

6.4.2 Machining methods

The threads may be cut in a lathe with a tool having trapezoidal form having edges in the cutting plane which match the profile of the thread space in a plane normal to the reference helix of the thread space.

This is equivalent to placing the tool as for A type threads, then to rotate it around an axis matching its symmetrical axis up to an angle equal to the reference lead angle γ_{m1} (Figure 13).

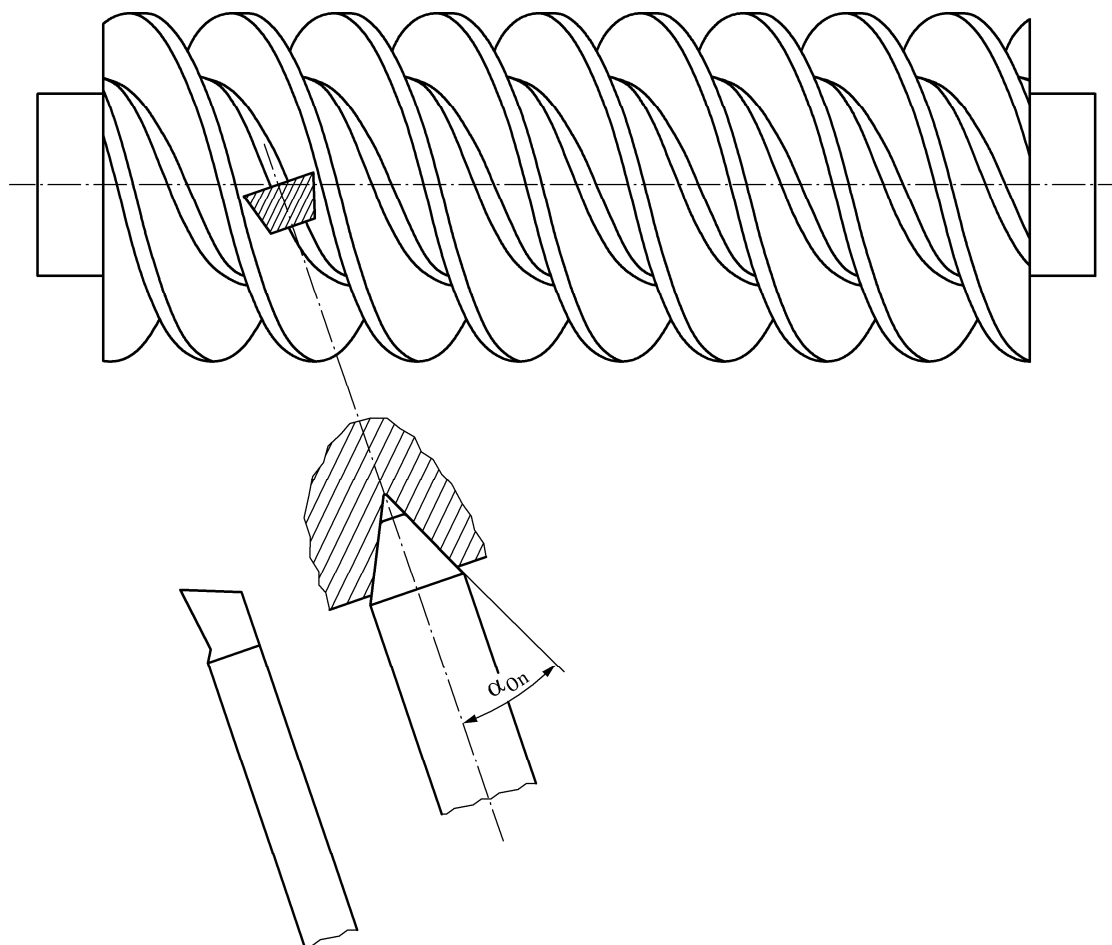


Figure 13 — Profile N - Machining methods

Type N profiles are slightly concave in an axial plane.

The development of the formulae for the axial profile of type N worm is in Annex A, A.4.

6.5 General formulae for A, I and N profiles

6.5.1 General formula of axial profile for A, I, N profiles

The general formula of the axial profile of the worm for A, I, N profiles can be given by a single formula as follows, with x_x as the axial abscissa of the axial profile point function of the radius of the circle y_r at this point:

$$x_x(y_r) = a_1 \cdot \arctan\left(\frac{\sqrt{y_r^2 - a_2^2}}{a_2}\right) + a_3 \cdot \sqrt{y_r^2 - a_2^2} + a_4 \quad (48)$$

$$y_x(y_r) = y_r \quad (49)$$

The values for the coefficients are given in Table 4.

Table 4 — Coefficients a_1 to a_4 for A, I and N profiles

Coefficient	Profile		
	A Profile	I Profile	N Profile
a_1	$a_{1A} = 0$	$a_{1I} = -p_{zu1}$	$a_{1N} = p_{zu1}$
a_2	$a_{2A} = 0$	$a_{2I} = r_{bI}$	$a_{2N} = \frac{A \cdot \sin(\gamma_{m1}) \cdot \tan(\alpha_{0n})}{\sqrt{1 + (\sin(\gamma_{m1}) \cdot \tan(\alpha_{0n}))^2}}$ $A = r_{m1} - \frac{s_{mx1} \cdot \cos(\gamma_{m1})}{2 \cdot \tan(\alpha_{0n})}$
a_3	$a_{3A} = \frac{\tan(\alpha_{0n})}{\cos(\gamma_{m1})}$	$a_{3I} = \tan(\gamma_{b1})$	$a_{3N} = \frac{a_{2N}}{A \cdot \tan(\gamma_{m1})}$
a_4	0	0	0

In Formula (48), a_4 coefficient allows to translate the axial profile and to set up it at the correct position needed in the calculation.

To set up axial profile at the pitch point in the mid plane the a_4 coefficient has to be determined by the application of the general formula for the pitch radius r_{w1}

$$a_4 = -x_x(r_{w1}) = -a_1 \cdot \arctan\left(\frac{\sqrt{r_{w1}^2 - a_2^2}}{a_2}\right) - a_3 \cdot \sqrt{r_{w1}^2 - a_2^2} \quad (50)$$

In complement to the coordinate of the axial point it possible to define at this point the axial pressure angle at the y_r circle which is defined by the derivate of x_x function according to y_r :

$$dx_x(y_r) = \left(\frac{a_1 \cdot a_2}{y_r} + a_3 \cdot y_r \right) \cdot \frac{1}{\sqrt{y_r^2 - a_2^2}} \quad (51)$$

$$dy_x(y_r) = 1 \quad (52)$$

$$\tan \alpha_x(y_r) = \frac{dx_x(y_r)}{dy_x(y_r)} \quad (53)$$

6.6 Type K

6.6.1 Geometrical definition and method

Unlike those of types A, I and N, the thread flanks of type K worms do not have straight line generatrices. The thread spaces of type K worms are generated with a biconical grinding wheel or disc type milling cutter having straight cone generatrices (Figure 14).

The common perpendicular to the tool spindle and worm axes lies in the line (Δ) of intersection of the median plane (M) of the tool and a transverse plane of the worm (R). The angle between the two planes is equal to the worm reference lead angle of the worm γ_{m1} . The straight generatrix of each tool cone and the median plane of the tool forms an angle equal to the normal pressure angle α_{on} of the tool.

The worm is turned uniformly with simultaneous axial translation of threads so that a point on the common perpendicular, distant r_1 (r_1 : reference radius of worm) from the worm axis, describes the reference helix.

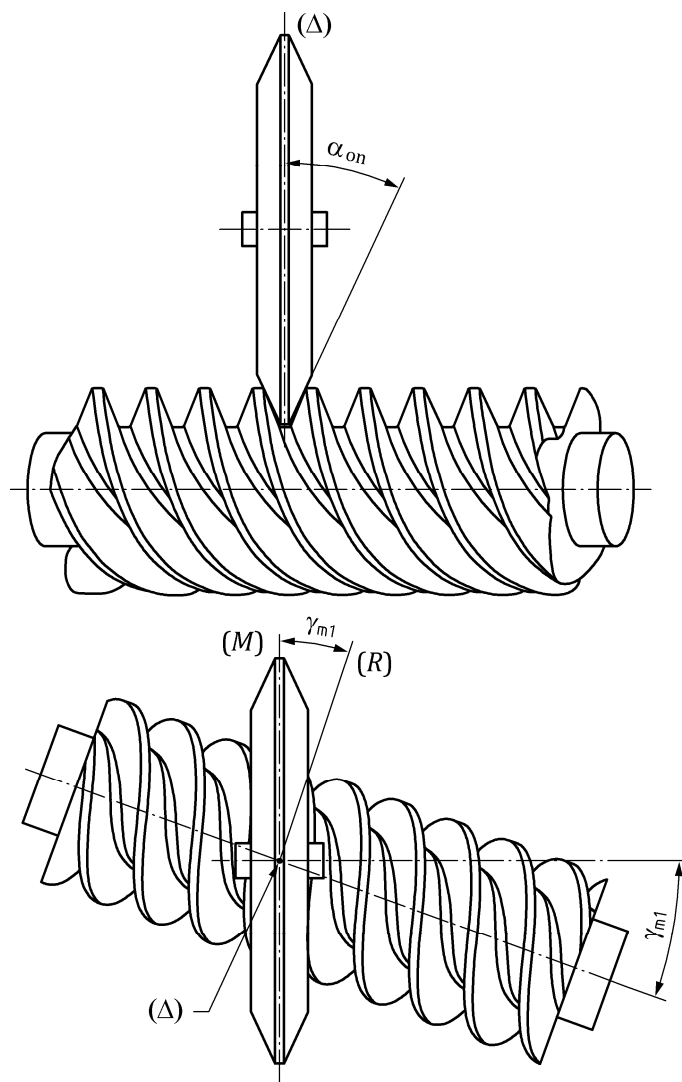
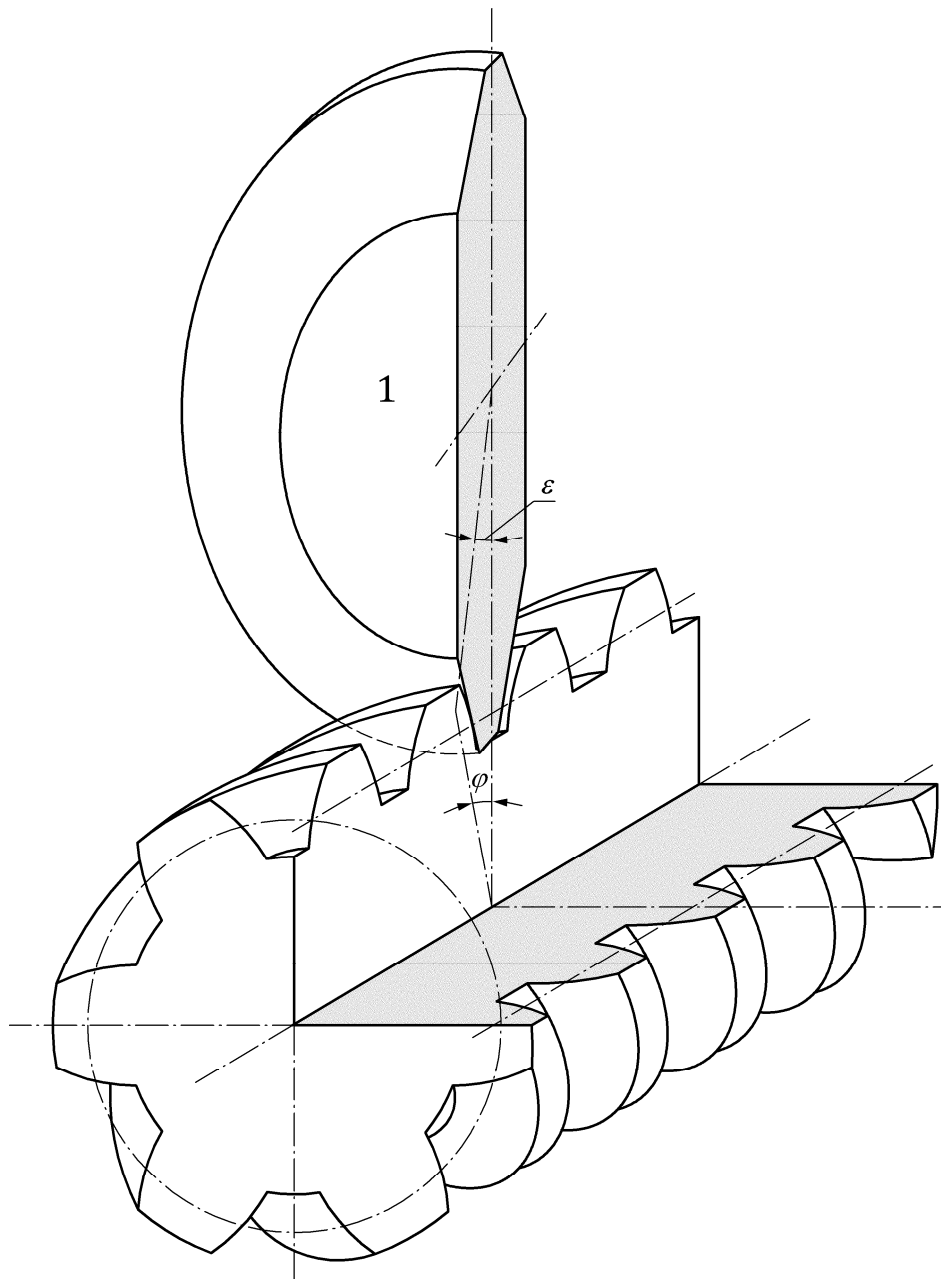


Figure 14 — Profile K - Machining method

The helicoidal flanks of the worm are generated by the conical sides of the tool and the profile form is influenced by the change of helix angle with change of thread height and points on the tool flanks which contact the worm threads lie on a curve and not on any one cone generatrix (Figure 15).



Key

1 grinding wheel

Figure 15 — Profile K - System of coordinates

Like type I profile, profile K is convex in axial planes.

The development of the formulae for the axial profile of grinding wheel for type K worm is in Annex B, B.1.