

# Dynamic Characterization of Intelligent Vehicle Systems Theory and Experiment

**M. G. Mehrabi**  
University of Detroit Mercy

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## ABSTRACT

This paper deals with controller design for path tracking control of intelligent vehicle systems. A two degrees of freedom (DOF) model is developed to represent the motion of a vehicle which is used to formulate the path tracking problem of vehicles. By using optimal control theory, the structure of a controller is obtained for vehicles with front and rear steering systems. In designing the controller, a quadratic measure of performance consisting of the integral of the errors (in position and orientation of the vehicle) and the input (front and rear wheel steering angles) are minimized. This controller is implemented to an experimental vehicle and the experimental results of its path following are presented.

## INTRODUCTION

The problem of guidance and motion control of intelligent vehicles have drawn the attention of many researchers as can be observed from the latest studies reported in the literature [1-2]. As a first step toward the stability analysis and design of the path tracking controller, it is necessary to represent the motion of a vehicle by a reasonably accurate mathematical model obtained from kinematic or dynamic equations. Regarding their kinematic modeling, several studies reported in the literature where for various wheelbase configurations [3-4] or different types of wheels, kinematic models are developed and the posture (position and orientation) of the vehicle are obtained from the wheels position. Regarding their dynamics, literature survey reveals that there have been two basic approaches followed by researchers [5-11]. In the first approach,

conventional Lagrange equations of motion for nonholonomic systems have been used. Based on this methodology, it is possible to formulate the problem and derive the equations of motion of the vehicle. However, the procedure is very long and the final forms of equations appear in a nonlinear fashion that makes it difficult to obtain more insights to dynamic behavior of the system.

In the other approach, the equations of motion are obtained by using Newton's law of motion and the expressions of the forces are obtained by using certain concepts of vehicle dynamics [7-8]. The remarkable advantage of this approach is that while it takes into account the stiffness properties of tires, it also makes possible to obtain a linearized version of this which model appears to be very useful in predicting the dynamic and response properties of the system. In this report, the latter approach is used for derivations of equations of motion [9-10] and a 2-DOF model is developed to represent the vehicles motion which takes into account the lateral and yaw motions. This model is utilized in design of an optimal controller for path tracking control of the vehicle. In designing the controller, a quadratic measure of performance consisting of the integral of the errors (in position and orientation of the vehicle) and the input (front and rear wheel steering angles) are minimized. The optimal path-tracking controller is implemented into a prototype vehicle and the experimental results of path following of the vehicle are presented.

## DEVELOPMENT OF THE DYNAMIC MODEL

The development of the dynamic model of the system has been carried out by a number of

researchers over time. Elaborate vehicle models which take into consideration all the forces as well as other effects acting on a vehicle have been developed along with simpler linear models which are adequate for most studies such as this one.

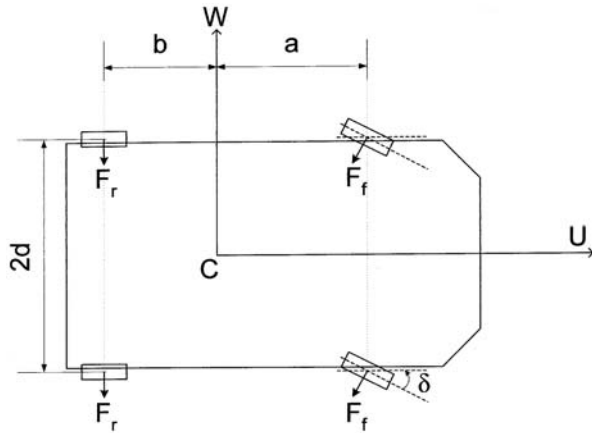


FIGURE 1. PARAMETERS AND DIMENSIONS OF THE AGV

Figure 1 schematically shows the freebody diagram (vehicle with front steering angle) and the pertinent parameters and dimensions of the vehicle which are relevant to our analysis. Lines 'CU' and 'CW' are the attached body axes of the vehicle in the X and Y directions, respectively. 'δ' is the steering angle and 'F<sub>f</sub>' and 'F<sub>r</sub>' are the tire side forces of the front and rear wheels respectively. For our analysis, a telescopic vehicle model is developed by collapsing the four wheels into two sets of wheels at the front and rear. (See Figure 2)

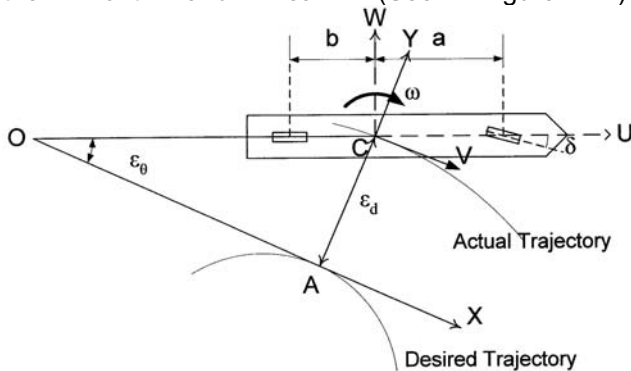


FIGURE 2. VEHICLE ERRORS ABOUT THE TRAJECTORY

Figure 2 shows the actual and desired trajectories of the vehicle. The position error 'ε<sub>d</sub>', and orientation error 'ε<sub>θ</sub>' are defined on the basis of the desired and the actual trajectories of the vehicle. 'ε<sub>d</sub>', the position error is the perpendicular distance between the center of gravity of the

vehicle and the tangent to the desired path, and 'ε<sub>θ</sub>', the orientation error is the angle between vehicle orientation and the tangent to the desired path. The tangent to the trajectory at point 'A' and the normal at that point are taken as the momentary X and Y axes, respectively. Thus, these errors can be directly read off the two sets of coordinate axes and the desired and actual trajectories. If there were to be no errors in either the orientation or the position, then the two sets of axes would coincide and the vehicle would move along the desired trajectory. Also, 'Ω' is defined as the yaw velocity of the vehicle.

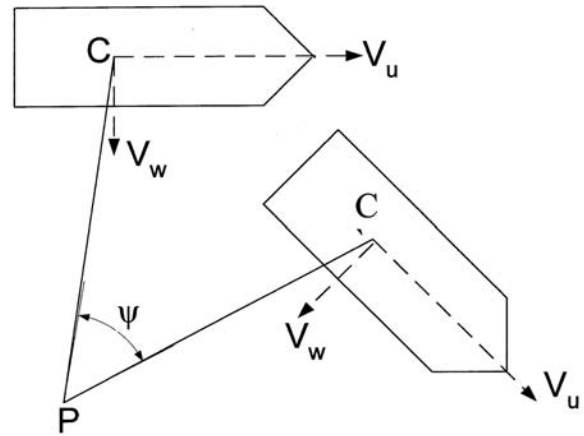


FIGURE 3. NEW POSITION OF THE VEHICLE AFTER A SMALL PERIOD OF TIME

Figure 3 shows the new position of the vehicle after a small interval of time, when it has rotated about point 'P' (instantaneous center of rotation), by an angle 'ψ'. The instantaneous lateral velocity, 'V<sub>w</sub>' and longitudinal velocity, 'V<sub>u</sub>' of the different positions of the vehicle at their center of gravity are also shown. Thus any new position of the vehicle can be considered to be obtained by a forward translation with velocity 'V<sub>u</sub>', a lateral translation with velocity 'V<sub>w</sub>' and a rotation by an angle 'ψ'.

From coordinate transformations one can get the expression for the straight line path of the vehicle to be:

$$\dot{\epsilon}_d = V_w \cos \psi + V_u \sin \psi \quad (1)$$

Where 'ψ' is the orientation of the vehicle, which is obtained from:

$$\dot{\psi} = \Omega \quad (2)$$

Using trigonometric approximations, (1) can be simplified as: