

Uncertainty Propagation in Multi-Disciplinary Design Optimization of Undersea Vehicles

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ABSTRACT

In this paper the development of statistical metamodels and statistical fast running models is presented first. They are utilized for propagating uncertainties in a multi-discipline design optimization process. Two main types of uncertainty can be considered in this manner: uncertainty due to variability in design variables or in random parameters; uncertainty due to the utilization of metamodels instead of the actual simulation models during the optimization process. The value of the new developments and their engagement in multi-discipline design optimization is demonstrated through a case study. An underwater vehicle is designed under four different disciplines, namely, noise radiation, self-noise due to TBL excitation, dynamic response due to propulsion impact loads, and response to an underwater detonation. The case study also demonstrates the value of the multi-discipline design optimization in identifying a system level optimum, and it emphasizes the importance of including uncertainties in the design optimization process.

INTRODUCTION

Computational methods and simulation based methodologies can be combined for optimizing the design of undersea vehicles with respect to cost and performance. In the past design tools have been developed in order to engage multiple engineering disciplines along with a cost analysis that targets the total ownership cost in undersea vehicle design [1]. Engineering simulations such as: assessing the warhead performance and the shaping of the charge using a code like CTH [2,3]; structural-acoustic simulations for self-noise considerations; sonar performance assessment; guidance and control simulations; electromagnetic and thermal analyses associated with electric motor performance; computational fluid dynamic simulations associated with achieving high speeds through supercavitation; and structural dynamic analyses for assessing the performance of the structure during supercavitation; comprise the multiple engineering

disciplines which must be considered during undersea vehicle design. Specialized simulation tools such as the Advanced Collaborative Engagement Model (ACEM) and the Torpedo Optimization and Design (TOAD) [4] can also be integrated in the undersea vehicle design process. ACEM comprises a computer model which provides torpedo designers the ability to perform submarine on submarine engagement simulations and a torpedo performance assessment for the probability of kill and the probability of counter kill. TOAD allows relating physical design variables for the torpedo with its performance attributes and size (i.e. weight, buoyancy, range, velocity, etc.) [4].

Research has been performed in developing design optimization tools and incorporating them into undersea vehicles design and optimization [1]. Work was completed in designing the guidance and control system simultaneously with the warhead system [5]. These two disciplines are mutually competing in the torpedo design process. The guidance and control system provides an obstruction in delivering the high velocity metal jet formed from the Munroe effect of a shaped charge [6], but at the same time any modifications in the size and the geometry of the guidance and control system will deteriorate the acoustic performance of the vehicle, thus, creating a very challenging optimization problem [5]. Response surface methodologies were created for the thermal analysis simulations and were incorporated into the TOAD system. Second order quadratic equations were utilized for constructing the response surfaces [7]. An implementation of neural networks has been presented for torpedo design and optimization. It was demonstrated that predictive analysis can identify combinations of design variables and tactical parameters for producing a highly effective vehicle. In addition counter intuitive relationships can be discovered through this process [8]. A multi-objective Multi-Disciplinary Optimization (MDO) approach was presented for maximizing the probability of kill under multiple possible uses of the same vehicle while considering the warhead length, the hull diameter, the material of the hull, the warhead type, the first explosive type and the second explosive type as design variables [9]. An individual

discipline feasible approach was applied for designing an autonomous underwater vehicle by considering five subsystems during the design process: guidance and control, payload, powering, machinery, and hydrodynamics/propulsion. A system level coordination was established for ensuring compatibility between the subsystems [10]. An approach for exploring the performance space of a torpedo and then mapping it directly to the design space and the design variables was presented, allowing for concurrent optimization of the torpedo and its tactical utilization [4]. An attribute based multi-disciplinary optimization was presented for automating the design synthesis of undersea vehicles [11]. A contributing synthesis approach was employed for relating high level design requirements to low level design rules during undersea vehicle conceptual design [12]. All these efforts establish the necessity of employing an organized MDO process for undersea vehicles design. This area is the focus of the MDO effort presented in this paper.

Due to the large number of iterations required during a MDO process and in an effort to include high fidelity simulations (which require a significant computational time even for a single analysis), theories for developing metamodels (mathematically sophisticated interpolation models which are generated from a limited number of time consuming simulations and are utilized subsequently as a replacement for the time consuming simulations) have been established. Polynomials, interpolating and smoothing splines [13], neural network [14-17], radial basis functions [18], and wavelets [19] have been investigated in the past. References [20-22] introduced the concept of the Kriging method. The Kriging method provides an efficient predictor for a given set of data and constitutes a Gaussian process type of metamodel since it utilizes Gaussian kernels for expressing the spatial correlation functions. It treats the deterministic output as the realization of a stochastic process and provides a statistical basis for efficient prediction [23-26]. Kriging (i.e. Gaussian process metamodels) have been utilized in variable fidelity optimization strategies [27]; for managing system level uncertainty during conceptual design [28]; for approximating deterministic computer models [29]; and for design optimization [30]. Metamodels are employed in this paper for expediting the optimization computations.

In any optimal design the final system operates close to several of the constraints since the optimal point is driven against the active constraints by the optimization strategy. Deterministic optimization methods do not account for variability introduced due to manufacturing, material properties, and equipment performance, nor for uncertainties in the mission profile, human operators, and the environment. Thus, safety factors are introduced during the design process in all disciplines leading to highly conservative and thus costly designs. Government and Industry interests are focusing on savings for the total ownership cost when designing complex systems with a long life. In order to achieve such objectives it is important to bring information and knowledge of uncertainty into the early design cycle

when most of the cost commitment is made [31]. The need for including uncertainty in MDO process has been recognized by NASA for aerospace vehicle applications [32]. Several publications have been presented for including uncertainty in optimization processes through Reliability Based Design Optimization (RBDO) algorithms [33-38]. Currently these methods are either potentially inaccurate or computationally expensive. The utilization of Kriging models for managing uncertainty during system level conceptual design has been presented [39]. Due to the small computational resources required by the Kriging models, Monte Carlo simulations are enabled during the optimization process. A similar approach was utilized in [40] for employing Kriging models during a probabilistic MDO. Metamodels are employed in this paper for evaluating the probabilistic constraints and for considering uncertainty in the optimization.

The variability which originates from the random variables and from the random parameters comprises one source of uncertainty in a MDO environment. When scalar or time dependent metamodels are employed instead of the actual simulation models, then an additional source of uncertainty is introduced. In this work a Bayesian analysis via a Markov chain Monte Carlo method [41] is employed for assessing the latter source of uncertainty as suggested in [28, 39], thus creating statistical Kriging models (for scalar responses) and statistical Fast Running Models (FRM) (for time dependent responses). In the past a MDO under Uncertainty (MDO-U) process has been developed [42] based on the Target Cascading method [43-44] and a single-loop RBDO algorithm [45]. In this work the evaluation of the probabilistic constraints is performed through a Monte Carlo simulation by using statistical Kriging models and statistical FRM. In this manner the computations are expedited and at the same time the uncertainty from utilizing metamodels rather than the actual simulation models is accounted. In this paper the theory for developing the statistical Kriging models and the statistical FRM is presented first. Aspects of the MDO-U methodology utilized in this work are discussed. Finally, the application of the new MDO-U approach in an undersea vehicle design case study is presented. Multi-discipline optimizations are performed without and with uncertainty. The results identify the importance of engaging a multi-disciplinary design optimization and the impact of the uncertainty in the optimal solution.

DEVELOPMENT OF STATISTICAL METAMODELS

Statistical Kriging Models

The Kriging model is based on treating $Z(\tilde{x})$, the difference between the actual performance variable $y(\tilde{x})$ and a regression model prediction $\hat{F}(\tilde{x})$, as a stochastic process:

$$y(\tilde{x}) = \hat{F}(\tilde{x}) + Z(\tilde{x}) \quad (1)$$

where $\tilde{x}$ is the d-dimensional vector of the variables that defines the point where the performance variable is evaluated, and "d" is the number of variables. A

regression model which is a linear combination of “m” selected functions $f(\tilde{x})$ is used here:

$$\hat{F}(\tilde{x}) = \beta_1 f_1(\tilde{x}) + \dots + \beta_m f_m(\tilde{x}) = f(\tilde{x})^T \beta \quad (2)$$

where $\beta^T = \{\beta_1, \beta_2, \dots, \beta_m\}$ are regression parameters.

$Z(\tilde{x})$ is considered as a normal process with zero mean and a covariance that can be expressed as:

$$\text{cov}(Z(\tilde{x}_i), Z(\tilde{x}_j)) = \sigma^2 R(\tilde{x}_i, \tilde{x}_j) \quad (3)$$

where σ^2 is the process variance and $R(\tilde{x}_i, \tilde{x}_j)$ is the spatial correlation function. The equation used for the spatial correlation function is a Gaussian spatial correlation function:

$$R(\tilde{x}_i, \tilde{x}_j) = \prod_{k=1}^d \exp(-\theta_k (\tilde{x}_{i,k} - \tilde{x}_{j,k})^2) \quad (4)$$

and it indicates a process with infinitely differentiable paths in the mean square sense. θ_k is the correlation parameter that corresponds to the k^{th} component of the d-dimensional vector of the random variables $\tilde{x}$, i.e.

$k=1, 2, \dots, d$; and θ represents the vector of the θ_k

parameters. For a set $\tilde{x}_s$ comprised by “n” number of sample points

$$\tilde{x}_{si}, i=1, 2, \dots, n$$

$$\tilde{x}_s^T = \{\tilde{x}_{s1}, \tilde{x}_{s2}, \dots, \tilde{x}_{sn}\} \quad (5)$$

The corresponding performance variable $\tilde{y}_s$ is considered known and its values are defined as:

$$\tilde{y}_s^T = \{y(\tilde{x}_{s1}), y(\tilde{x}_{s2}), \dots, y(\tilde{x}_{sn})\} \quad (6)$$

The vector of correlations between the sample points $\tilde{x}_s$ and the evaluation point $\tilde{x}$ can be expressed as:

$$\tilde{r}^T(\tilde{x}) = \{R(\tilde{x}, \tilde{x}_{s1}), R(\tilde{x}, \tilde{x}_{s2}), \dots, R(\tilde{x}, \tilde{x}_{sn})\} \quad (7)$$

The correlation matrix $[R]$ is also defined among all the sample points:

$$[R] = [R(\tilde{x}_{si}, \tilde{x}_{sj})]_{n \times n} \quad (8)$$

The spatial correlation function in Equations (7) and (8) has been defined by Equation (4). In the Kriging method the value of the performance function evaluated by the metamodel at the evaluation point $\tilde{x}$ is treated as a random variable. The computation of β and $Z(\tilde{x})$ in Equation (1) is based on minimizing the mean square error (MSE) in the response:

$$\text{MSE}[\hat{y}(\tilde{x})] = E[\hat{y}(\tilde{x}) - y(\tilde{x})]^2 \quad (9)$$

subjected to the unbiased constraint:

$$E[\hat{y}(\tilde{x})] = E[y(\tilde{x})] \quad (10)$$

The matrix R and the parameters β and σ^2 depend on θ . Once θ is determined, the regression parameter β and the variance σ^2 can be computed as:

$$\hat{\beta} = (\tilde{F}^T R^{-1} \tilde{F})^{-1} \tilde{F}^T R^{-1} \tilde{y}_s \quad (11)$$

$$\hat{\sigma}^2 = \frac{1}{n} (\tilde{y}_s - \tilde{F} \hat{\beta})^T R^{-1} (\tilde{y}_s - \tilde{F} \hat{\beta}) \quad (12)$$

where the matrix $\tilde{F}$ is defined as $\tilde{F} = [f_j(\tilde{x}_{si})]_{n \times m}$. The value for the response of interest is computed as:

$$\hat{y}(\tilde{x}) = f(\tilde{x})^T \hat{\beta} + \tilde{r}^T(\tilde{x}) R^{-1} (\tilde{y}_s - \tilde{F} \hat{\beta}) \quad (13)$$

In the traditional Kriging method, the optimal value of θ is computed as the maximum likelihood estimator of the likelihood function:

$$L(\theta, \beta, \sigma^2 | \tilde{y}_s) = p(\tilde{y}_s | \theta, \beta, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2 \cdot \det(R)}} \exp\left(-\frac{(\tilde{y}_s - \tilde{F} \hat{\beta})^T R^{-1} (\tilde{y}_s - \tilde{F} \hat{\beta})}{2\sigma^2}\right) \quad (14)$$

where $p(\tilde{y}_s | \theta, \beta, \sigma^2)$ is the multivariate normal distribution for the “n” observations $\tilde{y}_s$ given the model parameters θ , β and σ^2 . In the ordinary Kriging method this is accomplished by minimizing the product $\left[(\det(R))^{1/n} \cdot \sigma^2\right]$ while neglecting the variations in the model parameters θ , β and σ^2 .

In this work the Martin and Simpson approach for considering the uncertainties in the development of a statistical Kriging model is followed. In order to estimate the effect of randomness in the model parameters, the Bayesian analysis via the Markov chain Monte Carlo (MCMC) is employed. The posterior distribution of the model parameters given the observations $\tilde{y}_s$ is defined as:

$$\pi(\theta, \beta, \sigma^2 | \tilde{y}_s) = \frac{p(\tilde{y}_s | \theta, \beta, \sigma^2) \pi(\theta, \beta, \sigma^2)}{\int_{\gamma} p(\tilde{y}_s | \theta, \beta, \sigma^2) \pi(\theta, \beta, \sigma^2) d\gamma} \quad (15)$$

where $\pi(\theta, \beta, \sigma^2)$ is the prior distribution of the model parameters and γ represents the model parameters. In general, the prior distribution has the form of:

$$\pi(\theta, \beta, \sigma^2) \propto \frac{\pi(\theta)}{(\sigma^2)^a} \quad (16)$$

where a is a real constant, and typical values $\pi(\theta) = 1$

and $a = \frac{3}{2}$ are used. Hence, $\pi(\theta, \beta, \sigma^2) \propto \frac{1}{(\sigma^2)^{\frac{3}{2}}}$.

In this work, the Metropolis-Hastings algorithm is employed to realize the MCMC method. Suppose the current chain is $\{\gamma_1, \gamma_2, \dots, \gamma_k\}$, a random candidate γ' is generated using a candidate-generating density function $q(\gamma_k, \gamma')$. This new candidate γ' is only related to the last entry γ_k in the current chain. In this study, the candidate-generating function is defined as a normal distribution $N(\gamma_k, s^2)$ with γ_k being the mean value and